Renormalization group flows in holographic model for light/heavy quarks

Based on a joint work with I.Ya. Arefeva, A. Hajilou, P. Slepov arXiv:2402.14512v2

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Quarks - 2024

AdS/CFT-correspondence for AdS$_5$

Gravity in 5-dimensional $AdS_5 \Leftrightarrow$ 4d conformal field theory

The 5-dimensional Einstein-Maxwell-scalar action

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{\mathfrak{f}_0(\varphi)}{4}F^2 - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \mathcal{V}(\varphi)\right],$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Anzatz for the metric:

$$ds^2 = B^2(z)\left[-g(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{g(z)}\right],$$
$$B(z) = \frac{e^{A(z)}}{z}, \quad \varphi = \varphi(z), \quad A_\mu = \left(A_t(z)\ , \vec{0}, 0\right)$$

Light quarks

$$A_{LQ}(z) = -a\log\left(bz^2+1\right)$$
$$f_{LQ}(z) = e^{-c\,z^2 - A(z)}$$

fitting with experimental data: ↘

$a = 4.046, b = 0.01613GeV^2, c = 0.227GeV^2$

(Li et.al.'17; Aref'eva et. al.'21, '22)

Heavy quarks

$$A_{HQ}(z) = -\frac{\mathrm{s}}{3}z^2 - p\,z^4$$
$$f_{HQ}(z) = e^{-\mathrm{s}\,z^2 - A(z)}$$

↘

$\mathrm{s} = 1.16GeV^2, p = 0.273GeV^4$

(Yang & Yuan, '16; Aref'eva et. al.'20, '23)

Equations of motion:

$$\varphi'' + \left(\frac{g'}{g} + 3A' - \frac{3}{z}\right)\varphi' + \left(\frac{z^2 e^{-2A} A_t' f_{0,\varphi}}{2g} - \frac{e^{2A} V_\varphi}{z^2 g}\right) = 0,$$

$$A_t'' + \left(\frac{f_0'}{f_0} + A' - \frac{1}{z}\right) A_t' = 0,$$

$$A'' - A'^2 + \frac{2}{z}A' + \frac{\varphi'^2}{6} = 0,$$

$$g'' + \left(3A' - \frac{3}{z}\right) g' - e^{-2A} z^2 f_0 A_t'^2 = 0,$$

$$A'' + 3A'^2 + \left(\frac{3g'}{2g} - \frac{6}{z}\right) A' - \frac{1}{z}\left(\frac{3g'}{2g} - \frac{4}{z}\right) + \frac{g''}{6g} + \frac{e^{2A} V}{3z^2 g} = 0$$

Applied boundary conditions:

$$A_t(0) = \mu, \quad A_t(z_h) = 0, \quad g(0) = 1, \quad g(z_h) = 0,$$

and for the dilaton field: $\boxed{\varphi(z, z_0)\Big|_{z=z_0} = 0, \quad \text{where} \quad z_0 = \mathfrak{z}(z_h)}$

General solutions:

$$\phi'(z) = \sqrt{-6\left(A'' - A'^2 + \frac{2}{z}A'\right)}, \quad A_t(z) = \mu\,\frac{e^{cz^2} - e^{cz_h^2}}{1 - e^{cz_h^2}}$$

$$V(z) = -3z^2 g e^{-2A}\left[A'' + 3A'^2 + \left(\frac{3g'}{2g} - \frac{6}{z}\right)A' - \frac{1}{z}\left(\frac{3g'}{2g} - \frac{4}{z}\right) + \frac{g''}{6g}\right]$$

Domain wall solution:

$$ds^2 = e^{2A(w)}\eta_{ij}dx^i dx^j + dw^2, \quad \phi = \phi(w)$$

E. Akhmedov'98; de Boer, Verlinde, Verlinde'98, Skenderis et. al.'98; Skenderis'99

Solution under consideration:

$$ds^2 = B(z)^2\left(\eta_{ij}dx^i dx^j + dz^2\right), \quad B(z) = \frac{e^{A(z)}}{z}, \quad \varphi = \varphi(z)$$

We have the following holographic dictionary:

- B corresponds to the energy scale E of the dual field theory
- $\lambda = e^{\varphi(z)}$ must be identified as running coupling of the field theory
- Connection with β-function in this background (DeWolfe et. al. '14, Kiritsis et.al.'14)

$$\beta = \left.\frac{d\lambda}{d\log E}\right|_{QFT} = \lambda\left.\frac{d\varphi}{d\log B}\right|_{Holo}$$

- The holographic RG is geometrized

Dynamical system for the case: $T = 0$, $\mu = 0$

New dynamical variables:

$$X(z) = \frac{\dot{\varphi} B}{3\dot{B}}$$

Kiritsis et.al. '08

Physical meaning of the dynamical system

- Variable X is associated with overstretched β-funtion:

$$\beta = 3\,\alpha\,X$$

- $B(z)$ increases along the flow

Holographic RG flows equation:

$$\frac{d\mathcal{X}}{d\varphi} \;=\; -\frac{4}{3}\left(1 - \frac{3}{8}\mathcal{X}^2\right)\left(1 + \frac{1}{\mathcal{X}}\frac{\partial_\varphi \mathcal{V}(\varphi)}{\mathcal{V}(\varphi)}\right)$$

I. Ya. Aref'eva, K. Rannu '18

where we consider $\quad X(z) = \mathcal{X}(\varphi(z))$

HRG flows equation is invariant with respect to change of b.c.

After subtraction of the scale factors into EOM:

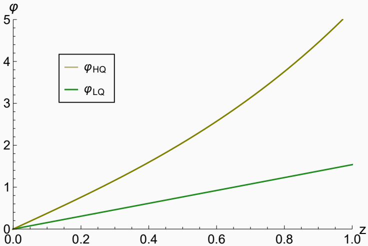
$$\varphi_{LQ}(z,\varphi_0) = \varphi_0 + 2\sqrt{3a}\left[\sqrt{2a+1}\,\text{arcsinh}\left(\sqrt{\frac{b(2a+1)}{3}}\,z\right) - \sqrt{2(a-1)}\,\text{arctanh}\left(\sqrt{\frac{2b(a-1)}{(2a+1)bz^2+3}}\,z\right)\right]$$

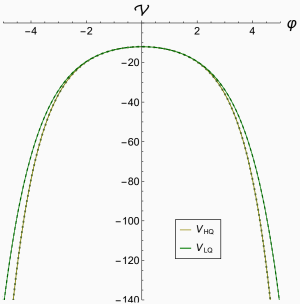
$$\varphi_{HQ}(z,\varphi_0) = \varphi_0 + \sqrt{6}\int_0^z d\xi\sqrt{\left(-4\,p\,\xi^3 - \frac{2\,\text{s}\,\xi}{3}\right)^2 + 2\left(4\,p\,\xi^2 + \frac{2\,\text{s}}{3}\right) + 12\,p\,\xi^2 + \frac{2\,\text{s}}{3}}$$

$$V_{LQ}(z) = -6\left(bz^2+1\right)^{2a-2}\left[bz^2\left((a(6a+7)+2)bz^2+5a+4\right)+2\right],$$

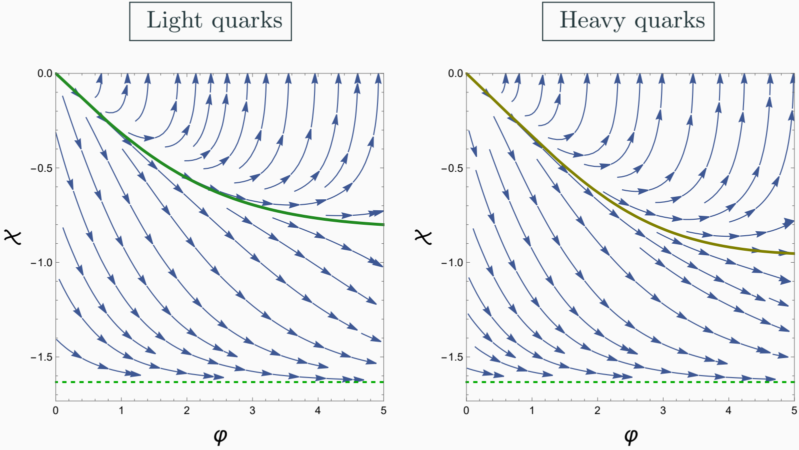
$$V_{HQ}(z) = -2\,e^{\frac{1}{3}(6\,p\,z^4+2\,\text{s}\,z^2)}\left[6+5\,\text{s}\,z^2+2(9\,p+\text{s}^2)z^4+24\,p\,\text{s}\,z^6+72\,p^2 z^8\right]$$

Reconstruction of the potentials: $\boxed{\text{V}(\varphi) = \sum_{i=0}^{18} c_i\varphi^i}$ $\quad -13.5 \le \varphi \le 13.5$





Comparison of the flows



Green dashed lines are fixed (attractor) lines: $\mathcal{X} = -\sqrt{8/3}$.
Solid green and olive lines depict an exact solution $\mathcal{X}$.

- simple hRG flows for light and heavy quarks models are almost the same!

Change of variables

$$X = \frac{\dot{\varphi}}{3}\frac{B}{\dot{B}}, \quad Y = \frac{1}{4}\frac{\dot{g}}{g}\frac{B}{\dot{B}}, \quad H = \frac{\dot{A}_t}{B^2}$$

$$\frac{d\mathcal{X}}{d\varphi} = -\frac{4}{3}\left(1 - \frac{3}{8}\mathcal{X}^2 + \mathcal{Y}\right)\left(1 + \frac{1}{\mathcal{X}}\frac{2\partial_\varphi \mathcal{V}(\varphi) - \mathcal{H}^2 \partial_\varphi \mathfrak{f}_0}{2\mathcal{V}(\varphi) + \mathcal{H}^2 \mathfrak{f}_0}\right),$$

$$\frac{d\mathcal{Y}}{d\varphi} = -\frac{4\mathcal{Y}}{3\mathcal{X}}\left(1 - \frac{3}{8}\mathcal{X}^2 + \mathcal{Y}\right)\left(1 + \frac{3}{2\mathcal{Y}}\frac{\mathcal{H}^2 \mathfrak{f}_0}{2\mathcal{V}(\varphi) + \mathcal{H}^2 \mathfrak{f}_0}\right),$$

$$\frac{d\mathcal{H}}{d\varphi} = -\left(\frac{1}{\mathcal{X}} + \frac{\partial_\varphi \mathfrak{f}_0}{\mathfrak{f}_0}\right)\mathcal{H}$$

where $\quad X = \mathcal{X}(\varphi(z)), \quad Y = \mathcal{Y}(\varphi(z)), \quad H = \mathcal{H}(\varphi(z))$

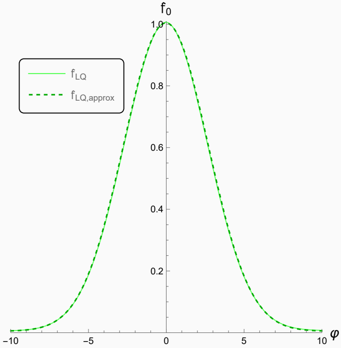
Gauge kinetic functions

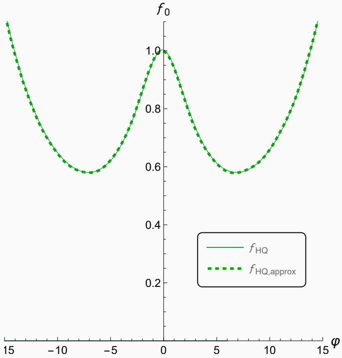
$$f_{LQ}(z) = e^{-cz^2 + a\log(bz^2+1)}, \qquad f_{HQ}(z) = e^{-\frac{2}{3}\mathrm{s}\, z^2 + p\, z^4}$$

Reconstruction:

$$\mathfrak{f}_{LQ,approx}(\varphi) = \frac{1}{0.39931\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\varphi - \varphi_0}{2.70895}\right)^2\right]$$

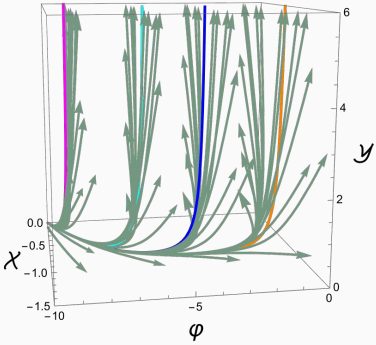
$$\mathfrak{f}_{HQ,approx}(\varphi) = \sum_{i=0}^{30} w_i\, \varphi^i$$



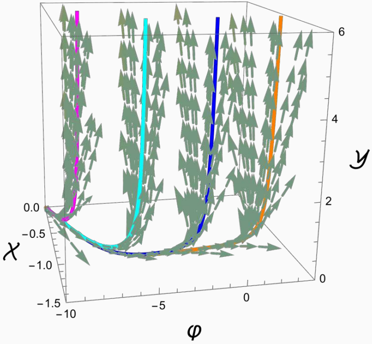


hRG flows respected T and μ

Light quarks



Heavy quarks

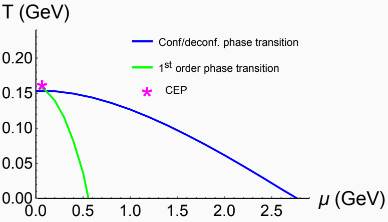


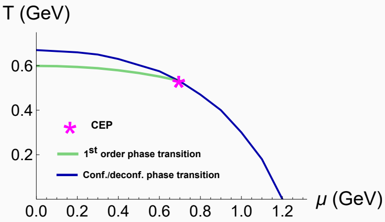
- hRG flows in $\varphi, \mathcal{X}, \mathcal{Y}$-phase space for light and heavy quarks models including $T \neq 0$, $\mu \neq 0$ also are almost the same!

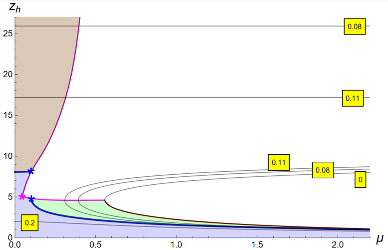
Phase structure

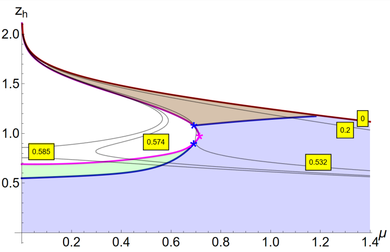
Light quarks

Heavy quarks

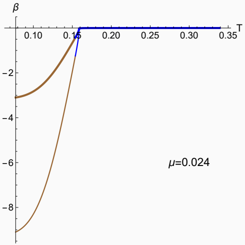


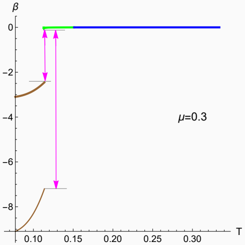


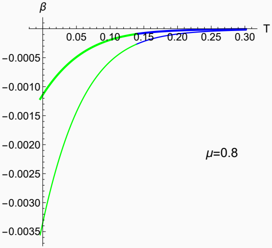


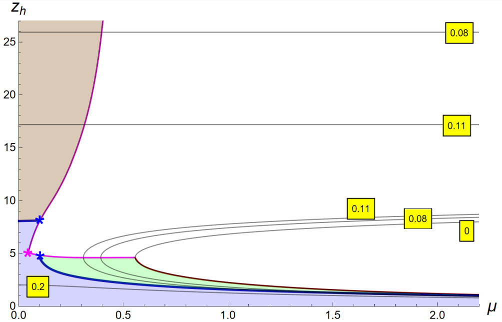


Light quarks model: β-function respected the phase structure

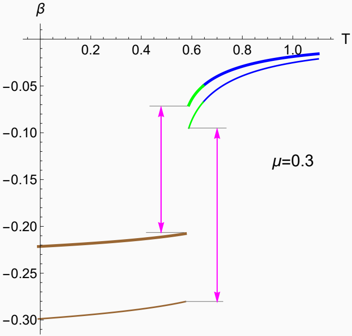


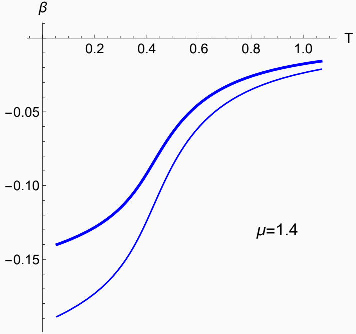


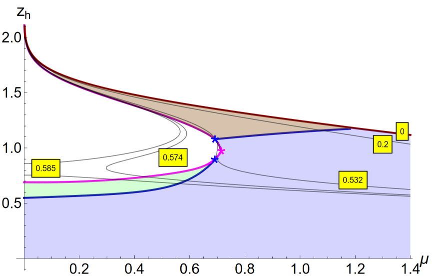




Heavy quarks model: β-function respected the phase structure

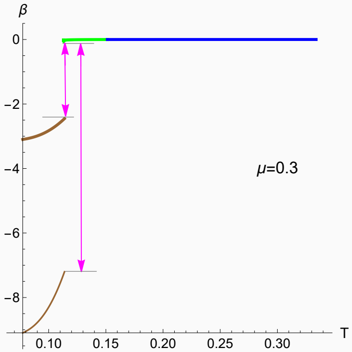




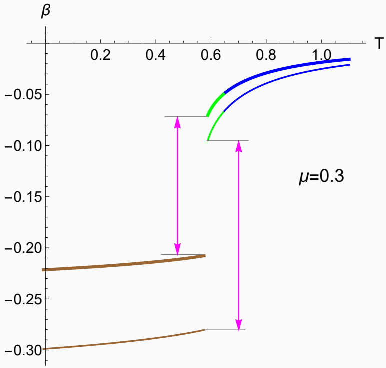


Comparison of β-functions

Light quarks



Heavy quarks



Conclusion

Results for hRG

- HRG flows for LQ and HQ are approximately the same although the associated warp factor is completely different
- The choice of different boundary conditions simply shifts the RG both for heavy and light quarks, without leading to significant changes

Result for β-function

- in all regions β is negative
- on the 1-st order transition line, β has a jump
  - magnitude of the jump is zero at the CEP
  - this magnitude increases by decreasing the probe energy scale E
- both for hadronic and QGP regimes at fixed μ and energy scale z, β increases with increasing T - the speed of growth is different for LQ and HQ

Prospective questions

- Exact numerical correspondence of β_{holo} with β_{QCD}

Thank you for your attention!

Questions?

Agreement of our holographic model β-function with 2-loop QCD

S. He et. al.'11; T. van Ritbergen et.al.'97

The QCD β-function at 2-loop level has the following form

$$\beta(\alpha) = -b_0\,\alpha^2 - b_1\,\alpha^3$$

where $b_0 = \frac{1}{2\pi}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$ and $b_1 = \frac{1}{8\pi^2}\left(\frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{N_f}\right)\right)$.