



Renormalization group flows in holographic model for light/heavy quarks

Based on a joint work with I.Ya. Arefeva, A. Hajilou, P. Slepov

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AdS/CFT-correspondence for AdS₅

Gravity in 5-dimensional AdS₅, 4d conformal field theory

The 5-dimensional Einstein-Maxwell-scalar action

$$S = \frac{1}{16 G_5} \int d^5 x \sqrt{-g} \left[R - \frac{f_0(')}{4} F^2 - \frac{1}{2} @ ' @ ' - V(') \right];$$

$$F = @ A @ A$$

Ansatz for the metric:

$$ds^2 = B^2(z) \left[g(z) dt^2 + dx^2 + \frac{dz^2}{g(z)} \right];$$

$$B(z) = \frac{e^{A(z)}}{z}; \quad ' = '(z); \quad A = A_t(z); \quad \theta; 0$$

Light quarks

$$A_{LQ}(z) = a \log bz^2 + 1$$

$$f_{LQ}(z) = e^{-cz^2} A(z)$$

fitting with experimental data:

$$a = 4.046; b = 0.01613 \text{ GeV}^2; c = 0.227 \text{ GeV}^2$$

(Li et.al.'17; Aref'eva et. al.'21, '22)

Heavy quarks

$$A_{HQ}(z) = \frac{s}{3} z^2 + pz^4$$

$$f_{HQ}(z) = e^{-sz^2} A(z)$$

$$s = 1.16 \text{ GeV}^2; p = 0.273 \text{ GeV}^4$$

(Yang & Yuan, '16; Aref'eva et. al.'20, '23) ²

Equations of motion:

$$\begin{aligned}
 \ddot{A}^0 + \frac{g^0}{g} + 3A^0 \frac{3}{z} \dot{A}^0 + \frac{z^2 e^{2A} A_t^0 f_0}{2g} \dot{A}^0 - \frac{e^{2A} V}{z^2 g} &= 0; \\
 A_t^0 + \frac{f_0^0}{f_0} + A^0 \frac{1}{z} \dot{A}_t^0 &= 0; \\
 A^{00} - A^{02} + \frac{2}{z} A^0 \dot{A}^0 + \frac{\dot{A}^0{}^2}{6} &= 0; \\
 g^{00} + 3A^0 \frac{3}{z} g^0 - e^{2A} z^2 f_0 A_t^0{}^2 &= 0; \\
 A^{00} + 3A^{02} + \frac{3g^0}{2g} \frac{1}{z} \dot{A}^0 - \frac{1}{z} \frac{3g^0}{2g} \frac{4}{z} \dot{A}^0 + \frac{g^{00}}{6g} + \frac{e^{2A} V}{3z^2 g} &= 0
 \end{aligned}$$

Applied boundary conditions:

$$A_t(0) = \dots; \quad A_t(z_h) = 0; \quad g(0) = 1; \quad g(z_h) = 0;$$

and for the dilaton field:

$$\dot{\phi}(z; z_0)_{z=z_0} = 0; \quad \text{where } z_0 = z(z_h)$$

General solutions:

$$\begin{aligned}
 A^0(z) &= \frac{1}{6} \left(A^{00} - A^{02} + \frac{2}{z} A^0 \dot{A}^0 \right); \quad A_t(z) = \frac{e^{cz^2} - e^{cz_h^2}}{1 - e^{cz_h^2}} \\
 V(z) &= \frac{3z^2 g e^{2A}}{3z^2 g e^{2A}} \left(A^{00} + 3A^{02} + \frac{3g^0}{2g} \frac{1}{z} \dot{A}^0 - \frac{1}{z} \frac{3g^0}{2g} \frac{4}{z} \dot{A}^0 + \frac{g^{00}}{6g} \right)
 \end{aligned}$$

Domain wall solution:

$$ds^2 = e^{2A(w)} g_{ij} dx^i dx^j + dw^2; \quad ' = '(w)$$

E. Akhmedov'98; de Boer, Verlinde, Verlinde'98, Skenderis et. al.'98; Skenderis'99

Solution under consideration:

$$ds^2 = B(z)^2 g_{ij} dx^i dx^j + dz^2; \quad B(z) = \frac{e^{A(z)}}{z}; \quad ' = '(z)$$

We have the following holographic dictionary:

- B corresponds to the energy scale E of the dual field theory
- $' = e^{A(z)}$ must be identified as running coupling of the field theory
- Connection with β -function in this background (DeWolfe et. al. '14, Kiritsis et.al.'14)

$$' = \frac{d}{d \log E}_{QFT} = \frac{d'}{d \log B}_{Holo}$$

- The holographic RG is geometrized

Dynamical system for the case: $T = 0, \quad = 0$

New dynamical variables:

$$X(z) = \frac{B}{3B}$$

Kiritsis et.al. '08

Physical meaning of the dynamical system

Variable X is associated with overstretched -function:

$$= 3 X$$

$B(z)$ increases along the flow

Holographic RG flows equation:

$$\frac{dX}{d'} = \frac{4}{3} X - \frac{3}{8} X^2 + 1 + \frac{1}{X} \frac{V'(X)}{V(X)}$$

I. Ya. Aref'eva, K. Rannu '18

where we consider $X(z) = X(X(z))$

HRG flows equation is invariant with respect to change of b.c.

Scalar fields and potentials

After subtraction of the scale factors into EOM:

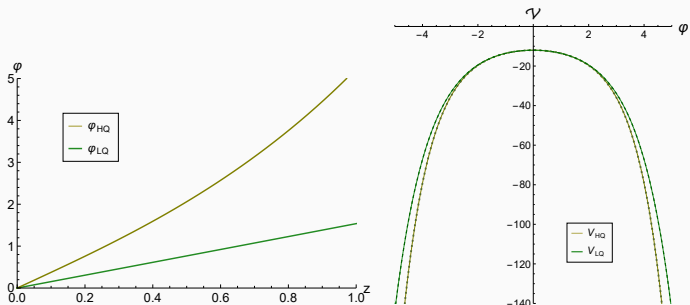
$${}_{LQ}(z; ' 0) = ' 0 + 2^p \frac{2}{3a} 4^p \frac{1}{2a+1} \operatorname{arcsinh} @ \frac{O_s}{3} \frac{b(2a+1)}{zA} \frac{1}{2(a-1)} \operatorname{arctanh} @ \frac{O_y}{t} \frac{2b(a-1)}{(2a+1)bz^2+3} \frac{1}{zA^5}$$

$${}_{HQ}(z; ' 0) = ' 0 + \frac{p}{6} \frac{z^2}{d} \frac{z^2}{t} \frac{1}{4p^3} \frac{2s}{3} \frac{1}{2} + 2 \frac{4p}{2} + \frac{2s}{3} + 12p^2 + \frac{2s}{3}$$

$$V_{LQ}(z) = \frac{1}{6} \frac{bz^2+1}{2a} \frac{2^h}{bz^2} (a(6a+7)+2)bz^2 + 5a+4 + 2^i;$$

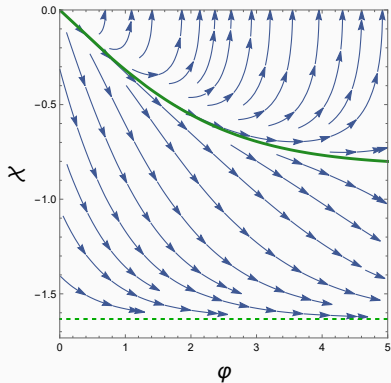
$$V_{HQ}(z) = \frac{1}{2} e^{\frac{1}{3}(6pz^4+2sz^2)} \frac{h}{6+5sz^2+2(9p+s^2)z^4+24psz^6+72p^2z^8}$$

Reconstruction of the potentials: $V(') = \sum_{i=0}^{18} C_i '^i$ 13:5 ' 13:5

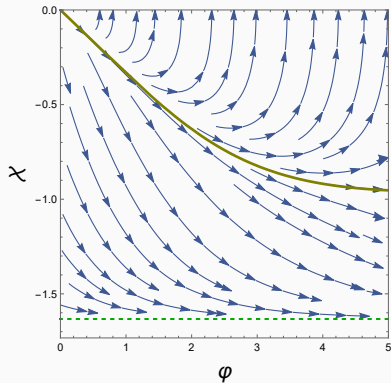


Comparison of the flows

Light quarks



Heavy quarks



Green dashed lines are fixed (attractor) lines: $\chi = \sqrt[3]{8/3}$.
Solid green and olive lines depict an exact solution χ .

- simple hRG flows for light and heavy quarks models are almost the same!

Change of variables

$$X = \frac{r}{3} \frac{B}{B}; \quad Y = \frac{1}{4} \frac{g}{g} \frac{B}{B}; \quad H = \frac{A_t}{B^2}$$

$$\frac{dX}{d'} = \frac{4}{3} \frac{1}{1 + \frac{3}{8} X^2 + Y} \left(1 + \frac{1}{X} \frac{2 \partial_r V(r)}{2V(r) + H^2 f_0} \right) \frac{H^2 \partial_r f_0}{2V(r) + H^2 f_0};$$

$$\frac{dY}{d'} = \frac{4Y}{3X} \frac{1}{1 + \frac{3}{8} X^2 + Y} \left(1 + \frac{3}{2Y} \frac{H^2 f_0}{2V(r) + H^2 f_0} \right);$$

$$\frac{dH}{d'} = \frac{1}{X} + \frac{\partial_r f_0}{f_0} H$$

where $X = X(r(z)); \quad Y = Y(r(z)); \quad H = H(r(z))$

Gauge kinetic functions

$$f_{LQ}(z) = e^{cz^2 + a \log(bz^2 + 1)};$$

$$f_{HQ}(z) = e^{\frac{2}{3}sz^2 + pz^4}$$

Reconstruction:

$$f_{LQ;approx}(z) = \frac{1}{0.39931} p^{\frac{1}{2}} \exp\left(\frac{1}{2} \frac{z^2}{2.70895}\right)^{\#}$$

$$f_{HQ;approx}(z) = \sum_{i=0}^{\infty} w_i z^i$$

Light quarks

Heavy quarks

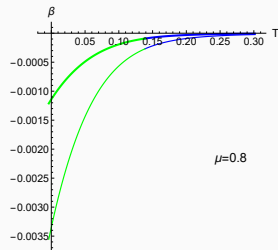
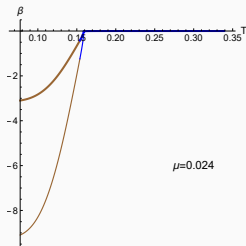
- hRG flows in $(X; Y)$ -phase space for light and heavy quarks models including $T \neq 0$, $\epsilon \neq 0$ also are almost the same!

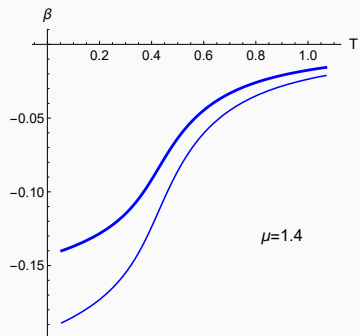
Light quarks

Heavy quarks

Light quarks model:

β -function respected the phase structure





Light quarks

Heavy quarks

Results for hRG

- HRG flows for LQ and HQ are approximately the same although the associated warp factor is completely different
- The choice of different boundary conditions simply shifts the RG both for heavy and light quarks, without leading to significant changes

Result for β -function

- in all regions β is negative
- on the 1-st order transition line, β has a jump
 - magnitude of the jump is zero at the CEP
 - this magnitude increases by decreasing the probe energy scale E
- both for hadronic and QGP regimes at fixed μ and energy scale Z , β increases with increasing T - the speed of growth is different for LQ and HQ

Prospective questions

- Exact numerical correspondence of $holo$ with QCD

Thank you for your attention!

Questions?

The QCD β -function at 2-loop level has the following form

$$\beta(\alpha) = -b_0 \alpha^2 - b_1 \alpha^3$$

where $b_0 = \frac{1}{2} \left(\frac{11}{3} N_c - \frac{2}{3} N_f \right)$ and $b_1 = \frac{1}{8} \left(\frac{34}{3} N_c^2 - \frac{13}{3} N_c N_f - \frac{1}{N_f} \right)$.

