Non-topological solitons in effective theories of vector fields

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Introduction

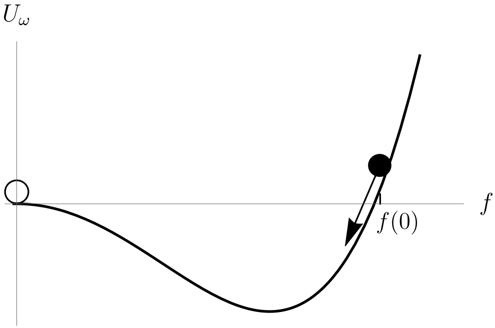
DM candidates in modern cosmology which can be (?) described by classical field theory (*at which scales*?)

Axions, Bose stars, Q-balls.

Stability issue: U(1)-invariance (in approximate limit)

Introduction

The simplest scalar theory $\phi = f(r)e^{i\omega t}$, i.e. stationary solution



Rosen(1968), Coleman's condidions (1985). Overshoot and undershoot. Unrenormalizable potential. UV compleetion is needed?

Introduction

In real life — Gross-Pitaevskii equation
Ideal Bose gas can be localized in the harmonic trap:

$$\Psi \sim \exp(-\omega^2 x^2)$$

This system can be studied both experimentally and theoretically, as classical theory of the complex field Ψ with Lagrangian

$$\mathrm{i}\Psi^* \frac{d}{dt}\Psi - \frac{1}{2m}|\nabla\Psi|^2 - U(x)|\Psi|^2 + \lambda_0|\Psi|^4$$

Negative dimensionless combination $\lambda = m^2\lambda_0$ is not small!

Introduction

In more real life

$$\Psi \to \Psi_i,$$

which describes Rb atoms (neutral!) at $T < T_c$. Harmonic trap is provided by magnetic field B_i.
Nonlinear term is just a correction to potential — dilute gas approximation.
Although Rb is a metall at usual temperature.

Setup

Vector DM. Interesting -ph involving magnetic field
Is it possible to construct effective theory with Lorentz invariance, healthy NR limit and trivial vacuum? What about C,P-invariance?

$$\mathcal{L} = -\frac{1}{2} V^*_{\mu\nu} V^{\mu\nu} - U(V^\nu, V^{*\mu}) \ ,$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. Let us choose:

$$U = -M^2 V^*_\mu V^\mu - \frac{\alpha}{2} (V^*_\mu V^\mu)^2 - \frac{\beta}{2} (V^*_\mu V^{*\mu})(V_\nu V^\nu)$$

General P-invariant Lagrangian with global U(1). α and β are dimensionless. Problems in QFT? (Glashow, 1959)

Ansatz for localized configuration

Switch to the dimensionless units $\tilde{V}_\mu, r, \tau$ as follows,

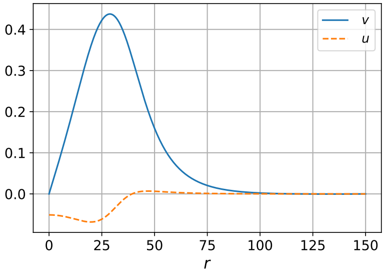
$$V^\nu = \frac{M\tilde{V}^\nu}{\sqrt{|\alpha|}} \ , \ \ \kappa = \frac{\beta}{|\alpha|} \ , \ \ x_i x_i = \frac{r^2}{M^2} \ , \ \ t = \frac{\tau}{M} \ . \qquad (1)$$

We use the following ansatz (Loginov, PRD, 2015):

$$\tilde{V}_0 = iu(r)\mathrm{e}^{-i\mathrm{w}\tau} \ , \quad \tilde{V}^i = n^i v(r)\mathrm{e}^{-i\mathrm{w}\tau}.$$

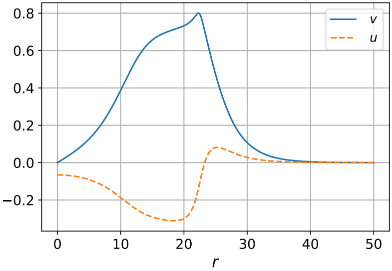
Reparametrization is typical for semiclassical analysis of theory with quartic couplings. Natural assumption $\alpha, \beta = O(1)$.

Perfect soliton profiles



Vector soliton at $\kappa = -0.9$ and $\mathrm{w} = 0.998$. NR limit. This solution is kinematically stable, $E < MQ$.

Ugly soliton profiles



Vector soliton at $\kappa = -0.55$ and $\mathrm{w} = 0.96$. This solution is kinematically stable, $E < MQ$. Check $\frac{\partial E}{\partial Q} = M\mathrm{w}$ holds...

Math origin of the cusp

E.O.M. for our anzatz

$$u'' + \frac{2}{r}u' - \mathrm{w}(v' + \frac{2}{r}v) - u - (u^2 - v^2)u - \kappa(u^2 + v^2)u = 0 \,,$$
$$\mathrm{w}u' + (1 - \mathrm{w}^2)v + (u^2 - v^2)v - \kappa(u^2 + v^2)v = 0 \,, \tag{2}$$

in terms of $u, v, \chi = v'$ has form:

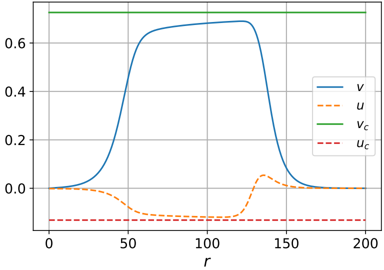
$$v' = \frac{\mathrm{w}}{1 + (u^2 - v^2) - \kappa(u^2 + v^2) - 2v^2(1+\kappa)} \times \left(-\frac{2}{\mathrm{w}}(1-\kappa)uv\chi + \frac{2(\chi - \mathrm{w}v)}{r} - u - u(u^2 - v^2) - \kappa u(u^2 + v^2) \right) . \tag{3}$$

and denominator can vanish.

The reason of cusp

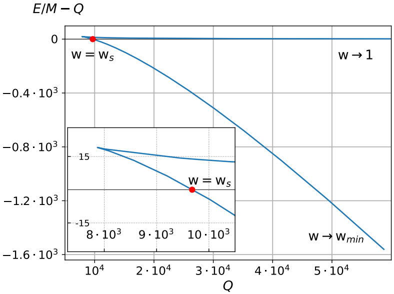
This problem was considered for time evolution (Mou and Zhang, PRL,2022) and connected to constraints in massive vector theory with selfinteraction. However, in phase space near the vacuum $V_\mu = 0$ this problem is absent. So, the theory is effective even on the classical level. The strong coupling scale of the theory can be estimated as $\Lambda \sim M\sqrt{4\pi}/\alpha^{1/4}$ (Porrati, Rahman, Nucl. Phys., 2008).

Thin-wall profiles



Vector soliton at $\kappa = -0.9$ and $\mathrm{w} = 0.99$. This solution is kinematically stable, $E < MQ$.

Kinematic stability of the solitons



$E/M - Q$ as a function of Q for vector solitons with $\tilde{\alpha} = 1$ and $\kappa = \tilde{\beta}/|\tilde{\alpha}| = -0.55$.

Conclusion

- We considered effective theory of vector fields in $(3+1)$ dimensions with good NR limit
- Single dimensional parameter M allows simple estimation of cutoff
- We obtained a kinematically stable branch of configurations of localized non-topological solitons

Thanks for your attention!

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