Secular effects in de Sitter space

QUARKS-2024

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Motivation

- It is believed that in early Universe quantum corrections played the dominant role;
- De Sitter space is the simplest example to investigate effects of QFT in early Universe;
- Non-conformal quantum fields in dS show special IR behaviour: loops are not suppressed in comparison with tree-level contributions;
- The dream is to solve the problem of backreaction of quantum matter onto the ambient expanding background;
- We need to research the infrared effects from loops and their interrelation with stability of dS-invariant states, stability of dS itself and its isometries.

Types of secular effects

- The secular growth of the first kind for massless scalars and tachyonic fields:

$$W_{\text{tree+loops}} = \langle \phi^2(t,x) \rangle \simeq tA_0 + \lambda t^3 A_1 + \dots, \\ ds^2 = dt^2 - e^{2t} d\boldsymbol{x}^2. \tag{1}$$

[Starobinsky, Yokoyama(82,94), Linde(82), etc.]

- Here A_0 – tree-level contribution, A_1 – the first loop correction, λ – self-coupling constant of the scalar;
- Methods to approach this secular growth work only in EPP for small enough perturbations over the Bunch-Davies state.

Types of secular effects

- The secular growth of the second kind for scalars of arbitrary mass:

$$
\begin{gathered}
W_{\text{loop}}(t_1, t_2|p) \simeq \lambda^2 (t_1 - t_2) B, \\
W_{\text{loop}}(t_1, t_2|p) = \int d^{D-1}\boldsymbol{x}\, e^{i\boldsymbol{p}\boldsymbol{x}} \left\langle \phi(t_1, \boldsymbol{x}) \phi(t_2, 0) \right\rangle .
\end{gathered} \tag{2}
$$

[Boyanovsky, Vega, Holman(94), Vega, Salgado (97), etc.]

- Such terms become strong in the limit $|t_1 - t_2| \to \infty$ and usually leads to a mass renormalization or to contribution to the imaginary part of the self-energy;
- This effect cannot be definitely attributed to the IR effects.

Types of secular effects

- The secular growth of the third kind for scalars of arbitrary mass:

$$W_{\text{loop}}(t_1, t_2|p) \simeq \lambda^2 (t_1 + t_2) C \sim \lambda^2 \log(\sqrt{\eta_1 \eta_2}), \\ \text{where } \eta = e^{-t} \text{ in EPP and } \eta = e^{t} \text{ in CPP.} \tag{3}$$

[Krotov, Polyakov(11), Akhmedov(12,13), Akhmedov, Burda(12), etc.]

- This effect is crucial in the natural limit $|\frac{t_1+t_2}{2}| \to \infty, \; t_1 - t_2 = \text{const}$.

Types of secular effects

- Finally, the fourth type is secular divergence, which takes place in CPP and in Global dS:

$$W_{\text{loop}}(t_1, t_2|p) \simeq \lambda^2 (t - t_0) F \sim \lambda^2 \log(\frac{\eta}{\eta_0}),$$
$$\text{where } t = \frac{t_1 + t_2}{2}, \ \eta = \sqrt{\eta_1 \eta_2}. \tag{4}$$

[Krotov, Polyakov(11), Akhmedov(13), Akhmedov, Moschella, Pavlenko, Popov(17)]

- The dS isometry group is broken at loop level, the limit $t_0 \to -\infty$ is impossible;
- One question is the possibility of $t_0 \to -\infty$ after the resummation;
- The 3rd and 4th types have the same origin, but the resummation in these two cases are physically distinct problems.

Example 1: $\lambda\phi^3$, light fields in EPP

- First, let us consider scalar field of mass $m < \frac{D-1}{2}$ with cubic self-interaction:

$$S[\phi] = \int d^D x \sqrt{|g|} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 - \frac{\lambda}{6} \phi^3 \right]. \tag{5}$$

- Quantization:

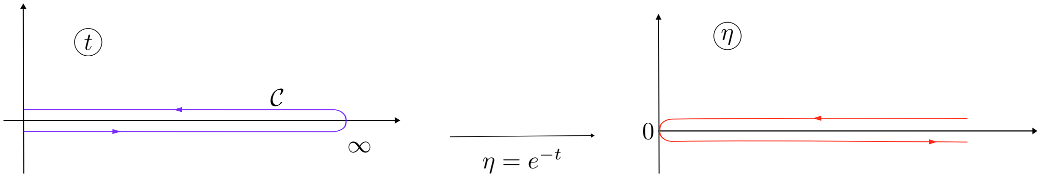
$$\phi(\eta, \boldsymbol{x}) = \int \frac{d^{D-1}\boldsymbol{p}}{(2\pi)^{D-1}} \left[\hat{a}_{\boldsymbol{p}} f_{\boldsymbol{p}}(\eta) e^{i\boldsymbol{px}} + \hat{a}^\dagger_{\boldsymbol{p}} f^*_{\boldsymbol{p}}(\eta) e^{-i\boldsymbol{px}} \right], \quad \left[\hat{a}_{\boldsymbol{p}}, \hat{a}^\dagger_{\boldsymbol{q}} \right] = (2\pi)^{D-1} \delta(\boldsymbol{p} - \boldsymbol{q}),$$

$$f_{\boldsymbol{p}}(\eta) = \eta^{\frac{D-1}{2}} h_\nu(p\eta), \; p \equiv |\boldsymbol{p}|,$$

$$h_\nu(p\eta) = \frac{\sqrt{\pi}}{2} H^{(1)}_\nu(p\eta), \; \nu = \sqrt{\frac{(D-1)^2}{4} - m^2}. \tag{6}$$

- Keldysh rotation:

$$\phi_{cl} = \frac{\phi_+ + \phi_-}{2}, \ \phi_q = \phi_+ - \phi_-, \tag{7}$$



Example 1: $\lambda\phi^3$, light fields in EPP

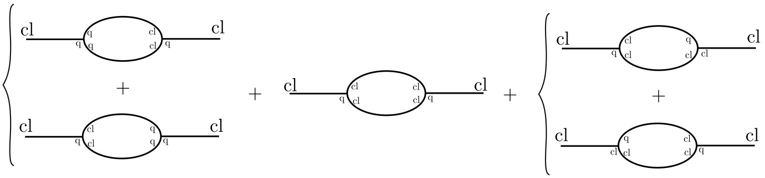
- Propagators:

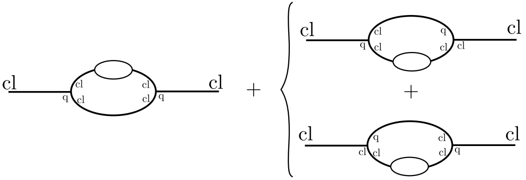
$$
\begin{aligned}
iG^K(t_1,\boldsymbol{x}_1|t_2,\boldsymbol{x}_2) &= \overbracket{\phi_{cl}(t_1,\boldsymbol{x}_1)\phi}_{cl}(t_2,\boldsymbol{x}_2) = \frac{1}{2}\left\langle \mathrm{BD}\middle|\left\{\phi(t_1,\boldsymbol{x}_1),\phi(t_2,\boldsymbol{x}_2)\right\}\middle|\mathrm{BD}\right\rangle,\\
iG^R(t_1,\boldsymbol{x}_1|t_2,\boldsymbol{x}_2) &= \overbracket{\phi_{q}(t_1,\boldsymbol{x}_1)\phi}_{cl}(t_2,\boldsymbol{x}_2) = -\theta(t_2-t_1)\rho(t_1,\boldsymbol{x}_1|t_2,\boldsymbol{x}_2),\\
\rho(t_1,\boldsymbol{x}_1|t_2,\boldsymbol{x}_2) &= \left\langle \mathrm{BD}\middle|\left[\phi(t_1,\boldsymbol{x}_1),\phi(t_2,\boldsymbol{x}_2)\right]\middle|\mathrm{BD}\right\rangle.
\end{aligned}
\tag{8}
$$

- In order to find the evolution of the state of the system we calculate loop corrections to the Keldysh propagator:

$$
G^K(p|\eta,\eta) = f^*_{\boldsymbol{p}}(\eta_1)f_{\boldsymbol{p}}(\eta_2)n_p(\eta) + f_{\boldsymbol{p}}(\eta_1)f_{\boldsymbol{p}}(\eta_2)\varkappa_p(\eta) + \text{c.c.} \tag{9}
$$

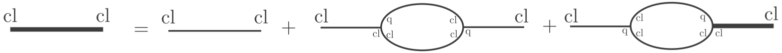
Example 1: $\lambda\phi^3$, light fields in EPP

$\Delta G_K =$  $\sim \quad \lambda^2 \log(p\eta)$

$\Delta^{(2)} G_K \underset{\text{leading part}}{\sim} 2 \times$  $\sim \quad \lambda^4 \log(p\eta)$

Example 1: $\lambda\phi^3$, light fields in EPP

- Resummation via Dyson-Schwinger equation for the leading logarithms:



- We use the following ansatz:

$$G^K(\boldsymbol{p}|\eta_1,\eta_2) = A_-^2\eta^{D-1}\frac{N(p\eta)}{(p\eta)^{2\nu}},\ \eta = \sqrt{\eta_1\eta_2}. \tag{10}$$

- Here we can assume an initial perturbation of BD state $N_0 = 1 + 2n(P_0) - 2\mathrm{Re}\{\varkappa(P_0)\}$ on the initial surface of physical momentum $P_0 = (p\eta)_0$.

Example 1: $\lambda\phi^3$, light fields in EPP

- The equation for $N(p\eta)$ can be cast into the form:

$$\frac{\partial N(p\eta)}{\partial \log(p\eta)} \simeq 4N_0\lambda^2 A_- \mathrm{Im}(A_+) \int_1^{\frac{\nu}{p\eta}} \frac{dv}{v^{2\nu+1}} \mathrm{Im}\left(F(v)\right) \left[N(p\eta\sqrt{v}) + N_0\right]. \tag{11}$$

- The solution of type $N(p\eta) = C(p\eta)^\alpha$ with $\alpha < 0$ exists only when the initial state is such that $N_0 = -|N_0| < 0$!
- In this case:

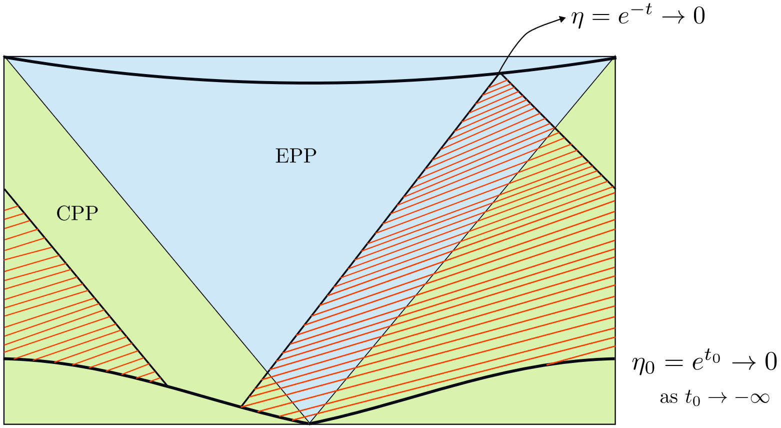
$$G^K(\mathbf{p}|\eta,\eta) \Big/ G_0^K(\mathbf{p}|\eta,\eta) \sim (p\eta)^{-\frac{\lambda^2|N_0|\cdot r}{(D-1)^2-4\nu^2}} \to \infty \text{ as } p\eta \to 0. \tag{12}$$

Example 2: $\lambda\phi^3$, heavy fields in Global dS

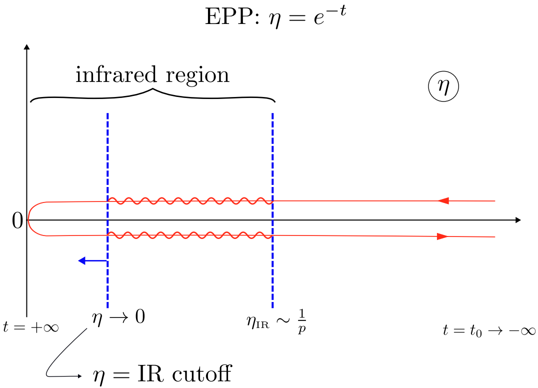
- Above we showed how to deal with 3rd type of secular effects on the example of light fields $m < \frac{D-1}{2}$ ("complementary series");
- Now we show why in Global dS the secular divergence can play a significant role on the example of heavy fields $m > \frac{D-1}{2}$ ("principal series");
- The dominant contribution to the loop corrections comes from the infrared region, where $p_i \eta_i \ll 1$ for each momentum p_i and time η_i in a loop;
- In the vicinity of these "infrared regions" we can approximate the harmonics by the expressions in EPP and CPP:

$$f_p(t) \simeq \begin{cases} \eta_+^{\frac{D-1}{2}} h_+ (p\eta_+) \,, & \eta_+ = e^{-t}, \quad t \to +\infty \\ \eta_-^{\frac{D-1}{2}} h_- (p\eta_-) \,, & \eta_- = e^{t}, \quad t \to -\infty \end{cases} \tag{13}$$

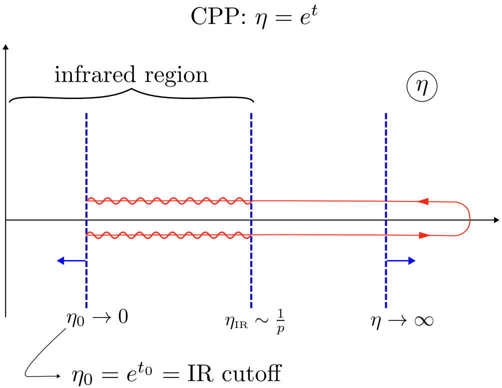
Global dS



EPP vs CPP



EPP: $\eta = e^{-t}$
infrared region
η
0
$t = +\infty$
$\eta \to 0$
$\eta_{\rm IR} \sim \frac{1}{p}$
$t = t_0 \to -\infty$
$\eta = $ IR cutoff



CPP: $\eta = e^{t}$
infrared region
η
$\eta_0 \to 0$
$\eta_{\rm IR} \sim \frac{1}{p}$
$\eta \to \infty$
$\eta_0 = e^{t_0} = $ IR cutoff

Example 2: $\lambda\phi^3$, heavy fields in Global dS

- Therefore, after some calculation the following result in one loop is expected (the same for $\varkappa_p(t)$):

$$n_p(t) \sim \lambda^2 \log\left(p^2 \eta\eta_0\right). \tag{14}$$

- Additional internal loops will bring additional powers of $\log(\eta_0)$, hence we must account for this contributions in Dyson-Schwinger equation (here in CPP):

$$\begin{aligned}\frac{dn_p(\eta)}{d\log(\eta/\eta_0)} = \frac{\lambda^2 S_{D-2}|A|^2}{(2\pi)^{D-1}} \int_0^\infty dq\eta(q\eta)^{\frac{D-1}{2}} \int_0^\infty d\eta' q\,(q\eta')^{\frac{D-1}{2}} \times \\ \times \left\{\mathrm{Re}\left[(q\eta)^{-i\mu} V(q\eta)\,(q\eta')^{i\mu}\, V^*(q\eta')\right] \left\{[1+n_p]\, n_q^2 - n_p\,[1+n_q]^2\right\}(\eta) + \right. \\ +2\,\mathrm{Re}\left[(q\eta)^{i\mu} W(q\eta)\,(q\eta')^{-i\mu}\, W(q\eta')\right] \{n_q\,[1+n_q]\,[1+n_p] - [1+n_q]\,n_q n_p\}(\eta) + \\ \left. +\,\mathrm{Re}\left[(q\eta)^{i\mu} V(q\eta)\,(q\eta')^{-i\mu}\, V^*(q\eta')\right] \left\{[1+n_q]^2\,[1+n_p] - n_q^2 n_p\right\}(\eta)\right\}.\end{aligned} \tag{15}$$

Example 2: $\lambda\phi^3$, heavy fields in Global dS

- In the cases of mild a) $n_p \ll 1$ and strong b) $n_p \gg 1$ initial perturbations, (15) reduces to

$$a)\ \frac{dn_p(\eta)}{d\log(\eta/\eta_0)} \simeq -\Gamma_1 n_p(\eta) + \Gamma_2; \quad b)\ \frac{dn_p(\eta)}{d\log(\eta/\eta_0)} \simeq \bar{\Gamma} n_p^2(\eta). \qquad (16)$$

- The corresponding solutions are

$$a)\ n_p(\eta) \simeq \frac{\Gamma_2}{\Gamma_1}; \quad b)\ n_p(\eta) \simeq \frac{1}{\bar{\Gamma}\log(\eta_\star/\eta)}. \qquad (17)$$

- In the case a) the dS invariance is restored. In contrast, in the case b) the backreaction must violate dS geometry.

What have we learned?

- The secular effects of 3rd and 4th type can significantly change tree-level picture after the resummation of the leading contributions;
- The evolution of the system at late times $\frac{t_1+t_2}{2} \to +\infty$ strongly depends on its initial state on some Couchy surface at the moment t_0;
- In CPP and Global dS the isometry invariance is broken at loop level, but it is restored after the resummation for the mild perturbations over the BD-state. The strong perturbations lead to explosive behaviour of $n_p(t)$.

What is next?

- The above-mentioned phenomena is extremely important for understanding of how the ambient space is distorted by the quantum fluctuations:

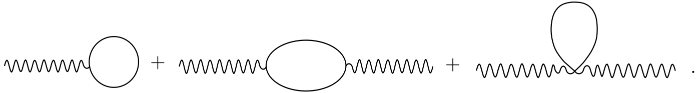
$$\text{Einstein tensor}_{\mu\nu} = \left\langle T_{\mu\nu} \right\rangle_{\text{matter}}. \tag{18}$$

- Another way is to explore effective action of cosmological perturbations after the integration over the matter fields:

$$\begin{aligned} h_{00} &= 2\Phi, \\ h_{0k} &= ik_k Z + Z_k^T, \\ h_{kl} &= -2\Psi\delta_{kl} - 2k_k k_l E + i(k_k W_j^T + k_l W_i^T) + h_{kl}^{TT}. \end{aligned} \tag{19}$$

Graviton mass in dS

$\Gamma_{\text{eff}} = S_{\text{cl}} +$

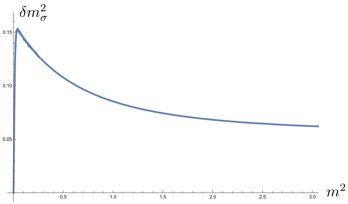


- Effective mass of the external field's perturbation is the simplest component of the effective action in the long-wave expansion;
- It was shown that in BD state for the free fields the mass term vanishes in the tensor sector despite the natural Gibbons-Hawking temperature $T_{\text{dS}} = \frac{H}{2\pi}$:

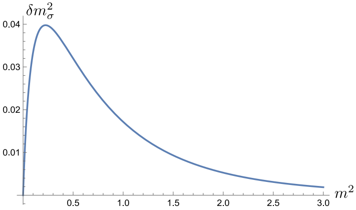
$$m_{TT}^2 \equiv 0. \tag{20}$$

[Sadekov(24)]

- In scalar sector the situation is not trivial already for free fields. For example, in two dimensions for the mass term of Liouville field:



(a)



(b)

- The problem of backreaction of self-interacting fields onto the ambient geometry due to infrared secular effects is still open.

Thank You