Renormalization group and basis invariants in Two-Higgs-Doublet Model

Alexander Bednyakov

Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research

Quarks-2024 Seminar

24 May 2024

work in progress

based on [JHEP11(2018) 154], and [JHEP04(2021) 233]

Outline

- Two-Higgs-Doublet Model: scalar sector
  - Basis rotations and basis invariants
  - Bilinear formalism
  - Counting invariants: Hilbert series
- RG calculations: methods & results
  - Challenges in RG computation
  - Structure of RG equations
  - Beta functions for basis invariants
  - From 3 to 6 loops
- Conclusions and Outlook

Some motivation: higgs discovery, vacuum stability, ...

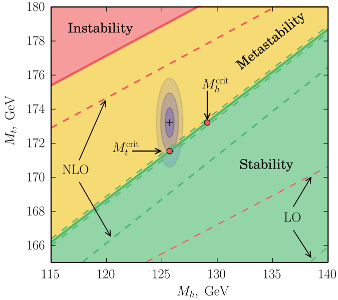
- Three loop beta-functions
  for gauge, Yukawa, and self-coupling
  [Mihaila,Salomon,Steinhauser'12]
  [AB,Pikelner,Velizhanin'13;Chetyrkin&Zoller'13]
- Two loop full $\mathcal{O}(\alpha^2)$ threshold corrections
  [Buttazzo,...'13;Kniehl,Pikelner,Veretin'15]

$$V_{\text{eff}}(\phi \gg v) \simeq \frac{\lambda(\mu = \phi)}{4}\phi^4$$

$$(4\pi)^2 \frac{d\lambda}{d\ln\mu^2} = 12\lambda + 6y_t^2\lambda - 3y_t^4 + \ldots$$

$$(4\pi)^2 \frac{dy_t}{d\ln\mu^2} = \frac{9}{4}y_t^3 - 4g_s{}^2 y_t + + \ldots$$

[AB,Kniehl,Pikelner,Veretin'15]



- 3 loops in BSM, e.g.,
  in extended Higgs sector?

Scalar sector of 2HDM

- 2HDM [Lee'73] is one of the simplest SM extensions

  (see, e.g., [Branco...'12,Ivanov'17])
- Predicts three more scalar states ($H_0, A_0, H^{\pm}$) in the spectrum
- Additional sources for CP (and flavor) violation

$$V_H = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \mathrm{h.c.} \right), \quad \Phi_{1,2} \text{ are higgs doublets}$$

$$+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right)$$

$$+ \left[\frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \lambda_6 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_1^\dagger \Phi_2\right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2\right)\left(\Phi_1^\dagger \Phi_2\right) + \mathrm{h.c.} \right]$$

m_{11}^2, m_{22}^2, $\lambda_{1,2,3,4}$ are real, m_{12}^2 and $\lambda_{5,6,7}$ can be complex

- Freedom to redefine higgs basis $\Phi_a \to U_{ab}\Phi_b, \quad U \in \mathrm{SU}(2)$

  reduces the number of physical parameters in the Higgs sector:

  14 (parameters in V_H) - 3 (broken generators) = 11

Yukawa sector of 2HDM

Most general Type III Yukawa interactions:

$$-\mathcal{L}_Y = \bar{Q}(\Phi^a Y_d^a) d_R + \bar{Q}(\tilde{\Phi}^a Y_u^a) u_R \\ + \bar{L}(\Phi^a Y_l^a) l_R + \left[\bar{L}(\tilde{\Phi}^a Y_\nu^a)\nu_R\right] + \mathrm{h.c.}$$

- $Q = (u, d)_L$, and $L = (\nu, l)_L$ are $\mathrm{SU}(2)_L$ quark and lepton doublets
- u_R, d_R, l_R $[\nu_R]$ are right-handed $\mathrm{SU}(2)_L$ singlets
- Φ^a, $\tilde{\Phi}^a \equiv -i\sigma_2(\Phi^a)^*$ — Higgs doublets with opposite hypercharge

NB: Dangerous tree-level Flavour-Changing-Neutral-Current transitions are possible in Type III model. A way out: Z_2-symmetry, e.g.,

- Type I: all fermions are coupled to a single Higgs boson
- Type II: up-type and down-type fermions couple to different doublets

NB: In what follows we neglect gauge and Yukawa interactions!

Restricting V_H by global symmetries

One can impose global symmetries to (further) reduce the number of parameters and obtain symmetry-constrained models

- Higgs-family symmetries

$$\Phi_a \to S_{ab}\Phi_b, \quad S \text{ - unitary}$$

- Generalized CP symmetries

$$\Phi_a \to X_{ab}\Phi_b^*, \quad X \text{ - unitary}$$

NB: Due to reparametrization freedom the same symmetry may look different in bases related by $U \in \mathrm{SU}(2)$:

$$\Phi \to U\Phi, \quad S \to USU^\dagger, \quad X' \to UXU^T$$

Igor Ivanov [Ivanov'05-07] demonstrated that there exists six different symmetries, dubbed in [Ferreira,Haber,Silva'09] as

$$\mathrm{CP1}, \mathrm{Z_2}, \mathrm{CP2}, \mathrm{U(1)}, \mathrm{CP3}, \mathrm{SU(2)}$$

Six global symmetries of V_H

Higgs-family symmetries:

$$\Phi_a \to S_{ab}\Phi_b$$

$$S_{SU(2)} = \begin{pmatrix} e^{-i\alpha}\cos\theta & e^{-i\beta}\sin\theta \\ -e^{i\beta}\sin\theta & e^{+i\alpha}\cos\theta \end{pmatrix}$$

$$S_{U(1)} = \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{+i\alpha} \end{pmatrix}$$

$$S_{Z_2} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Generalized CP symmetries:

$$\Phi_a \to X_{ab}\Phi_b^*$$

$$X_{CP3} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$X_{CP2} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$X_{CP1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

See,e,g., [Bento,Boto,Silva,Trautner,'21, Ferreira,Grzadkowski, Ogreid,Osland'23]

Six global symmetries of V_H

Higgs-family symmetries:

$$\Phi_a \to S_{ab}\Phi_b$$

$$S_{SU(2)}: m_{12}^2 = 0, \lambda_{5,6,7} = 0$$
$$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2$$
$$\lambda_4 = \lambda_1 - \lambda_3$$

$$S_{U(1)}: m_{12}^2 = 0, \lambda_{5,6,7} = 0$$

$$S_{Z_2}: m_{12}^2 = 0, \lambda_{6,7} = 0$$

Generalized CP symmetries:

$$\Phi_a \to X_{ab}\Phi_b^*$$

$$X_{CP3}: m_{12}^2 = 0, \lambda_{6,7} = 0$$
$$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2$$
$$\lambda_5 = \lambda_1 - \lambda_3 - \lambda_4$$

$$X_{CP2}: m_{12}^2 = 0, \lambda_6 + \lambda_7 = 0$$
$$m_{11}^2 = m_{22}^2, \lambda_1 = \lambda_2$$

$X_{CP1}: m_{12}^2, \lambda_{5,6,7}$ - real

Conditions in particular basis....Basis-independent?...RG stable?...

See,e.g., [Bento,Boto,Silva,Trautner,'21, Ferreira,Grzadkowski, Ogreid,Osland'23]

Basis rotations and basis invariants

- Convenient (favourite) basis choice can simplify computations
- Reparametrization freedom should not affect physical observables
- Equivalence of results obtained in different bases can be obscured
- Observables in terms of reparametrization invariants
- How to construct basis invariants? [Botella,Silva'95,Davidson,Haber'05]

One can rewrite V_H in a more general form

$$V_H = \frac{1}{2}\lambda_{ab,cd}(\Phi_a^\dagger\Phi_b)(\Phi_c^\dagger\Phi_d) + m_{ab}^2(\Phi_a^\dagger\Phi_b)$$

$$\lambda_{ab,cd} = \lambda^\dagger_{dc,ba}, \quad m_{ba}^2 = m_{ab}^{\dagger 2},$$

and consider various contractions ("traces") of $\lambda_{ab,cd}$ and m_{ab}^2, together with their products. Natural questions:

- How many invariants? Infinite! At each order in λ? Finite!
- Dependencies between invariants? Ring structure?

This parameterization is not very convenient...too many indices:)

Basis rotations and basis invariants: bilinear formalism

- Convenient (favourite) basis choice can simplify computations
- Reparametrization freedom should not affect physical observables
- Equivalence of results obtained in different bases can be obscured
- Observables in terms of reparametrization invariants [Davidson,Haber'05]
- Bilinear formalism [Ivanov'05-07]: $2 \otimes \bar{2} = 3 \oplus 1$

$$\Phi_a \Phi_b^\dagger = \frac{1}{2}\left[\Phi^\dagger \Phi\right]\delta_{ab} + \frac{1}{2}\left[\Phi^\dagger \vec{\sigma}\Phi\right]\vec{\sigma}_{ab}, \qquad V_H = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

$$r_\mu = \{r_0, \vec{r}\}$$

Λ_{00} (top-left element), $\vec{\Lambda}$ (rest of first row and first column), Λ (lower-right 3×3 block):

$$\Lambda_{\mu\nu} = \begin{pmatrix} \frac{\lambda_1+\lambda_2}{2}+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \frac{\lambda_1-\lambda_2}{2} \\ \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) \\ -\mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6-\lambda_7) \\ \frac{\lambda_1-\lambda_2}{2} & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \frac{\lambda_1+\lambda_2}{2}-\lambda_3 \end{pmatrix}$$

M_0 (first component), $\vec{M}$ (remaining components):

$$M_\mu = \{m_{11}^2 + m_{22}^2, \; -2\mathrm{Re}\, m_{12}^2, 2\mathrm{Im}\, m_{12}^2, m_{11}^2 - m_{22}^2\}$$

Basis rotations and basis invariants: bilinear formalism

- Convenient (favourite) basis choice can simplify computations
- Reparametrization freedom should not affect physical observables
- Equivalence of results obtained in different bases can be obscured
- Observables in terms of reparametrization invariants [Davidson,Haber'05]
- Bilinear formalism [Ivanov'05-07]: nice geometric interpretation

$$\Phi_a\Phi_b^\dagger = \frac{1}{2}\left[\ r_0\ \right]\delta_{ab} + \frac{1}{2}\left[\ \vec{r}\ \right]\vec{\sigma}_{ab}, \qquad V_H = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

$$r_\mu = \{r_0, \vec{r}\}$$

Λ_{00} $\vec{\Lambda}$

$$\Lambda_{\mu\nu} = \begin{pmatrix} \frac{\lambda_1+\lambda_2}{2}+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \frac{\lambda_1-\lambda_2}{2} \\ \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) \\ -\mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6-\lambda_7) \\ \frac{\lambda_1-\lambda_2}{2} & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \frac{\lambda_1+\lambda_2}{2}-\lambda_3 \end{pmatrix}$$

M_0 $\vec{M}$ Λ

$$M_\mu = \{m_{11}^2 + m_{22}^2, -2\mathrm{Re}\, m_{12}^2, 2\mathrm{Im}\, m_{12}^2, m_{11}^2 - m_{22}^2\}$$

Bilinear formalism: Cayley-Hamilton theorem and invariants

- $U \in \mathrm{SU}(2)$ is mapped to rotations $\mathrm{SO}(3)$, $R_{ij} = 1/2\mathrm{Tr}(U^\dagger \sigma_i U \sigma_j)$:

$$\Lambda_{00} \to \Lambda_{00}, M_0 \to M_0, \quad \vec{\Lambda} \to R\vec{\Lambda}, \vec{M} \to R\vec{M}, \quad \Lambda \to R\Lambda R^T$$

$$\begin{aligned} \Lambda_{00}, M_0 \; [\text{ and } \mathrm{tr}\Lambda \;] \quad &— \quad \text{singlets}, \\ \vec{\Lambda}, \text{ and } \vec{M} \quad &— \quad \text{triplets (vectors)}, \\ \tilde{\Lambda} \equiv \Lambda - \tfrac{1}{3}\mathrm{tr}\Lambda \quad &— \quad \text{five-plet (traceless symmetric matrix)} \end{aligned}$$

NB: Higgs-family symmetries: $\det R = +1$
GCP-symmetries: $\det R = -1$

- Advantage: we need to deal with at most two indices instead of four
- Advantage: Cayley-Hamilton theorem for 3x3 matrix

$$\Lambda^3 = \mathrm{tr}\Lambda\Lambda^2 - \frac{1}{2}\left(\mathrm{tr}^2\Lambda - \mathrm{tr}\Lambda^2\right)\Lambda + \frac{1}{3!}\left(\mathrm{tr}^3\Lambda - 3\mathrm{tr}\Lambda\mathrm{tr}\Lambda^2 + 2\mathrm{tr}\Lambda^3\right),$$

can be used to get rid of high powers Λ^n ($n \geq 3$)

Counting invariants and Hilbert Series

- In bilinear formalism one can easily construct (low-order) invariants:

$$I_{1,1} = \Lambda_{00}, \qquad I_{1,2} = \mathrm{tr}\Lambda, \qquad I_{x,y} \quad y\text{-th invariant of order } x$$

$$I_{2,1} = \vec{\Lambda}\cdot\vec{\Lambda}, \qquad I_{2,2} = \mathrm{tr}\Lambda^2, \qquad x \text{ always counts } \lambda\text{'s}$$

$$I_{3,1} = \vec{\Lambda}\cdot\Lambda\cdot\vec{\Lambda}, \qquad I_{3,2} = \mathrm{tr}\Lambda^3,$$

$$I_{4,1} = \vec{\Lambda}\cdot\Lambda^2\cdot\vec{\Lambda}, \quad \mathcal{I}_{6,1} = \vec{\Lambda}\cdot\left[(\Lambda\cdot\vec{\Lambda})\times(\Lambda^2\cdot\vec{\Lambda})\right] \qquad \text{CP-odd}$$

- Algebraically independent?
- Generating set of a ring of ("massless") invariants?
- Hilbert series see [Hanany...'09,Jenkins...'09,Hanany...'10,Lehman...'15]

$$H(t) = \sum_{n=0}^{\infty} c_n t^n, \qquad c_n \text{ — number of invariants of degree } n,\ [c_0 = 1].$$

NB: Here order = degree

Counting invariants and Hilbert Series

- Neglecting mass terms ($M_0 = 0$, $\vec{M} = 0$) [AB'18]

$$H(t) = \frac{1+t^6}{(1-t)^2(1-t^2)^2(1-t^3)^2(1-t^4)}.$$

- Multi-graded Hilbert series (one variable t_i for each irrep)

$$H(\underbrace{t_1}_{\Lambda_{00}}, \underbrace{t_2}_{\mathrm{tr}\Lambda}, \underbrace{t_3}_{\vec{\Lambda}}, \underbrace{t_4}_{\tilde{\Lambda}}) =$$

$$\frac{1 + {t_3}^3 {t_4}^3}{\underbrace{(1-t_1)}_{I_{1,1}}\underbrace{(1-t_2)}_{I_{1,2}}\underbrace{(1-{t_3}^2)}_{I_{2,1}}\underbrace{(1-{t_4}^2)}_{I_{2,2}}\underbrace{(1-{t_4}^3)}_{I_{3,2}}\underbrace{(1-t_4{t_3}^2)}_{I_{3,1}}\underbrace{(1-{t_3}^2{t_4}^2)}_{I_{4,1}}}.$$

- Denominator: primary invariants, their degree, and their number
- Numerator: additional invariant of degree six:
  $\mathcal{I}_{6,1} \equiv \vec{\Lambda} \cdot \left[(\tilde{\Lambda} \cdot \vec{\Lambda}) \times (\tilde{\Lambda}^2 \cdot \vec{\Lambda})\right]$, which is algebraically related (determined up to a sign) to those presented above.

Counting invariants and Hilbert Series

- Full scalar sector [AB'18]

$$H(t) = \frac{1 + t^3 + 4t^4 + 2t^5 + 4t^6 + t^7 + t^{10}}{(1-t)^3(1-t^2)^4(1-t^3)^3(1-t^4)}.$$

- Multi-graded Hilbert series (one variable t_i for each irrep)

$$H(\underbrace{t_1}_{\Lambda_{00}}, \underbrace{t_2}_{\mathrm{tr}\Lambda}, \underbrace{t_3}_{M_0}, \underbrace{t_4}_{\vec{\Lambda}}, \underbrace{t_5}_{\vec{M}}, \underbrace{t_6}_{\tilde{\Lambda}}) =$$

$$\underbrace{(1-t_1)}_{I_{1,1}}{}^{-1}\underbrace{(1-t_2)}_{I_{1,2}}{}^{-1}\underbrace{(1-t_3)}_{I_{0,1}}{}^{-1} \cdot \frac{N(t_4, t_5, t_6)}{D(t_4, t_5, t_6)}$$ [Trautner'19]

- Additional invariants [linear in mass parameters]:

$$I_{0,1} = M_0, \qquad I_{1,3} = \vec{\Lambda} \cdot \vec{M}, \qquad I_{2,3} = \vec{\Lambda} \cdot \Lambda \cdot \vec{M}, \qquad I_{3,3} = \vec{\Lambda} \cdot \Lambda^2 \cdot \vec{M}$$

$$\mathcal{I}_{3,4} = (\Lambda \cdot \vec{\Lambda}) \cdot (\vec{\Lambda} \times \vec{M}), \quad \mathcal{I}_{4,2} = (\Lambda^2 \cdot \vec{\Lambda}) \cdot (\vec{\Lambda} \times \vec{M}) \quad \mathcal{I}_{5,1} = (\Lambda^2 \cdot \vec{\Lambda}) \cdot (\Lambda \cdot \vec{\Lambda} \times \vec{M})$$

NB: Here order (number of λ) $\neq$ degree (number of contracted reps)

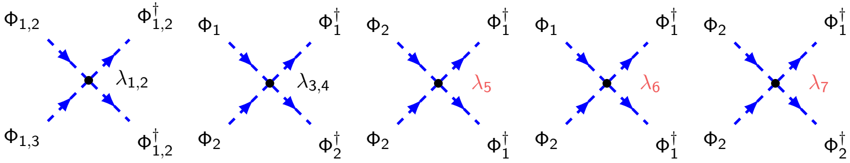
Higgs sector of 2HDM: RG equations

Scale-dependence of parameters in the $\overline{\mathrm{MS}}$ scheme can be written

- In terms of λ_i and m_{ij}^2, e.g., $[t = \ln\mu,\ h^{-1} = 16\pi^2]$

$$\frac{d}{dt}\lambda_{1,2} = h\left[12\lambda_{1,2}^2 + 2\left[\lambda_4^2 + |\lambda_5|^2\right] + 4\left[\lambda_3\lambda_4 + \lambda_3^2\right] + 24|\lambda_{6,7}|^2\right]$$

- Many similar vertices:



- Proliferation of Feynman diagrams at high orders

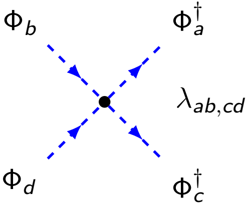
Higgs sector of 2HDM: RG equations (II)

Scale-dependence of parameters in the $\overline{\text{MS}}$ scheme can be written

- In terms of $\lambda_{ab,cd}$ and m^2_{ab}, e.g., $[t = \ln\mu,\ h^{-1} = 16\pi^2]$

$$\frac{d\lambda_{ab,cd}}{dt} = h\left[2\lambda_{ab,ij}\lambda_{ji,cd} + \lambda_{ab,ij}\lambda_{ci,jd} + \lambda_{ai,jb}\lambda_{ij,cd} + \lambda_{ai,cj}\lambda_{ib,jd} + \lambda_{ai,jd}\lambda_{ic,jb}\right]$$

- A "single" vertex



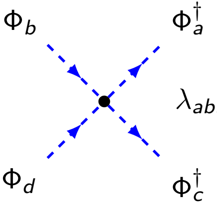
- Proliferation of "tensor structures" built from $\lambda_{ab,cd}$ at high orders, c.f., with [AB,Pikelner'21]

$$\frac{d}{dt}\lambda_{abcd} = \sum_{l=1}^{6} h^l \sum_{i=1}^{n_l} T^{(l)}_{i,abcd} C^{(l)}_i, \qquad n_l = \{1, 2, 7, 23, 110, 571\}$$

Higgs sector of 2HDM: RG equations (III)

Scale-dependence of parameters in the $\overline{\text{MS}}$ scheme can be written

- in bilinear formalism [Ivanov'06,AB'18] ($t = \ln\mu^2$, $h^{-1} = 16\pi^2$):



$$\lambda_{ab,cd} = \frac{1}{2}\left[\Lambda_{00}\delta_{ab}\delta_{cd} + \vec{\Lambda}_i \cdot \left(\sigma^i_{ab}\delta_{cd} + \delta_{ab}\sigma^i_{cd}\right) + \Lambda_{ij}\sigma^i_{ab}\sigma^j_{cd}\right]$$

$$\frac{d\Lambda_{\mu\nu}}{dt} \equiv \beta_{\Lambda_{\mu\nu}}, \qquad \frac{dM_\mu}{dt} \equiv \gamma_{M_\mu}$$

$$\beta_{\Lambda_{00}} = a_0,$$

$$\beta_{\vec{\Lambda}} = b_0\,\vec{\Lambda} + b_1\,\Lambda\cdot\vec{\Lambda} + b_2\,\Lambda^2\cdot\vec{\Lambda},$$

[explicitly confirmed at 3 loop]

$$\beta_{\Lambda} = c_0 + c_1\,\Lambda + c_2\,\Lambda^2 + c_3\,\vec{\Lambda}\otimes\vec{\Lambda} + c_4\left(\Lambda\cdot\vec{\Lambda}\otimes\vec{\Lambda} + \vec{\Lambda}\otimes\Lambda\cdot\vec{\Lambda}\right) + c_5(\Lambda\cdot\vec{\Lambda}\otimes\Lambda\cdot\vec{\Lambda}),$$

$$\beta_{M_0} = d_0 M_0 + d_1(\vec{\Lambda}\cdot\vec{M}) + d_2\vec{\Lambda}\cdot\Lambda\cdot\vec{M} + d_3\vec{\Lambda}\cdot\Lambda^2\cdot\vec{M}$$

$$\beta_{\vec{M}} = e_0\vec{M} + e_1\Lambda\cdot\vec{M} + e_2\Lambda^2\cdot\vec{M} + e_3 I_M\vec{\Lambda} + e_4 I_M\Lambda\cdot\vec{\Lambda} + e_5 I_M\Lambda^2\cdot\vec{\Lambda}$$

Higgs sector of 2HDM: RG equations for invariants

- "Massless" invariants (N_n - number of such invariants at order n)

$$\frac{d}{dt} I_{x,y} = \sum_{l=1}^{\infty} h^l \sum_{k=x+1}^{N_{x+l}} c^{(x,y)}_{x+l,k} \cdot I_{x+l,k}$$

- Invariants, linear in mass ($\mathcal{N}_n$ - number of such invariants at order n)

$$\frac{d}{dt} I_{x,y} = \sum_{l=1}^{\infty} h^l \sum_{k=x}^{\mathcal{N}_{x+l}} c^{(x,y)}_{x+l,k} \cdot I_{x+l,k}$$

- Both N_n and $\mathcal{N}_n$ can be determined from Hilbert Series
- At 3 loops RGE were derived from that $\Lambda_{\mu\nu}$ and M_μ [AB'18]

Higgs sector of 2HDM: RG equations for invariants

- "Massless" invariants (N_n - number of such invariants at order n)

$$\frac{d}{dt} I_{x,y} = \sum_{l=1}^{6} h^l \sum_{k=x+1}^{N_{x+l}} c_{x+l,k}^{(x,y)} \cdot I_{x+l,k}$$

- Invariants, linear in mass ($\mathcal{N}_n$ - number of such invariants at order n)

$$\frac{d}{dt} I_{x,y} = \sum_{l=1}^{6} h^l \sum_{k=x}^{\mathcal{N}_{x+l}} c_{x+l,k}^{(x,y)} \cdot I_{x+l,k}$$

- Both N_n and $\mathcal{N}_n$ can be determined from Hilbert Series
- NEW: 6-loop result based on [Kompaniets,Panzer'17,AB,Pikelner'21]: all needed $c_{m,n}^{(x,y)}$ are determined from a systems of linear equations, obtained by expressing LHS and RHS in terms of λ_i and m_{ij}^2.

- "Massless" invariants: number of coefficients N_n

	$I_{1,1}$	$I_{2,1}$	$I_{3,1}$								
	$I_{1,2}$	$I_{2,2}$	$I_{3,2}$	$I_{4,1}$							
n	2	3	4	5	6	7	8	9	10	11	12
even	5	10	19	32	54	84	129	190	275	386	536
odd	0	0	0	0	1	2	5	10	19	32	54
total	5	10	19	32	55	86	134	200	294	418	590
						$\mathcal{I}_{6,1}$					

- Invariants, linear in mass: number of coefficients $\mathcal{N}_n$

	$I_{0,1}$	$I_{1,3}$	$I_{2,3}$	$I_{3,3}$							
n	1	2	3	4	5	6	7	8	9	10	11
even	3	8	18	36	66	115	189	299	457	678	980
odd	0	0	1	3	8	18	36	66	115	189	299
total	3	8	9	39	74	133	225	365	572	867	1279
				$\mathcal{I}_{3,4}$	$\mathcal{I}_{4,2}$	$\mathcal{I}_{5,1}$					

Some results: structure of RG equations

$$\frac{d}{dt}\mathcal{I}_{6,1} = \mathcal{I}_{6,1}\left[h\cdot(54I_{1,1}+27I_{1,2})+\ldots+h^6(\ldots)\right]$$ 3 loop [AB'18]

$$\frac{d}{dt}I_{0,1} = I_{0,1}\left[h\cdot(5I_{1,1}+I_{1,2})+\ldots+h^6(\ldots)\right]$$ "massive" CP-even

$$+ I_{1,3}\left[h\cdot 6+\ldots+h^6(\ldots)\right]$$

$$+ I_{2,3}\left[h^2\cdot(-15)+\ldots+h^6(\ldots)\right]$$

$$+ I_{3,3}\left[h^3\cdot(\frac{441}{2}+\ldots+h^6(\ldots)\right],$$ new 6 loop

$$\frac{d}{dt}\mathcal{I}_{3,4} = \mathcal{I}_{3,4}\left[h\cdot(31I_{1,1}+3I_{1,2})+\ldots+h^6(\ldots)\right]$$ "massive" CP-odd

$$+ \mathcal{I}_{4,2}\left[h\cdot 14+\ldots+h^6(\ldots)\right]$$

$$+ \mathcal{I}_{5,1}\left[h^2\cdot 55+\ldots+h^6(\ldots)\right]$$

$$+ \mathcal{I}_{6,1}I_{0,1}\left[h^2\cdot(-216)+\ldots+h^6(\ldots)\right]$$ new 6 loop

Higgs sector of 2HDM: results and consequences

- Results:
  - 3-loop RGE for all parameters of 2HDM potential (both for $\Lambda_{\mu\nu}, M_\nu$, and λ_i, m^2_{ij}) [AB'18]
  - Hilbert series for 2HDM [AB'18]
  - Update: 6-loop RGE for λ_i, m^2_{ij} [AB,Pikelner'21]
  - New: 6-loop RGE for reparametrization invariants
  - New: Conditions imposed by symmetries are RG stable upto 6 loops
- Impact:
  - Systematic construction of basis invariants in 2HDM [Trautner'18] (and NHDM [Bento'21]), see also [Ivanov,Nishi,Trautner'19]
  - 3-loop RGE in 2HDM influence derivation of 3-loop RGE in general scalar theory [Steudtner'20] (updated to 6 loops in [AB,Pikelner,'21]) (see also [Steudner'21,Jack,Osborn,Steudtner'23])
  - All public codes for 2-loop running were corrected [Schienbein...'18]
- TODO:
  - New renormalization-group (and basis) invariants? New symmetries?
  - Include gauge couplings and Yukawa couplings...
  - Phenomenological analysis...

Thank you for attention!

Backup: Hilbert Series

- Plethystic exponent

$$\mathrm{PE}[z, t; \mathrm{r}] = \exp\left[\sum_{i=1}^{\infty} \frac{t^i \chi_{\mathrm{r}}(z^i)}{i}\right]$$

- Characters of $[\mathrm{SU}(2)]$ representations $\mathrm{r} = \{3, 5\}$

$$\chi_3(z) = z^2 + 1 + \frac{1}{z^2}, \quad \chi_5(z) = z^4 + z^2 + 1 + \frac{1}{z^2} + \frac{1}{z^4}$$

- (Multi-graded) Hilbert Series (via Molien-Weyl formula)

$$H(t_1, t_1, t_2) = \int d\mu_{\mathrm{SU}(2)}(z) \cdot \underbrace{\mathrm{PE}[z, t_1, 3]}_{\vec{\Lambda}} \cdot \underbrace{\mathrm{PE}[z, t_2, 3]}_{\vec{M}} \cdot \underbrace{\mathrm{PE}[z, t_3, 5]}_{\tilde{\Lambda}}$$

$$\int d\mu_{\mathrm{SU}(2)} = \frac{1}{2\pi i} \oint_{|z|=1} \frac{dz}{z}(1 - z^2)$$