

On the possibility of an avalanche of particle creation

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[1405.5285](#), [0901.0424](#)

Main problem of interest

Our eventual goal is to determine whether an **avalanche** of **particle creation** is possible for a sufficiently strong and long-lasting background electric field: **the current of created pairs grows so rapidly** in proper time **that one cannot maintain the external field as constant**.

We have no doubt that such an avalanche is possible: for a sufficiently strong and long-lasting background field, the created pairs radiate photons, and these photons subsequently decay into more pairs — **a process that can amplify the current**. Our aim, therefore, is to establish the conditions on the strength, duration, and spatial extent of the background pulse required for such an avalanche to occur.

In the **very early Universe** — unlike conditions created at the LHC — **there were certainly strong background gravitational fields** and possibly other strong fields. A potential avalanche of particle creation in such fields **could induce significant backreaction on the background gravitational field**, which may play a key role in matter and structure formation.

Setup of the problem

To simplify the situation without losing the physical content we consider **scalar electrodynamics**:

$$S[\phi, \phi^\dagger; A^\mu] = \int d^4x \left[\left| \partial_\mu \phi + ieA_\mu \phi \right|^2 - m^2 |\phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu^{\text{cl}} A^\mu \right], \quad (1)$$

and divide the vector potential A^μ into the **classical and quantum parts**, $A^\mu = A_{\text{cl}}^\mu + a^\mu$, where

$$A_{\text{cl}}^\mu = (0, A_1(t), 0, 0), \quad A_1(t) = ET \tanh \frac{t}{T}, \quad (2)$$

is due to j_μ^{cl} . We consider the **lengthy pulse**: $eET^2 \gg 1$. **EM field** is quantized in the standard way in the **Lorentz gauge**. At the same time, the **scalar field** operator:

$$\hat{\phi}(t, \mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(\hat{a}_{\mathbf{p}} e^{i\mathbf{p}\mathbf{x}} f_{\mathbf{p}}(t) + \hat{b}_{\mathbf{p}}^\dagger e^{-i\mathbf{p}\mathbf{x}} f_{-\mathbf{p}}^*(t) \right); \quad (3)$$

Quantization

Where

$$\left(\partial_t^2 + [\mathbf{p} + e\mathbf{A}(t)]^2 + m^2 \right) f_{\mathbf{p}}(t) = 0. \quad (4)$$

We take the modes to be **single waves** at past infinity (**in-modes**):

$$f_{\mathbf{p}}^{\text{in}}(t/T \rightarrow -\infty) \simeq \frac{e^{-i\omega_- t}}{\sqrt{2\omega_-}}, \quad \omega_{\pm} = \sqrt{[\mathbf{p} + \mathbf{A}(\pm\infty)]^2 + m^2} \quad (5)$$

The asymptotic form of these modes in **the future infinity** is:

$$f_{\mathbf{p}}^{\text{in}}(t/T \rightarrow +\infty) \simeq C_+(\mathbf{p}) \frac{e^{i\omega_+ t}}{\sqrt{2\omega_+}} + C_-(\mathbf{p}) \frac{e^{-i\omega_+ t}}{\sqrt{2\omega_+}}, \quad (6)$$

There is a clear physical explanation of such a behaviour of the modes due to **QM scattering**. In a **weak field** $m^2/e \gg E$ one can use the **semiclassical approximation** for the modes. But we can calculate even for **strong field**.

The tree-level current

For the **Fock space in-ground state**, $\hat{a}_{\mathbf{p}} |in\rangle = 0$, the tree-level current is:

$$j_1^{\text{tree}}(t) \simeq \frac{E^2 e^3 T}{2\pi^3 \mu} \cdot \exp[-\mu^2], \quad (7)$$

where $\mu = m\sqrt{\pi}/\sqrt{eE}$.

This expression has a clear physical explanation: during the pulse the pairs of charged particles are created (when $eEL = 2m$) with the Schwinger's probability rate per unit four-volume

$$w_s \sim E^2 e^2 \exp[-\pi m^2/eE].$$

Hence, by the end of the “effective” pulse duration one obtains the current density of the form $j_1^{\text{tree}} \sim e w T/\mu$. Such a current is totally due to the **amplification of the zero point fluctuations**.

Photon propagator for a generic state

For a generic state the photon's Keldysh propagator can be represented as follows:

$$G_{\mu\nu}^K(\mathbf{x}, \mathbf{y}; \tau_1, \tau_2) \equiv \frac{1}{2} \left\langle \left\{ a_\mu(\mathbf{x}, \tau_1), a_\nu(\mathbf{y}, \tau_2) \right\} \right\rangle = \int \frac{d^3\mathbf{q} d^3\mathbf{p}}{(2\pi)^6} e^{i\mathbf{q}\mathbf{x} - i\mathbf{p}\mathbf{y}} \tilde{G}_{\mu\nu}^K(\mathbf{q}, \mathbf{p}; \tau_1, \tau_2), \quad (8)$$

Where:

$$\tilde{G}_{\mu\nu}^K = \left(\frac{1}{2} \eta_{\mu\nu} \delta(\mathbf{p} - \mathbf{q}) + \langle \alpha_{\mu, \mathbf{p}}^+ \alpha_{\nu, \mathbf{q}} \rangle \right) \frac{e^{-i|\mathbf{q}|(\tau_1 - \tau_2)}}{2|\mathbf{q}|} + \langle \alpha_{\mu, \mathbf{p}} \alpha_{\nu, \mathbf{q}} \rangle \frac{e^{-i|\mathbf{q}|(\tau_1 + \tau_2)}}{2|\mathbf{q}|} + \text{h.c.}, \quad (9)$$

where $\alpha_{\mu, \mathbf{p}}^+$ and $\alpha_{\nu, \mathbf{q}}$ are creation and annihilation operators for photons.

Red - is due to zero point fluctuations;

Brown - is due to higher energy levels and anomalous averages.

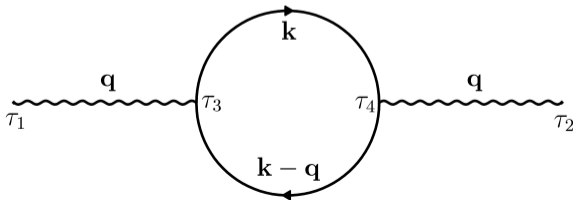
Loop corrections (story of photons)

For a **spatially homogeneous state** loop diagrams in the limit $|\tau_1 - \tau_2| \ll \mathcal{T} = (\tau_1 + \tau_2)/2$ correct the propagator as

$$\Delta_{loops} G_{\mu\nu}^K(\mathbf{q}; \tau_1, \tau_2) \simeq n_{\mu\nu}(\mathbf{q}, \mathcal{T}) \frac{e^{-i|\mathbf{q}|(\tau_1 - \tau_2)}}{2|\mathbf{q}|} + \kappa_{\mu\nu}(\mathbf{q}, \mathcal{T}) \frac{e^{-i2|\mathbf{q}|\mathcal{T}}}{2|\mathbf{q}|} + \text{h.c.}, \quad (10)$$

where $\langle \alpha_{\mu, \mathbf{p}}^+ \alpha_{\nu, \mathbf{q}} \rangle = n_{\mu\nu}(\mathbf{q}, \mathcal{T}) \delta(\mathbf{p} - \mathbf{q})$ and $\langle \alpha_{\mu, \mathbf{p}} \alpha_{\nu, \mathbf{q}} \rangle = \kappa_{\mu\nu}(\mathbf{q}, \mathcal{T}) \delta(\mathbf{p} + \mathbf{q})$.

In the **Schwinger-Keldysh technique** one has a sum of such diagrams in the **first loop correction** to the photon propagator:



From **one loop** when $\mathcal{T} < T$ the photon **level-population**, $n_{\mu\nu}(\mathbf{q}, \mathcal{T})$, receives the secular correction:

$$\begin{aligned}
 n_{\mu\nu}(\mathbf{q}, \mathcal{T}) = & 2e^2(\mathcal{T} + T) \int_{-\infty}^{+\infty} d\tau \frac{e^{-2i|\mathbf{q}|\tau}}{2|\mathbf{q}|} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \times \\
 & \times f_{\mathbf{k}_\perp}(k_1 + eE\tau) \overleftrightarrow{D}_\mu f_{\mathbf{k}_\perp - \mathbf{q}_\perp}(k_1 - q_1 + eE\tau) \times \\
 & \times f_{\mathbf{k}_\perp}^*(k_1 - eE\tau) \overleftrightarrow{D}_\nu^\dagger f_{\mathbf{k}_\perp - \mathbf{q}_\perp}^*(k_1 - q_1 - eE\tau),
 \end{aligned} \tag{11}$$

where $\overleftrightarrow{D}_\mu = \overleftarrow{D}_\mu - \overrightarrow{D}_\mu^\dagger$, while **anomalous averages**, $\kappa_{\mu\nu}(\mathbf{q}, \mathcal{T})$, do not receive secularly growing corrections. **In the absence of an external electric field** $n_{\mu\nu}(\mathbf{q}, \mathcal{T})$ is also vanishing.

This has a clear physical meaning: RHS of the last equation is the **collision integral** describing creation of photons together with charged particles under the action of the background strong electric field, while the ground state for the photon field does not change. Thus, under the action of the strong electric field there is not only the **electrical breakdown** of the vacuum, but also this vacuum acts as a kind of **medium for a laser** that radiates photons.

To clarify our point consider e.g.:

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4} \phi^4 \right], \quad (12)$$

with **non-stationary initial state**: $\hat{\rho}_0 \neq \frac{e^{-\beta \hat{H}_0}}{Z}$.

Then at tree-level

$$G_0^K(t_1, t_2, p) = \left(\frac{1}{2} + n_p^0 \right) \frac{\cos(\omega_p \Delta t)}{\sqrt{2} \omega_p}, \quad \omega_p = \sqrt{p^2 + m^2},$$

where $\Delta t = t_1 - t_2$, and $\text{Tr}[\hat{\rho}_0 \hat{a}_{\vec{p}\vec{a}\vec{q}}^+] = n_p^0 \delta^{(3)}(\vec{p} - \vec{q})$. In the second loop (rising sun) in the limit $t = \frac{t_1+t_2}{2} \gg |t_1 - t_2|$ the corrected propagator has the same form with $n_p(t)$ instead of n_p^0 , where:

$$n_p(t) = (t - t_0) I[n_p^0],$$

where t_0 is the position of the initial Cauchy surface and $I[n]$ is the Boltzmann's collision integral:

$$I[n] \propto \lambda^2 \int \frac{d^3 q_1 d^3 q_2 d^3 q_3}{\sqrt{\omega_p \omega_1 \omega_2 \omega_3}} \delta^{(4)}(\underline{p} + \underline{q}_1 - \underline{q}_2 - \underline{q}_3) \left[(1+n_p) (1+n_1) n_2 n_3 - n_p n_1 (1+n_2) (1+n_3) \right]$$

Loop corrections (story of changed particles)

Somewhat similar story there is a for **charged scalars**, but with some peculiarities. For a **generic spatially homogeneous state** the Fourier transformed Keldysh propagator can be represented as follows:

$$\begin{aligned} D^K(\mathbf{p}; \tau_1, \tau_2) &\equiv \frac{1}{2} \left\langle \left\{ \phi^+(\mathbf{p}, \tau_1), \phi(-\mathbf{p}, \tau_2) \right\} \right\rangle = \\ &= \left(\frac{1}{2} + n_p \right) f_{\mathbf{p}}(\tau_1) f_{\mathbf{p}}^*(\tau_2) + \kappa_p f_{\mathbf{p}}(\tau_1) f_{\mathbf{p}}(\tau_2) + \text{h.c.}, \end{aligned} \quad (13)$$

where $\langle \hat{a}_{\mathbf{p}}^+ \hat{a}_{\mathbf{q}} \rangle = n_{\mathbf{q}} \delta(\mathbf{p} - \mathbf{q})$, $\langle \hat{a}_{\mathbf{p}} \hat{b}_{\mathbf{q}} \rangle = \kappa_{\mathbf{q}} \delta(\mathbf{p} + \mathbf{q})$ and $\langle \hat{b}_{\mathbf{p}}^+ \hat{b}_{\mathbf{q}} \rangle = n_{\mathbf{q}}^* \delta(\mathbf{p} - \mathbf{q})$.

Here $1/2$ - is due to zero point fluctuations;

While $n_{\mathbf{q}}$ and $\kappa_{\mathbf{q}}$ - are due to higher energy levels and anomalous averages.

Tree-level vs. loop corrections to the current of created particles

The current is calculated with the use of the Keldysh function:

$$j_1 \equiv \left\langle : \phi^+(\underline{x}) \overleftrightarrow{D}_\mu \phi(\underline{x}) : \right\rangle = \overleftrightarrow{D}_\mu D^K(\underline{x}, \underline{y}) \Big|_{\underline{x}=\underline{y}}$$

The calculated above tree-level current $j_1^{\text{tree}} \sim ewT$ is totally due to **zero point fluctuations**. For a **very long electric pulse** the loop corrections to the current gained during the pulse can be encompassed into the expression

$$\Delta j_1^{\text{loops}}(t) \approx 2e \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (p_1 + eET) \left[|f_{\mathbf{p}}(t)|^2 n_{\mathbf{p}}(t) + f_{\mathbf{p}}^2(t) \kappa_{\mathbf{p}}(t) + h.c. \right], \quad (14)$$

If $n_{\mathbf{p}}$ and $\kappa_{\mathbf{p}}$ are **generated in the loops**, then the current under consideration receives corrections. Meanwhile if they grow with time, then the corrections to the current also grow with time.

Loop corrections to the current

Where from the first two loops we have that

$$n_{\mathbf{p}}(t) \simeq 2e^2 \int_{t_0}^t d\tau_1 \int_{t_0}^t d\tau_2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} G_{\mu\nu}^K(\mathbf{q}; \tau_1, \tau_2) \times \quad (15)$$
$$\times f_{\mathbf{p}}^*(\tau_1) \overleftrightarrow{D}^{\mu\dagger} f_{\mathbf{p}-\mathbf{q}}^*(\tau_1) \cdot f_{\mathbf{p}}(\tau_2) \overleftrightarrow{D}^{\nu} f_{\mathbf{p}-\mathbf{q}}(\tau_2)$$

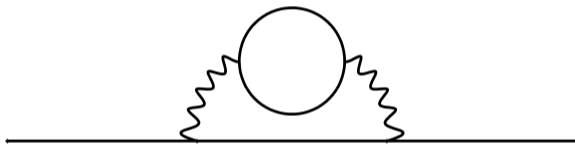
and the anomalous average

$$\kappa_{\mathbf{p}}(t) \simeq -2e^2 \int_{t_0}^t d\tau_1 \int_{t_0}^{\tau_1} d\tau_2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} G_{\mu\nu}^K(\mathbf{q}; \tau_1, \tau_2) \times \quad (16)$$
$$\times f_{\mathbf{p}}(\tau_1) \overleftrightarrow{D}^{\mu\dagger} f_{\mathbf{p}-\mathbf{q}}^*(\tau_1) \cdot f_{\mathbf{p}}(\tau_2) \overleftrightarrow{D}^{\nu} f_{\mathbf{p}-\mathbf{q}}(\tau_2).$$

First vs. second loop correction

Interestingly enough, **the first loop correction** to the current Δj_1^{loops} **does not grow with time** faster than the tree-level contribution.

But the second loop correction due to the Schwinger-Keldysh diagrams of the form:



does grow much faster than the first loop.

The reason we must proceed to the **second-loop correction** to observe a modification of Schwinger's current for a sufficiently long pulse has a clear physical meaning: only at the second loop can we account for how radiated by pairs photons also decay into pairs, a process that can amplify the current.

The second loop correction looks as (that is due to the growth of $n_{\mu\nu}$ in $G_{\mu\nu}^K$ in the first loop):

$$\Delta j_1(T) = -j_1^{(\text{tree,osc})}(T) \frac{2e^4}{\pi} \cdot \frac{T^3}{3} \cdot \frac{e^2 E^2 T}{8\pi^3 \mu} \exp\left[-\frac{\pi m^2}{eE}\right]. \quad (17)$$

$$j_1^{(\text{tree,osc})}(t \sim T) \simeq \frac{e^2 E \sqrt{eE}}{4\pi \sqrt{\pi} \left[\frac{t^2}{T^2} + 1\right] \ln \frac{eET}{m}} \cdot \exp\left[-\frac{\pi m^2}{2eE} \left(1 + \frac{t^2}{T^2}\right)\right] \cos\left[\psi_{E,m,T}(t)\right], \quad (18)$$

where $\psi_{E,m,T}(t)$ is some phase function, which depends on t and E, m, T .

Thus, for a long enough pulse, when the pulse duration T compensates for the smallness of the fine-structure constant and the Schwinger factor $\exp[-\pi m^2/eE]$, loop corrections become of the same order as the tree-level contribution to the current. For a strong field E , this should happen even faster. Therefore, the Schwinger result for the current is drastically modified. The question is whether we can obtain an avalanche of particle creation for a strong and sufficiently long pulse. To answer this question one has to perform a resummation of the leading corrections from all loops preferably for generic initial conditions.

Other non-stationary backgrounds

We observe similar secular corrections to the stress-energy flux and electric currents in other strong background fields. E.g.:

- FRW expanding and contracting universes, including de Sitter space-time;
- Black hole collapse;
- Strong constant electric field;
- Dynamical Casimir effect due to the moving mirrors;
- Strong scalar field background

Strong background fields are more similar to the condensed matter physics rather than to high energy particle physics.

In accelerators one pumps out vacuum and scatters single particles. That is the reason to consider stationary Poincare invariant correlation functions to construct amplitudes and cross-sections. In strong background fields, in the very early universe and in the background of collapsing microscopic black holes there is no reason to assume the state to be vacuum or even thermal.

THANKS!