Four-vector deformations of IIB supergravity solutions

work based on works
[2302.08749, 2011.11424]

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Holographic interpretation

Bivector transformation - hidden symmetry of solutions space of 10d supergravity

$$
\begin{aligned}
(G+B)^{-1} &= (g+b)^{-1} + \beta, \\
\beta^{mn} &= k_a{}^m k_b{}^n \boxed{r^{ab}}
\end{aligned}
\tag{1}
$$

Backgrounds looks like $AdS_D \ltimes M_{10-D}$: duals to CFT deformations

$AdS_5 \times \mathbb{S}^5 \Longleftrightarrow \mathcal{N} = 4$ SYM

Deformed background $\Longleftrightarrow$

- adding of operators ($\mathrm{Tr}[\Phi^3]$);
- changing of RG behaviour (AdS cut-off);
- non-commutativity ;

Holographic interpretation

- There are three possibilities:
  1. All isometrics were taken through M: **marginal deformations**;
  2. All isometrics were taken through AdS: non-commutativity;
  3. Mixed case: dipole deformations.
- In case of using of basic hidden symmetry of space of solutions of supergravity, isometrics of M_{10-D} must to be commutative, thus acceptable only abelian deformations

$$[k_a, k_b] = 0$$

- U-duality (in following advanced) hidden symmetry of supegravity, allow us expand acceptable view of deformation
- Advanced hidden symmetry allow non-abelian isometrics of compact space M_{10-D}

$$[k_a, k_b] = f_{ab}{}^c k_c$$

Holographic interpretation

Solution $\mathrm{AdS}_{\mathrm{D}+1} \times \mathrm{M}$ dual to D-dimensional gauge theory with symmetries:

- Conformal group SO(D,2): symmetry of AdS space
- Group of internal R-symmetry G: symmetry of M space

Examples:

- $\mathrm{AdS}_5 \times \mathbb{S}^5 \quad \Longleftrightarrow \quad \mathcal{N} = 4,\ \mathrm{D} = 4$ SYM
- $\mathrm{AdS}_7 \times \mathbb{S}^4 \quad \Longleftrightarrow \quad \mathcal{N} = (2,0),\ \mathrm{D} = 6$ SCFT (non-lagrangian theory)

Polyvector deformations break symmetries, through that they taken:

- through AdS case: breaking of space-time symmetry of dual theory (non-commutativity and non-locality)
- through compact M space: breaking of super-symmetry ($\equiv$ adding of new terms into lagrangian)

Non-abelian deformations

- YB bi-vector transformation of vary solution with b-field
  [Bakhmatov, Colgain, Sheikh-Jabbari, Yavatanoo (2018)]

$$(G+B)^{-1} = (g+b)^{-1} + \beta \tag{2}$$

- Necessary to define

$$\begin{aligned}
[k_a, k_b] &= f_{ab}{}^c k_c && \text{(algebra of symmetries)}\\
\beta^{mn} &= k_a{}^m k_b{}^n r^{ab} && \text{(bi-vector anzatz);}\\
r^{b_1[a_1} r^{|b_2|a_2} f_{b_1 b_2}{}^{a_3]} &= 0 && \text{(classical YB equation);}\\
r^{b_1 b_2} f_{b_1 b_2}{}^a k_a{}^m &= I^m = 0 && \text{(unimodularity);}
\end{aligned} \tag{3}$$

In case of compact isometrics:

- Abelian $\mathfrak{u}(1)^n$: $f_{ab}{}^c = 0 \implies \forall r_{ab}$

- Non-abelian (SU(N), SO(N), ...): $r_{ab} \equiv 0$
  [Lichnerowicz, Medina (1988), Pop, Stolin (2007)]

Generalization of the classical Yang–Baxter equation in 11d case

- In previously works were showed, that advanced hidden symmetry is part of space of solutions of 11d supergravity equations [Hohm, Samtleben]
- It's parameterized by trivector, spanned on Killing vectors of init solution

$$\Omega^{mnk} = \rho^{[a1a2a3]} k^m_{a_1} k^n_{a_2} k^k_{a_3}, \quad r^{b_1 b_2 b_3} f_{b_2 b_3}{}^a k_a{}^m k_{b_1}{}^n = I^{mn} = 0$$
$$\rho^{a_1[a_2|a_6|} \rho^{a_3 a_4|a_5|} f_{a_5 a_6}{}^{a_7]} - \rho^{a_2[a_1|a_6|} \rho^{a_3 a_4|a_5|} f_{a_5 a_6}{}^{a_7]} = 0. \tag{4}$$

[Sakatani, Blair, Malek, Thompson, Colgain, Deger, Sheikh-Jabbari, Bakhmatov, Gubarev, Musaev]

- Turn out, that in front of bi-vector case, exists non-trivial solutions in case of compact isometrics:

$$\hat{\Omega}_1 = a\, E_2 \wedge F_2 \wedge (H_1 - H_2) + a\, E_4 \wedge F_4 \wedge (H_1 + H_2) \tag{5}$$

[Musaev, Petrov]

- Such success gave inspiring for us to try find how advanced hidden symmetry looks like into case of more interesting in holographic context case 10d supergravity

Generalization of the Yang–Baxter equation in IIB case

- In previously works were showed, that advanced hidden symmetry is part of space of solutions of 10d supergravity equations [Hohm, Samtleben]
- In case of 10d IIB supergravity turn out, that advanced hidden symmetry in case of IIB supergravity parameterized by full-antisymmetrised four-vector:

$$\Omega^{mnkl} = \rho^{i_1 i_2 i_3 i_4} k^m_{i_1} k^n_{i_2} k^k_{i_3} k^l_{i_4} \qquad (6)$$

- Enough conditions on coordinates of four-vector for generation of IIB solution from IIB solution

Linear conditions: IIB analogue of unimodularity condition

$$\rho^{[a_1 a_2 |a_3 a_4|} f_{a_3 a_4}{}^{a_5]} = 0. \qquad (7)$$

Quadratic condition:Generalization of the classical Yang–Baxter equation in case of four-vector

$$\rho^{[a_1 a_2 |a_3 a_4|} \rho^{a_5 a_6 a_7] a_8} f_{a_3 a_8}{}^{a_9} - 3 \rho^{[a_1 a_2 |a_3 a_4|} \rho^{a_5 a_6 |a_9 a_8|} f_{a_3 a_8}{}^{a_7]} = 0. \qquad (8)$$

Deformation of $AdS_5 \times \mathbb{S}^5$

Consider solution in view $M_5 \times N_5$ with

$$\begin{aligned} ds^2 &= e^{\phi(y)} \underbrace{g_{\mu\nu} dx^\mu dx^\nu}_{AdS_5} + \underbrace{g_{mn} dy^m dy^n}_{\mathbb{S}^5}, \\ C_{\mathbb{S}_5} &= \frac{1}{4!} C_{mnkl}(y) dy^m \wedge dy^n \wedge dy^k \wedge dy^l, \\ C_{AdS_5} &= \frac{1}{4!} c_{\mu\nu\kappa\lambda}(x) dx^\mu \wedge dx^\nu \wedge dx^\kappa \wedge dx^\lambda \end{aligned} \tag{9}$$

- Examples of supersymmetric deformations of $AdS_5 \times \mathbb{S}_5$ through AdS isometrics were found early in [Maldacena, Lunin (2005)]
- Solution $AdS_5 \times \mathbb{S}^5$ corresponds to anzats and dual to $\mathcal{N} = 4, D = 4$ SYM

Results for SO(6) group

- gCYBE on SO(6) is very difficult object, and SO(6) were reduced to SO(4) $\times$ U(1)
- Were found family of real solutions of four-vector gCYBE on SO(4) $\times$ U(1) :

$$\hat{\Omega} = a\,(\,E_2 \wedge F_2 \wedge (H_1 - H_2) + \;E_4 \wedge F_4 \wedge (H_1 + H_2)) \wedge H_3 \tag{10}$$

where E, F, H – abstract vectors of SO(4)$\times$U(1)

- View in terms of Killing vectors of $\mathbb{S}^5$ part of initial metric:

$$\Omega = (a_1 + a_2)\left(\rho^2 + \sigma^2\right)\left(\frac{1}{\rho}\partial_\rho - \frac{1}{\sigma}\partial_\sigma\right) \wedge \partial_\phi \wedge \partial_\psi \wedge \partial_\alpha$$

where were used following coordinates:

$$\begin{aligned} y^1 &= \rho\cos\phi, & y^3 &= \sigma\cos\psi \\ y^2 &= \rho\sin\phi, & y^4 &= \sigma\sin\psi, \\ y^5 &= \sqrt{r^2 - \rho^2 - \sigma^2}\cos\alpha, & y^6 &= \sqrt{r^2 - \rho^2 - \sigma^2}\sin\alpha, \end{aligned} \tag{11}$$

where $\phi, \psi, \alpha \in [0, 2\pi]$ and $\rho^2 + \sigma^2 \leq r^2$.

Results

Four-vector deformation of $AdS_5 \times \mathbb{S}^5$ on sphere isometries:

$$ds^2 = K\lambda^2 r_+^6 \left(r_+^2 - 1\right)^2 R^2 \Bigg(-d\alpha^2 + \frac{1}{2} r_-^2 d\psi_- d\psi_+ + \frac{1}{4} r_+^2 d\psi_-^2 + \frac{1}{4} r_+^2 d\psi_+^2 + \\ + \frac{r_-^2 r_+^2 \left(r_+^2 - 1\right)}{r_-^4 - r_+^4} dr_-^2 - \frac{4\left(400 r_+^2 \left(r_+^4 - r_-^4\right)\psi_+^2 + \left(r_+^2 - 1\right) r_-^4\right)}{\left(r_+^4 - r_-^4\right)\left(r_+^2 - 1\right)^2} dr_+^2 - \\ - \frac{8 r_-^3 r_+}{\left(r_+^2 - 1\right)\left(r_+^4 - r_-^4\right)} dr_+ dr_- + 40 \frac{dr_+ \psi_+ \left(d\psi_- r_-^2 + d\psi_+ r_+^2\right)}{r_+^3 \lambda \left(r_+^2 - 1\right)^2} \Bigg) + K^2 ds^2(AdS^5) \quad (12)$$

$$F_5\left(M^5\right) = \frac{4}{5R} K \left(20K - \lambda r_+^2 (3K - 2)\left(4r_+^2 - 3\right)\right) dVol\left(S_5\right) \quad (13)$$

$$F_5\left(AdS_5\right) = \frac{K^3}{20}\left(20K - \frac{\sqrt{1-K}(3K-2)\left(4r_+^2 - 3\right)}{r_+\sqrt{r_+^2 - 1}}\right) dVol\left(AdS_5\right) \quad (14)$$

where $r_\pm^2 = \frac{1}{R^2}(\rho^2 \pm \sigma^2)$, $\psi_\pm = \phi \pm \psi$, $K = 1 + \frac{\lambda^2}{R^8}\left(\rho^2 + \sigma^2\right)^3 \left(R^2 - \rho^2 - \sigma^2\right)$

Summary and discussion

- For IIB supergravity were found new type of deformation of space of solutions by four vector, and found conditions on it
- In following try to find full solution of conditions on four-vector on $\mathbb{S}^5$ isometries
- Find precisely view of new deformations of $\mathrm{AdS}_5 \times \mathbb{S}^5$ that will be corresponds to non-supersymmetric conform manifold

Thanks for your attention!