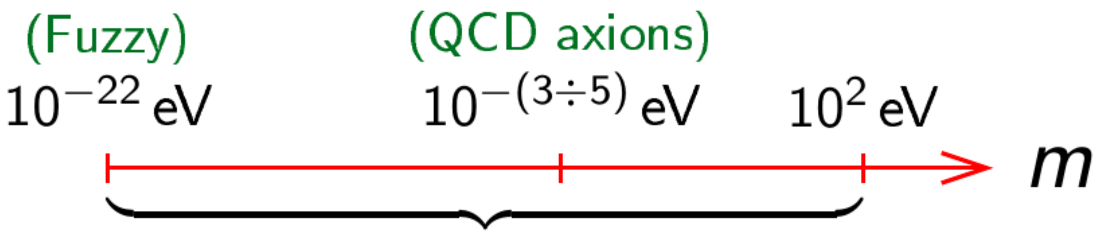
# Bose stars

A. Dmitriev, D. Levkov, Alexander Panin, I. Tkachev

INR RAS, Moscow

# Light bosonic dark matter




- **Fits into the galaxy:**

$$(mv)^{-1} \lesssim R \;\Rightarrow\; \boxed{m \gtrsim 10^{-22}\,\mathrm{eV}}$$

- **Can be only bosonic:**

$$f \sim \frac{\rho/m}{(mv)^3} \gtrsim 1 \;\Leftarrow\; \boxed{m \lesssim 100\,\mathrm{eV}}$$

- **Forms BE condensate:**

$$T < T_c \;\Rightarrow\; mv^2 \lesssim n^{2/3}/m \;\Leftarrow\; \boxed{m \lesssim 100\,\mathrm{eV}}$$

**any dwarf galaxy**

Kinematics is known!

$\rho \sim 0.1\,\mathrm{M}_\odot/\mathrm{pc}^3$

$R \sim \mathrm{kpc}$ $\quad v \sim 10\,\mathrm{km/s}$

See D. Levkov's and A. Dmitriev's talks

**Gravitational Interaction** $\Rightarrow$ **Bose-Einstein condensation!**

# Describing ALP DM

1 $f \gg 1$ ⇨ classical field $a(t, \mathbf{x})$: $\hbar = c = k_B = 1$

$$\Box a + m^2 a - \frac{g_4^2}{6} \frac{m^2}{f_a^2} a^3 + \ldots = 0$$

self-interaction

2 $v \ll 1$ ⇨ nonrelatisitic axions $E_a \simeq m$:

$$a = \frac{f_a}{\sqrt{2}} \left[ \psi(t, \mathbf{x})\, \mathrm{e}^{-imt} + \mathrm{h.\,c.} \right], \text{ where } \partial_{t,\mathbf{x}} \psi \ll m\psi$$

$$i\partial_t \psi = -\frac{\Delta\psi}{2m} + m \left( \Phi - \frac{g_4^2}{8} |\psi|^2 \right) \psi$$

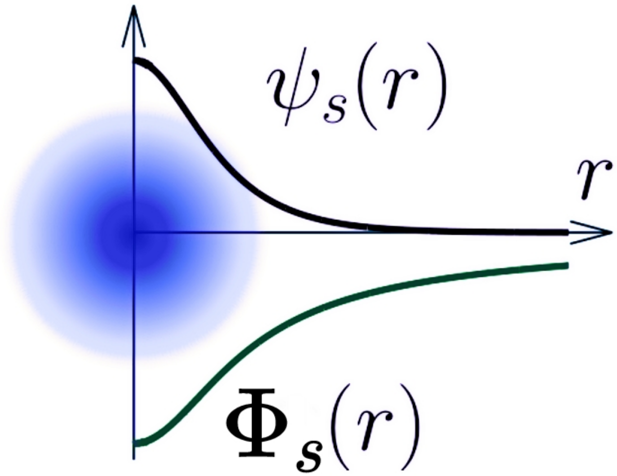
3 Newtonian gravity: $\Phi(t, \mathbf{x}) \ll 1$

$$\Delta\Phi(t, \mathbf{x}) = 4\pi G m^2 f_a^2 |\psi|^2$$

**Schrödinger-Poisson system: $\psi(t, \mathbf{x})$, $\Phi(t, \mathbf{x})$**

# Bose star

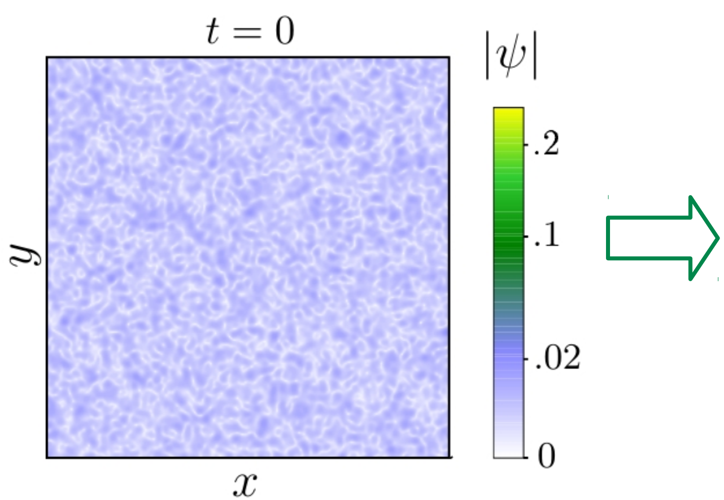
**Bose star = BE condensate in state $\psi_s(r)$**

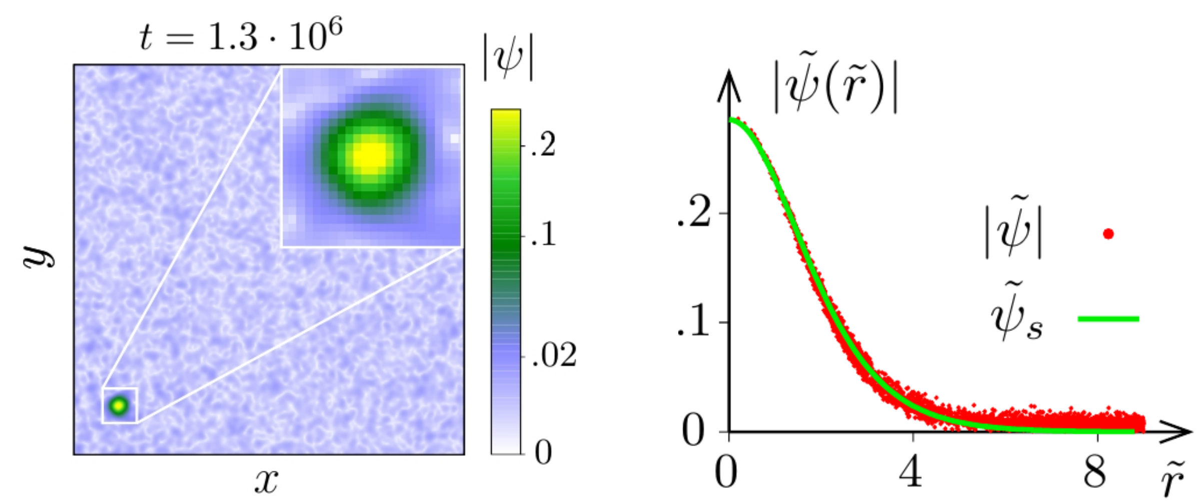



### Solitonic solution

- $\psi = \psi_s(r)e^{-i\omega_s t}$ - ground state of $\Phi$
- $\Phi = \Phi_s(r)$ - potential of $\psi_s(r)$
- $\omega_s < 0$ - energy level
- $M_s = M(\psi_s)$ - parameter

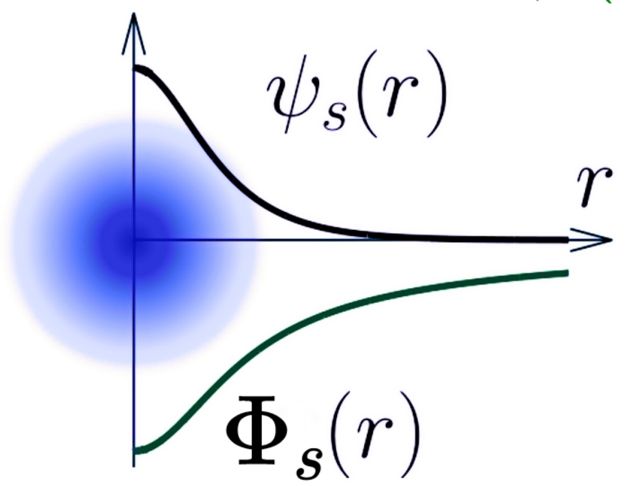
*[Ruffini, Bonazolla '69; Tkachev '86]*



# Bose star

Bose star = BE condensate in state $\psi_s(r)$




## Solitonic solution

- $\psi = \psi_s(r)\mathrm{e}^{-i\omega_s t}$ - ground state of $\Phi$
- $\Phi = \Phi_s(r)$ - potential of $\psi_s(r)$
- $\omega_s < 0$ - energy level
- $\mathrm{M}_s = M(\psi_s)$ - parameter

[Ruffini, Bonazolla '69; Tkachev '86]

Form in popular DM models via BE condensation:

**Fuzzy DM ($m \sim 10^{-22}$ eV)**

In (dwarf) galaxies during structure formation:

$M_{\rm halo} \sim 5 \times 10^9 M_\odot$

$M_s \sim 10^8 M_\odot$

[Schive, Chiueh, Broadhurst '14]

[Veltmaat, Niemeyer '16]

**QCD axions ($m \sim 10^{-5}$ eV)**

In axion miniclusters after RD/MD:

$M_{\rm halo} \sim 10^{-13} M_\odot$

$M_s \sim 10^{-15} M_\odot$

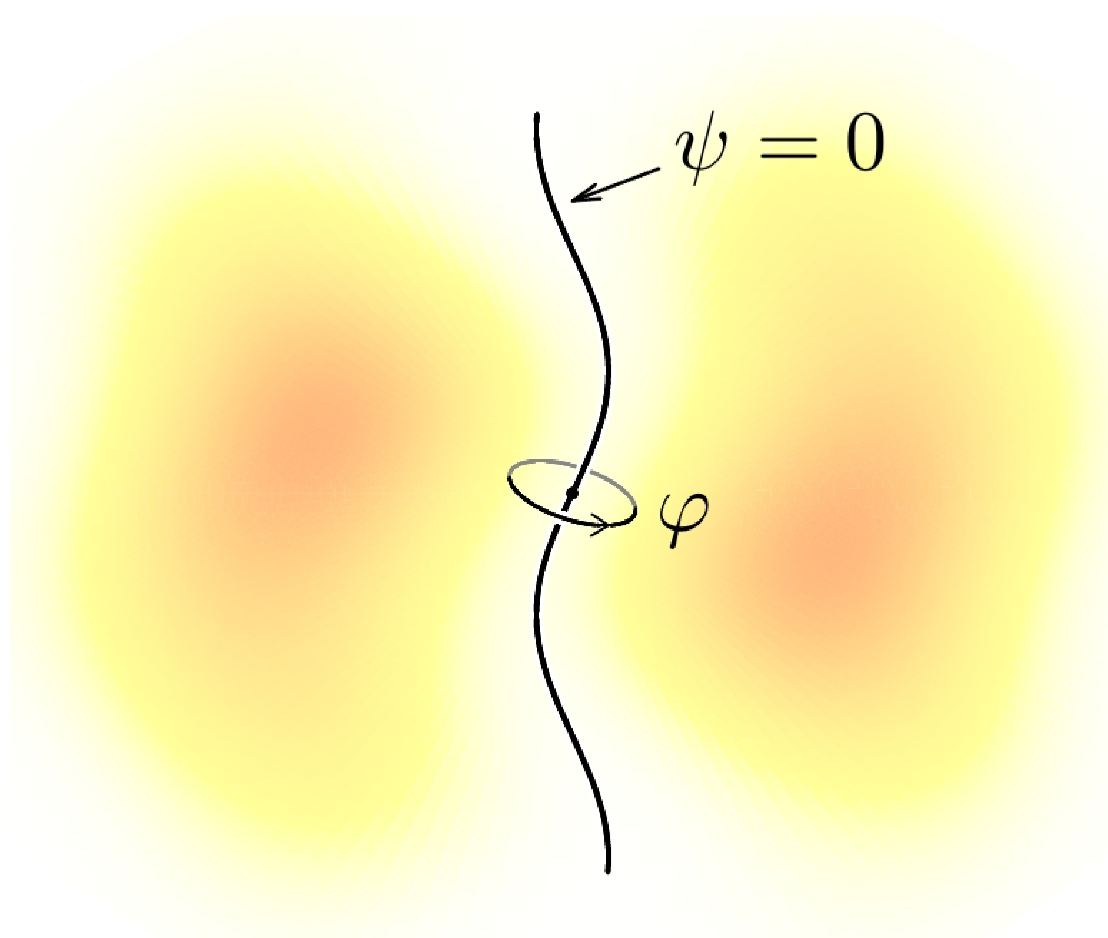
[Levkov, Panin, Tkachev '18]

[Eggemeier, Niemeyer '19]

# Can the Bose stars rotate?

BEC velocity is irrotational:

$$v = \frac{\mathrm{Im}(\psi^* \nabla \psi)}{m|\psi|^2} = \nabla \arg \psi / m$$




$$\boxed{\mathrm{rot}\, v = 0}$$

$\Downarrow$

Rotation ↔ **vortices**!

$$\oint v\, dx = l$$

$l$ rotation quanta

$\Leftarrow$

$$\psi \propto r^l e^{il\phi}$$

This costs energy!

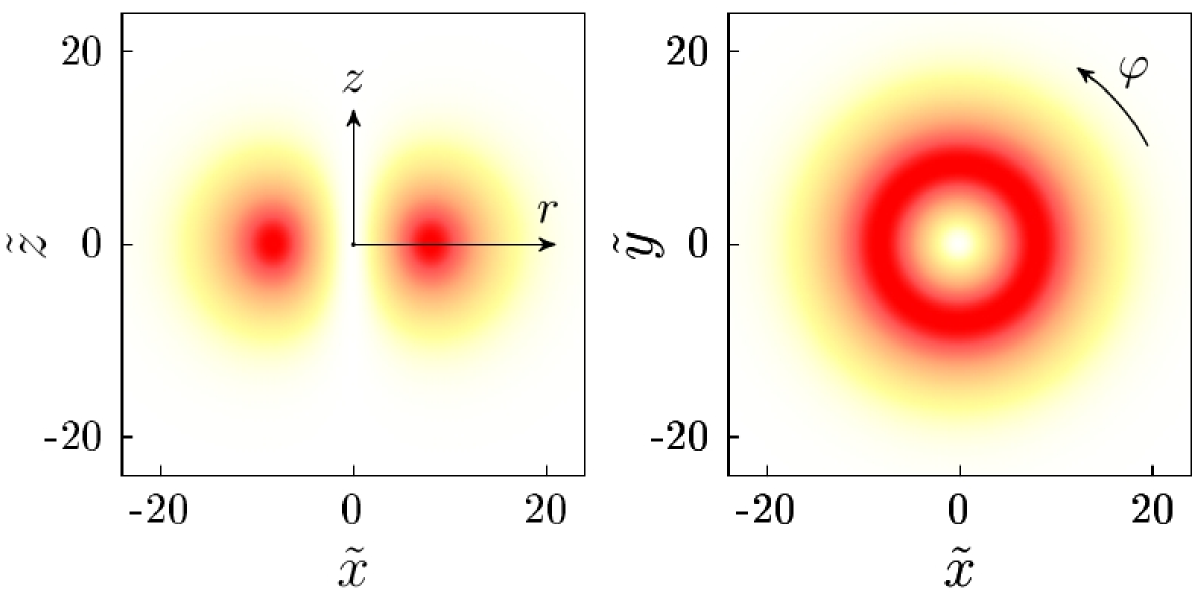
# Rotating Bose stars

Axially-symmetric Ansatz:

$$\begin{cases} \psi = \psi_s(r,z)\mathrm{e}^{-i\omega_s t + il\phi} \\ \Phi = \Phi_s(r,z) \end{cases} \qquad \psi_s \propto r^l \text{ as } r \to 0$$

BEC of particles with angular momenta l: $L_z = M_s l/m$

Solve equations numerically at $l = 1,\ g_4 = 0$: (project to $R_{\pi/2}\psi = \mathrm{e}^{i\pi/2}\psi$ )



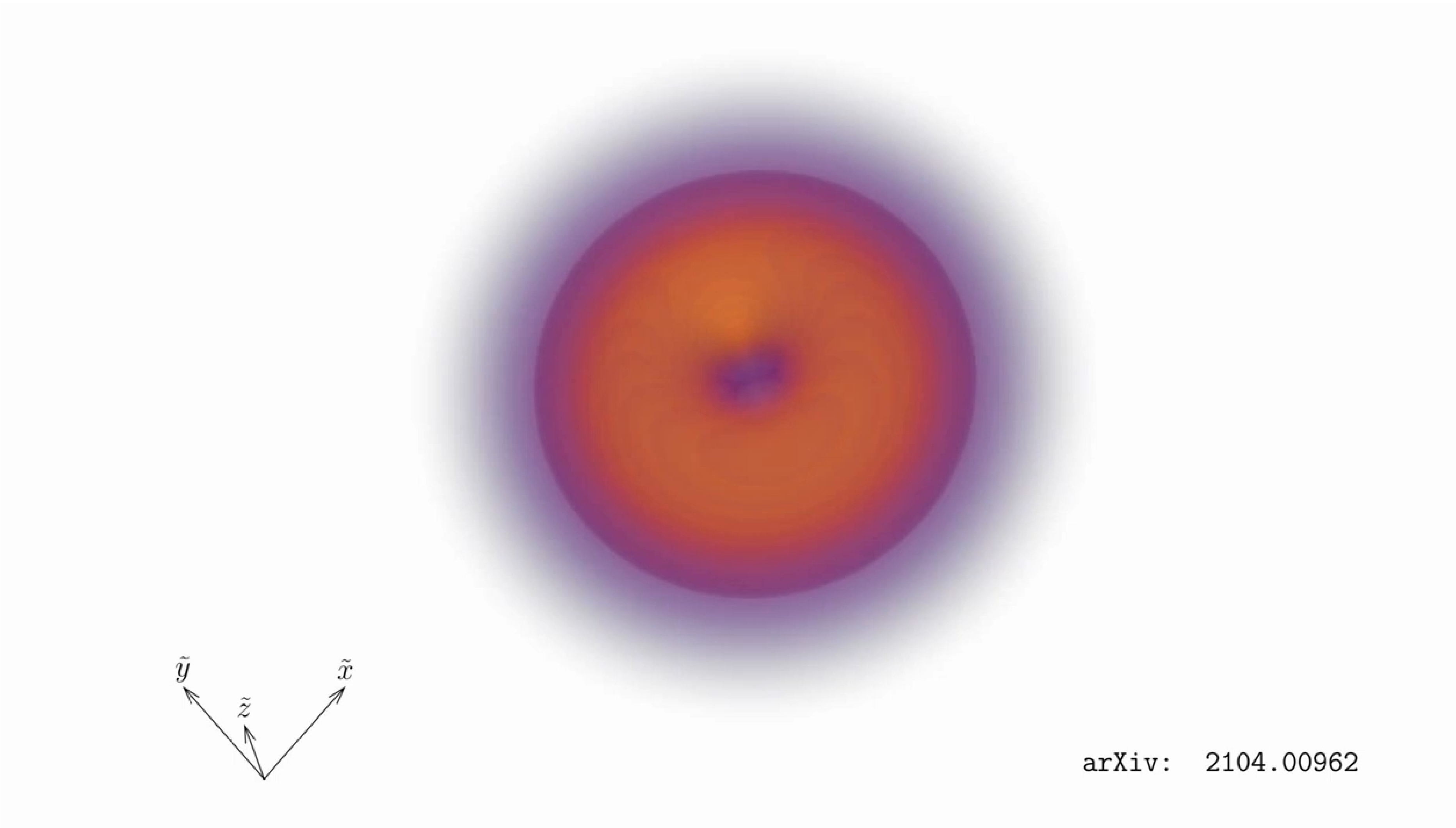

Two-parametric family:

$l,\ M_s$

# Evolve the l = 1 star numerically

$$t = 0: \qquad \psi = \psi_s + \delta\psi$$

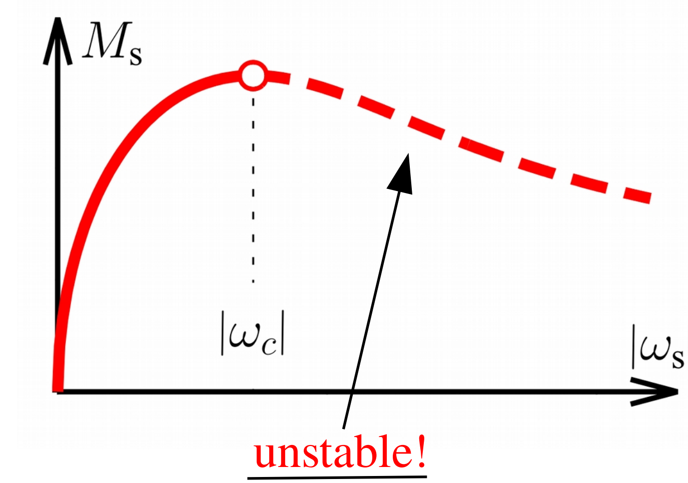
small



# Bosenova

$$\begin{cases} i\partial_t \psi = -\dfrac{\Delta\psi}{2m} + m\left(\Phi - \dfrac{g_4^2}{8}|\psi|^2\right)\psi \\ \Delta\Phi(t,\mathbf{x}) = 4\pi G m^2 f_a^2 |\psi|^2 \end{cases}$$

**attractive selfinteraction**




[Vakhitov, Kolokolov '71; Chavanis '11]

⇩

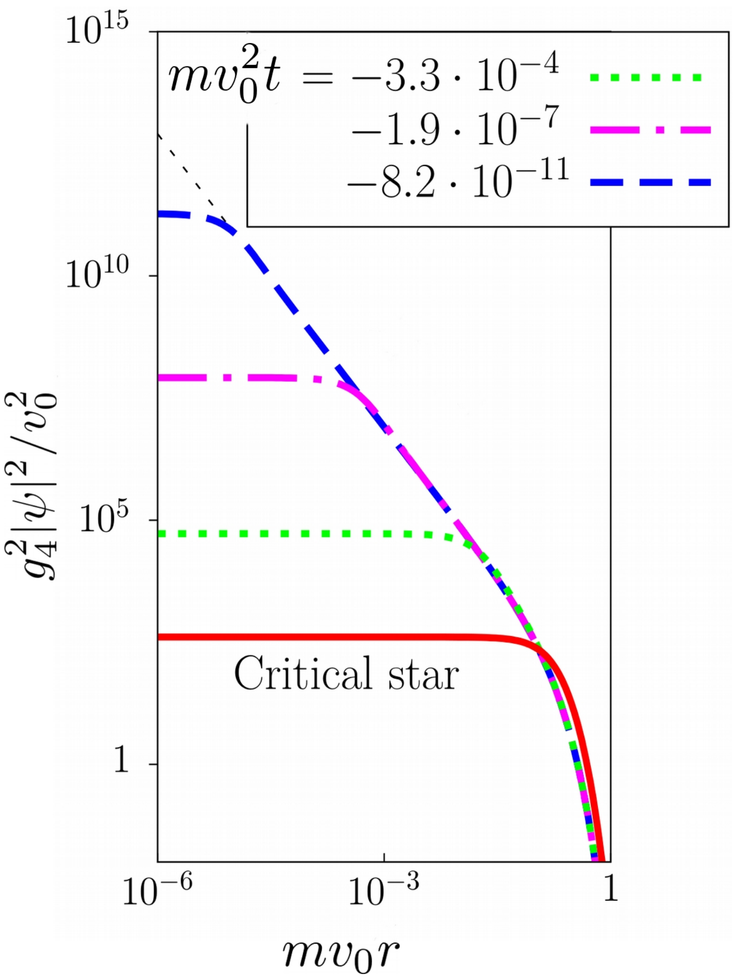
**Large-mass Bose stars are unstable!**

$$M_{cr} \approx 10\frac{M_{Pl} f_a}{m g_4} \approx \mathbf{5 \times 10^{-12}\, M_\odot}$$

$$R_{cr} \approx 0.18\frac{g_4 M_{Pl}}{m f_a} \approx \mathbf{70\,km}$$

QCD axion

[Levkov, Panin, Tkachev '17]



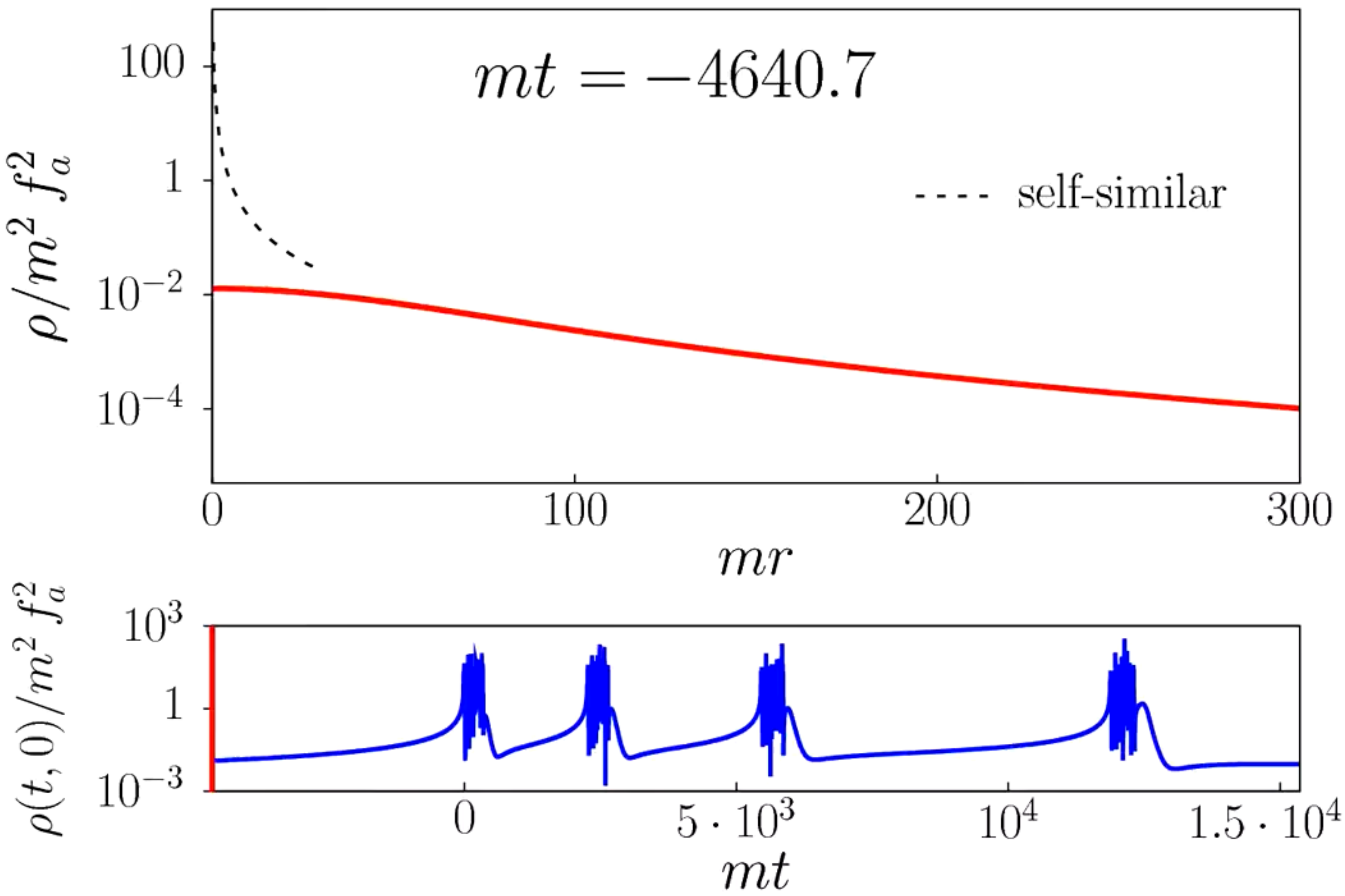

**Q: What will happen to an unstable Bose star?**

**A: It's collapse!**

# Bosenova: full relativistic simulation

Solve numerically relativistic equation: $\Box a = -(1+2\Phi)\,\mathcal{V}'(a/f_a)/f_a$

$\mathcal{V}(\theta) = -m_a^2 f_a^2\,(1+1/z)\,\sqrt{1+z^2+2z\cos\theta}$, where $z \equiv m_u/m_d \approx 0.56$




**Relativistic axions are emitted!**

*[Levkov, Panin, Tkachev '17]*

# Interaction with photons

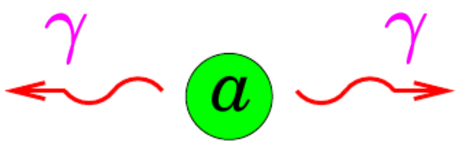
$$\partial_\mu \left( F_{\mu\nu} + g_{a\gamma\gamma} a \tilde{F}_{\mu\nu} \right) = 0$$

$$a = \frac{f_a}{\sqrt{2}} \left[ \psi \, \mathrm{e}^{-imt} + \mathrm{h.c.} \right]$$

coherently oscillates

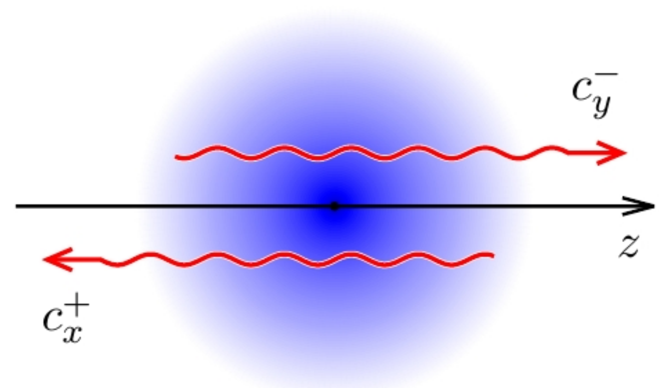
⇓

**parametric resonance**




$E_a \approx m$ ⇒ $E_\gamma = p_\gamma \simeq m/2$

⇓




$$A_i = \underbrace{c_i^+(t,x)\,\mathrm{e}^{im(z+t)/2}}_{\text{left-moving}} + \underbrace{c_i^-(t,x)\,\mathrm{e}^{im(z-t)/2}}_{\text{right-moving}} + \mathrm{h.c.}$$

⇓

$$\partial_t c_x^+ = \partial_z c_x^+ + i g_{a\gamma\gamma} f_a m \psi^* c_y^- / 2^{3/2}$$

$$\partial_t c_y^- = -\partial_z c_y^- - i g_{a\gamma\gamma} f_a m \psi c_x^+ / 2^{3/2}$$

⟵ $\partial_{t,x} c_i^\pm \ll m c_i^\pm$

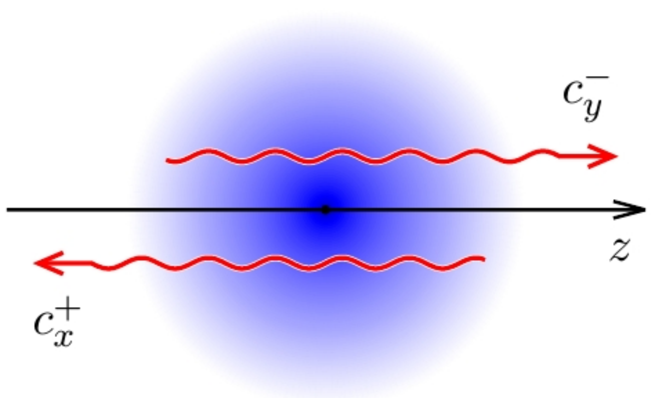
# Simple equations for photons

$$\mu \cancel{\partial_t} c_x^+ = \partial_z c_x^+ + i g_{a\gamma\gamma} f_a m \psi^* c_y^- /2^{3/2}$$

$$\mu \cancel{\partial_t} c_y^- = -\partial_z c_y^- - i g_{a\gamma\gamma} f_a m \psi c_x^+ /2^{3/2}$$

- Boundary conditions:

$c_i^\pm \to 0$ as $z \to \pm\infty$




- Quasi-stationary approximation:

$$\begin{matrix} t_\gamma & \ll & t_a \\ \wr & & \wr \\ R & & R/v \end{matrix} \quad \Rightarrow \quad \begin{cases} c_i^\pm \propto e^{\int^t \mu(t')dt'} \\ vR^{-1} \ll \mu \ll R^{-1} \end{cases}$$

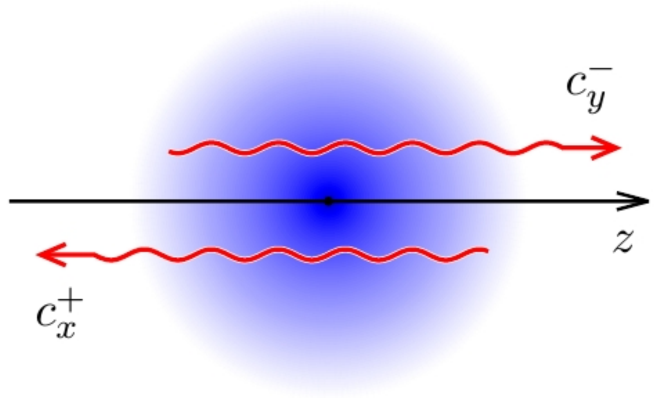
(start of resonance)

Restoring the solution:

$$A_i = \int d\mathbf{n}_z c_i^{(\mathbf{n}_z)}(x) e^{\int^t \mu(t')dt' + im(\mathbf{n}_z \mathbf{x} + t)/2}$$

# Static coherent axions (Bose stars)

$$v = 0, \quad \mu \ll R^{-1}$$




1 Analytic solution for any $\psi$!

$$\left.\begin{aligned} c_x^+ &= A e^{\mu z} \cos(D(z)) \\ c_y^- &= -iAe^{-\mu z} \sin(D(z)) \end{aligned}\right\} \times e^{\int^t \mu dt}$$

$$D(z) = g_{a\gamma\gamma} f_a m 2^{-3/2} \int_{-\infty}^{z} \psi dz'$$

2 Growth exponent: $\mu = \dfrac{D(+\infty) - \pi/2}{\int dz \sin(2D(z))}$

3 Condition for resonance: $D(+\infty) \geq \pi/2$

Need massive Bose stars! $M \geq M_0 = 7.66 M_{Pl}/(m g_{a\gamma\gamma})$

4 QCD axions: $M \leq M_{cr} \Rightarrow g_{a\gamma\gamma} \geq 0.31/f_a$

or collapse

# Applications

[Levkov, Panin, Tkachev '20]

$$D(z) = g_{a\gamma\gamma} f_a m 2^{-3/2} \int_{-\infty}^{z} \psi dz'$$

- **One Bose star**

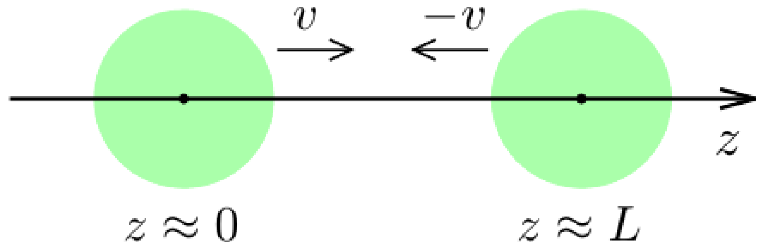
  Resonance at $D(+\infty) \geq \pi/2$

  In the center of collapsing BS: $g_{a\gamma\gamma} \geq 0.13/f_a$

- **Two Bose stars**

  Resonance is easier: $2D(\infty) \geq \pi/2$

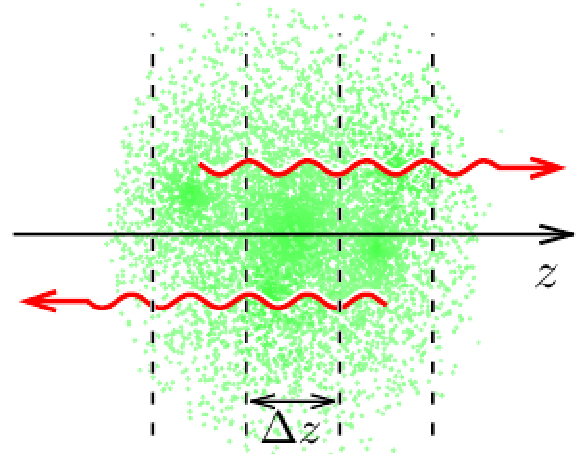
  But: $v \leq (mR)^{-1}, \quad L > R$




- **Diffuse axions**

  Coarse-graining ⇨ kinetic eq. for $n_\gamma$

$$D_{\text{diff}} \equiv \frac{g^2_{a\gamma\gamma}}{8} \int \rho(z) l_{\text{coh}}(z) dz \geq 1$$

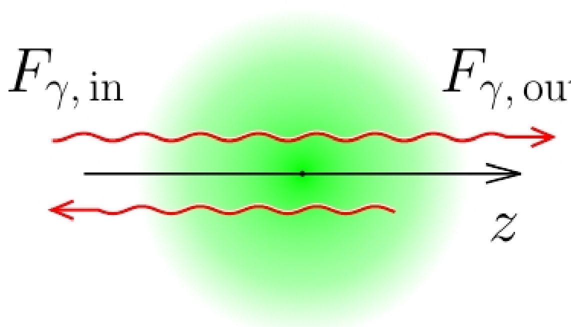



- **Radio amplification**, $\omega_\gamma \simeq m/2$:

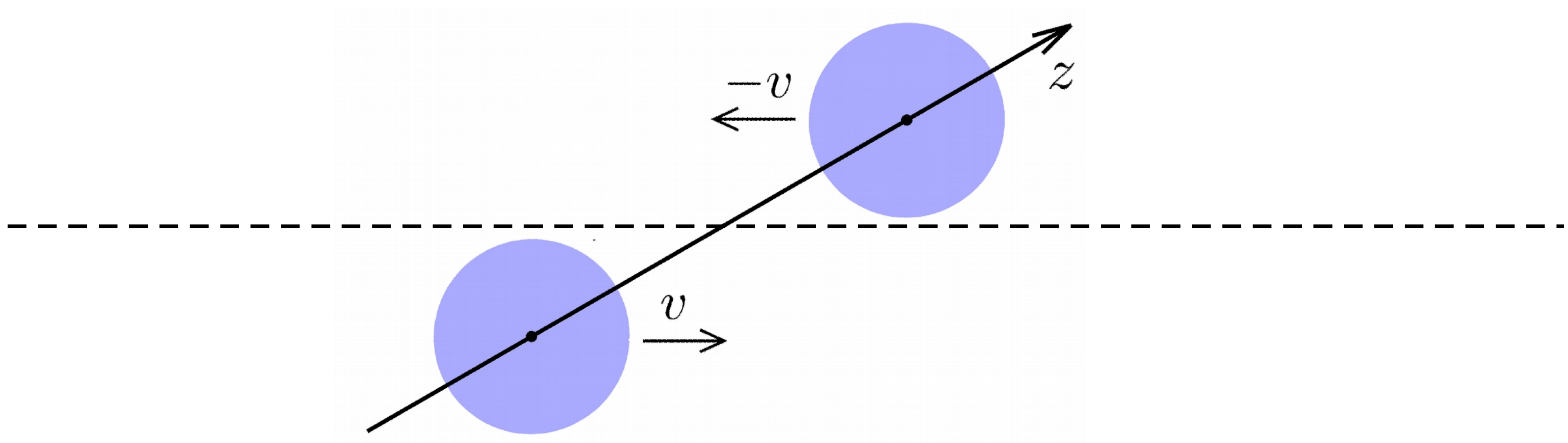
  $F_{\gamma,\text{out}} = F_{\gamma,\text{in}} / \cos^2 D$ - Bose star

  (dominant if $\rho_{\text{star}}/\rho_{\text{diff}} > 10^{-4}$)

  $F_{\gamma,\text{out}} = F_{\gamma,\text{in}} / (1 - D_{\text{diff}})$ - diffuse axions



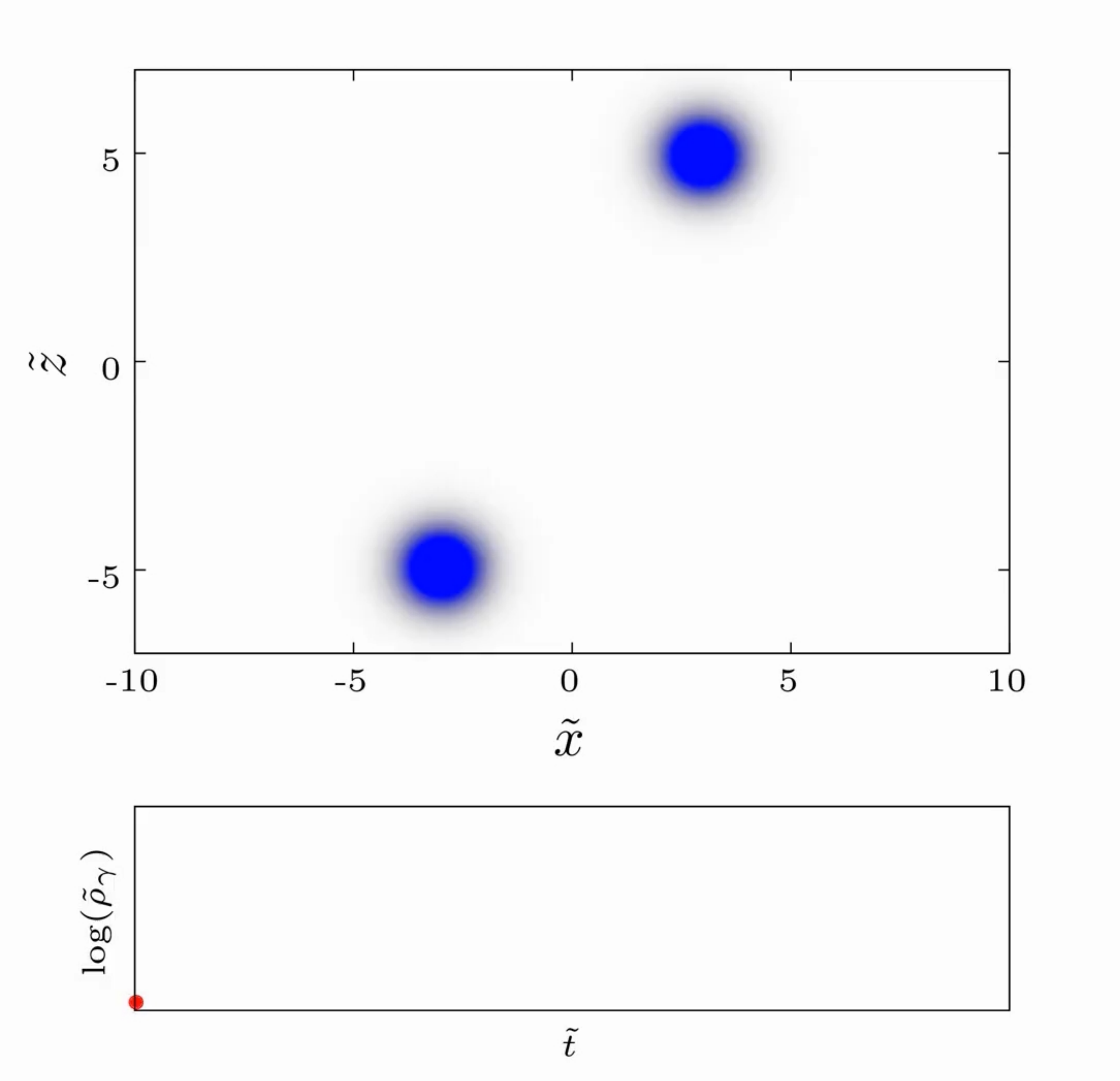
# Two moving Bose stars: numerical simulation




$$
\begin{cases}
\partial_t c_x^+ = \partial_z c_x^+ - \dfrac{i}{m}(\partial_x^2 + \partial_y^2) c_x^+ + i g_{a\gamma\gamma} f_a m \psi^* c_y^- / 2^{3/2} \\
\partial_t c_y^- = -\partial_z c_y^- + \dfrac{i}{m}(\partial_x^2 + \partial_y^2) c_y^- - i g_{a\gamma\gamma} f_a m \psi c_x^+ / 2^{3/2} \\
i\partial_t \psi = -\dfrac{\Delta\psi}{2m} + m\left(\Phi - \dfrac{g_4^2}{8}|\psi|^2\right)\psi - \dfrac{m g_{a\gamma\gamma}}{2^{3/2} f_a} \epsilon_{\alpha\beta} c_\alpha^- c_\beta^{+*} \\
\Delta\Phi = 4\pi G(m^2 f_a^2 |\psi|^2 + \rho_\gamma)
\end{cases}
$$

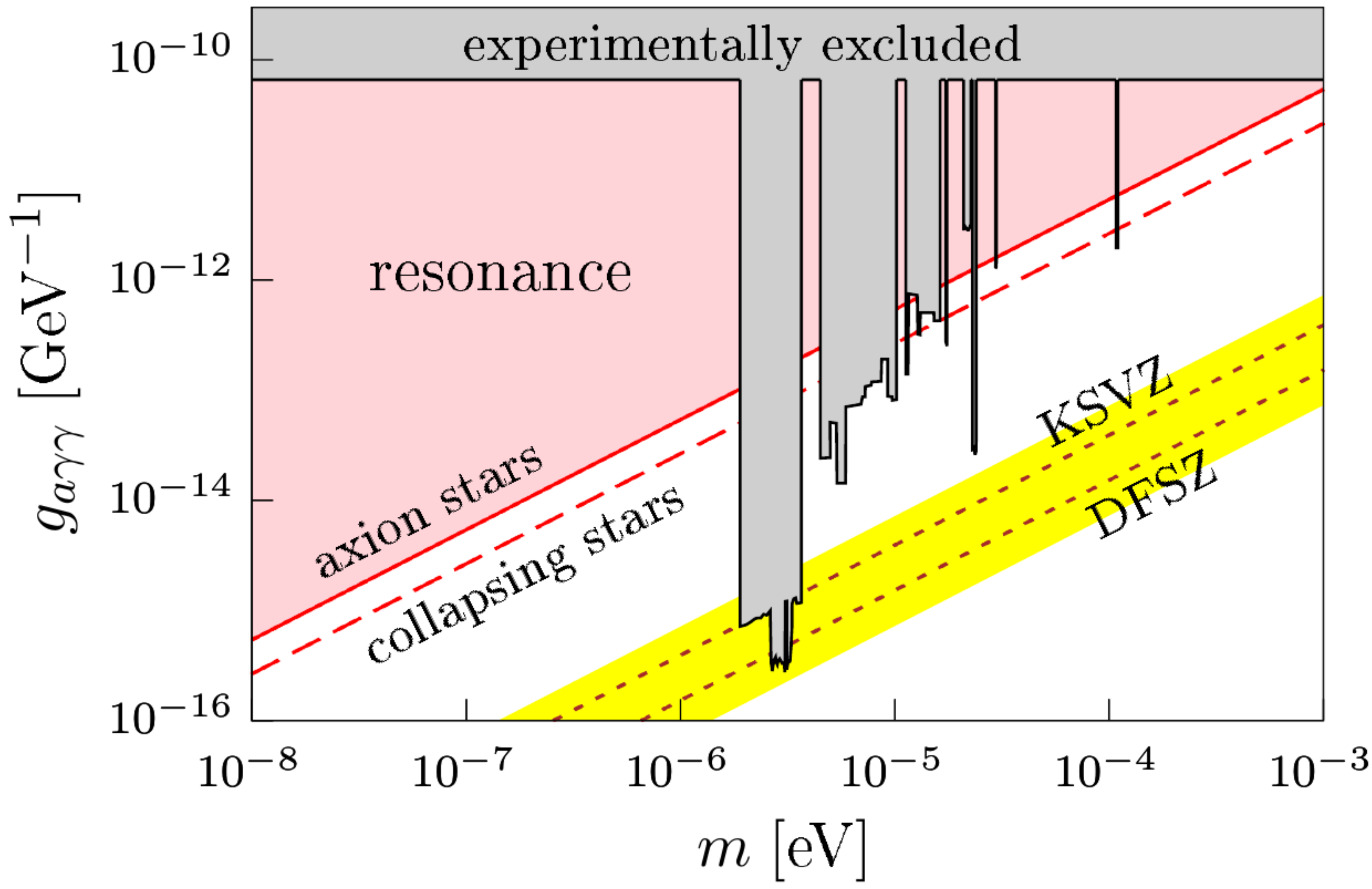
backreaction

# Two moving Bose stars: numerical simulation



# Exclusion plot for QCD axions

$g_{a\gamma\gamma} \geq 0.31/f_a$




*[Hertzberg, Schiappacasse '18; Levkov, Panin, Tkachev '18]*

# Conclusions: Implications of Bose stars in axion cosmology

- Less diffuse DM ⇒ weaker signals in DM detectorts
- Radio lines from transient axion stars

  *[Witte et al ’22]*

- Parametric resonance: radio explosions of heavy stars — explain FRB?

  *[Levkov, Panin, Tkachev ’20; Chung-Jukko et al ’22]*

- Radio-emitting stars heat the cosmological medium

  *[Escudero et al ’23]*

- Bosenovas: additional flux of axions in DM detectors

  *[Levkov, Panin, Tkachev ’17; Eby et al ’22]*

**THANK YOU FOR ATTENTION!**

This work was supported by the RSF grant 22-12-00215