Phase transition for HQGP in magnetic field: magnetic catalysis for heavy quarks

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with I.Ya. Aref'eva, A. Hajilou, P. Slepov
Eur.Phys.J.C **83** 12 (2023); K.R. arXiv:2405.07881 [hep-th]

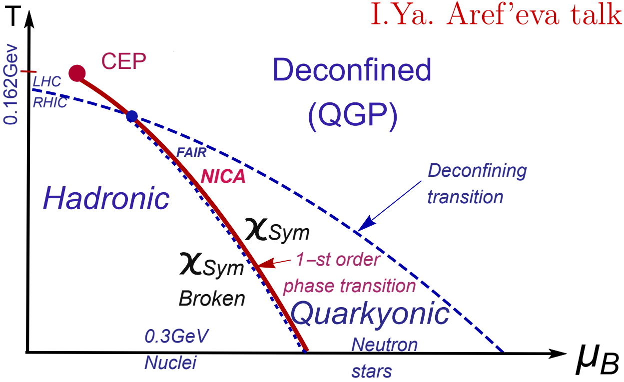
The expected QCD phase diagram

Goal of Holographic QCD — describe QCD phase diagram

Requirements:

- reproduce the QCD results from perturbative theory at short distances
- reproduce Lattice QCD results at large distances (~ 1 fm) and **small** μ_B

I.Ya. Aref'eva talk



Twice anisotropic background

$$\mathcal{L} = R - \frac{f_0(\phi)}{4} F_0^2 - \frac{f_1(\phi)}{4} F_1^2 - \frac{f_3(\phi)}{4} F_3^2 - \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$A_\mu^0 = A_t(z)\delta_\mu^0$ $\qquad F_1 = q_1\ dx^2 \wedge dx^3$ $\qquad F_3 = q_3\ dx^1 \wedge dx^2$

$A_t(0) = \mu$ $\qquad g(0) = 1$ $\qquad$ *Dudal et al., (2019)*

$A_t(z_h) = 0$ $\qquad g(z_h) = 0$ $\qquad \phi(z_0) = 0 \ \rightarrow\ \sigma_{\text{string}}$

$$ds^2 = \frac{L^2}{z^2}\,\mathfrak{b}(z)\left[-\,g(z)\ dt^2 + dx_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_2^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_3^2 + \frac{dz^2}{g(z)}\right]$$

I.A., A.G. (2014), Giataganas (2013) $\qquad$ *Gürsoy, Järvinen et al., (2019)*

$\mathfrak{b}(z) = e^{2\mathcal{A}(z)} \ \rightarrow\$ quarks mass $\qquad$ "Bottom-up approach"

$\mathcal{A}(z) = -\,cz^2/4 \ \rightarrow\$ heavy quarks background (b, t) $\qquad$ *Andreev, Zakharov (2006)*

$\mathcal{A}(z) = -\,a\ln(bz^2+1) \ \rightarrow\$ light quarks background (d, u) $\qquad$ *Li, Yang, Yuan (2020)*

"Heavy" quarks warp factor extensions

$$\mathcal{A}(z) = -\,cz^2/4$$

Aref'eva, K.R., Slepov
*JHEP **07** 161 (2021) arXiv:2011.07023 [hep-th]*

$\Downarrow$ $\Downarrow$

$$\mathcal{A}(z) = -\,R_{gg}z^2/3 - pz^4$$

Hea, Yang, Yuan
arXiv:2004.01965 [hep-th]

$$f_0 = e^{-(R_{gg}+\frac{c_B q_3}{2})z^2}\,\frac{z^{-2+\frac{2}{\nu}}}{\sqrt{\mathfrak{b}}}$$

$R_{gg} = 1.16$ GeV^2, $p = 0.273$ GeV^2

No magnetic catalysis

$$\mathcal{A}(z) = -\,az^2 - dB^2z^5$$

Bohra, Dudal, Hajilou, Mahapatra
*PRD **103** 086021 (2021)*

$$f_0 = e^{-(c+q_3^2)z^2}\,\frac{z^{-2+\frac{2}{\nu}}}{\sqrt{\mathfrak{b}}}$$

$a = 0.15$ GeV^2, $c = 1.16$ GeV^2

$d > 0.05$

“Heavy” quarks warp factor extensions

$$\mathcal{A}(z) = -\, cz^2/4$$

Aref'eva, K.R., Slepov
*JHEP **07** 161 (2021) arXiv:2011.07023 [hep-th]*

$\Downarrow$ $\Downarrow$

$$\mathcal{A}(z) = -cz^2/4 - (p - c_B\, q_3)z^4$$

$$f_0 = e^{-(R_{gg} + \frac{c_B q_3}{2})z^2}\, \frac{z^{-2+\frac{2}{\nu}}}{\sqrt{\mathfrak{b}}}$$

$c = 4R_{gg}/3$, $R_{gg} = 1.16$ GeV^2,
$p = 0.273$ GeV^2

Aref'eva, Hajilou, K.R., Slepov
*Eur.Phys.J.C **83** 12 (2023)*

A. Hajilou's talk

$$\mathcal{A}(z) = -\, az^2 - dq_3^2 z^5$$

$$f_0 = e^{-(c+q_3^2)z^2}\, \frac{z^{-2+\frac{2}{\nu}}}{\sqrt{\mathfrak{b}}}$$

$a = 0.15$ GeV^2, $c = 1.16$ GeV^2
$d > 0.05$?

K.R.
arXiv:2405.07881 [hep-th]

Solution for "heavy" quarks for pz^4

$$A_t(z) = \mu\left(1 - \frac{1 - e^{\left(R_{gg} + \frac{c_B(q_3-1)}{2}\right)z^2}}{1 - e^{\left(R_{gg} + \frac{c_B(q_3-1)}{2}\right)z_h^2}}\right) = \mu - \rho\, z^2 + \ldots, \;\; \rho = -\frac{\mu\left(2R_{gg} + c_B(q_3-1)\right)}{2\left(1 - e^{\left(R_{gg} + \frac{c_B(q_3-1)}{2}\right)z_h^2}\right)}$$

$$g(z) = e^{c_B z^2}\left[1 - \frac{I_1(z)}{I_1(z_h)} + \frac{\mu^2\left(2R_{gg} + c_B(q_3-1)\right)I_2(z)}{L^2\left(1 - e^{\left(R_{gg} + \frac{c_B(q_3-1)}{2}\right)z_h^2}\right)^2}\left(1 - \frac{I_1(z)}{I_1(z_h)}\,\frac{I_2(z_h)}{I_2(z)}\right)\right]$$

$$I_1(z) = \int_0^z e^{\left(R_{gg} - \frac{3c_B}{2}\right)\xi^2 + 3p\xi^4}\xi^{1+\frac{2}{\nu}}\, d\xi \quad I_2(z) = \int_0^z e^{\left(R_{gg} + \frac{c_B}{2}\left(\frac{q_3}{2} - 2\right)\right)\xi^2 + 3p\xi^4}\xi^{1+\frac{2}{\nu}}\, d\xi$$

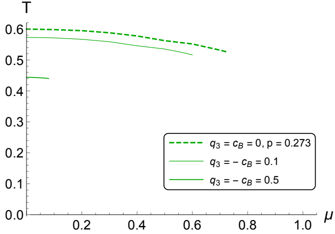
$$T = \left| -\frac{e^{\left(R_{gg} - \frac{c_B}{2}\right)z_h^2 + 3pz_h^4}\, z_h^{1+\frac{2}{\nu}}}{4\pi\, I_1(z_h)} \times \right.$$

$$\left. \times\left[1 - \frac{\mu^2\left(2R_{gg} + c_B(q_3-1)\right)\left(e^{\left(R_{gg} + \frac{c_B(q_3-1)}{2}\right)z_h^2}\, I_1(z_h) - I_2(z_h)\right)}{L^2\left(1 - e^{\left(R_{gg} + \frac{c_B(q_3-1)}{2}\right)z_h^2}\right)^2}\right]\right|$$

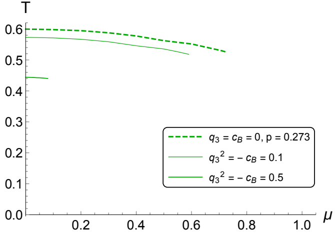
$$s = \frac{1}{4}\left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}} e^{-\left(R_{gg} - \frac{c_B}{2}\right)z_h^2 - 3pz_h^4}$$

1-st order phase transition for pz^4

$\nu = 1$

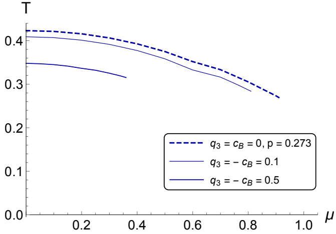


$c_B = \pm 0.5$

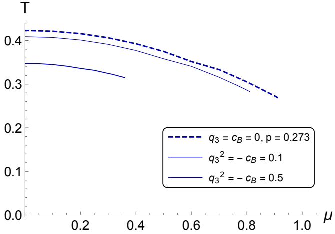


$q_3 = 0.5$

$\nu = 4.5$



$q_3 = \pm c_B$



$q_3^2 = \pm c_B$

Solution for “heavy” quarks for $(p - c_B q_3) z^4$

$$g(z) = e^{c_B z^2}\left[1 - \frac{I_1(z)}{I_1(z_h)} + \frac{\mu^2\big(2R_{gg} + c_B(q_3 - 1)\big) I_2(z)}{L^2\left(1 - e^{(R_{gg} + \frac{c_B(q_3-1)}{2}) z_h^2}\right)^2}\left(1 - \frac{I_1(z)}{I_1(z_h)}\,\frac{I_2(z_h)}{I_2(z)}\right)\right]$$

$$I_1(z) = \int_0^z e^{(R_{gg} - \frac{3c_B}{2})\xi^2 + 3(p - c_B q_3)\xi^4}\, \xi^{1+\frac{2}{\nu}}\, d\xi$$

$$I_2(z) = \int_0^z e^{(R_{gg} + \frac{c_B}{2}(\frac{q_3}{2} - 2))\xi^2 + 3(p - c_B q_3)\xi^4}\, \xi^{1+\frac{2}{\nu}}\, d\xi$$

$$T = \left| - \frac{e^{(R_{gg} - \frac{c_B}{2}) z_h^2 + 3(p - c_B q_3) z_h^4}\, z_h^{1+\frac{2}{\nu}}}{4\pi\, I_1(z_h)} \times \right.$$

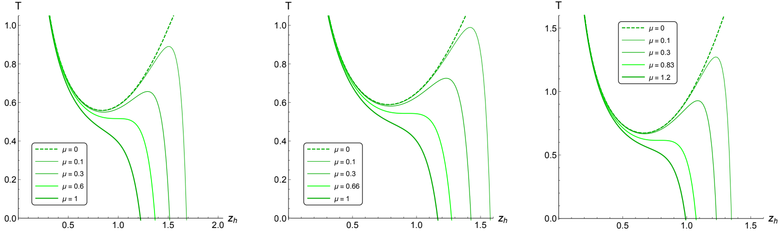
$$\left. \times \left[1 - \frac{\mu^2\big(2R_{gg} + c_B(q_3 - 1)\big)\left(e^{(R_{gg} + \frac{c_B(q_3-1)}{2}) z_h^2}\, I_1(z_h) - I_2(z_h)\right)}{L^2\left(1 - e^{(R_{gg} + \frac{c_B(q_3-1)}{2}) z_h^2}\right)^2}\right]\right|$$

$$s = \frac{1}{4}\left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}} e^{-(R_{gg} - \frac{c_B}{2}) z_h^2 - 3(p - c_B q_3) z_h^4}$$

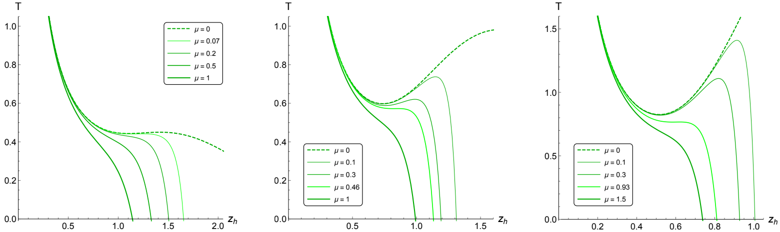
*Aref'eva et al. Eur.Phys.J.C **83** 12 (2023) arXiv:2305.06345 [hep-th]*

Temperature $T(z_h)$, $\nu = 1$

$c_B = -0.1$



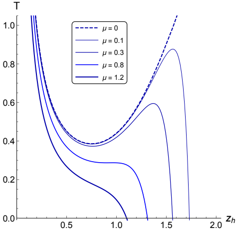
$c_B = -0.5$

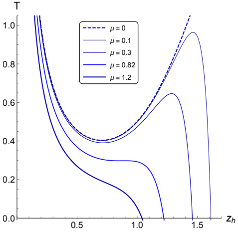


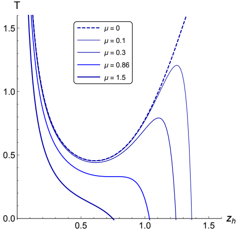
$q_3 = 0$ $q_3 = 1$ $q_3 = 5$

Temperature $T(z_h)$, $\nu = 4.5$

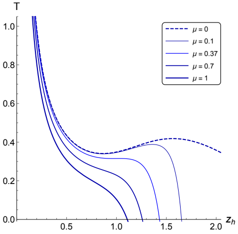
$c_B = -0.1$

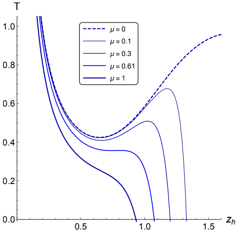


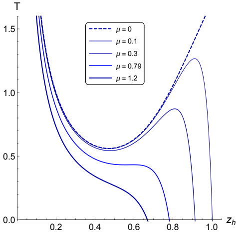




$c_B = -0.5$



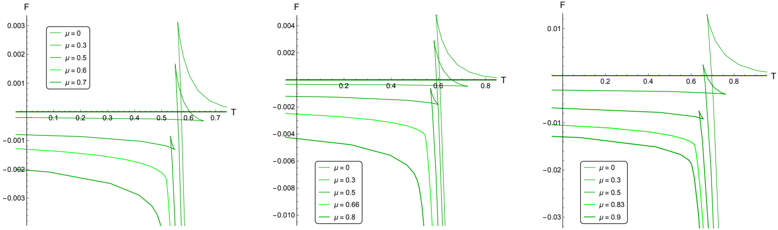




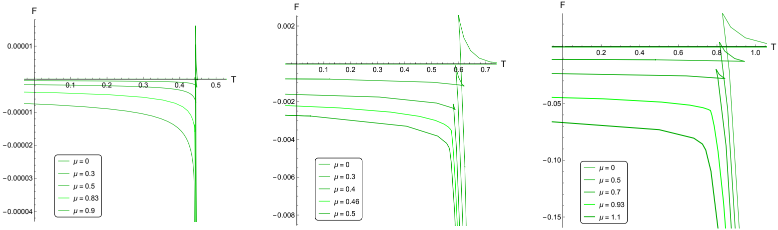
$q_3 = 0$ $\qquad$ $q_3 = 1$ $\qquad$ $q_3 = 5$

Free energy $F(T)$, $\nu = 1$

$c_B = -0.1$



$c_B = -0.5$



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Free energy $F(T)$, $\nu = 4.5$

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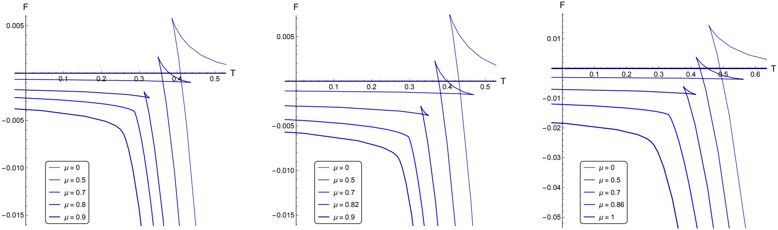
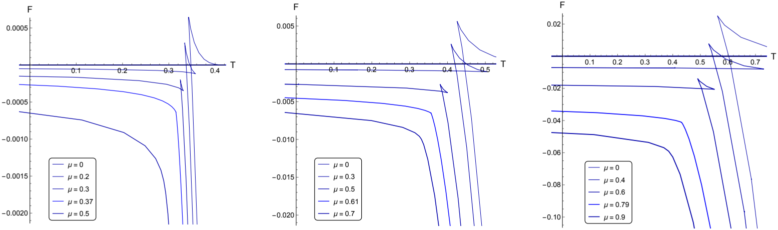


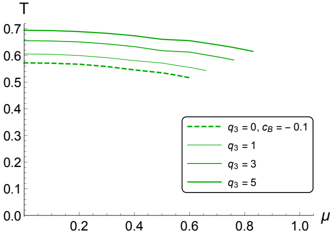
$c_B = -0.5$

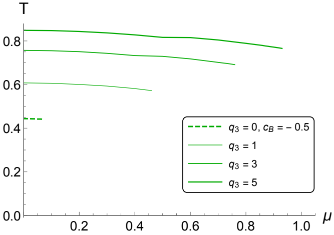


$q_3 = 0$ $q_3 = 1$ $q_3 = 5$

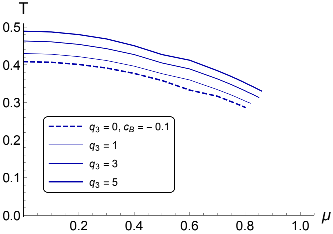
1-st order phase transition $T(\mu)$

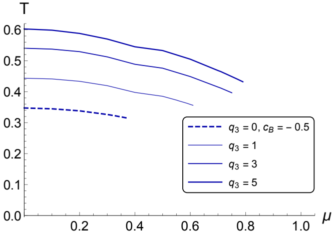
$\nu = 1$





$\nu = 4.5$





$c_B = -0.1$ $\qquad$ $c_B = -0.5$

To be continued... and published

Solution for "heavy" quarks for $dq_3^2 z^5$-term

$$A_t(z) = \mu \left(1 - \frac{1 - e^{(c - \frac{c_B}{2} + q_3^2) z^2}}{1 - e^{(c - \frac{c_B}{2} + q_3^2) z_h^2}}\right) = \mu - \rho\, z^2 + \ldots, \quad \rho = - \frac{\mu \left(2c - c_B + 2q_3^2\right)}{2 \left(1 - e^{(c - \frac{c_B}{2} + q_3^2) z_h^2}\right)}$$

$$g(z) = e^{c_B z^2} \left[1 - \frac{I_1(z)}{I_1(z_h)} + \frac{\mu^2 \left(2c - c_B + 2q_3^2\right) I_2(z)}{L^2 \left(1 - e^{(c - \frac{c_B}{2} + q_3^2) z_h^2}\right)^2} \left(1 - \frac{I_1(z)}{I_1(z_h)} \frac{I_2(z_h)}{I_2(z)}\right)\right],$$

$$I_1(z) = \int_0^z e^{3\left(a - \frac{c_B}{2} + dq_3^2 \xi^3\right) \xi^2} \xi^{1 + \frac{2}{\nu}}\, d\xi \qquad I_2(z) = \int_0^z e^{3\left(a + \frac{c - 2c_B + q_3^2}{2} + dq_3^2 \xi^3\right) \xi^2} \xi^{1 + \frac{2}{\nu}}\, d\xi$$

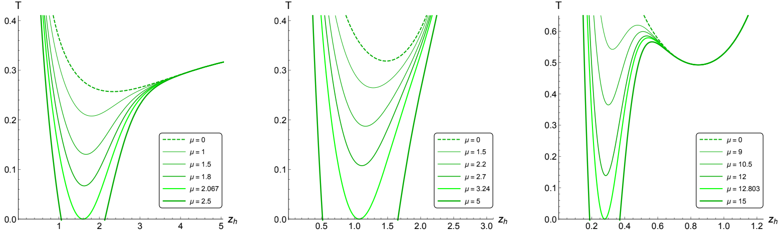
$$T = \left| - \frac{e^{3(c - \frac{c_B}{6} + dq_3^2) z_h^2}\, z_h^{1 + \frac{2}{\nu}}}{4\pi\, I_1(z_h)} \left[1 - \frac{\mu^2 \left(2c - c_B + 2q_3^2\right) \left(e^{(c - \frac{c_B}{2} + q_3^2) z_h^2}\, I_1(z_h) - I_2(z_h)\right)}{L^2 \left(1 - e^{(c - \frac{c_B}{2} + q_3^2) z_h^2}\right)^2}\right]\right|$$

$$s = \frac{1}{4} \left(\frac{L}{z_h}\right)^{1 + \frac{2}{\nu}} e^{-3(c - \frac{c_B}{6} + dq_3^2) z_h^2}$$

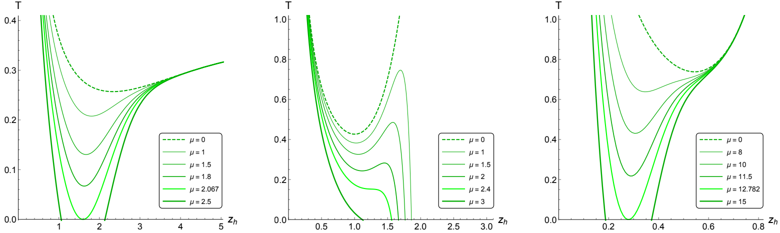
K.R. arXiv:2405.07881 [hep-th]

Temperature $T(z_h)$, $c_B = -0.01$, $\nu = 1$

$d = 0.01$

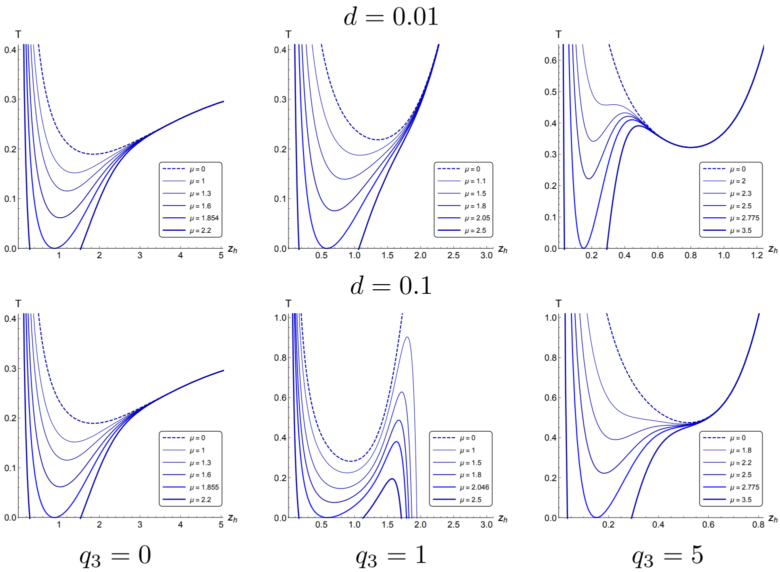


$d = 0.1$



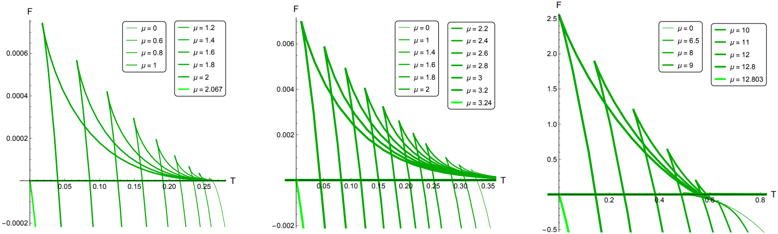
$q_3 = 0$ $\qquad$ $q_3 = 1$ $\qquad$ $q_3 = 5$

Temperature $T(z_h)$, $c_B = -0.01$, $\nu = 4.5$

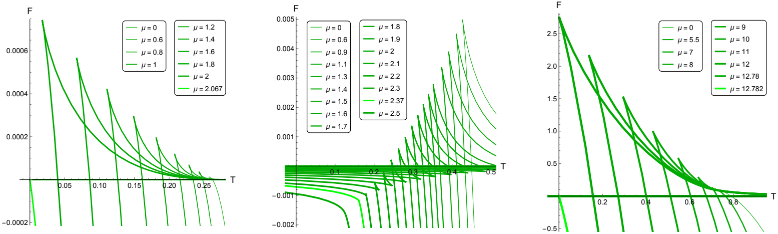


Free energy $F(T)$, $c_B = -0.01$, $\nu = 1$

$d = 0.01$



$d = 0.1$



$q_3 = 0$ $q_3 = 1$ $q_3 = 5$

Free energy $F(T)$, $c_B = -0.01$, $\nu = 4.5$

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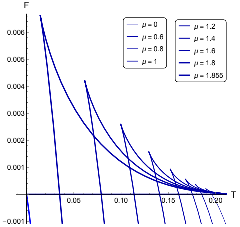
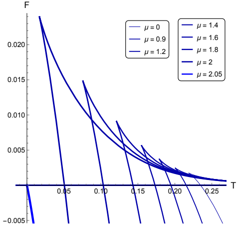
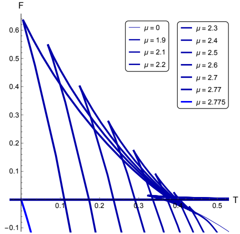
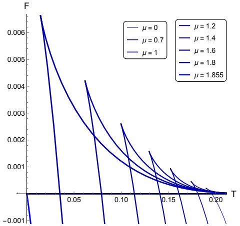
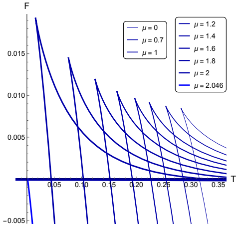


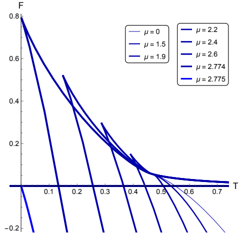




$d = 0.1$



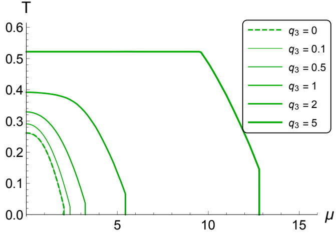


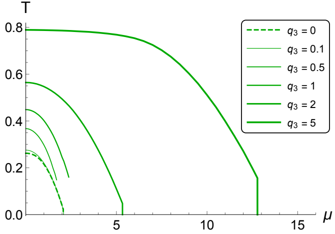


$q_3 = 0$ $q_3 = 1$ $q_3 = 5$

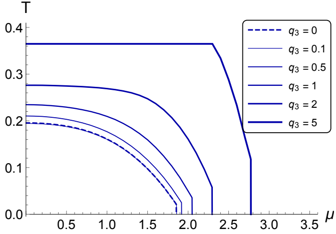
1-st order phase transition $T(\mu)$, $c_B = -0.01$

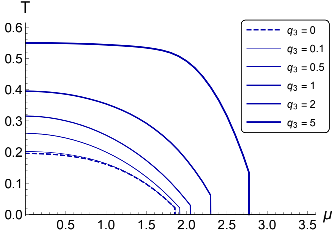
$\nu = 1$





$\nu = 4.5$



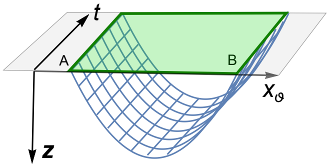


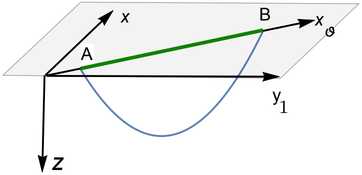
$d = 0.01$ $\qquad$ $d = 0.1$

Temporal Wilson loop

$$W[C_\vartheta] = e^{-S_{\vartheta,t}} \qquad \vec{n}: \qquad n_x = \cos\vartheta, \quad n_{y_1} = \sin\vartheta, \quad n_{y_2} = 0$$

ℓ x – collision axes





Two special cases:

- $\vartheta = 0$ WL (longitudinal)
- $\vartheta = \pi/2$ WT (transversal)

$\ell \to \infty \qquad S \sim \sigma_{DW}\ \ell$

$\swarrow$

the string tension

$$\sigma_{DW} = \frac{b(z)e^{\sqrt{\frac{2}{3}}\phi(z,z_0)}}{z^2}\ \sqrt{g(z)\left(z^{2-\frac{2}{\nu}}\sin^2(\vartheta)+\cos^2(\vartheta)\right)}\Bigg|_{z=z_{DW}}, \qquad \frac{\partial\sigma}{\partial z}\Bigg|_{z=z_{DW}} = 0$$

*Aref'eva, K.R., Slepov PLB **792** (2019) 470 arXiv:1808.05596 [hep-th]*

Temporal Wilson Loops for $dq_3^2 z^5$-term

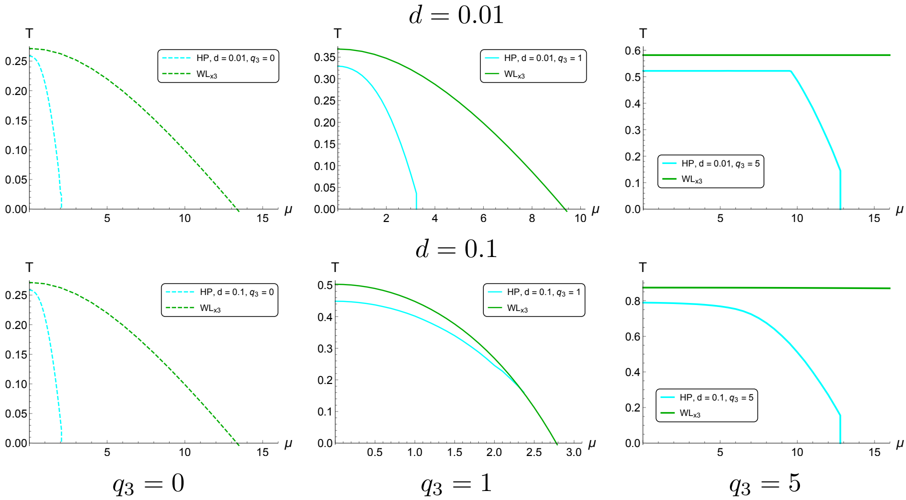
$$\text{WL}x_1: \quad -4az - 10dq_3^2 z^4 + \sqrt{\frac{2}{3}}\,\phi'(z) + \frac{g'}{2g} - \frac{2}{z}\,\Big|_{z=z_{DWx_1}} = 0$$

$$\text{WL}x_2: \quad -4az - 10dq_3^2 z^4 + \sqrt{\frac{2}{3}}\,\phi'(z) + \frac{g'}{2g} - \frac{\nu+1}{\nu z}\,\Big|_{z=z_{DWx_2}} = 0$$

$$\text{WL}x_3: \quad -4az - 10dq_3^2 z^4 + \sqrt{\frac{2}{3}}\,\phi'(z) + \frac{g'}{2g} - \frac{\nu+1}{\nu z} + c_B z\,\Big|_{z=z_{DWx_3}} = 0$$

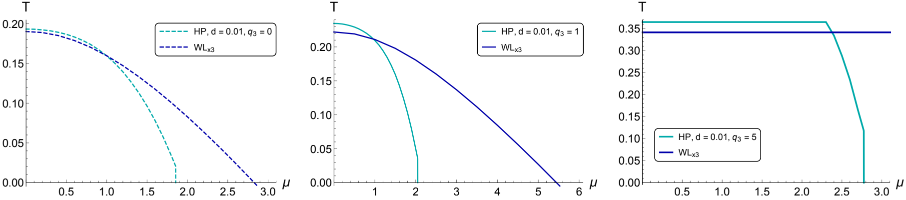
K.R. arXiv:2405.07881 [hep-th]

Phase diagram $T(\mu)$, $c_B = -0.01$, $\nu = 1$

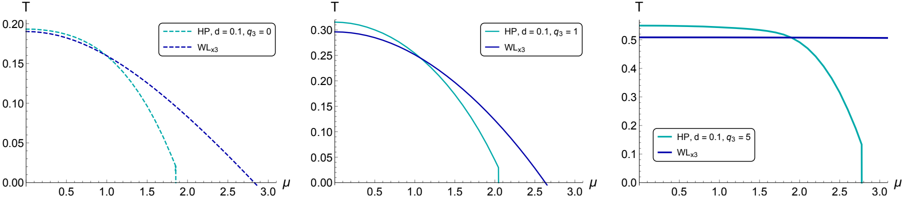


Phase diagram $T(\mu)$, $c_B = -0.01$, $\nu = 4.5$

$d = 0.01$



$d = 0.1$



$q_3 = 0$ $q_3 = 1$ $q_3 = 5$

Resuls for z^4 the warp factor term

- 1-st order phase transition exists on $\mu \in [0, \mu_{max}]$ and has CEP typical for previous neavy quarks models
- For zero and near-zero μ transition to thermal gas (HP) occurs, then it changes to black hole-black hole (BB) transition
- Increasing c_B absolute value enlarges the magnetic field influence on temperature of the 1-st order phase and CEP position μ_{max}
- Primary isotropisation rises temperature for HP/BB and the crossover, and also enlarges the magnetic field influence on CEP position μ_{max}

Resuls for z^5 the warp factor term

- Increasing d value rises temperature of the 1-st order phase transition (HP/BB) and the crossover (WL), but influence on μ_{max}: $T(\mu_{max}) = 0$ seems negligibly weak
- Primary isotropisation rises temperature for both HP and the crossover, but destabilises μ_{max} value
- In primary anisotropic case the confinement/deconfinement phase transition is determined by the crossover for small μ and by HP for large μ
- Magnetic field growth leads to nontrivial changes of the crossover behavior and increases the crossover μ-interval
- Isotropisation leaves HP/BB to determine the confinement/deconfinement phase transition

Conclusions

Terms z^4 and z^5 in the warp factor open a wide variety of thermodynamical behavior scenarios, realised via different sets of model parameters

- Stable solution needs fixed $c_B < 0$
- Higher order warp factor terms allow larger c_B absolute values
- MC behavior of the 1-st order phase transition can be provided for z^4 by the improved coefficient and for z^5 directly
- Isotropisation rises phase transition temperature and makes phase diagram more sensitive to higher order terms' model parameters

Thank you
for your attention

BACKUP. Relations between 5-dim backgrounds and 4-dim models

- Relations between parameters of the 5-dim background (black hole) and thermodynamical parameters are the following:
  - $T_{BH} = T_{QCD}$, where T_{BH} is the temperature of the 5-dim black hole;
  - $A_0(z) = \mu_B - \rho_B z^2 + \mathcal{O}(z)$, where $A_0(z)$ is the 0-component of the electromagnetic field $A_\mu(z)$, μ_B is the baryonic chemical potential, ρ_B is the density and z is the 5-dimentional coordinate;
  - $S_{BH} = s$, where S_{BH} is the entropy of the black hole, which as usual is defined by the square of the black hole horizon, s is the thermodynamical entropy;
  - $F_{BH} = -\,p$, where F_{BH} is the free energy of the black hole, p is the thermodynamical pressure.