Bilinear currents in the AdS_4 higher-spin theory

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Fronsdal equations (Fronsdal'78):

$$\Box\phi^{(\underline{a}_1\cdots\underline{a}_s)}(x)+\cdots=0\,. \tag{1.1}$$

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We study bilinear currents J_I resulting from the nonlinear HS theory.

Unfolded formalism

Frame-like fields:

$$\phi^{\underline{a}(s)} \longleftrightarrow \begin{array}{ll} \omega_{\alpha(s-1+m)\ \dot\alpha(s-1-m)}\,, & |m| \leqslant s-1\,, \\ C_{\alpha(2s+n)\ \dot\alpha(n)}\,, \quad C_{\alpha(n)\ \dot\alpha(2s+n)}\,, & n \geqslant 0\,. \end{array}$$

ω – 1-form, C – 0-form.

$$t^{\underline{a}(s)} \equiv t^{(\underline{a}_1 \cdots \underline{a}_s)}\,; \qquad \alpha = 1{,}2\,,\ \dot\alpha = 1{,}2\,;$$

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Moyal star product:

$$f(Y) \star g(Y) = f(Y) e^{i \frac{\overleftarrow{\partial}}{\partial y^\alpha} \epsilon^{\alpha\beta} \frac{\overrightarrow{\partial}}{\partial y^\beta} + c.c.} g(Y)\,.$$

Unfolded formalism

Free unfolded equations (Vasiliev'87)

$$\begin{cases} D_\Omega \omega(Y|x) = \Upsilon(\Omega,\Omega,C)\,, \\ D_\Omega C(Y|x) = 0\,. \end{cases}$$

ω – 1-form, C – 0-form.

$$D_\Omega = \mathrm{d} + [\Omega, \bullet]_\star\,;$$

$$\Omega \equiv \Omega(Y|x) = -\frac{i}{4}(\varpi_{\alpha\beta}(x)y^\alpha y^\beta + 2h_{\alpha\dot{\alpha}}(x)y^\alpha \bar{y}^{\dot{\alpha}} + \bar{\varpi}_{\dot{\alpha}\dot{\beta}}(x)\bar{y}^{\dot{\alpha}}\bar{y}^{\dot{\beta}})\,,$$

$\varpi_{\alpha\beta}$, $\bar{\varpi}_{\dot{\alpha}\dot{\beta}}$ – Lorentz connection, $h_{\alpha\dot{\alpha}}$ – vierbein.

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$\varpi_{\alpha\beta}$, $\bar\varpi_{\dot\alpha\dot\beta}$ – Lorentz connection, $h_{\alpha\dot\alpha}$ – vierbein.
AdS connection is flat:

$$\mathrm{d}\Omega + \Omega \star \Omega = 0 \Longleftrightarrow D_\Omega^2 = 0\,.$$

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Gauge transformations:

$$\begin{cases} \delta_\epsilon \omega(Y|x) = D_\Omega \epsilon(Y|x)\,, \\ \delta_\epsilon C(Y|x) = 0\,. \end{cases}$$

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TT-gauge:

$$\omega(Y|x) = h^{\alpha\dot\alpha}(x)\frac{\partial}{\partial y^\alpha}\frac{\partial}{\partial \bar y^{\dot\alpha}}\phi(Y|x) \qquad \text{– tracelessness}$$

$$D^{\alpha\dot\alpha}\frac{\partial}{\partial y^\alpha}\frac{\partial}{\partial \bar y^{\dot\alpha}}\phi(Y|x) = 0 \qquad \text{– transversality}$$

$\phi(Y|x)$ – 0-form, $D^{\alpha\dot\alpha}$ – Lorentz-covariant derivative, $h^{\alpha\dot\alpha}(x)$ – vierbein.

Nonlinear HS theory

Vasiliev equations (Vasiliev'92)

$$\begin{cases} \mathrm{d}W + W \star W = -i\theta^2(1 + \eta B \star \varkappa \star k) - i\bar{\theta}^2(1 + \bar{\eta} B \star \bar{\varkappa} \star \bar{k})\,, \\ \mathrm{d}B + W \star B - B \star W = 0\,. \end{cases} \tag{1.2}$$

$\eta = |\eta| e^{i\vartheta}$ – free complex parameter;

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2nd order of perturbation theory (Didenko, Gelfond, Korybut, Vasiliev):

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$$\Updownarrow$$

$$\Box \phi^{a(s)} + \dots = \sum_l g_l(\eta) J_l \,.$$

Conservation laws and consistency conditions

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Consistency condition:

$$D_\Omega^2 = 0 \Longrightarrow D_\Omega \,\mathrm{RHS} = 0\,. \tag{2.2}$$

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Terms that obey (2.2) separately = currents that conserve independently

Conservation laws and consistency conditions

$s_i + s_j \geqslant s_k + 1$	$\Upsilon(\omega,\omega) + \Upsilon(\Omega,\omega,C)$	$\#_{der} \leqslant S - 2s$
	$\Upsilon(\Omega,\Omega,C,C)$	$\#_{der} = S$
$s_2 = s_1 + s_3$	$\Upsilon(\Omega,\omega,C)$	$\#_{der} \leqslant S - 2s$
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	$\Upsilon(\Omega,\omega,C)$	$\#_{der} \leqslant S$
$s_1 \geqslant s_2 + s_3$	$\Upsilon(\Omega,\Omega,C,C)$	$\#_{der} = S - 2s$
	$\Upsilon(\Omega,\Omega,C,C)$	$\#_{der} = S$

s_1 – spin of the field in LHS, s_2, s_3 – spins of the fields in RHS;
$S = [s_1] + [s_2] + [s_3]\,, \qquad [\dots]$ – integer part$\,, \qquad s = \min\{s_1,s_2,s_3\}.$

Nontriviality

Central on-mass-shell theorem (Bychkov, Ushakov, Vasiliev'21):
currents are trivial $\Leftrightarrow$ exists a **local** change of variables $\Delta\omega, \Delta C$, such that

$$\begin{cases} D_\Omega(\omega + \Delta\omega) = \Upsilon(\Omega,\Omega, C + \Delta C) + 0\,, \\ D_\Omega(C + \Delta C) = 0\,. \end{cases} \tag{2.3}$$

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- Nontriviality of $\omega\omega$- and ωC-currents can be shown via **gauge invariance**.

$$\begin{cases} D_\Omega \omega = \Upsilon_\omega\,, \\ D_\Omega C = \Upsilon_C\,; \end{cases} \quad \Longrightarrow \begin{cases} \delta_\epsilon \omega = D_\Omega \epsilon + \left(\epsilon \cdot \frac{\partial}{\partial\omega}\right) \Upsilon_\omega\,, \\ \delta_\epsilon C = \left(\epsilon \cdot \frac{\partial}{\partial\omega}\right) \Upsilon_C\,. \end{cases} \tag{2.4}$$

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ΔC transforms correctly $\Longrightarrow$ ΔC – nonlocal

Result

$$J = |\eta|^2 J^{\text{even}}_{\text{min}} + |\eta|^2 \cos 2\vartheta \, J^{\text{even}}_{\text{max}} + |\eta|^2 \sin 2\vartheta \, J^{\text{odd}}_{\text{max}} \, ; \quad \eta = |\eta| e^{i\vartheta} \, . \qquad (3.1)$$

$$\text{min: } \#_{der} \leqslant [s_1] + [s_2] + [s_3] - 2 \min\{s_1, s_2, s_3\} \, ,$$
$$\text{max: } \#_{der} \leqslant [s_1] + [s_2] + [s_3] \, .$$

Example: Yang-Mills theory

$$J^{a\ \underline{m}} = |\eta|^2(2\eta^{\underline{ml}}\eta^{\underline{kn}} - \eta^{\underline{mk}}\eta^{\underline{nl}})(c^a_{bc} - c^a_{cb})\phi^b_{\underline{n}} D_{\underline{k}}\phi^c_{\underline{l}} + \\ + \frac{1}{3}|\eta|^2 \cos 2\vartheta \left(\eta^{\underline{kl}}\eta^{\underline{pq}} - \eta^{\underline{pl}}\eta^{\underline{qk}}\right)(c^a_{bc} - c^a_{cb})(D^{\underline{m}} D_{\underline{k}}\phi^b_{\underline{p}})(D_{\underline{l}}\phi^c_{\underline{q}}) - \\ - \frac{1}{3}|\eta|^2 \sin 2\vartheta \epsilon^{\underline{klpq}}(c^a_{bc} - c^a_{cb})(D^{\underline{m}} D_{\underline{k}}\phi^b_{\underline{p}})(D_{\underline{l}}\phi^c_{\underline{q}}) . \tag{3.2}$$

c^a_{bc} – structure constant, $D_{\underline{n}}$ – Lorentz-covariant derivative.

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- Without ϑ – standard Yang-Mills (F^2)

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- With $\sin 2\vartheta$ – $(*F)^3$-terms

Discussion

$$J = |\eta|^2 J^{\text{even}}_{\text{min}} + |\eta|^2 \cos 2\vartheta J^{\text{even}}_{\text{max}} + |\eta|^2 \sin 2\vartheta J^{\text{odd}}_{\text{max}}\,; \quad \eta = |\eta| e^{i\vartheta}\,. \tag{3.3}$$

min: $\#_{der} \leqslant [s_1] + [s_2] + [s_3] - 2\min\{s_1, s_2, s_3\}\,,$

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Lightcone cubic vertices (Bengtsson, Bengtsson, Linden'87 and Metsaev'18):

$$V_{\lambda_1,\lambda_2,\lambda_3} \propto \mathbb{P}^{\lambda_1+\lambda_2+\lambda_3} , \qquad \lambda_1 + \lambda_2 + \lambda_3 \geqslant 0 ,$$

$$\bar{V}_{\lambda_1,\lambda_2,\lambda_3} \propto \mathbb{P}^{-\lambda_1-\lambda_2-\lambda_3} , \qquad \lambda_1 + \lambda_2 + \lambda_3 \leqslant 0 .$$

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Holography (Maldacena, Zhiboedov'13):

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle \propto \cos^2\vartheta \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{bos} + \sin^2\vartheta \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{fer} + \\ + \sin\vartheta \cos\vartheta \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{odd}$$

Field structure

$$\omega(Y;-K) = \omega(Y;K)\,, \qquad C(Y;-K) = -C(Y;K)\,. \tag{4.1}$$

For field of spin s:

$$(\mathcal{N} + \bar{\mathcal{N}})\omega(Y;K) = (2s-2)\omega(Y;K)\,, \tag{4.2}$$

$$(\mathcal{N} - \bar{\mathcal{N}})C(Y;K) = \pm 2sC(Y;K)\,; \tag{4.3}$$

In TT-gauge:

$$\phi_{s+m+\{s\},s-m-\{s\}}(Y;K) = i^m(s-\{s\})\frac{(s-m-1-\{s\})!}{(s+m+\{s\})!}D^m(y,\bar{p})\phi_{s+\{s\},s-\{s\}}(Y;K)\,,$$

$$\phi_{s-m-\{s\},s+m+\{s\}}(Y;K) = i^m(s-\{s\})\frac{(s-m-1-\{s\})!}{(s+m+\{s\})!}D^m(p,\bar{y})\phi_{s-\{s\},s+\{s\}}(Y;K)\,;$$

$$C_{2s+n,n}(Y;K) = 2s\frac{i^{s-1-n-\{s\}}}{\bar{\eta}(2s-1)(2s+n)!n!}D^n(y,\bar{y})D^{s-\{s\}}(y,\bar{p})\phi_{s+\{s\},s-\{s\}}(Y;K)\bar{k}\,,$$

$$C_{n,2s+n}(Y;K) = 2s\frac{i^{s-1-n-\{s\}}}{\eta(2s-1)(2s+n)!n!}D^n(y,\bar{y})D^{s-\{s\}}(p,\bar{y})\phi_{s-\{s\},s+\{s\}}(Y;K)k\,.$$

Vertices

$$(p_i)_\alpha := -i\frac{\partial}{\partial y_i^\alpha}\,, \qquad (uv) = u_\alpha \epsilon^{\alpha\beta} u_\beta\,, \qquad h(a,\bar{a}) = h_{\alpha\dot{\alpha}} a^\alpha \bar{a}^{\dot{\alpha}}\,,$$

$$\bar{H}(\bar{\partial},\bar{\partial}) := \frac{1}{2}\bar{H}_{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial \bar{y}_{\dot{\alpha}}\partial \bar{y}_{\dot{\beta}}}\,, \qquad \bar{H}_{\dot{\alpha}\dot{\beta}} := h_{\gamma\dot{\alpha}} \wedge h^{\gamma}{}_{\dot{\beta}}\,.$$

Vertices in ω equation

$$\Upsilon(\Omega,\Omega,C) = \frac{i}{2}\eta \bar{H}(\bar{\partial},\bar{\partial}) C(0,\bar{y};K)k + c.c.\,;$$

$$\Upsilon(\omega,\omega) = -\exp\{-i(p_1p_2) + i(p_1y) + i(p_2y) - i(\bar{p}_1\bar{p}_2) + i(\bar{p}_1\bar{y}) + i(\bar{p}_2\bar{y})\}\omega(Y_1;K)\omega(Y_2;K)\Big|_{Y_i=0}\,;$$

$$\Upsilon(\Omega,\omega,C) = -\frac{i\eta}{2}\int_0^1 dt h(p_1, t\bar{p}_1 + \bar{p}_2)\exp\{-it(p_1p_2) + i(1-t)(p_1y) - i(\bar{p}_1\bar{p}_2) + i(\bar{p}_1\bar{y}) + i(\bar{p}_2\bar{y})\}\cdot$$

$$\cdot\left[\omega(Y_1;K)C(Y_2;K) - e^{-2i(\bar{p}_1\bar{y})}C(Y_2;K)\omega(-Y_1;K)\right]k\Big|_{Y_i=0} + c.c.\,;$$

$$\Upsilon(\Omega,\Omega,C,C) = -\frac{i\eta\bar{\eta}}{4}\int_{[0;1]^4}\frac{d^4t}{t_4^2}\delta(1-t_3-t_4)\delta'(1-t_1-t_2)\bar{H}(\bar{p},\bar{p})\cdot$$

$$\cdot\exp\{it_1(p_1y) - it_2(p_2y) - it_3(\bar{p}_1\bar{p}_2) + it_2t_4(\bar{p}_1\bar{y}) - it_1t_4(\bar{p}_2\bar{y})\}C(Y_1;K)C(Y_2;K)k\bar{k}\Big|_{Y_i=0} + c.c.\,;$$

Vertices

$$(p_i)_\alpha := -i\frac{\partial}{\partial y_i^\alpha}\,, \qquad (uv) = u_\alpha \epsilon^{\alpha\beta} u_\beta\,, \qquad h(a,\bar{a}) = h_{\alpha\dot{\alpha}} a^\alpha \bar{a}^{\dot{\alpha}}\,,$$

$$\bar{H}(\bar{\partial},\bar{\partial}) := \frac{1}{2}\bar{H}_{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\bar{y}_{\dot{\alpha}}\partial\bar{y}_{\dot{\beta}}}\,, \qquad \bar{H}_{\dot{\alpha}\dot{\beta}} := h_{\gamma\dot{\alpha}} \wedge h^{\gamma}{}_{\dot{\beta}}\,.$$

Vertices in C equation

$$\Upsilon(\omega, C) = -\exp\{-i(p_1p_2) + i(p_1y) + i(p_2y) - i(\bar{p}_1\bar{p}_2) + i(\bar{p}_1\bar{y}) + i(\bar{p}_2\bar{y})\}\cdot \\ \cdot\big[\omega(Y_1;K)C(Y_2;K) - C(Y_1;K)\omega(Y_2;K)\big]\Big|_{Y_i=0}\,;$$

$$\Upsilon(\Omega, C, C) = \frac{i\eta}{2}\int_0^1 dth(y,(1-t)\bar{p}_1 - t\bar{p}_2)\exp\{it(p_1y) - i(1-t)(p_2y) - i(\bar{p}_1\bar{p}_2) + i(\bar{p}_1\bar{y}) + i(\bar{p}_2\bar{y})\}\cdot \\ \cdot C(Y_1;K)C(Y_2;K)k\Big|_{Y_i=0} + c.c.$$

Structure constants

- Color indices: $(fg)^a \equiv c^a_{bc} f^b g^c \ \forall f,g \in A$
- $\phi^a(Y;K) = \phi^{a,0}(Y) + \phi^{a,1}(Y)k\bar{k}\,; \qquad \mathcal{A} = (a,0) \vee (a,1)\,;$

$$\begin{aligned}
c^{a,0}_{\mathcal{B}\mathcal{C}}(s_1,s_2,s_3) =&(\delta^{b,0}_{\mathcal{B}}\delta^{c,0}_{\mathcal{C}} + (-1)^{2s_3}\delta^{b,1}_{\mathcal{B}}\delta^{c,1}_{\mathcal{C}})c^a_{bc}+\\
&+(-1)^{s_2+s_3-s_1+4s_2s_3}(\delta^{b,0}_{\mathcal{B}}\delta^{c,0}_{\mathcal{C}} + (-1)^{2s_2}\delta^{b,1}_{\mathcal{B}}\delta^{c,1}_{\mathcal{C}})c^a_{cb}\,,\\
c^{a,1}_{\mathcal{B}\mathcal{C}}(s_1,s_2,s_3) =&(\delta^{b,0}_{\mathcal{B}}\delta^{c,1}_{\mathcal{C}} + (-1)^{2s_3}\delta^{b,1}_{\mathcal{B}}\delta^{c,0}_{\mathcal{C}})c^a_{bc}+\\
&+(-1)^{s_2+s_3-s_1+4s_2s_3}(\delta^{b,1}_{\mathcal{B}}\delta^{c,0}_{\mathcal{C}} + (-1)^{2s_2}\delta^{b,0}_{\mathcal{B}}\delta^{c,1}_{\mathcal{C}})c^a_{cb}\,;\\
f^{a,0}_{\mathcal{B}\mathcal{C}}(s_1,s_2,s_3) =&(\delta^{b,0}_{\mathcal{B}}\delta^{c,1}_{\mathcal{C}} + (-1)^{2s_3}\delta^{b,1}_{\mathcal{B}}\delta^{c,0}_{\mathcal{C}})c^a_{bc}+\\
&+(-1)^{s_1+s_2+s_3+4s_2s_3}(\delta^{b,1}_{\mathcal{B}}\delta^{c,0}_{\mathcal{C}} + (-1)^{2s_2}\delta^{b,0}_{\mathcal{B}}\delta^{c,1}_{\mathcal{C}})c^a_{cb}\,,\\
f^{a,1}_{\mathcal{B}\mathcal{C}}(s_1,s_2,s_3) =&(\delta^{b,0}_{\mathcal{B}}\delta^{c,0}_{\mathcal{C}} + (-1)^{2s_3}\delta^{b,1}_{\mathcal{B}}\delta^{c,1}_{\mathcal{C}})c^a_{bc}+\\
&+(-1)^{s_1+s_2+s_3+4s_2s_3}(\delta^{b,0}_{\mathcal{B}}\delta^{c,0}_{\mathcal{C}} + (-1)^{2s_2}\delta^{b,1}_{\mathcal{B}}\delta^{c,1}_{\mathcal{C}})c^a_{cb}\,.
\end{aligned}$$

Currents

$$J^{\mathcal{A}}_{s_1-2,s_1-2}(Y) = \sum_{s_2,s_3,m,n} \Big(\prod_{i,j,k} \theta(s_i+s_j-s_k-1)\Big) c^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_2,s_3)\mathrm{A}(s_1,s_2,s_3|m,n|\phi^{\mathcal{B}}_{s_2},\phi^{\mathcal{C}}_{s_3})\,;$$

$$\begin{aligned}
J^{\mathcal{A}}_{s_1,s_1}(Y) = &\sum_{s_2,s_3,m,n} \Big(\prod_{i,j,k} \theta(s_i+s_j-s_k-1)\Big) c^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_2,s_3)\mathrm{B}(s_1,s_2,s_3|m,n|\phi^{\mathcal{B}}_{s_2},\phi^{\mathcal{C}}_{s_3})+\\
&+\sum_{s_2,s_3,n} \theta(s_2+s_3-s_1-1)c^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_2,s_3)\mathrm{C}(s_1,s_2,s_3|n|\phi^{\mathcal{B}}_{s_2},\phi^{\mathcal{C}}_{s_3})+\\
&+\sum_{s_2,s_3,n} \theta(s_1-s_2-s_3)c^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_2,s_3)\mathrm{D}(s_1,s_2,s_3|n|\phi^{\mathcal{B}}_{s_2},\phi^{\mathcal{C}}_{s_3})+\\
&+\cos 2\vartheta \sum_{s_2,s_3,n} \theta(s_2-s_1-s_3-1)f^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_2,s_3)\frac{1}{2}\mathrm{E}(s_1,s_2,s_3|n|\phi^{\mathcal{B}}_{s_2},\phi^{\mathcal{C}}_{s_3})+\\
&+\cos 2\vartheta \sum_{s_2,s_3,n} \theta(s_3-s_1-s_2-1)f^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_3,s_2)\frac{1}{2}\mathrm{E}(s_1,s_3,s_2|n|\phi^{\mathcal{B}}_{s_3},\phi^{\mathcal{C}}_{s_2})+\\
&+\cos 2\vartheta \sum_{s_2,s_3,n} \theta(s_1+s_2-s_3)\theta(s_1+s_3-s_2)f^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_2,s_3)\mathrm{F}(s_1,s_2,s_3|n|\phi^{\mathcal{B}}_{s_2},\phi^{\mathcal{C}}_{s_3})+\\
&+\sin 2\vartheta \sum_{s_2,s_3,n} \theta(s_2-s_1-s_3-1)f^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_2,s_3)\frac{1}{2}\tilde{\mathrm{E}}(s_1,s_2,s_3|n|\phi^{\mathcal{B}}_{s_2},\phi^{\mathcal{C}}_{s_3})+\\
&+\sin 2\vartheta \sum_{s_2,s_3,n} \theta(s_3-s_1-s_2-1)f^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_3,s_2)\frac{1}{2}\tilde{\mathrm{E}}(s_1,s_3,s_2|n|\phi^{\mathcal{B}}_{s_3},\phi^{\mathcal{C}}_{s_2})+\\
&+\sin 2\vartheta \sum_{s_2,s_3,n} \theta(s_1+s_2-s_3)\theta(s_1+s_3-s_2)f^{\mathcal{A}}_{\mathcal{BC}}(s_1,s_2,s_3)\tilde{\mathrm{F}}(s_1,s_2,s_3|n|\phi^{\mathcal{B}}_{s_2},\phi^{\mathcal{C}}_{s_3})\,.
\end{aligned}$$