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Heavy quark holographic model for running coupling and magnetic catalysis

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My collaborators:

Running Coupling and Beta-functions for HQCD with Heavy and Light Quarks: Isotropic case **arXiv: 2402.14512**

Magnetic Catalysis in Holographic Model with two Types of Anisotropy for Heavy Quarks ***EPJC, 2023***

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Outline:

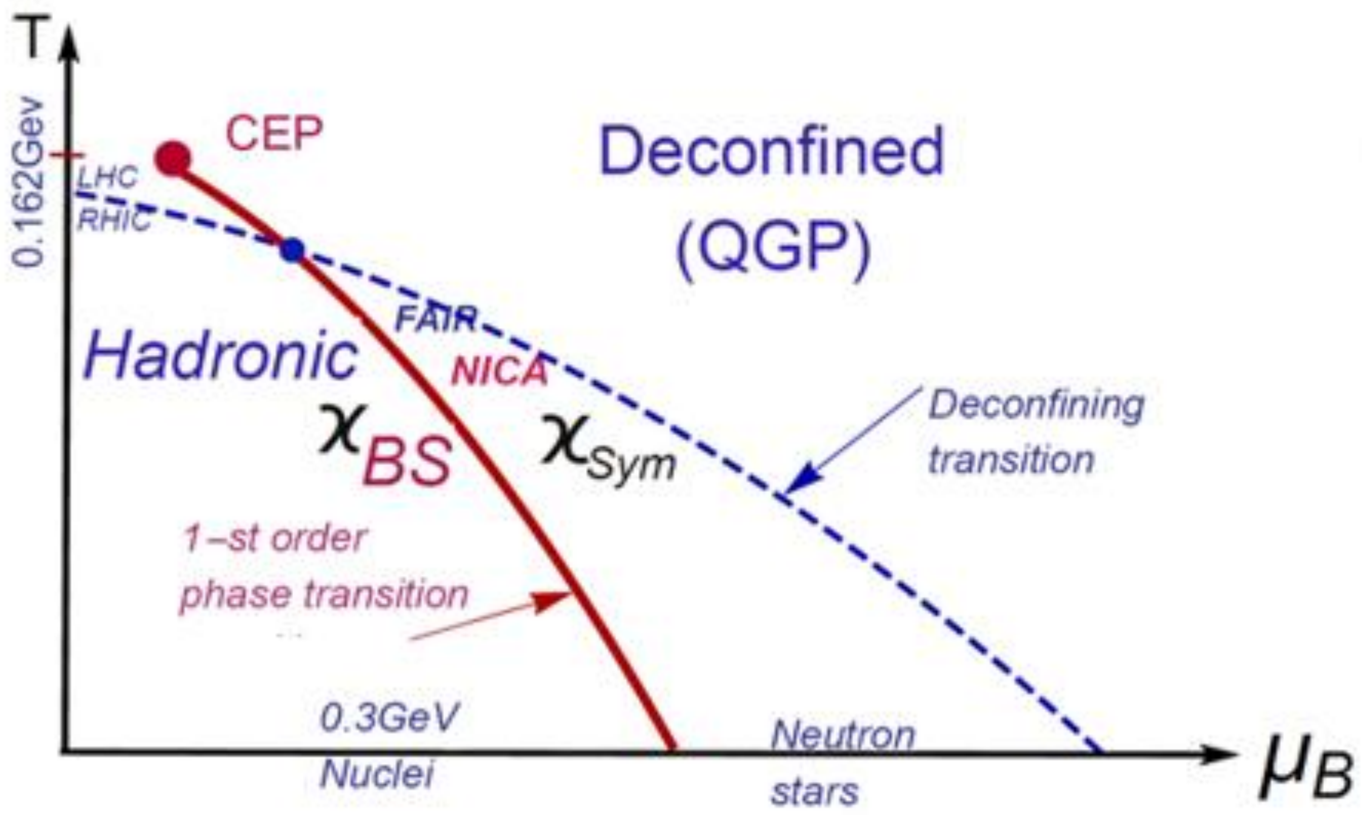
- Introduction (Motivation!!)
- Set up Two Questions
- Approach: AdS/CFT or Gauge/Gravity Duality
- Results
- Summary

Introduction: QCD phase diagram: Experiments **(Light quarks)**

RHIC (2000); LHC (2010)

FAIR (Facility for Antiproton and Ion Research) → Search for signs of the phase transition between hadronic matter and QGP

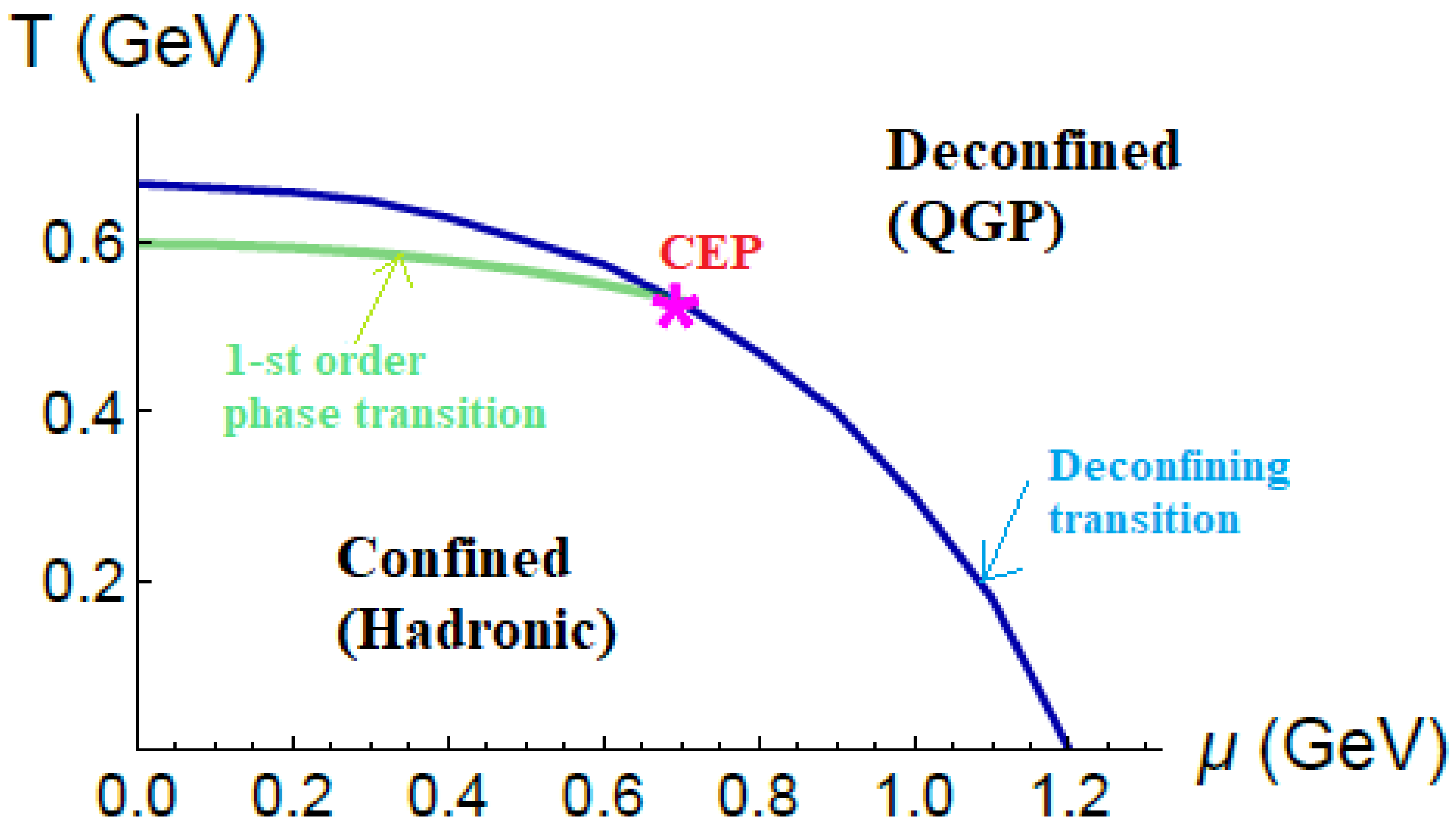
NICA (Nuclotron-based Ion Collider fAcility) → Search for new phases of baryonic matter



Introduction: phase diagram

(Heavy Quarks Model)

(Isotropic case)



Introduction: Running Coupling

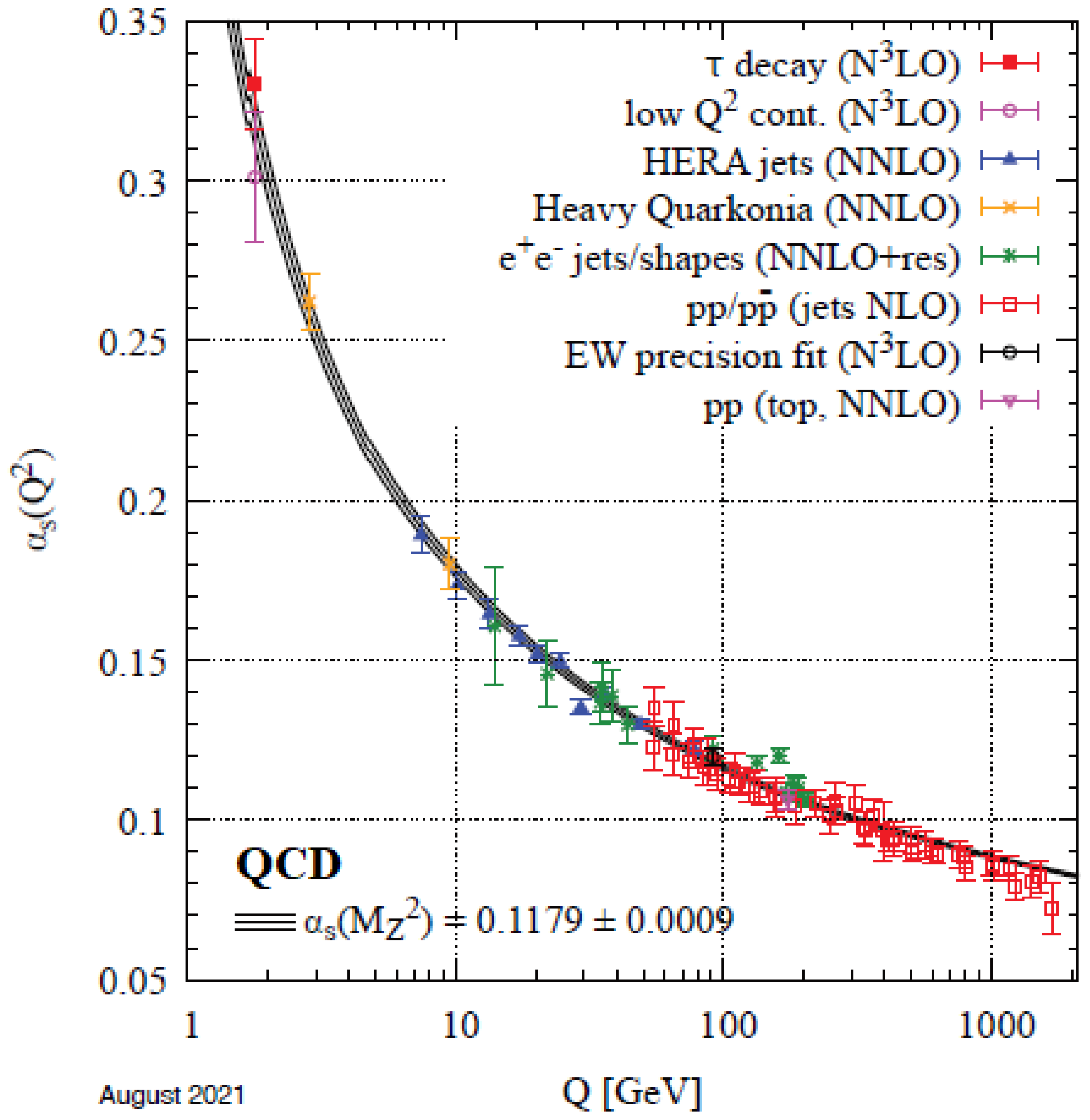
Defined by the Renormalization Group Equation:

$$\beta_{QFT}(\alpha) = \frac{\partial \alpha(Q)}{\partial\, ln(Q)}$$

β-function

Introduction: Running Coupling

Running coupling as a function of the energy scale Q



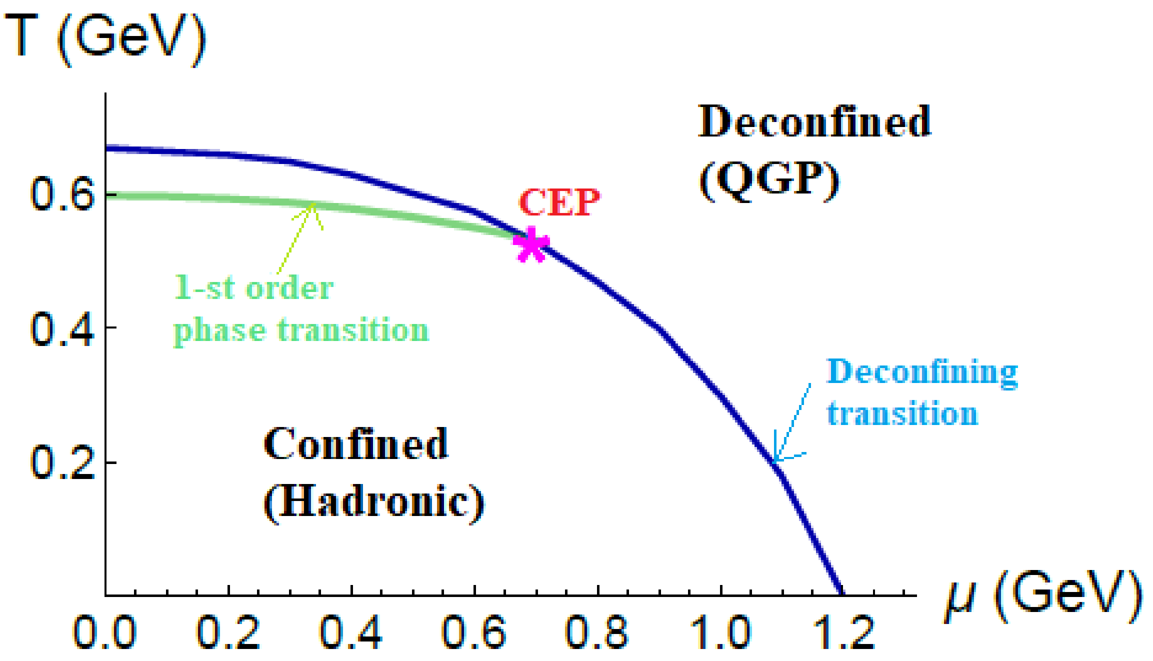
The respective degree of *QCD perturbation theory* used in the extraction of coupling is indicated in brackets (NLO: next-to-leading order, …)

R. L. Workman et al. (Particle Data Group) *PTEP* 2022 (2022) 083C01

1st Question

1st Question:

What is the dependence of running coupling on temperature and chemical potential at different phases?



Holographic Methods:

Top-down models:

(Directly constructed from string theory)

D3-D7 model

J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik , I. Kirsch,
M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters,…

D4-D8 model

T. Sakai and S. Sugimoto

Bottom-up models:

(phenomenological)

Introduce a dilaton field

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov,
A. Karch, B. Batell and T. Gherghetta, U. Gursoy, E. Kiritsis,…

Our Approach (bottom-up):

Classical gravity ⟺ Strongly coupled QFT

Anti-de Sitter Space (AdS) ⟷ Vacuum state

Black hole temperature ⟷ Temperature in QCD

Maldacena, Adv.Theor. Math.Phys. 1998; Witten, Adv.Theor.Math. Phys. 1998

Our Model: Einstein-Maxwell-dilaton action:

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{\mathfrak{f}_0(\varphi)}{4}F^2 - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \mathcal{V}(\varphi)\right]$$

F ⟷ Chemical potential

Equations of Motions (EOMs):

Einstein EOMs: $G_{\mu\nu} = T_{\mu\nu}$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\mathfrak{f}_0(\varphi)}{2}\left(F_{\mu\rho}F^{\rho}_{\nu} - \frac{1}{4}g_{\mu\nu}F^2\right) + \frac{1}{2}\left[\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}(\partial\varphi)^2 - g_{\mu\nu}\mathcal{V}(\varphi)\right]$$

Fields EOMs:

$$\nabla_\mu\left[\mathfrak{f}_0(\varphi)F^{\mu\nu}\right] = 0$$

$$\nabla^2\varphi = \frac{\partial\mathcal{V}}{\partial\varphi} + \frac{F^2}{4}\frac{\partial\mathfrak{f}_0}{\partial\varphi}$$

Our ansatzes for the fields:

Metric:

$$ds^2 = B^2(z)\left[-g(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{g(z)}\right]$$

$$B(z) = \frac{e^{A(z)}}{z}$$ Warp factor

Gauge field:

$$A_\mu = \left(A_t(z)\ ,\vec{0},0\right)$$

Dilaton field:

$$\varphi = \varphi(z)$$

Solving EOMs: (Potential reconstruction method)

Gauge field:

$$A_t'' + \left(\frac{f_0'}{f_0} + A' - \frac{1}{z}\right) A_t' = 0$$

Dilaton field:

$$A'' - A'^2 + \frac{2}{z} A' + \frac{\varphi'^2}{6} = 0$$

Blackening function:

$$g'' + \left(3A' - \frac{3}{z}\right) g' - e^{-2A} z^2 f_0 A_t'^2 = 0$$

Warp factor: $B(z) = \frac{e^{A(z)}}{z}$

Has very crucial effect on the physics in the QFT side

Light quark: $\mathcal{A}(z) = -a\ln(bz^2+1)$

Li, Yang, Yuan 2015

Heavy quark: $\mathcal{A}(z) = -cz^2/4$

Zakharov, Andreev, 2008

Warped factor: Heavy Quarks

Our choice:

$$A(z) = -\frac{\mathrm{s}}{3} z^2 - p\, z^4$$

Gauge coupling function:

$$f_0(z) = e^{-\mathrm{s}\, z^2 - A(z)}$$

By choosing this kinetic function our model can respect the Linear Regge trajectory for meson spectrum.

Thermodynamics: (Heavy quarks)

We need to find: g(z)

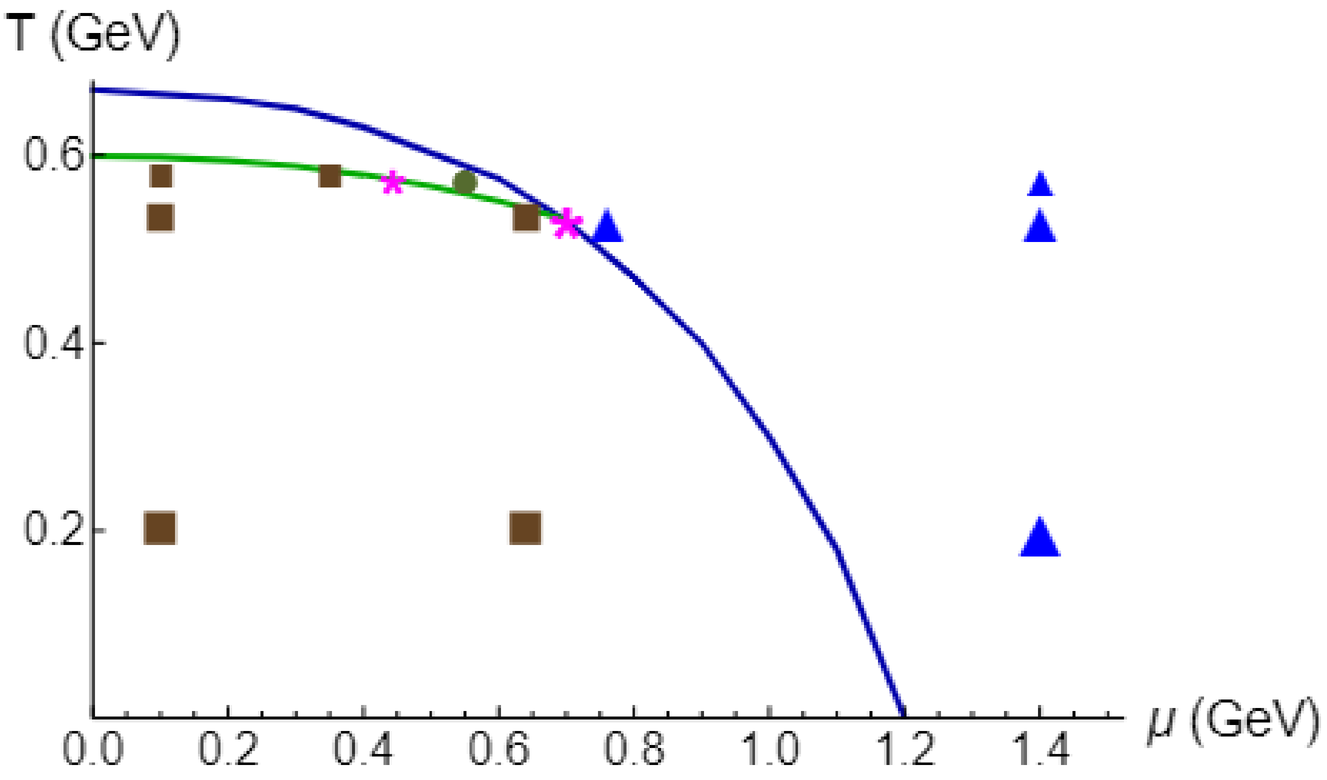
Temperature and Entropy:

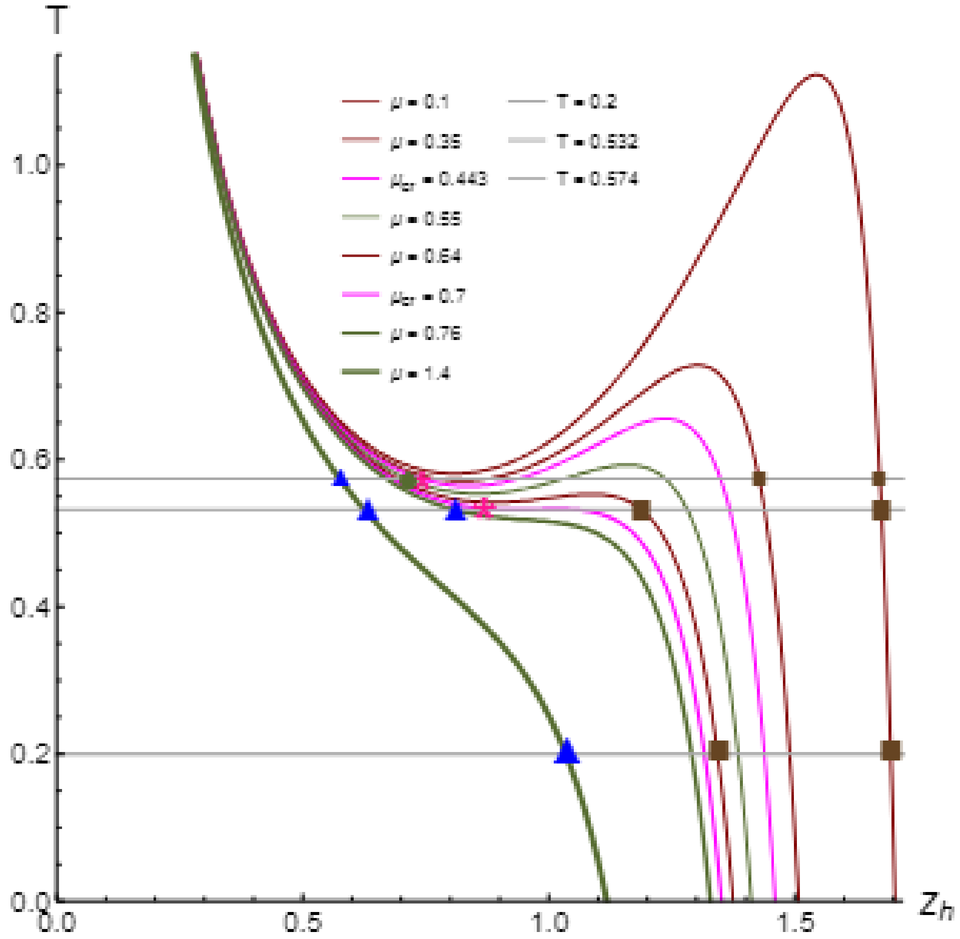
$$T = \frac{|g'|}{4\pi}\Big|_{z=z_h}$$

$$s = \frac{B^{3/2}(z_h)}{4z_h^3}$$

Free energy: $$F = -\int s\,dT = \int_{z_h}^{\infty} s\,T'dz.$$

Introduction: Heavy quarks

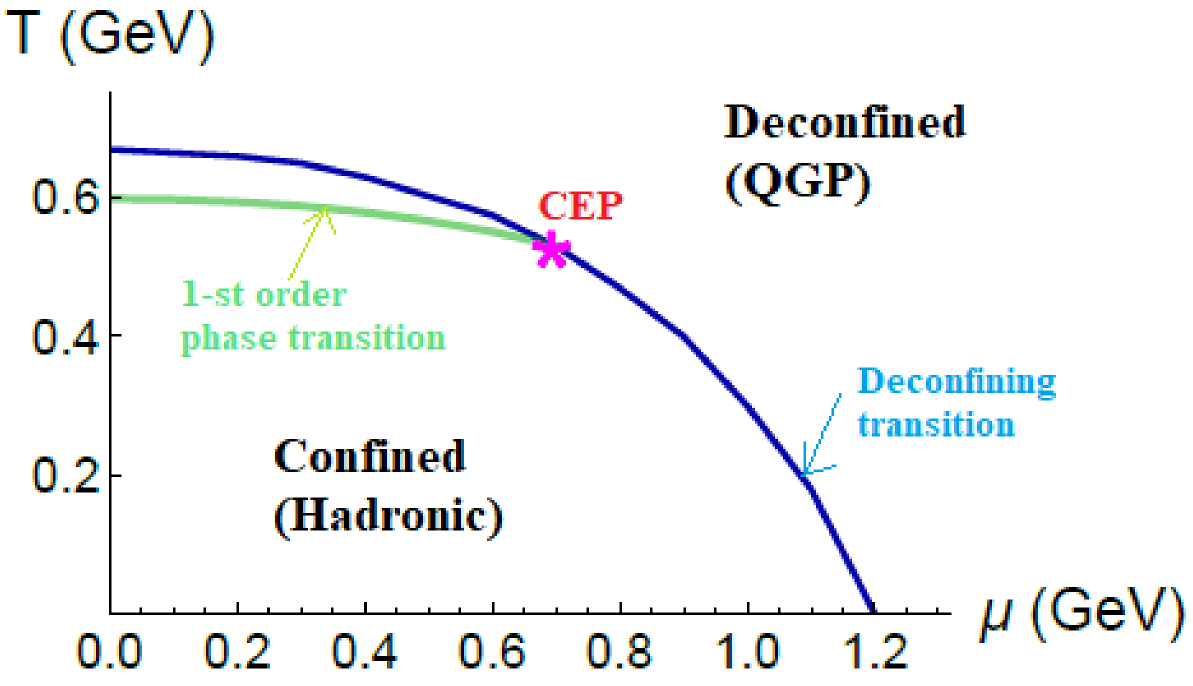


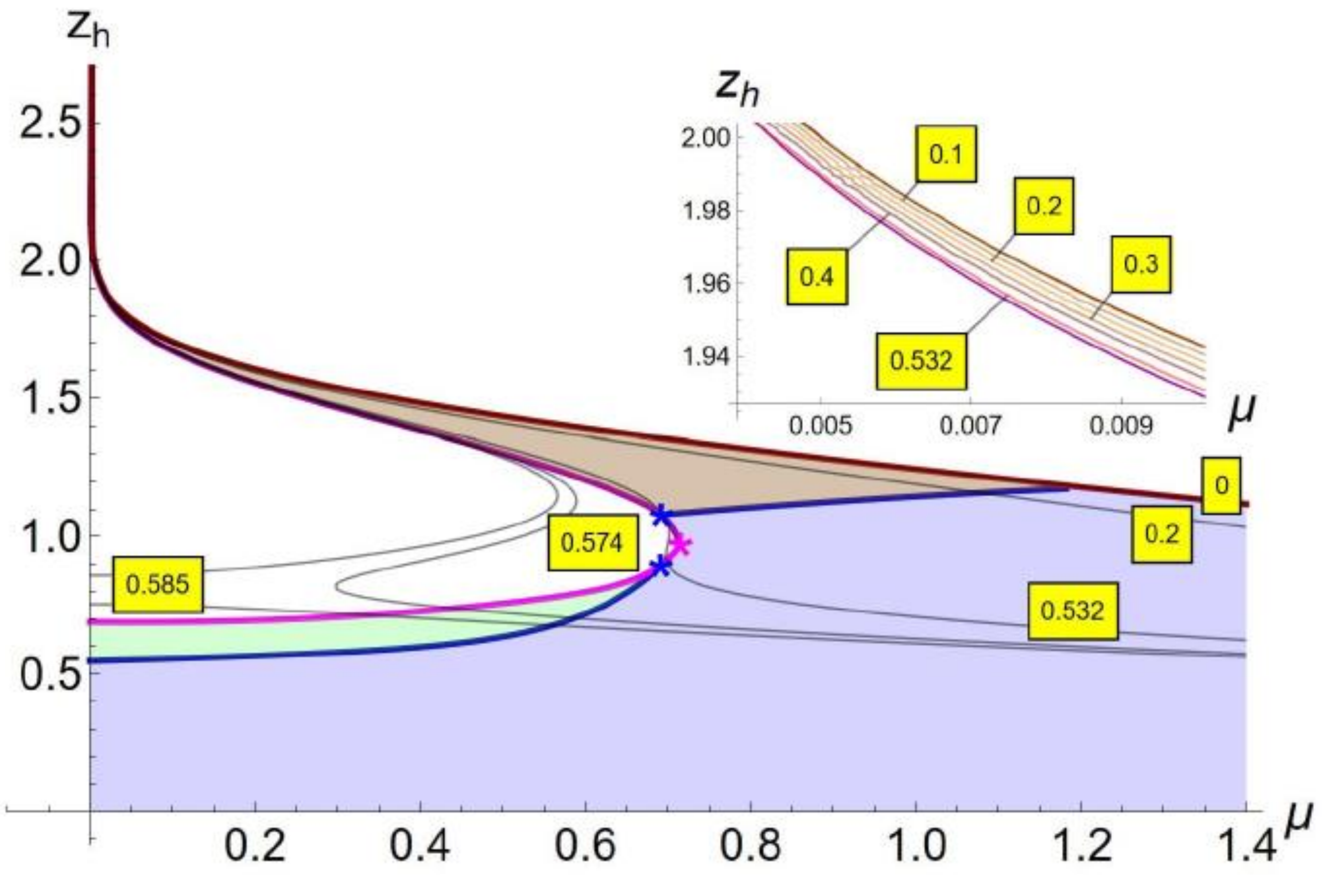


Introduction: phase diagram

(Heavy Quarks Model)

(Isotropic case)





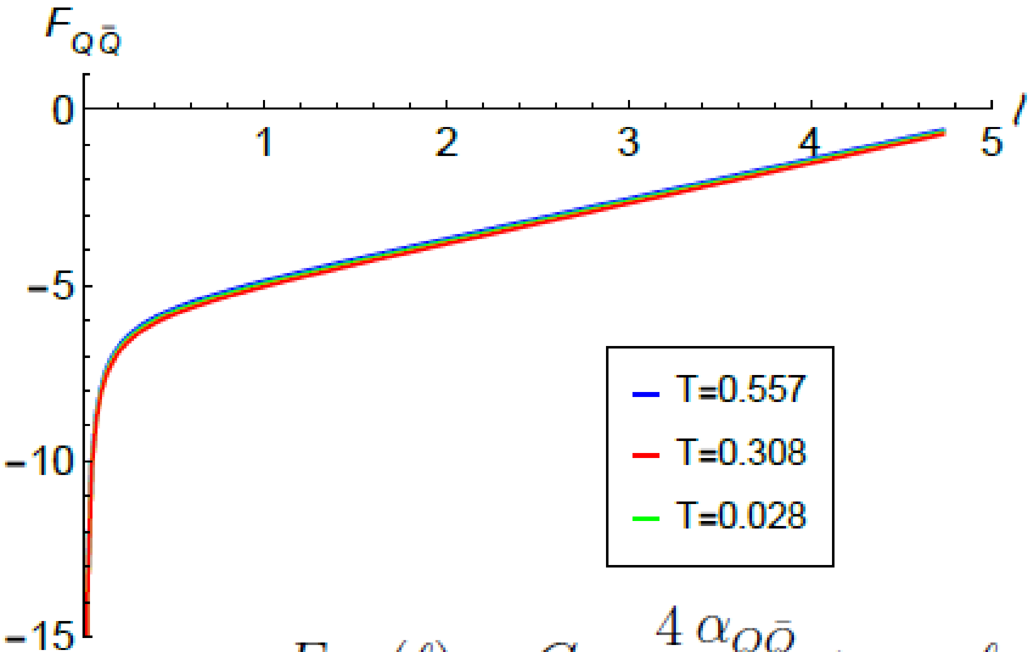
Boundary conditions:

Gauge field: $A_t(0) = \mu, \quad A_t(z_h) = 0$

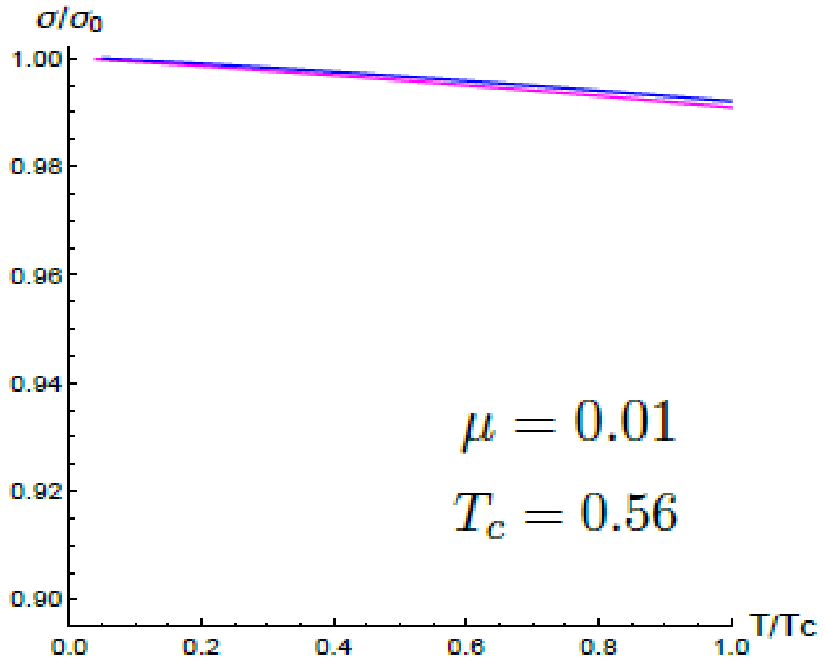
Blackening function: $g(0) = 1, \quad g(z_h) = 0$

Dilaton field: $\varphi(z, z_0)\Big|_{z=z_0} = 0 \quad \Longrightarrow \quad z_0 = 0, \quad z_0 = z_h, \quad z_0 = \mathfrak{z}(z_h)$

Physical boundary condition for dilaton:



$$F_{Q\bar{Q}}(\ell) \sim C - \frac{4}{3}\frac{\alpha_{Q\bar{Q}}}{\ell} + \sigma_{Q\bar{Q}}\ell$$



Magenta line: Asymptotic of effective potential

Blue line: Linear part of Cornell potential

Physical boundary condition:

$$z_0 = \mathfrak{z}_{HQ}(z_h) = e^{(-\frac{z_h}{4})} + 0.1$$

Holographic running coupling: $\alpha(z) = e^{\varphi(z)}$

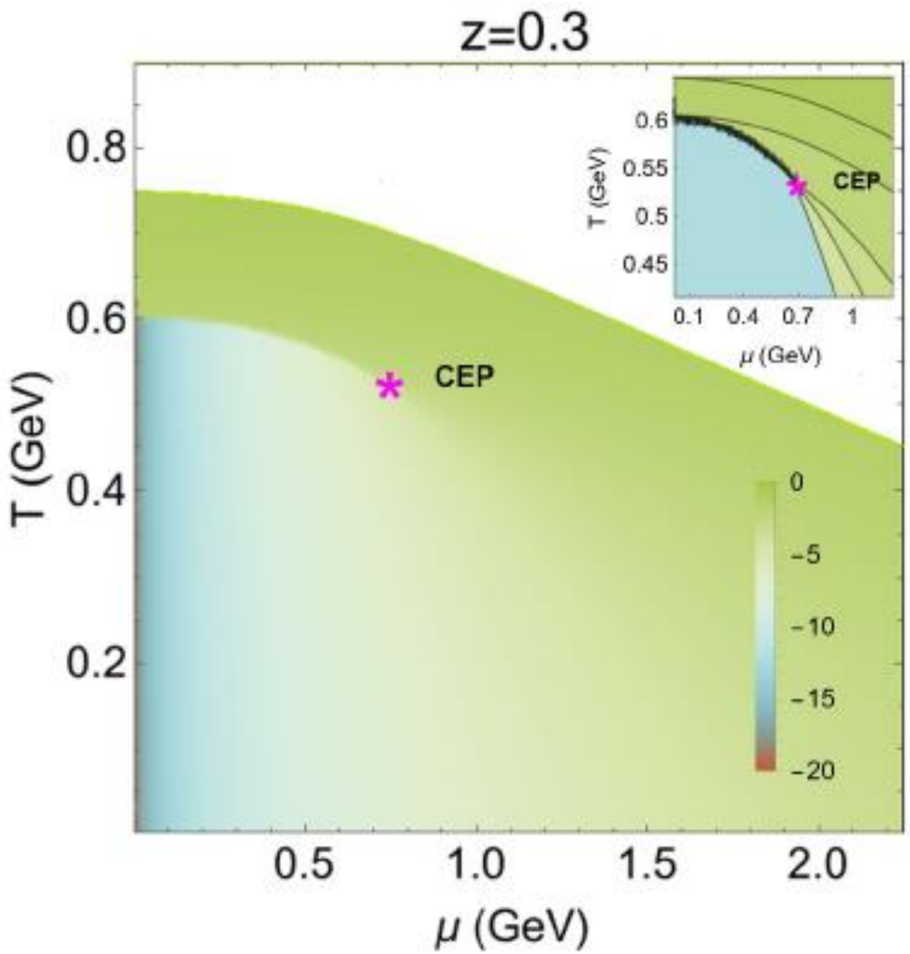
$$\varphi_{z_0}(z) = \varphi_0(z) - \varphi_0(z_0) \longrightarrow \varphi_0(z)\Big|_{z=0} = 0$$

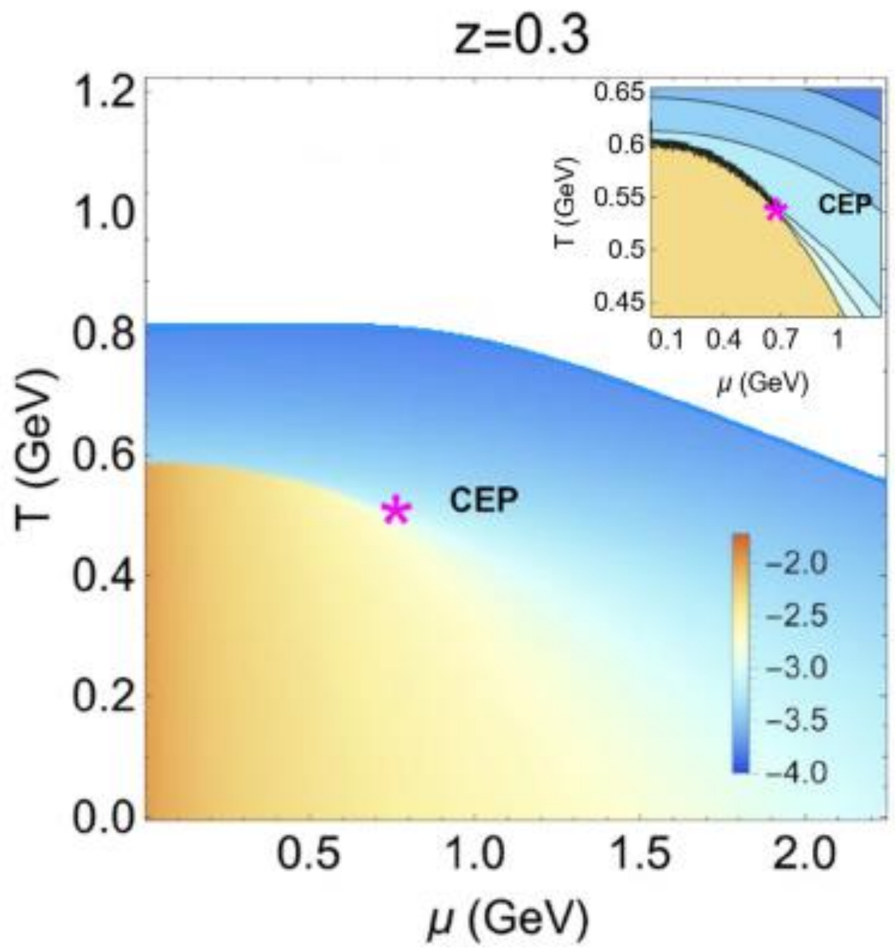
$$\Rightarrow \alpha_{z_0}(z) = \alpha_0(z)\,\mathfrak{G}(z_0) = e^{\varphi_0(z)} \quad e^{-\varphi_0(z_0)}$$

Choosing boundary condition:

$$\alpha_{\mathfrak{z}}(z;T,\mu) = \alpha_0(z)\,\mathfrak{G}(T,\mu) \quad \text{where} \quad \mathfrak{G}(T,\mu) = e^{-\varphi_0(\mathfrak{z}(z_h))}$$

Logarithm of running coupling: (phase diagram)



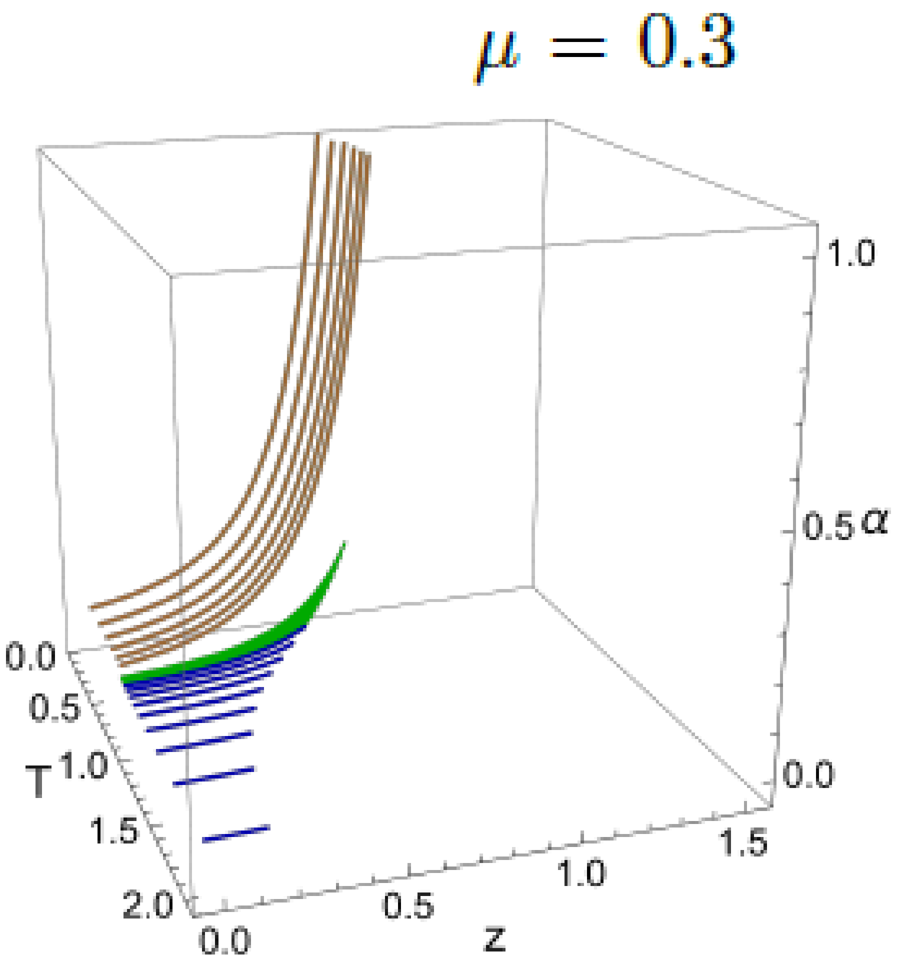


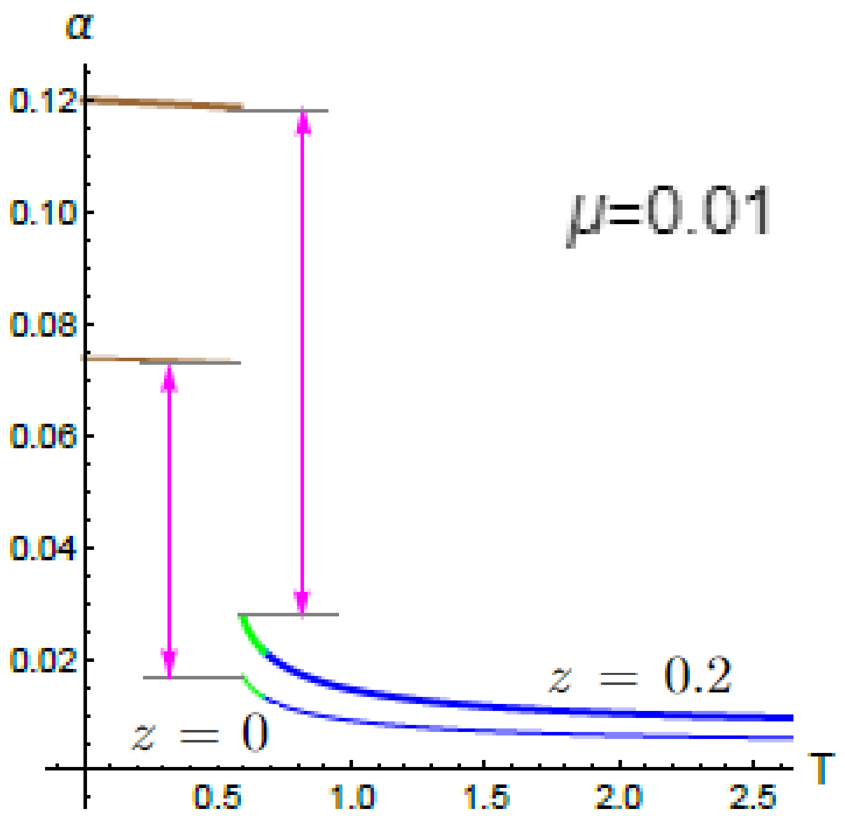
Boundary conditions: $z_0 = z_h$

$$z_0 = \mathfrak{z}_{HQ}(z_h) = e^{(-\frac{z_h}{4})} + 0.1$$

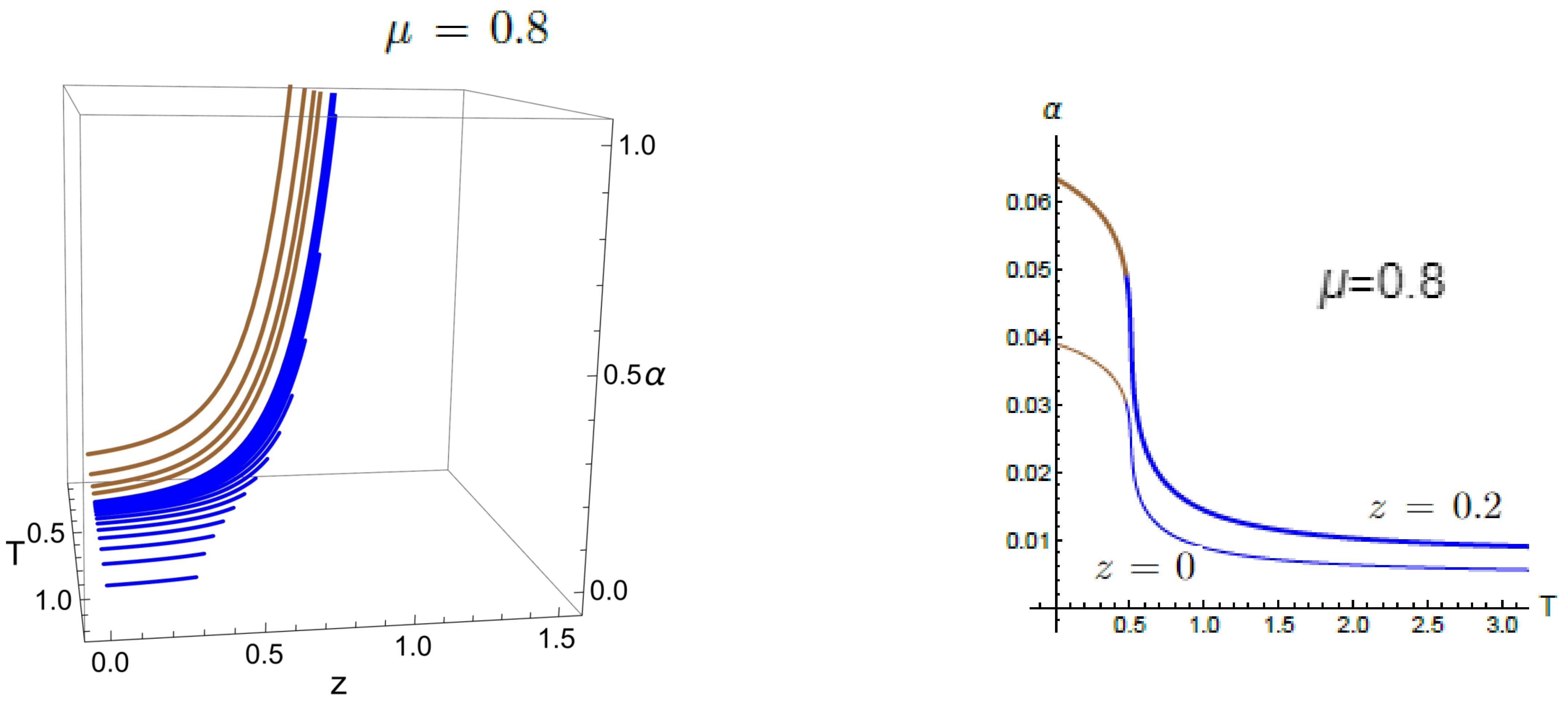
Running coupling vs T , z:

For heavy quarks the jump is larger in comparison to light quarks

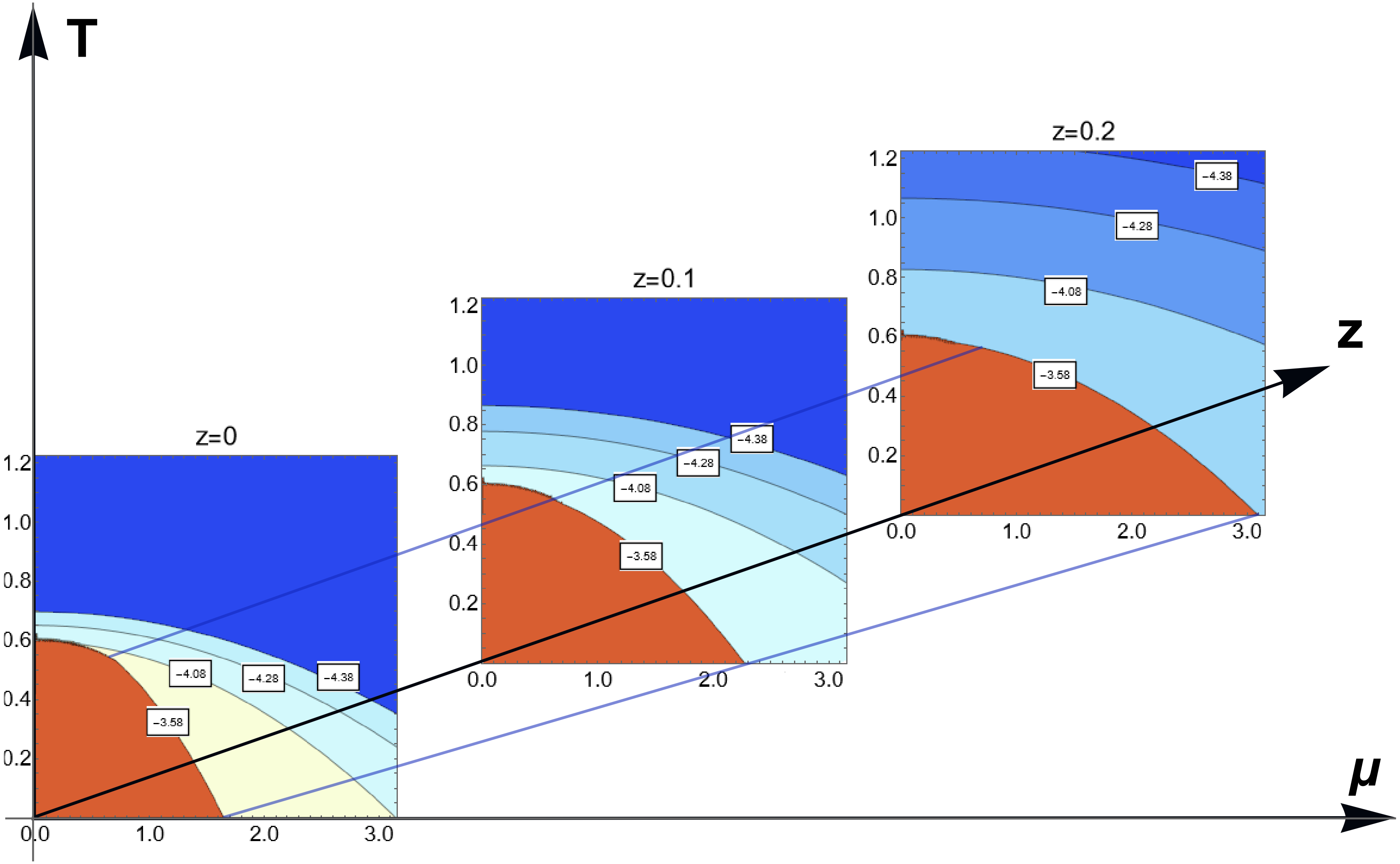




Running coupling vs T , z:



Logarithm of running coupling vs energy scale z:



$$z_0 = \mathfrak{z}_{HQ}(z_h) = e^{(-\frac{z_h}{4})} + 0.1$$

2nd Question

Towards the 2nd Question: Heavy ion collisions (HIC)

QGP Can teach us about properties of the high temperature phase of QCD.

Noncentral relativistic HIC → Anisotropic Plasma

Mateos, Trancanelli, *JHEP,* 2011;
Aref 'eva, Golubtsova, JHEP, 2014

There is a strong magnetic field at the early stages of relativistic HIC → $eB \sim 0.3\ GeV^2$

Skokov, Illarionov, Toneev, *IJMPA,* 2009;
Voronyuk, Toneev, Cassing, Bratkovskaya, Konchakovski, Voloshin, *PRC,* 2011

Towards the 2nd Question:

Complete description of the QCD phase diagram in a parameter space with:

"temperature,

chemical potential,

quark masses,

anisotropy,

magnetic field" etc.

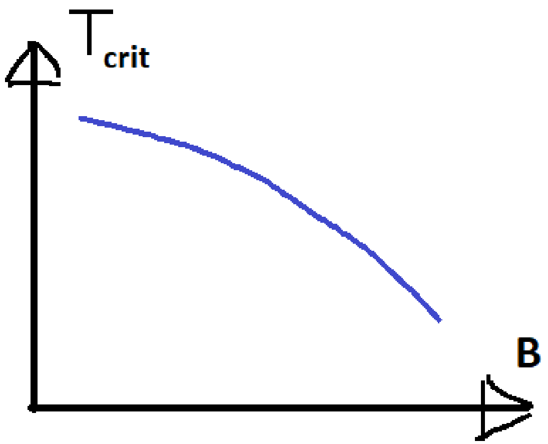
is a challenging and very important task in high energy physics.

2nd Question:

What is the effect of magnetic field on the phase transition temperature?

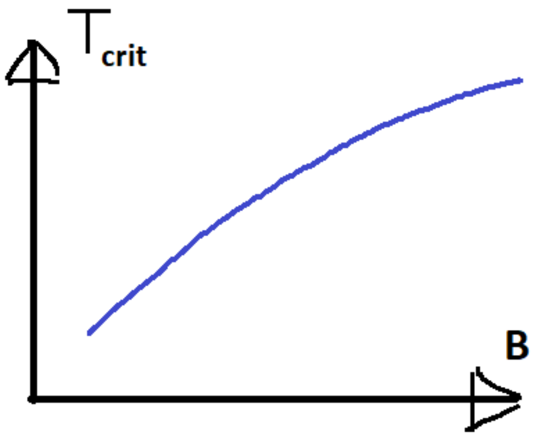
1- Inverse Magnetic Catalysis (IMC)

Mao, PLB, 2016;
Bohra, Dudal, Hajilou, Mahapatra, PLB, 2019;
Aref'eva, Rannu, Slepov, JHEP, 2020



2- Magnetic Catalysis (MC)

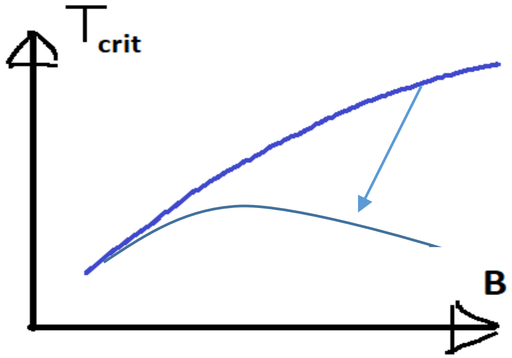
Miransky, Shovkovy, PRD, 2002;
He, Yang, Yuan, 2004.01965, 2020



2nd question:

How spatial anisotropy changes the effect of MC?

What is the effect of spatial anisotropy on the phase transition temperature?



Spatial anisotropy gives correct total multiplicity produced in HIC:

To produce total multiplicity by considering anisotropy: $\mathcal{M}_\nu \sim s^{\frac{1}{2+\nu}}$

$\Rightarrow$ $\nu = 4.45$

Aref 'eva, Golubtsova, JHEP, 2014

Our Model: Einstein-Maxwell-dilaton action

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{f_0(\phi)}{4}F_0^2 - \frac{f_1(\phi)}{4}F_1^2 - \frac{f_3(\phi)}{4}F_3^2 - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right]$$

$$\phi = \phi(z),$$

$$\text{Electric anzats } F_0\text{:} \quad A_0 = A_t(z), \quad A_{i=1,2,3,4} = 0,$$

$$\text{Magnetic anzats } F_k\text{:} \quad F_1 = q_1\, dx^2 \wedge dx^3, \quad F_3 = q_3\, dx^1 \wedge dx^2.$$

F_0 ⟷ Chemical potential

F_1 ⟷ Spatial anisotropy

F_3 ⟷ Magnetic field

Aref'eva, Rannu, Slepov, JHEP, 2021

Our ansatz for the metric:

$$ds^2 = \frac{L^2}{z^2}\mathfrak{b}(z)\left[-g(z)\,dt^2 + dx_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_2^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_3^2 + \frac{dz^2}{g(z)}\right]$$

$$\mathfrak{b}(z) = e^{2A(z)}$$

Warp factor

Isotropic $\nu = 1$

Anisotropic $\nu = 4.5$

Equations of Motions (EOMs):

Einstein EOMs: $G_{\mu\nu} = T_{\mu\nu}$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad , \quad \frac{\delta S_m}{\delta g^{\mu\nu}} = \frac{1}{2} T_{\mu\nu} \sqrt{-g}$$

Fields EOMs:

$$-\nabla_\mu \nabla^\mu \phi + V'(\phi) + \sum_{i=0,1,3} \frac{f_i'(\phi)}{4} F_{(i)}^2 = 0$$

$$\partial_\mu \left(\sqrt{-g}\, f_i\, F_{(i)}^{\mu\nu} \right) = 0$$

Solving EOMs: (Potential reconstruction method)

1st gauge field:

$$A_t'' + A_t'\left(\frac{\mathfrak{b}'}{2\mathfrak{b}} + \frac{f_0'}{f_0} + \frac{\nu - 2}{\nu z} + c_B z\right) = 0$$

Blackening function:

$$g'' + g'\left(\frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{\nu + 2}{\nu z} - c_B z\right) - 2g\left(\frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{2}{\nu z} + c_B z\right)c_B z - \left(\frac{z}{L}\right)^2 \frac{f_0 (A_t')^2}{\mathfrak{b}} = 0$$

In Search of Magnetic Catalysis (MC): Heavy Quarks

Our choice: $\mathcal{A}(z) = -cz^2/4 - pz^4$

Warp factor: $\mathrm{b}(z) = e^{2\mathcal{A}(z)} = e^{-cz^2/2 - 2pz^4}$

Gauge coupling function: $f_0 = e^{-(R_{gg} + \frac{c_B q_3}{2})z^2} \frac{z^{-2+\frac{2}{\nu}}}{\sqrt{\mathrm{b}}}$

By choosing this kinetic function our model can respect the Linear Regge trajectory for meson spectrum.

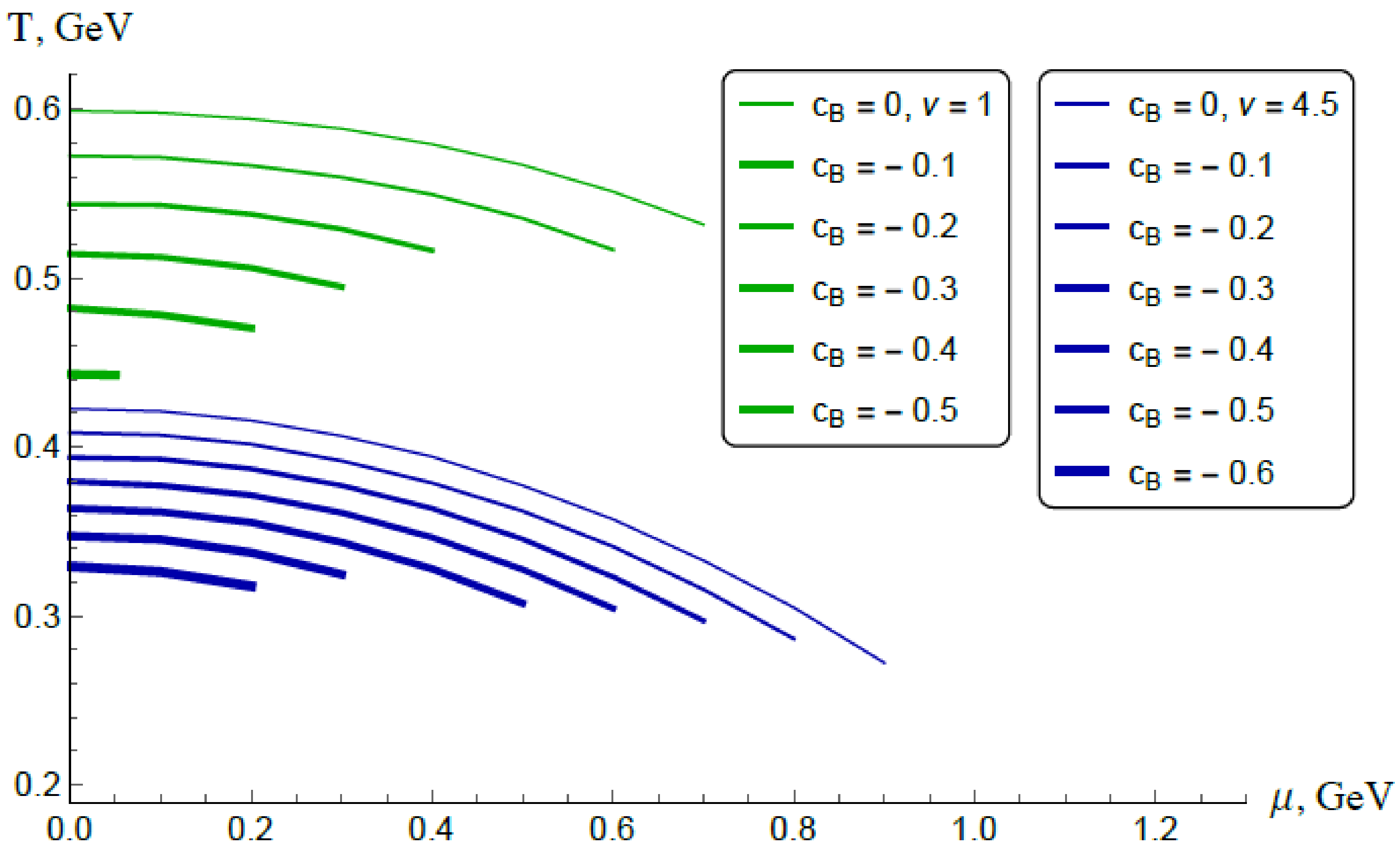
Boundary conditions:

1st gauge field: $A_t(0) = \mu, \quad A_t(z_h) = 0$

Blackening function: $g(0) = 1, \quad g(z_h) = 0$

Dilaton field: $\phi(z_0) = 0$

Phase diagram:



$$\mathfrak{b}(z) = e^{2A(z)} = e^{-cz^2/2 - 2pz^4}$$ → Inverse Magnetic Catalysis (IMC)

Warp factor: $$b(z) = e^{2A(z)} = e^{-cz^2/2 - 2pz^4}$$

NO MC phenomenon
was observed for this warp factor!!!!

In Search of Magnetic Catalysis (MC): Heavy Quarks

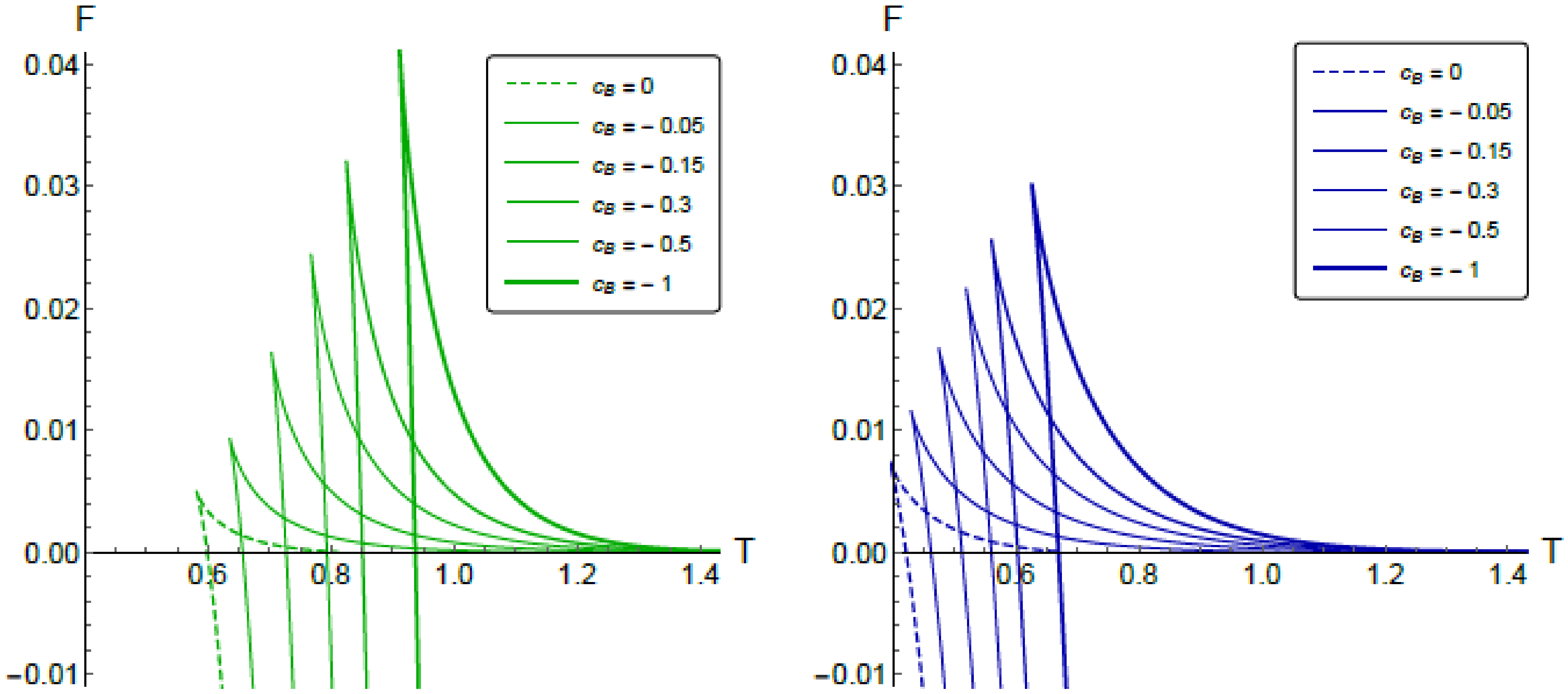
New choice: $\mathcal{A}(z) = -cz^2/4 - (p - c_B q_3)z^4$

$$ds^2 = \frac{L^2}{z^2}\mathfrak{b}(z)\left[-g(z)\,dt^2 + dx_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_2^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_3^2 + \frac{dz^2}{g(z)}\right]$$

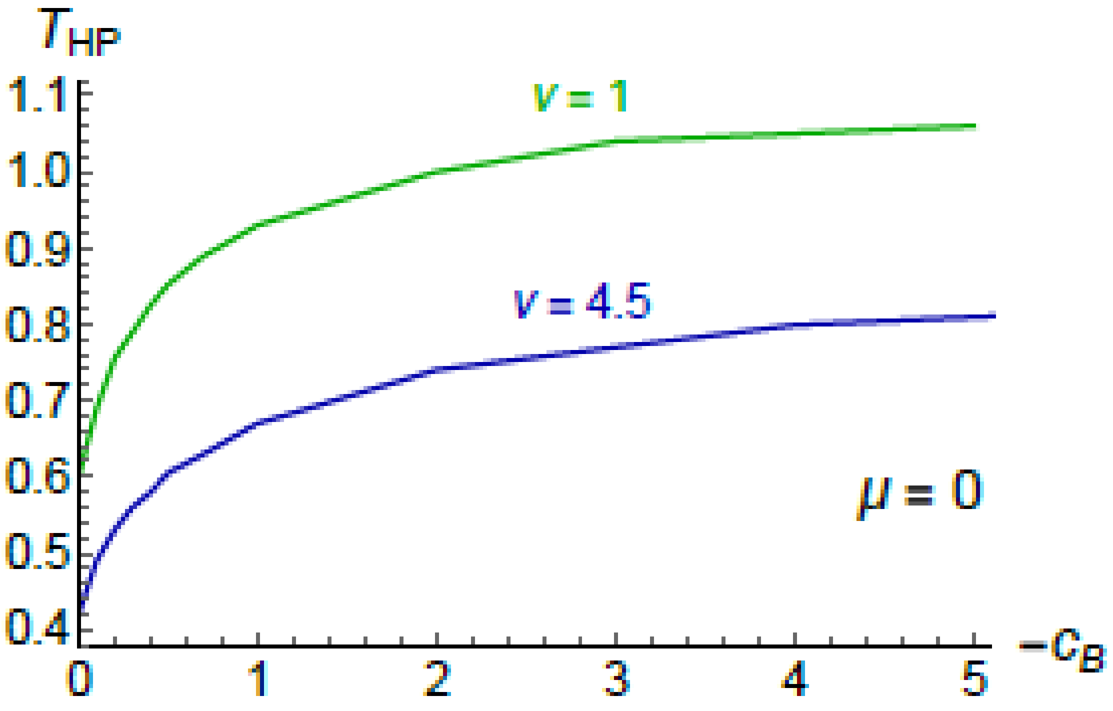
New Warp factor: $\mathfrak{b}(z) = e^{2\mathcal{A}(z)} = e^{-cz^2/2 - 2(p - c_B q_3)z^4}$

New term

Free energy: $\mu = 0$



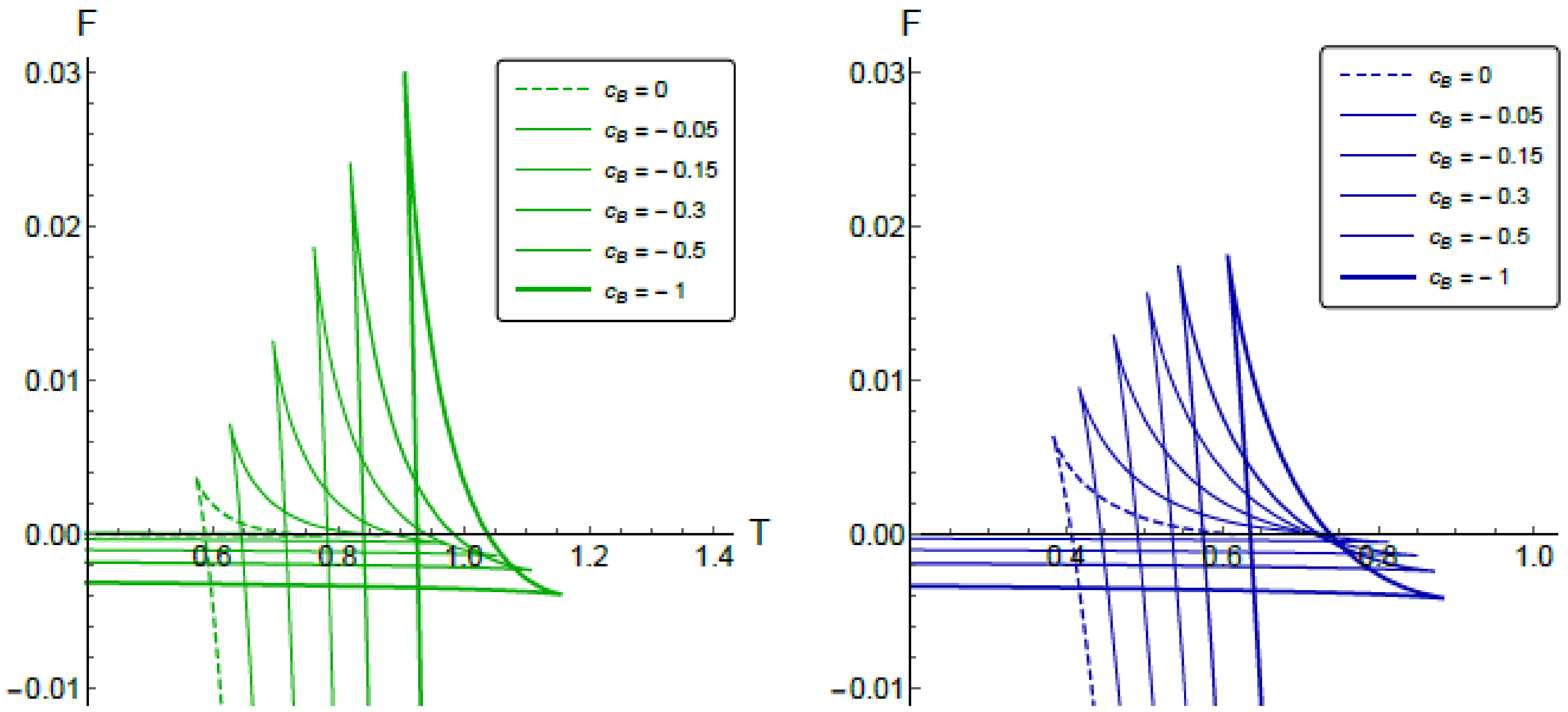
Critical temperature vs magnetic field



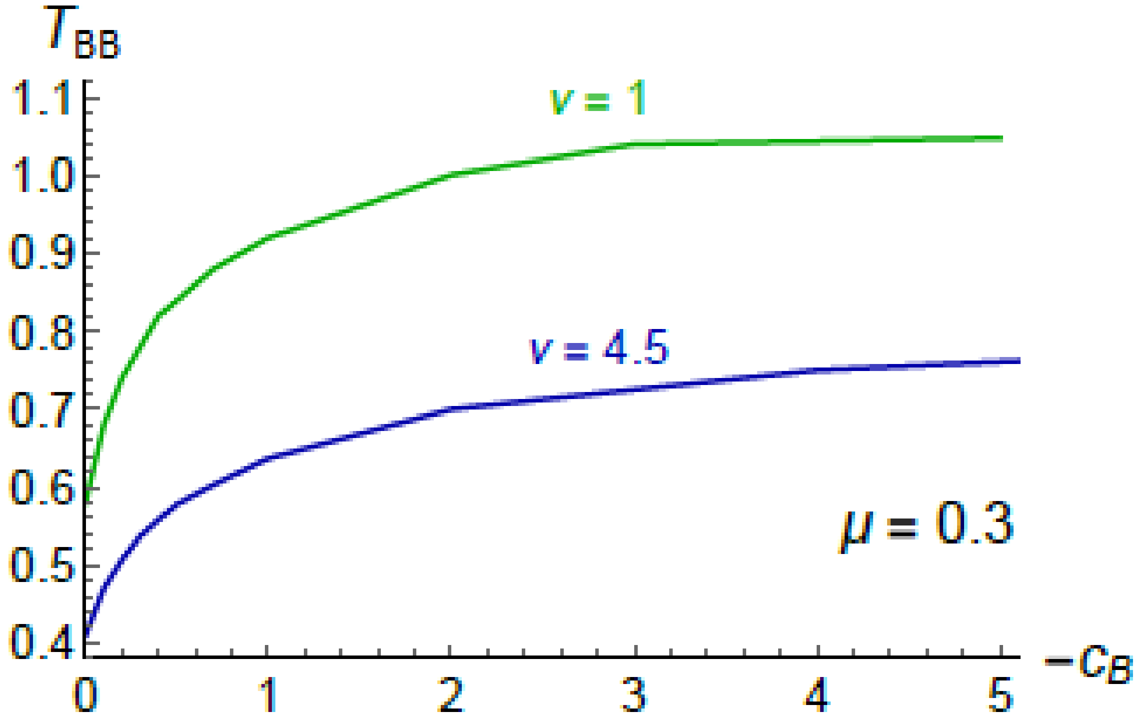
MC phenomenon is obtained!

It is found that primary anisotropy decreases for all values of magnetic field.

Free energy: $\mu = 0.3$



Critical temperature vs magnetic field

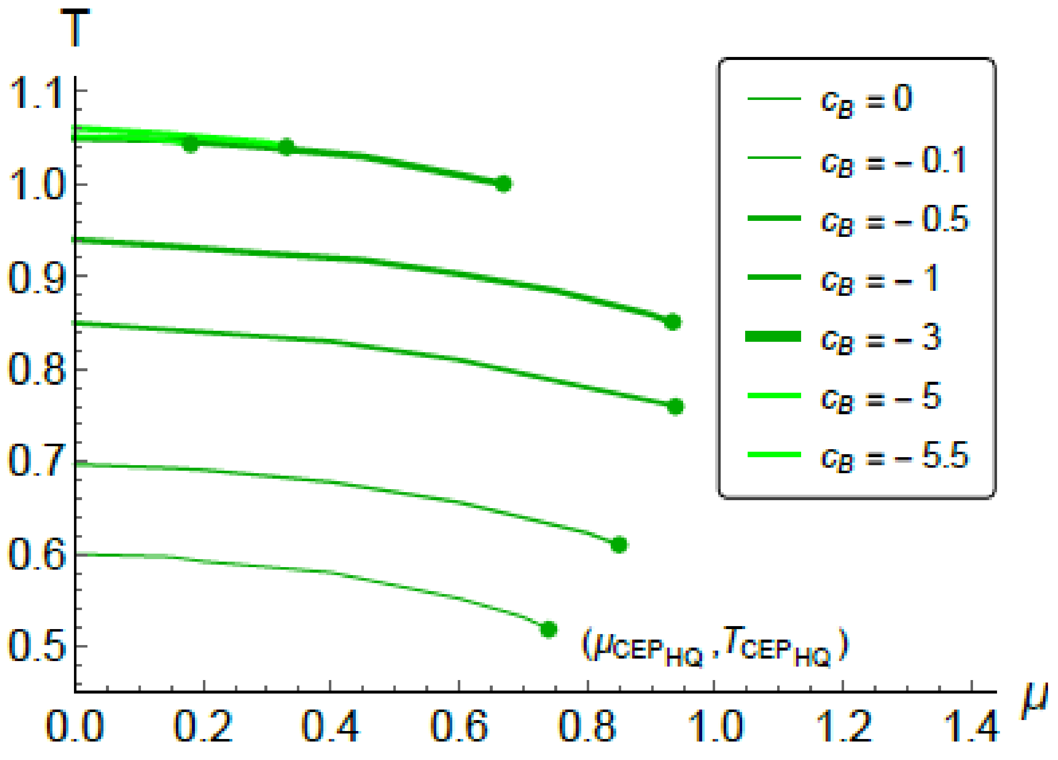


MC phenomenon is obtained!

It is found that primary anisotropy decreases for all values of magnetic field at fixed chemical potential.

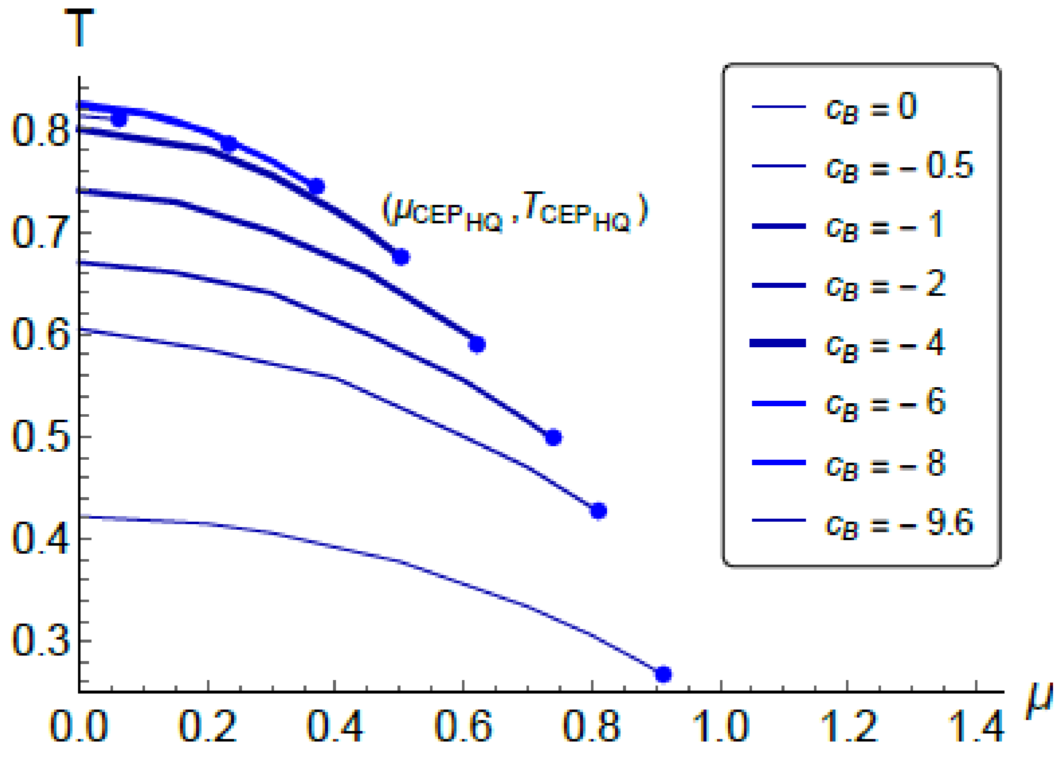
Phase diagram for different cases of anisotropy:

$\nu = 1$



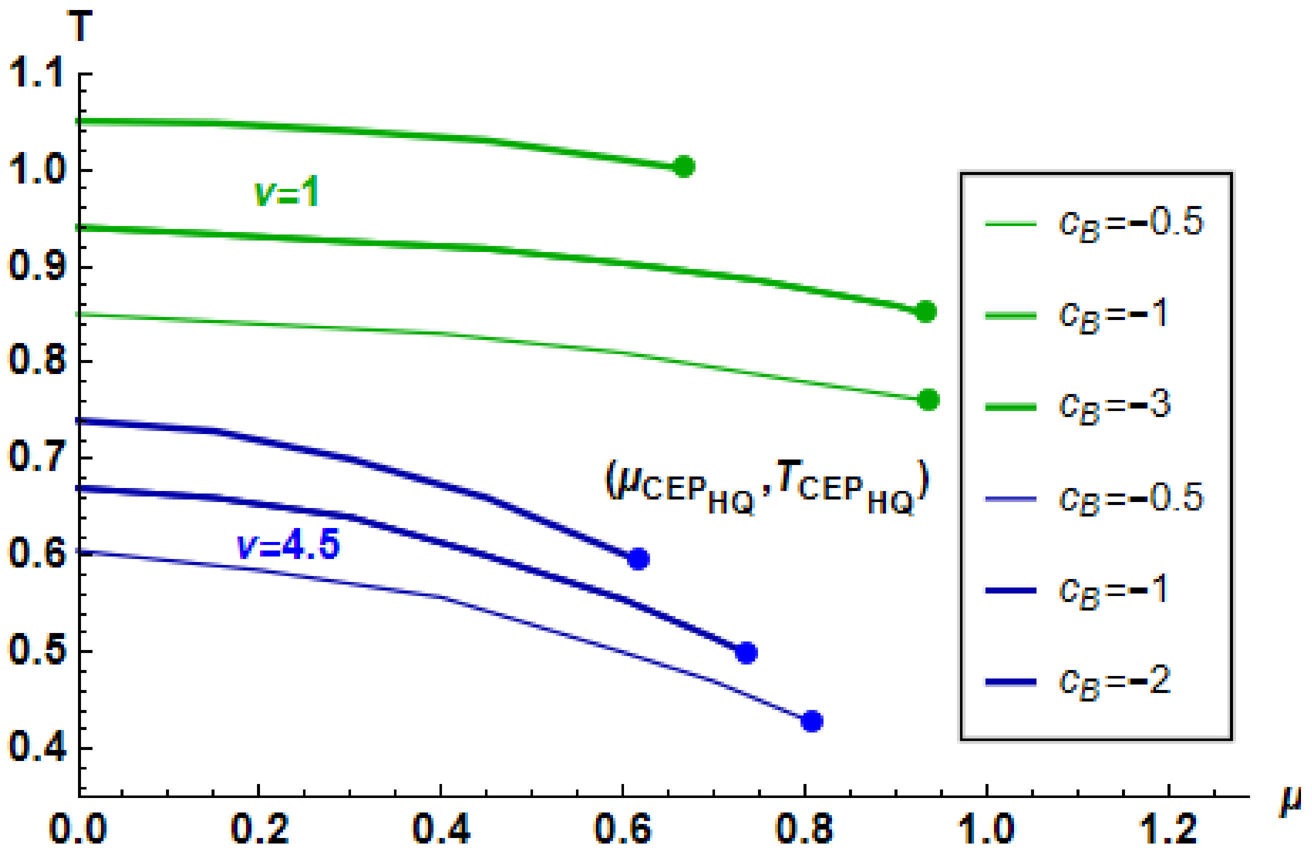
$5 < |c_B| \sim 6$

$\nu = 4.5$

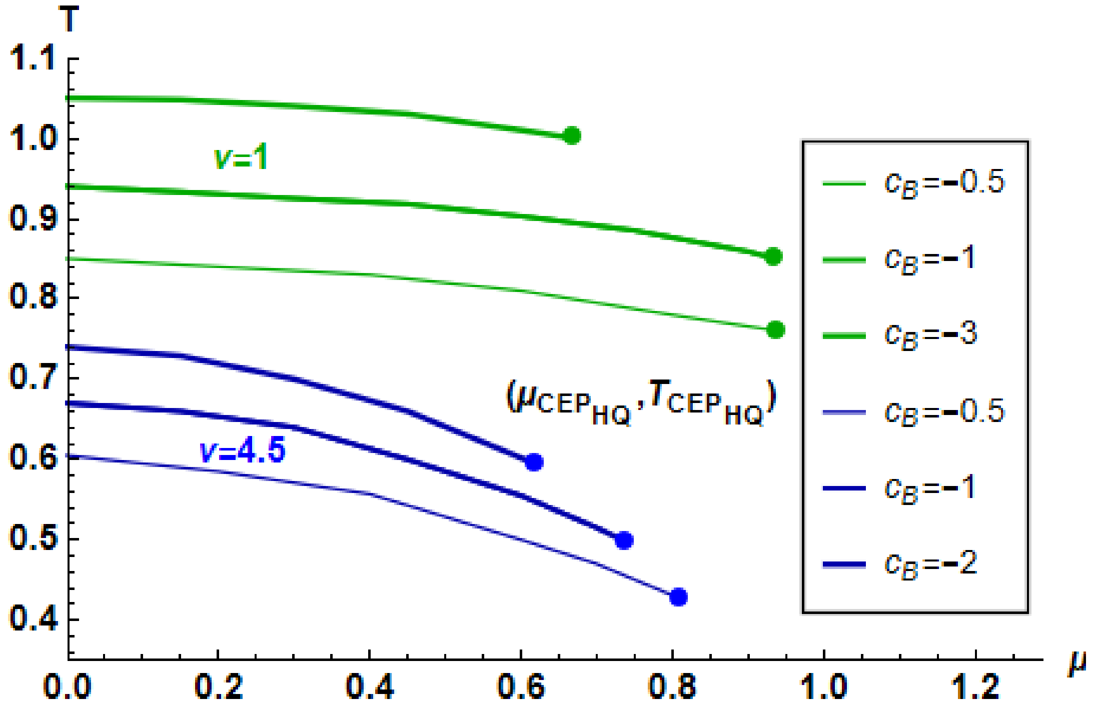


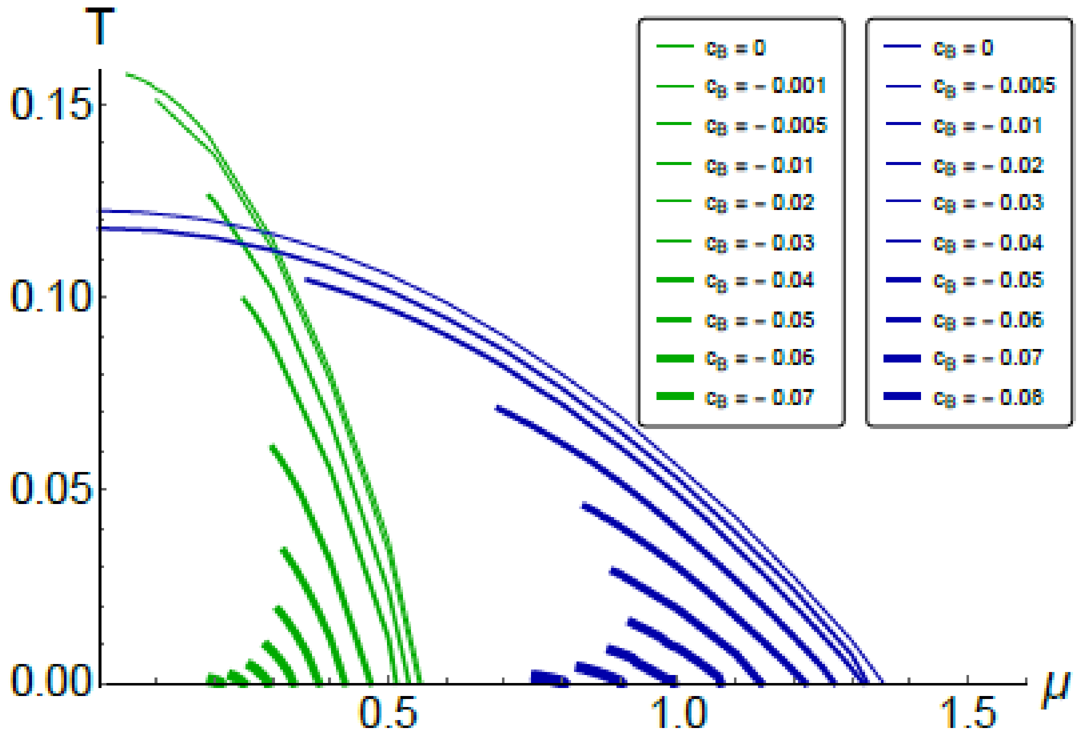
$8 < |c_B| \sim 10$

Phase diagram of heavy quarks:
(considering spatial anisotropy)



Heavy quarks vs Light quarks





Aref'eva, Ermakov, Rannu, Slepov, EPJC, 2023

MC vs IMC

Summary:

- Coupling senses the 1st order phase transition.
- Phase structure of QCD is independent of boundary conditions.
- A new 5-dim exact analytical solution for anisotropic holographic model of quark-gluon plasma reconstructed.
- The warp factor $\mathfrak{b}(z) = e^{2A(z)} = e^{-cz^2/2 - 2pz^4}$ Leads to IMC.
- The warp factor $\mathfrak{b}(z) = e^{2A(z)} = e^{-cz^2/2 - 2(p - c_B\, q_3)z^4}$ Leads to MC.
- Primary anisotropy decreases 1st order phase transition for all values of magnetic field.

Future plans:

- Investigating coupling constant of heavy quarks in the anisotropic model
- It would be interesting to study temporal and spatial Wilson loops in this background with the new corrected warp factor.
- Investigating energy loss and jet quenching in this background.
- Studying the chiral condensate in this background.

Thank you for your attention!

Complementarity

$$
\begin{aligned}
f_1(z) = &-\frac{2(\nu-1)}{q_1^2\,\nu^2 L^2}\,(\frac{L}{z})^{\frac{4}{\nu}}\,e^{\frac{-2}{3}z^2\left(-3c_B+R_{gg}+3(p-c_B\,q_3)z^2\right)}\Bigg[-2-2\,\nu\\
&+z^2\,\nu\left(3c_B-2R_{gg}\,-12(p-c_B\,q_3)z^2+\frac{\mu^2\,(c_B(-1+q_3)+2R_{gg})\,z^{\frac{2}{\nu}}\,e^{\frac{1}{2}z^2\left(4R_{gg}+6(p-c_B\,q_3)z^2-4c_B+c_B\,q_3\right)}}{(e^{\frac{1}{2}(c_B(-1+q_3)+2R_{gg})z_h^2}-1)^2\,L^2}\right)\\
&-\Bigg(\left(e^{\frac{1}{2}z^2\left(-3c_B\,2R_{gg}+6(p-c_B\,q_3)z^2\right)}\nu\,z^{2+\frac{2}{\nu}}+\left(-2+\nu\left(-2+(3c_B-2R_{gg})z^2-12\,(p-c_B\,q3)z^4\right)\right)\tilde{I}_1(z)\right)\\
&\times\left(\frac{1}{\tilde{I}_1(z_h)}\,+\frac{\mu^2\,(c_B(-1+q_3)+2R_{gg})}{(e^{\frac{1}{2}(c_B(-1+q_3)+2R_{gg})z_h^2}-1)^2\,L^2}\,\frac{\tilde{I}_2(z_h)}{\tilde{I}_1(z_h)}\right)\Bigg)\\
&+\frac{\mu^2\,(c_B(-1+q_3)+2R_{gg})\,(-2+\nu\,(-2+(3c_B-2R_{gg})z^2-12\,(p-c_B\,q_3)z^4))\,\tilde{I}_2(z)}{(e^{\frac{1}{2}(c_B(-1+q_3)+2R_{gg})z_h^2}-1)^2\,L^2}\Bigg]
\end{aligned}
$$

$$f_3(z) = -\frac{2\, c_B\, e^{c_B\, z^2 - \frac{2R_{gg}}{3}\, z^2 - 2(p - c_B\, q_3) z^4} \left(\frac{L}{z}\right)^{\frac{2}{\nu}}}{\left(e^{\frac{1}{2}(c_B(-1+q_3)+2R_{gg}) z_h^2} - 1\right)^2 L^2\, q_3^2\, \nu\, \tilde{I}_1(z_h)} \times \Bigg[\Big(e^{\frac{1}{2} z^2 \left(-3 c_B\, 2R_{gg} + 6(p - c_B\, q_3) z^2\right)}\, z^{2+\frac{2}{\nu}}\, \nu$$

$$+ \left(-2 + z^2\, \nu\, (3c_B - 2R_{gg} - 12(p - c_B\, q_3) z^2)\right) \tilde{I}_1(z) \Big) \Big(\Big(e^{\frac{1}{2}(c_B(-1+q_3)+2R_{gg}) z_h^2} - 1 \Big)^2 L^2$$

$$+ \mu^2 (c_B(-1 + q_3) + 2R_{gg})\, \tilde{I}_2(z_h) \Big) \tilde{I}_1(z_h) \Big(-e^{\frac{1}{2} z^2 \left(4R_{gg} + 6(p - c_B\, q_3) z^2 - 4c_B + c_B\, q_3\right)}\, \mu^2 (c_B(-1 + q_3) + 2R_{gg})$$

$$\times z^{2+\frac{2}{\nu}}\, \nu - \Big(2 - z^2\, \nu\, (3c_B - 2R_{gg} - 12(p - c_B\, q_3) z^2)\Big)$$

$$\times \Big(\Big(e^{\frac{1}{2}(c_B(-1+q_3)+2R_{gg}) z_h^2} - 1 \Big)^2 L^2 + \mu^2 (c_B(-1 + q_3) + 2R_{gg})\, \tilde{I}_2(z) \Big) \Big) \Bigg]$$

In search of Magnetic Catalysis: (MC)

We need to find: g(z)

$$T = \frac{\sqrt{g_{tt}{}' \, g^{zz}{}'}}{4\pi}\Big|_{z=z_h} = \frac{\sqrt{g_{00}{}' \, g^{55}{}'}}{4\pi}\Big|_{z=z_h} = \frac{|g'|}{4\pi}\Big|_{z=z_h}$$

Temperature and Entropy:

$$s = \frac{\sqrt{g_{xx}\, g_{y_1y_1}\, g_{y_2y_2}}}{4}\Big|_{z=z_h} = \frac{\sqrt{g_{11}\, g_{22}\, g_{33}}}{4}\Big|_{z=z_h}$$

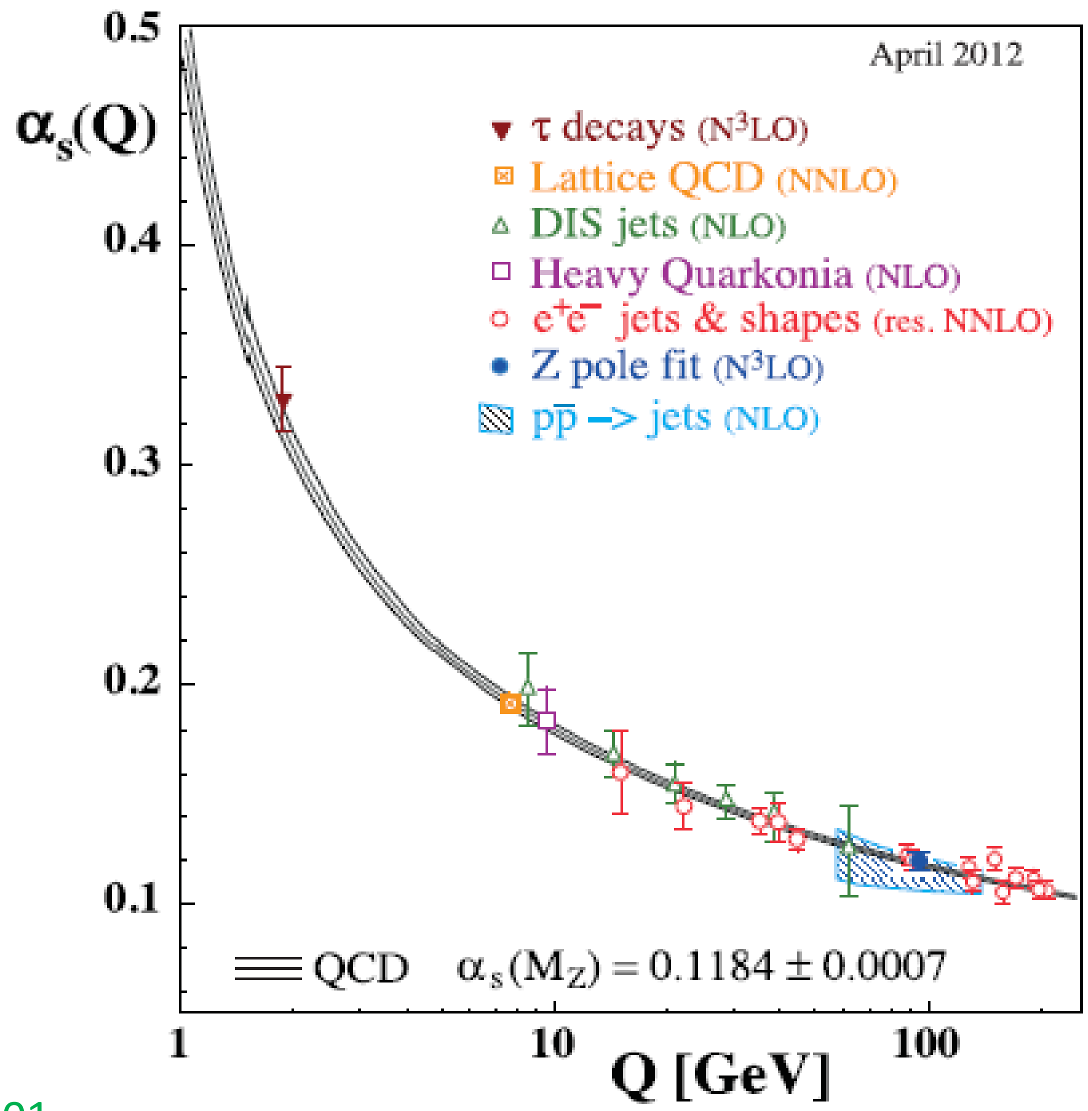
$$s = \frac{1}{4}\left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}} e^{-(2R_{gg}-c_B)\frac{z_h^2}{2}-3pz_h^4}$$

Free energy:

$$F = -\int s\, dT = \int_{z_h}^{\infty} s\, T' dz.$$

Introduction: OLD Running Coupling

Running coupling as a function of the energy scale Q



The respective degree of *QCD perturbation theory* used in the extraction of coupling is indicated in brackets (NLO: next-to-leading order, …)

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