Phase diagram of QCD and two colour QCD

Roman N. Zhokhov
IZMIRAN, IHEP

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Russian Science Foundation

БАЗИС
Фонд развития теоретической физики и математики

Group and Acknowlegements

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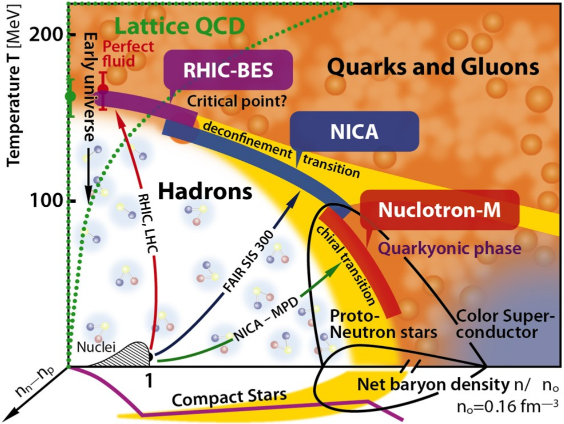
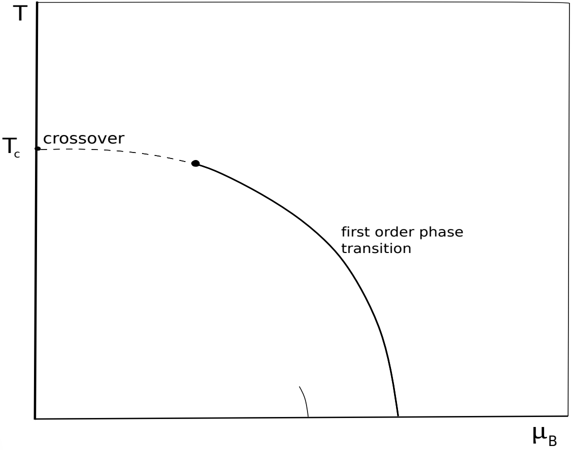
QCD Phase Diagram



Two main phase transitions

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

QCD Phase Diagram



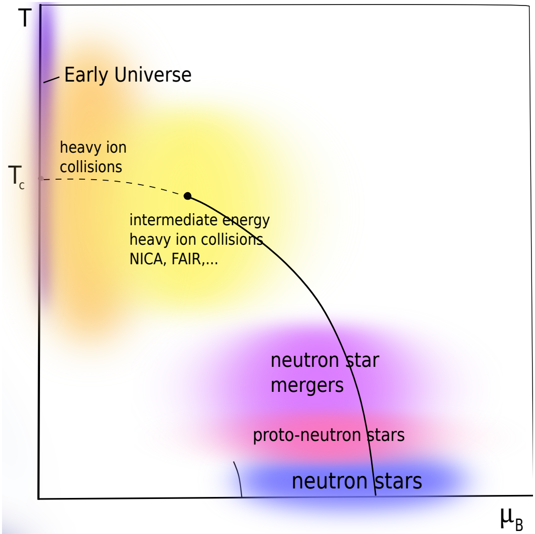
Two main phase transitions

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

QCD Dhase Diagram

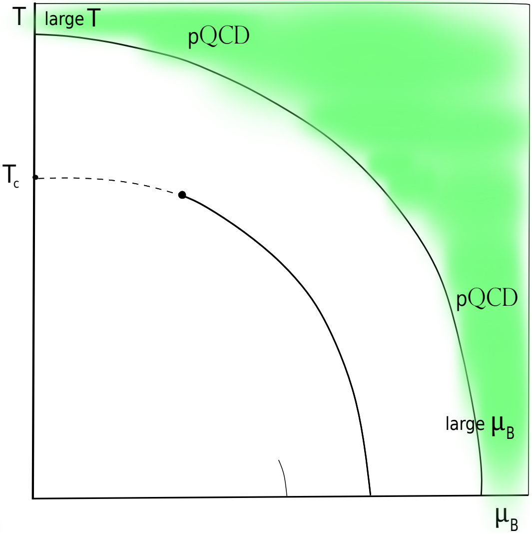
QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



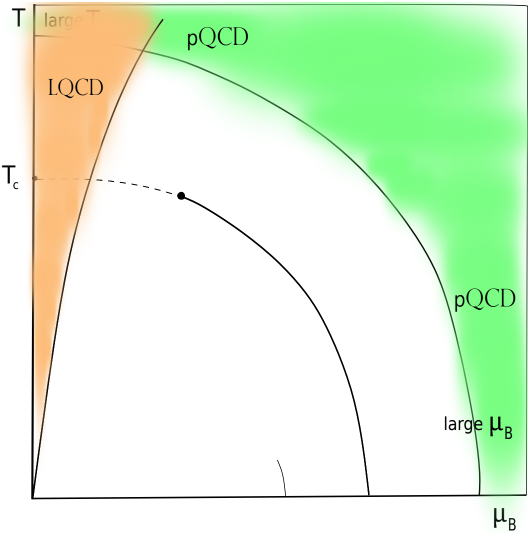
Methods of dealing with QCD

- Perturbative QCD



Methods of dealing with QCD

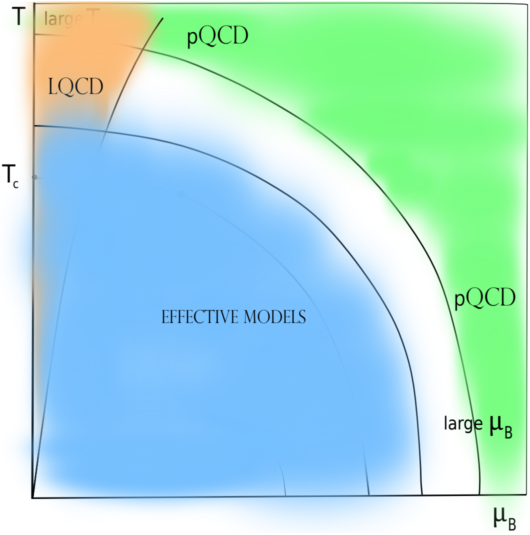
- Perturbative QCD
- First principle calculation – lattice QCD



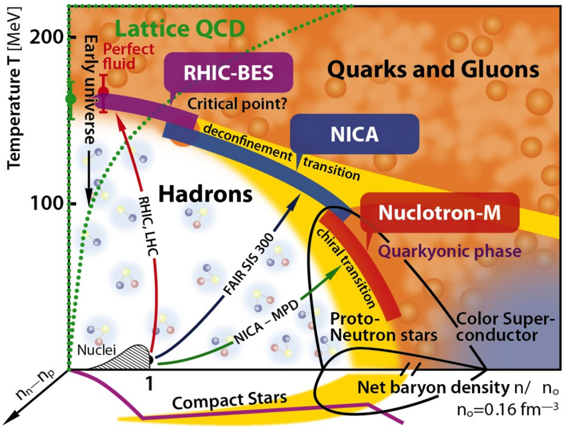
QCD Dhase Diagram and Methods

Methods of dealing with QCD

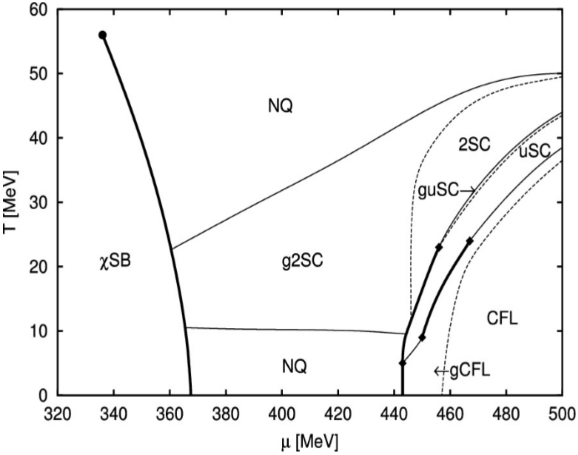
- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- Gauge/Gravity duality *(see talk by I. Aref'eva, A. Hajilou, K. Rannu)*
- .....



QCD Phase Diagram



QCD Phase Diagram



- **Isotopic chemical potential μ_I**

  Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

- Neutron stars, intermediate energy heavy-ion collisions, neutron star mergers

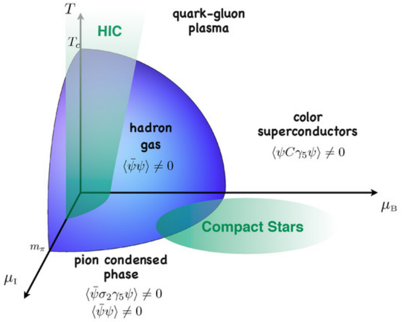


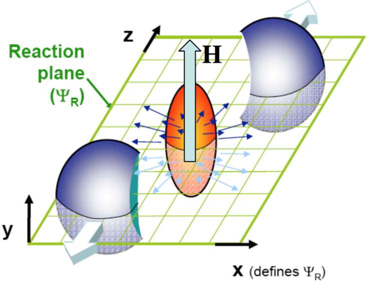
Figure: *taken from Massimo Mannarelli*

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right) \qquad n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

- **Chiral (axial) chemical potential**

Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

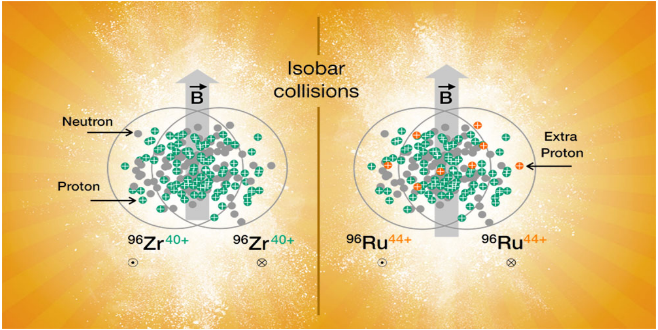
$$n_5 = n_R - n_L$$
$$\mu_5 = \mu_R - \mu_L$$



$$\vec{J} \sim \mu_5 \vec{B},$$

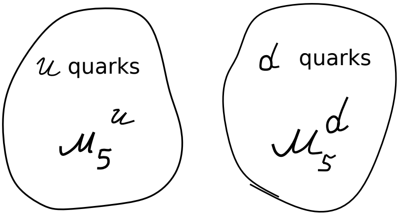
The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$



.

Even after Isobar run still no definitive conclusion on the presence or absence of CME in heavy-ion collisions



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \longleftrightarrow \nu_5$$

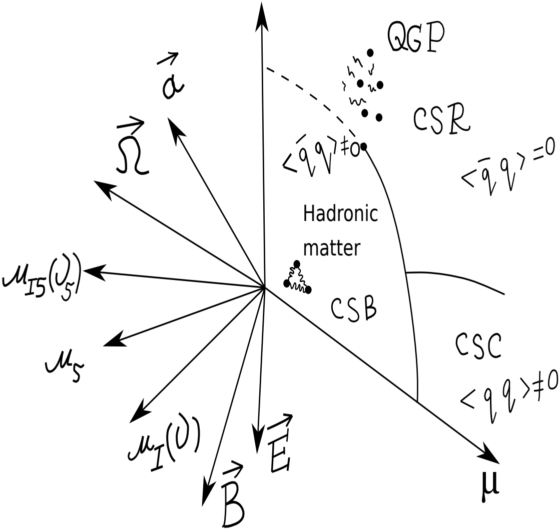
- Chiral isospin imbalance and chiral imbalance μ_{I5} and μ_5 can be generated in parallel magnetic and electric fileds $\vec{E} \,||\, \vec{B}$

- Chiral imbalance could appear in dense matter
    - Chiral separation effect
      (the idea of Igor Shovkovy)
    - Chiral vortical effect

More external conditions to QCD

More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields *(see talk by A. Hajilou)*
- rotation of the system $\vec{\Omega}$ *(see talk by G. Prokhorov)*
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



More external conditions to QCD

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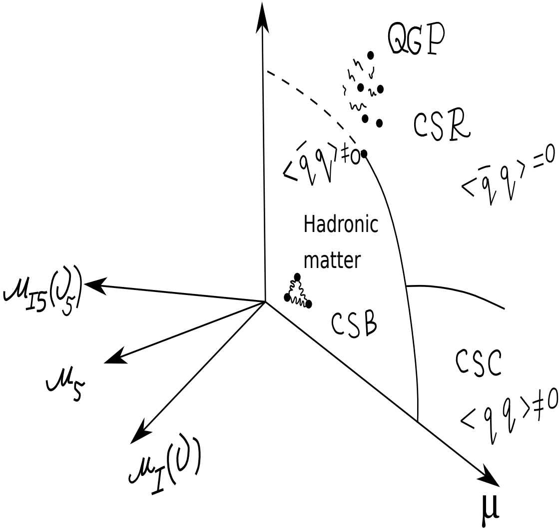
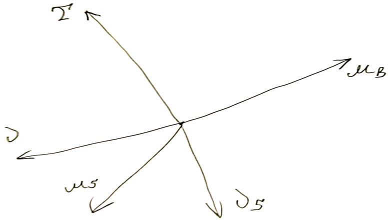


Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$



- **QCD phase diagram** has been considered in effective model of QCD, 3 color, without **diquark condensation** phenomenon

- **Duality between CSB and PC** has been found, 3 color, without **diquark condensation** phenomenon

- **Phase diagram of $\mathbf{QC_2D}$** has been investigated.

- **QCD phase diagram** has been studied and **color superconductivity phenomenon** and interesting features has been revealed

Recall that in NJL model **without color superconductivity phenomenon** there have been found **dualities**

(*It is not related to holography or gauge/gravity duality*)

Chiral symmetry breaking $\Longleftrightarrow$ pion condensation

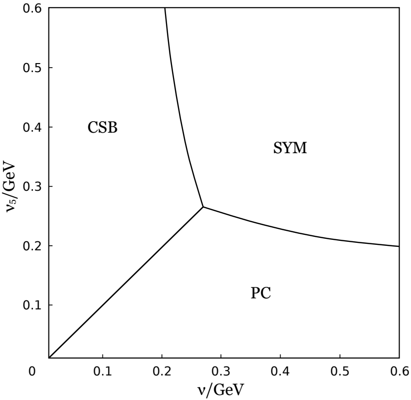
Isospin imbalance $\Longleftrightarrow$ Chiral imbalance

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle\bar{q}q\rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP

$$\Omega(T,\mu,\mu_i,...,\langle\bar{q}q\rangle,...) \qquad \Omega(T,\mu,\nu,\nu_5,...,M,\pi,...)$$



$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

- lattice results show the **catalysis** (*V. Braguta, A. Kotov, et al*)

  But unphysically large pion mass

Duality $\Rightarrow$ catalysis of chiral symmetry beaking

- Inhomogeneous phases (case)

$$\langle \sigma(x) \rangle = M(x), \quad \langle \pi_{\pm}(x) \rangle = \pi(x), \quad \langle \pi_3(x) \rangle = 0.$$

Inhomogeneous phases and phase diagram are obtained just from duality

Details on uses of duality in Particles 2020, 3(1), 62-79

Two colour QCD case

QC_2D

There are a lot similarities:

- similar phase transitions:

  confinement/deconfinement, chiral symmetry breaking/restoration at large T and μ

- A lot of physical quantities coincide with some accuracy

  Critical temperature, shear viscosity etc.

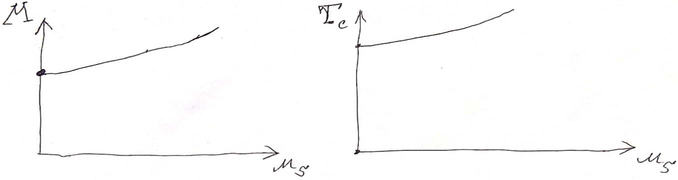
There is **no sign problem** in SU(2) case

$$(Det(D(\mu)))^\dagger = Det(D(\mu))$$

and lattice simulations at non-zero baryon density are possible

It is a great playground for studying dense matter

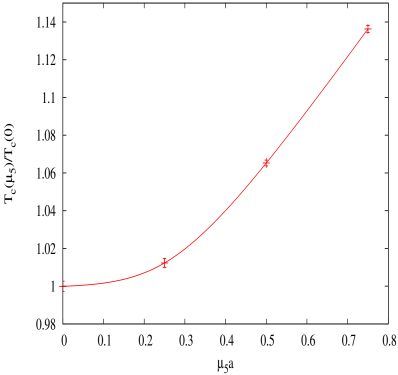
QCD at non-zero μ_5



catalysis of CSB by chiral imbalance:

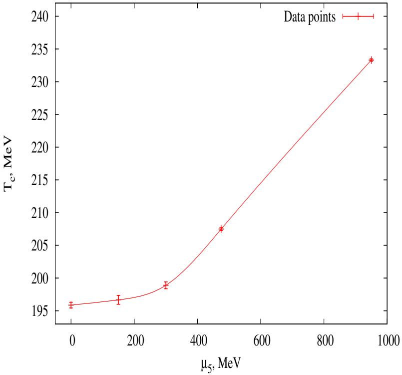
- increase of $\langle \bar{q}q \rangle$ as μ_5 increases
- increase of critical temperature T_c of chiral phase transition (crossover) as μ_5 increases

SU(2)



V. Braguta, A. Kotov et al, JHEP 1506, 094 (2015),PoS LATTICE 2014, 235 (2015)

SU(3)



V. Braguta, A. Kotov et al, Phys. Rev. D 93, 034509 (2016), arXiv:1512.05873 [hep-lat]

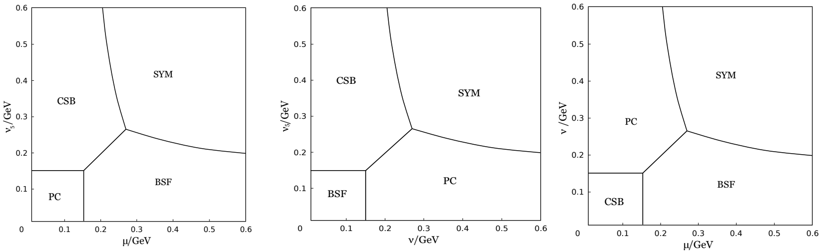
Phase diagram of QC_2D

Condensates and phases

$M = \langle \sigma(x) \rangle \sim \langle \bar{q}q \rangle,$ CSB phase: $M \neq 0,$

$\pi_1 = \langle \pi_1(x) \rangle = \langle \bar{q}\gamma^5 \tau_1 q \rangle,$ PC phase: $\pi_1 \neq 0,$

$\Delta = \langle \Delta(x) \rangle = \langle qq \rangle = \langle q^T C \gamma^5 \sigma_2 \tau_2 q \rangle,$ BSF phase: $\Delta \neq 0.$



$$(a) \qquad \mathcal{D}_1 : \quad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|, \qquad \text{PC} \longleftrightarrow \text{BSF}$$

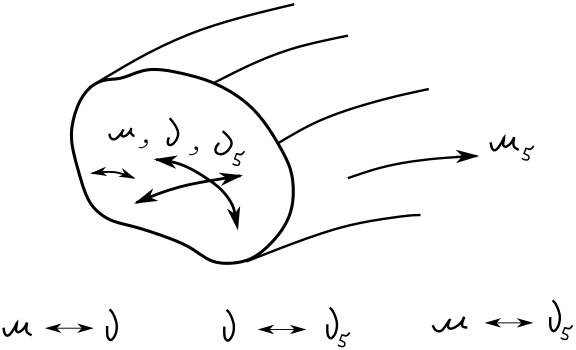
J. Andersen, T. Brauner, D. T. Son, M. Stephanov, J. Kogut, ...

$$(b) \qquad \mathcal{D}_3 : \quad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \qquad \text{PC} \longleftrightarrow \text{CSB}$$

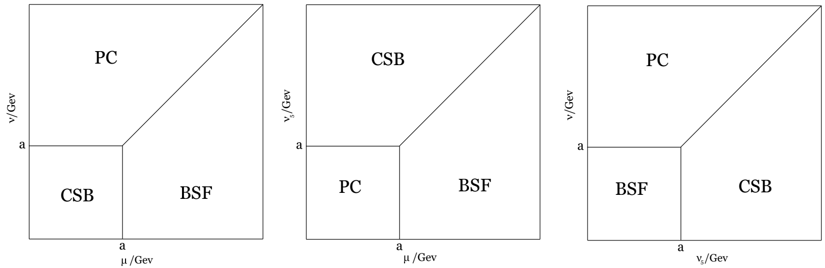
$$(c) \qquad \mathcal{D}_2 : \quad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|, \qquad \text{CSB} \longleftrightarrow \text{BSF}$$

The phase diagram of (μ, ν, μ_5, ν_5)

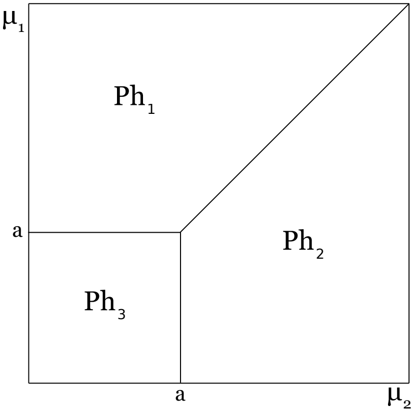
The phase diagram is foliation of dually connected cross-section of (μ, ν, ν_5) along the μ_5 direction

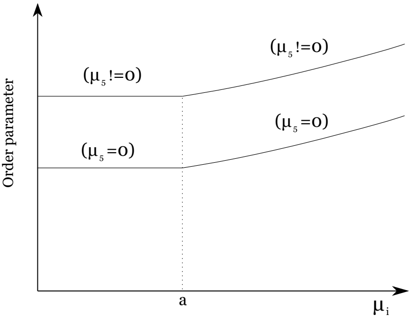


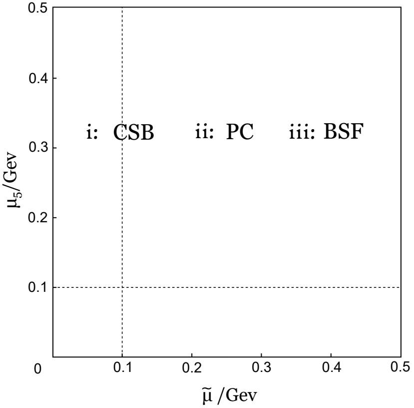
One to one correspondence

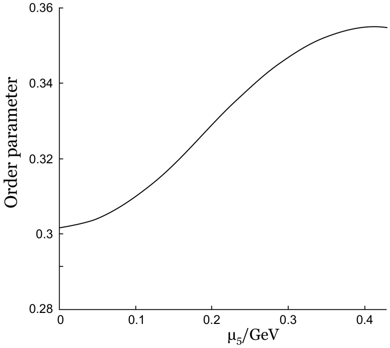


- Baryon density μ $\iff$ diquark condensation
- Isospin imbalance ν $\iff$ pion condensation
- Chiral imbalance ν_5 $\iff$ chiral symmetry breaking

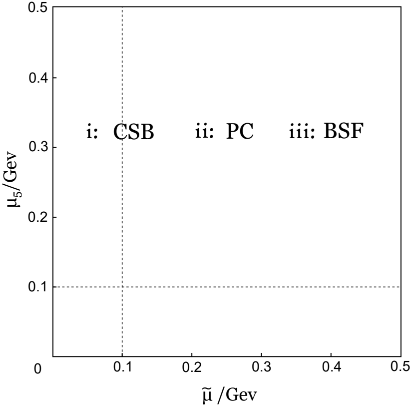

μ1
Ph1
a
Ph2
Ph3
a
μ2

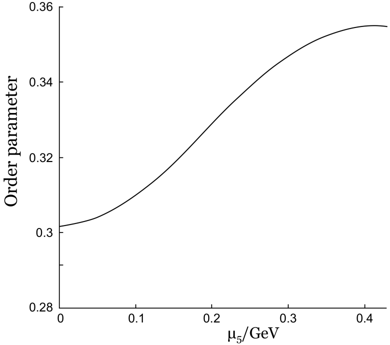

Order parameter
(μ5!=0)
(μ5!=0)
(μ5=0)
(μ5=0)
a
μi


0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
i: CSB
ii: PC
iii: BSF
μ_5/Gev
$\tilde{\mu}$ /Gev


0.36
0.34
0.32
0.3
0.28
0
0.1
0.2
0.3
0.4
Order parameter
μ_5/GeV

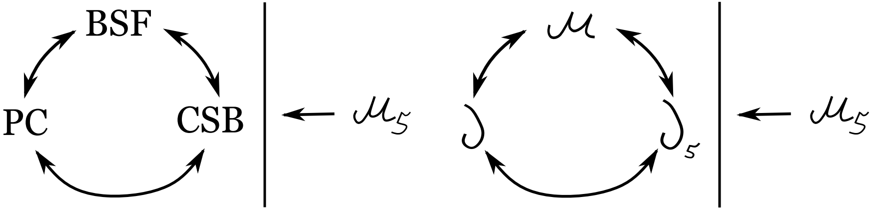
Chameleon property (mimicry) of μ_5





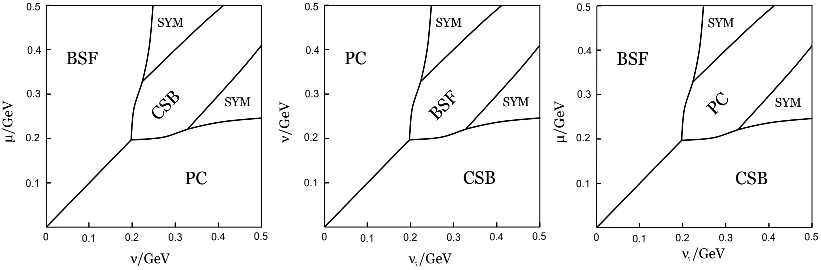
Chameleon nature of chiral imbalance μ_5

μ_5 mimics other chemical potentials μ, ν, ν_5



Chiral imbalance μ_5 does not participate in dual transformations

Dualities in the large values regime

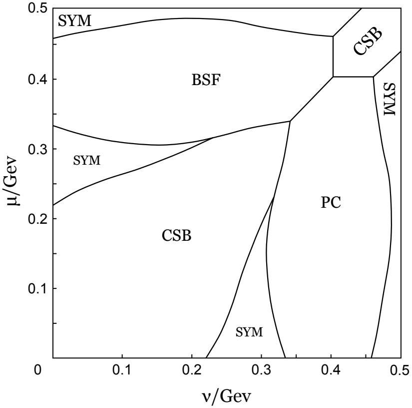


Chiral imbalance μ_5 could universally trigger all the phenomena

Diquark condensation at $\mu = 0$

Chiral imbalance μ_5 leads to several rather peculiar phases in the system, e. g. the **diquark condensation** in the region of the phase diagram at $\mu = 0$

It was known that μ_5 leads to pion condensation in dense quark matter with zero $\nu = 0$ in SU(3) case and in SU(2) as well



Phase diagram of three color QCD:

Color superconductivity
and
charged pion condensation in dense quark matter

Pion condensation history

In the early 1970s Migdal (Sawyer, Scalapino, Kogut, Manassah) suggested the possibility of **pion condensation in a nuclear matter**

A.B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2210 (1971) [Sov. Phys. JETP 36, 1052 (1973)]; A. B. Migdal, E. E. Saperstein, M. A. Troitsky and D. N. Voskresensky, Phys. Rept. 192, 179 (1990). R.F. Sawyer, Phys. Rev. Lett. 29, 382 (1972); J. Kogut, J.T. Manassah, Physics Letters A, 41, 2, 1972, Pages 129-131

pion condensation with zero momentum (s-wave condensation) is **highly unlikely to be realized** in nature in **matter of neutron star.**

A. Ohnishi D. Jido T. Sekihara, and K. Tsubakihara, Phys. Rev. C80, 038202 (2009) . .

PC in dense baryonic matter, chiral limit

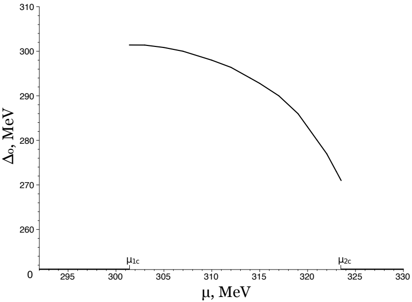
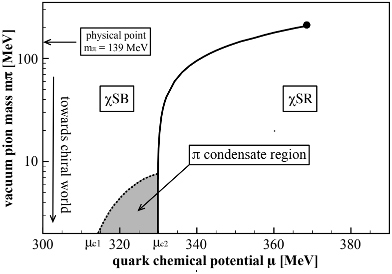


Figure: Pion condensate in dense quark matter in NJL model.

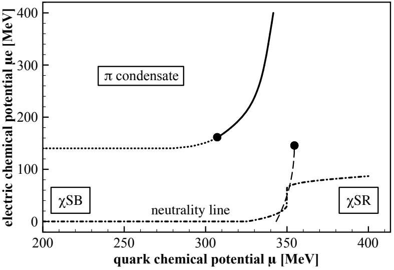
PC phenomenon is realized in **dense baryonic matter with isospin imbalance**

even in **charge neutral and β-equilibrated case**

K. G. Klimenko, D. Ebert
J.Phys. G32 (2006) 599-608;
Eur.Phys.J.C46:771-776,(2006)



(μ, m_0) phase portrait.



(μ, μ_e) phase portrait.

No PC condensation in the neutral case at the physical point

(H. Abuki, R. Anglani, M. Ruggieri etc.
Phys. Rev. D **79** (2009) 034032.

There are a number of **external parameters** such as **chiral imbalance** that can generate **PC in dense quark matter**.

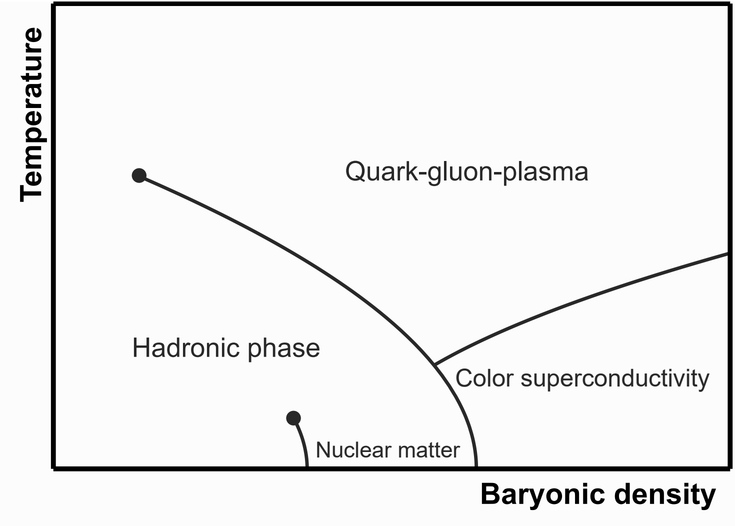
See small review

Symmetry 2019, 11(6), 778
arXiv:1912.08635 [hep-ph]

Special Issue "Nambu-Jona-Lasinio model and its applications" of symmetry

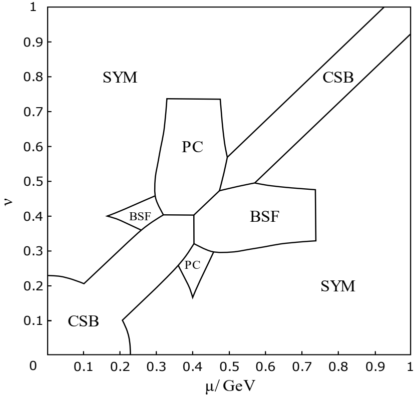
(Thanks to Tomohiro Inagaki)

Baryon density: Color superconductivity



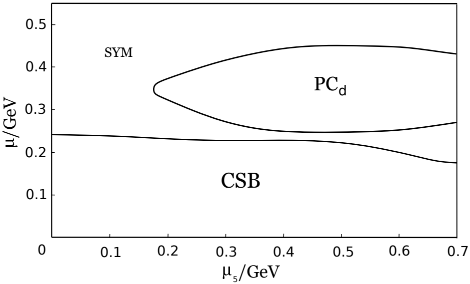
PC_d phase and diquark condensation

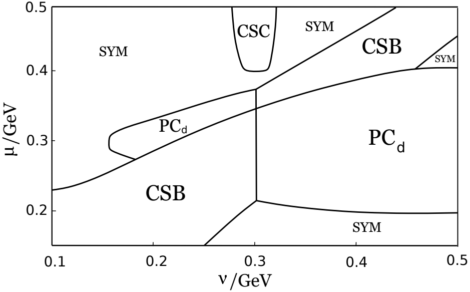
- PC_d phase has been predicted without possibility of diquark condensation
- Diquark condensation can take over the PC_d phase
- In two colour case diquark condensation is in a sense even stronger than in three colour case and starts from $\mu > 0$



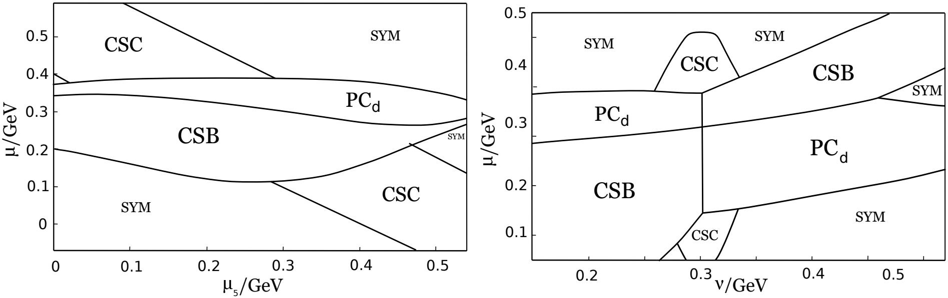
PC_d phase is unaffected by BSF phase in two color case.
Maybe one can infer that it is the case also for 3 color QCD

PC_d phase and color superconductivity phenomenon





PC_d phase is unaffected by color superconducting phase in three color QCD.



Even if one choose the chemical potentials in such a way that color superconducting phase and charged pion condensation with non-zero baryon phase overlap, PC_d phase is still ground state of the quark matter.

Phase diagram of three QCD and color superconductivity

the equations of motion for bosonic fields, which take the form

$$\sigma(x) = -2G(\bar{q}q), \quad \pi_a(x) = -2G(\bar{q}i\gamma^5\tau_a q),$$

$$\Delta_A(x) = -2H(\overline{q^c}i\gamma^5\tau_2\lambda_A q), \quad \Delta_A^*(x) = -2H(\bar{q}i\gamma^5\tau_2\lambda_A q^c)$$

the mesonic fields $\sigma(x), \pi_a(x)$ are real quantities, i. e. $(\sigma(x))^\dagger = \sigma(x), \quad (\pi_a(x))^\dagger = \pi_a(x)$, but all diquark fields $\Delta_A(x)$ are complex scalars, so $(\Delta_A(x))^\dagger = \Delta_A^*(x)$.

Clearly, the real $\sigma(x)$ and $\pi_a(x)$ fields are color singlets, whereas scalar diquarks $\Delta_A(x)$ form a color antitriplet $\bar{3}_c$ of the $SU(3)_c$ group. Note that the auxiliary bosonic field $\pi_3(x)$ corresponds to real $\pi^0(x)$ meson, whereas the physical $\pi^\pm(x)$-meson fields are the following combinations of the composite fields, $\pi^\pm(x) = (\pi_1(x) \mp i\pi_2(x))/\sqrt{2}$. If some of the scalar diquark fields have a nonzero ground state expectation value, i. e. $\langle\Delta_A(x)\rangle \neq 0$, the color symmetry of the model is spontaneously broken down.

the Lagrangian and the effective action are invariant under the color $SU(3)_c$ group, hence the TDP depends on the combination

$$\Delta_2\Delta_2^* + \Delta_5\Delta_5^* + \Delta_7\Delta_7^* \equiv \Delta^2,$$

where Δ is a real quantity.

There are only three order parameters

$$M = \langle\sigma(x)\rangle = -2G\langle\bar{q}q\rangle, \quad \pi = \langle\pi_1(x)\rangle = -2G\langle\bar{q}i\gamma^5\tau_1 q\rangle,$$

$$\Delta = \langle\Delta(x)\rangle = -2H\langle\overline{q^c}i\gamma^5\tau_2\lambda_2 q\rangle$$

Condensates and phases

$M = \langle \sigma(x) \rangle \sim \langle \bar{q}q \rangle \neq 0,$ CSB phase:

$\pi = \langle \pi_1(x) \rangle = \langle \bar{q}\gamma^5\tau_1 q \rangle \neq 0,$ PC phase: $\pi_1 \neq 0$

$\Delta = \langle \Delta(x) \rangle = \langle qq \rangle \neq 0,$ CSC phase: $\Delta \neq 0$

$$m_\pi,\ f_\pi,\ \langle \overline{q} q \rangle \longrightarrow \text{quark-antiquark coupling } G$$

H is not precisely determined

If the quark-antiquark interaction has been constrained empirically, the most natural solution is to determine the quark-quark coupling constants empirically, too. Unfortunately, the analog to the meson spectrum would be a diquark spectrum, which of course does not exist in nature

The most natural fit is

$$H = \frac{3}{4}G = 0.75G$$

- from Fiertz transform
- or from reasonable value of condensate

But we can use $0 < H < G$

If we one consider unphysical twice as strong diquark channel

$$H = \frac{3}{2}G = 1.5\,G$$

It will be very instructive later

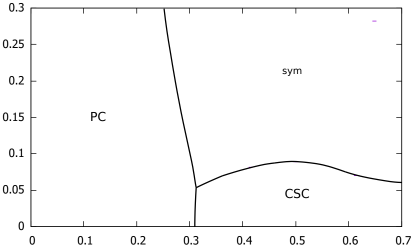


Figure: $\nu_5 = 0$

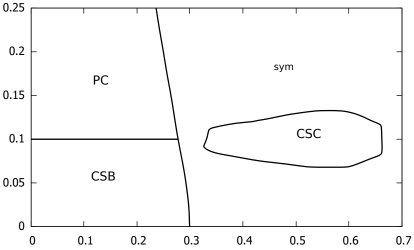
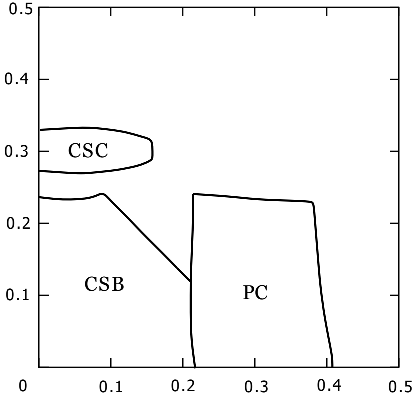


Figure: $\nu_5 = 0.1$

Color superconductivity phenomenon demonstrates qualitatively the same behaviour as BSF in two color case

Diquark condensation at $\mu = 0$ in SU(3)

Chiral imbalance μ_5 leads to the **diquark condensation** in the region of the phase diagram at $\mu = 0$ in three color case



Qualitative dual properties

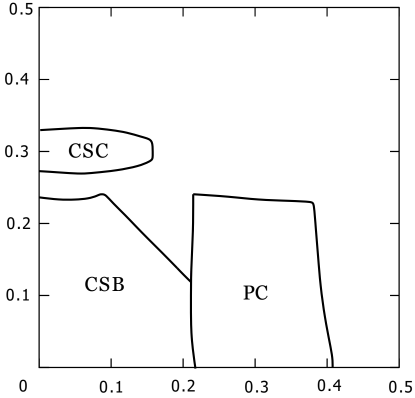
with color superconductivity phenomenon

in three color case

One can consider two regimes

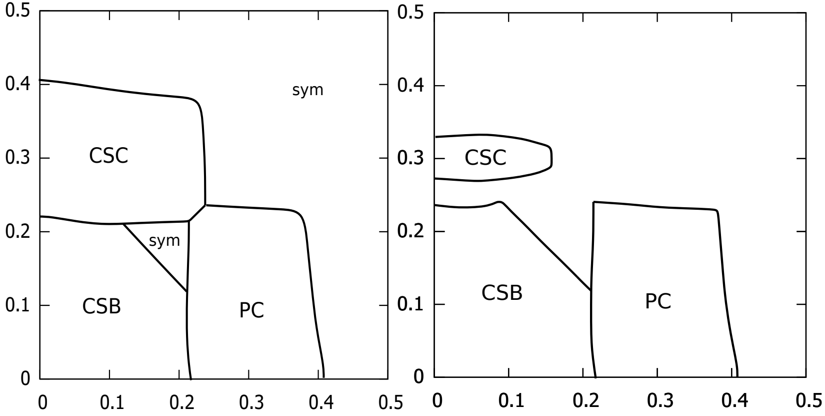
- **physical** $H = \frac{3}{4}G = 0.75G$ or around
- **unphysical** $H = \frac{3}{2}G = 1.5G$

Color superconductivity and **charged pion condensation** phenomena (PC and CSC phases) are **qualitativelu dual** to each other



One can consider two regimes

- **physical** $H = \frac{3}{4}G = 0.75G$ or around
- **unphysical** $H = \frac{3}{4}G = 1.5G$


0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
sym
CSC
sym
CSB
PC
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
CSC
CSB
PC

Gap equations are dual with respect to each other so the condensates

$$\frac{\partial F_1\left(M,\mu_i\right)}{\partial M}=0$$

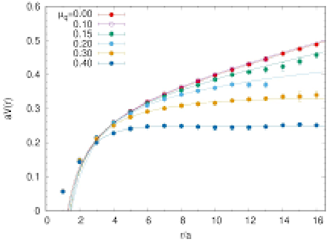
$$\frac{\partial F_2\left(\pi,\mu_i\right)}{\partial \pi}=0$$

$$\frac{\partial F_3\left(\Delta,\mu_i\right)}{\partial \Delta}=0$$

- Confinement/deconfinement transition was observed in two color case in lattice QCD

  (*Bornyakov, Braguta, Kotov et al.*)

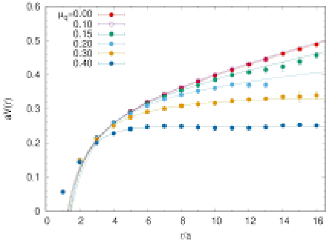
- Staggered fermions (tree level improved Symanzik gauge + rooted staggered fermion)



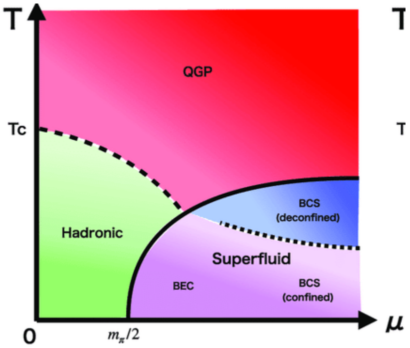
- Wilson fermions - **no deconfinement is found**

  (*Swansea group (Hands, Skullerud et al)*)

- confinement/deconfinement transition could be related to **finite temperature effects**



- Japan group (Iida, K.; Itou, E et al)
- confinement/deconfinement transition could be observed at some non-zero temperature



Dualities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ were found in

- In the framework of NJL model

- In the mean field approximation

Dualities are connected with Pauli-Gursey group

Dualities were found in

- In the framework of NJL model
beyond mean field

- In QC_2D non-pertubartively (at the level of
Lagrangian)

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Duality was found in

- In the framework of NJL model beyond mean field or at all orders of N_c approximation
- In QCD non-pertubartively (at the level of Lagrangian)

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram with color superconductivity was studied in three color color case
- It was shown that there exist dualities in QCD and QC_2D

  Richer structure of Dualities in the two colour case
- Qualitative dualities in three color case
- Dualities have been shown non-perturbetively in the two colour case
- Duality has been shown non-perturbarively in QCD