Schwinger-type process for baryons and the new non-perturbative phenomena in electric field

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Outline of the talk

- Worldline instanton approach for the non-perturbative pair production
- Composite worldline instantons
- Baryon pair production. Skyrmion and holographic baryon
- Possible corrections and generalizations
- New non-perturbative processes in QCD in electric field

Worldline instantons for pair production

$$S = \int d\tau m\sqrt{\dot{x}^2} + i\oint A$$

$$m\frac{d}{d\tau}\left(\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}}\right) = iF_{\mu\nu}\dot{x}_\nu$$

Worldline instanton — solution to the Euclidean equations of motion

$$S = 2\pi r_0 M - q\pi r_0^2 E.$$

Electric field in Minkowski= Magnetic field
In Euclid

$$r_0 = \frac{M}{qE}, \quad S = \frac{\pi M^2}{qE}.$$

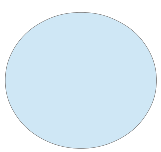
Larmour circles=worldline instantons

$$\Gamma \sim e^{-S}.$$

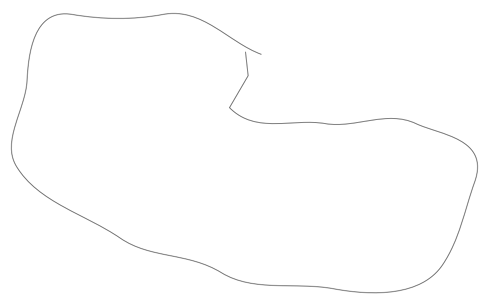
Affleck, Manton, Alvarez 81',82'

Worldline instantons for pair production

$$\omega = \frac{(eE)^2}{4\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{eE} \right)$$



Saddle point with negative mode



Sum over curves can be performed

It provides the preexponential factors

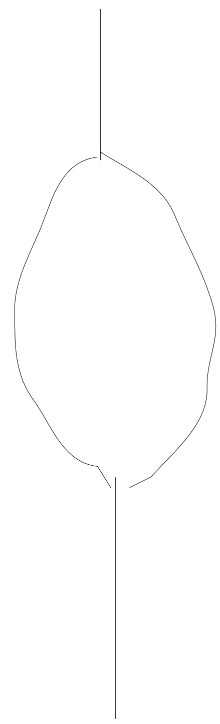
In two dimensions the pair production in electric field is equivalent to the false facuum decay in the scalar theory.

Bosonisation : kinks in the scalar theory=fermions in electric field

Fundamental versus composite particles. Induced processes

- Scwinger-pair production for elementary particles. Fermions, bosons, quarks etc
- Composite particles — some limitations of approach, monopoles(Affleck,Manton 82), baryons etc
- Take the corresponding solution with moduli. The size of the composite particle should be much smaller then the critical radius
- Interaction of the composite particle with some other modes should be small enough

Induced processes



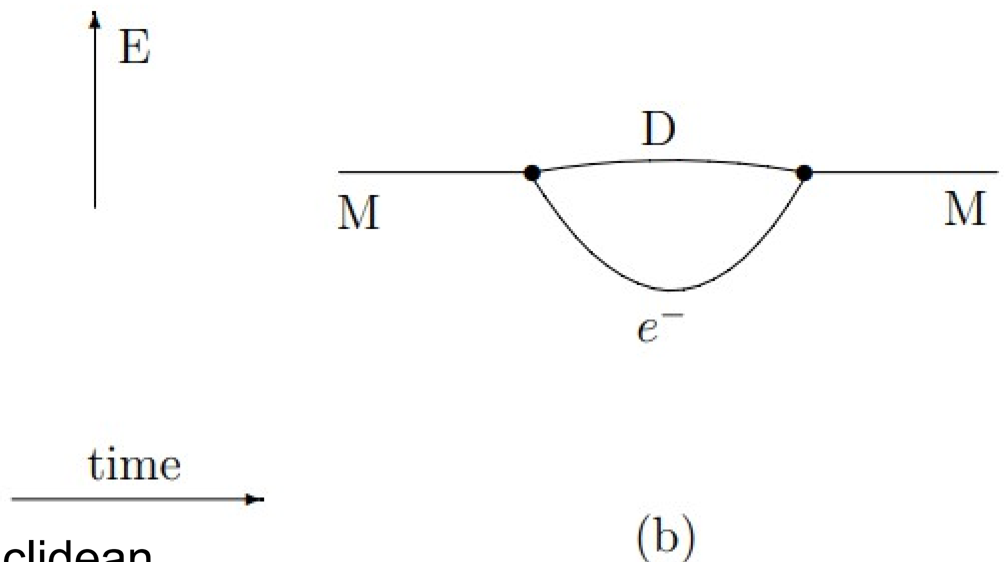
For Schwinger processes

Voloshin, Monin 2009-2011
Dunne 2009-2010

Some particle in initial state. It deforms the worldline instanton and increases the decay propability

Similar deformation due to the temperature. In the case of false vacuum decay
The induced processed have been considered as well
Affleck 79, Voloshin 84-85

Composite wordline instantons.



Euclidean

Simple example of the composite worldline instanton. Two arcs of the different radii. Equilibrium condition at the junction point Angle is fixed by the masses of the particles involved

Saraikin, Selivanov, A.G 2002

Holographic Schwinger pair production

Evaluation of the Wilson loop

Saraikin,Selivanov A.G 2002
Semenoff,Zarembo 2011

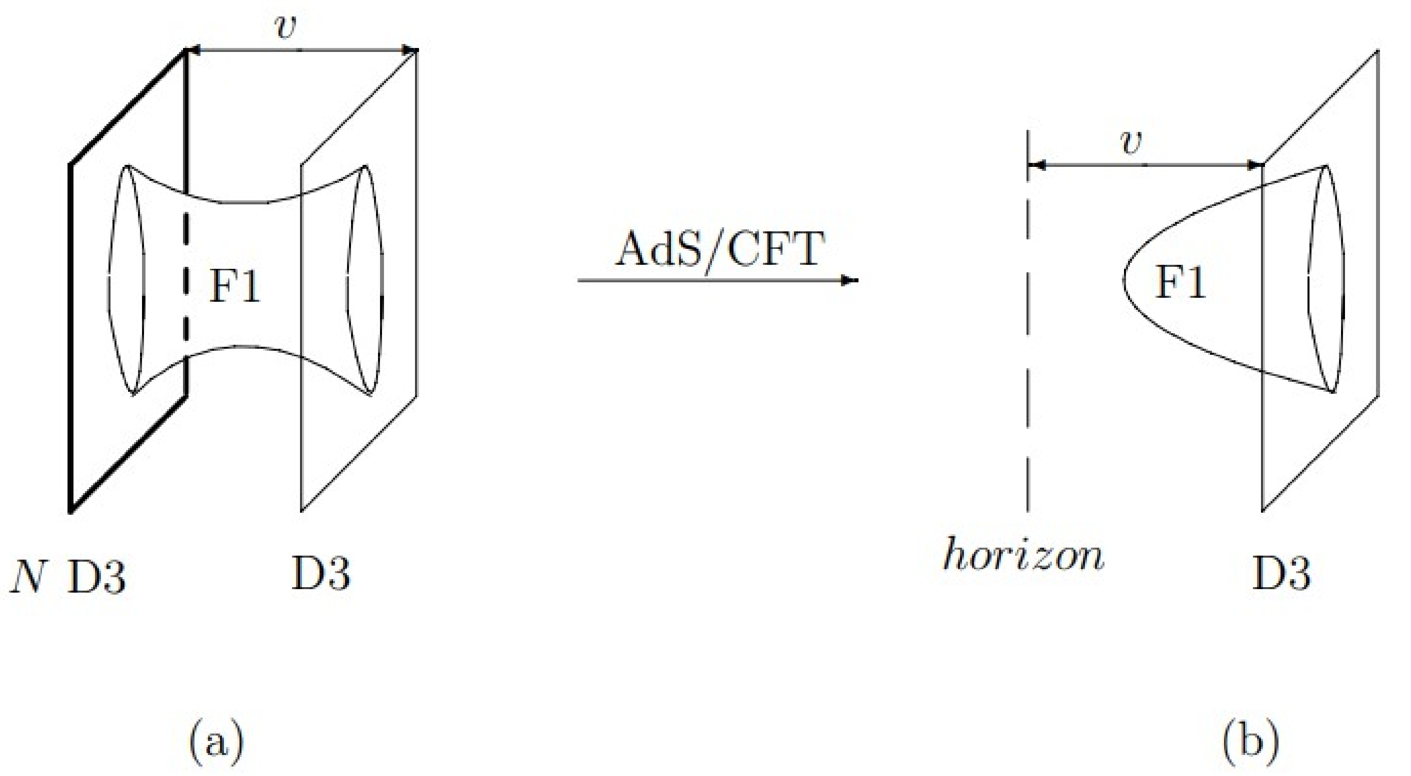
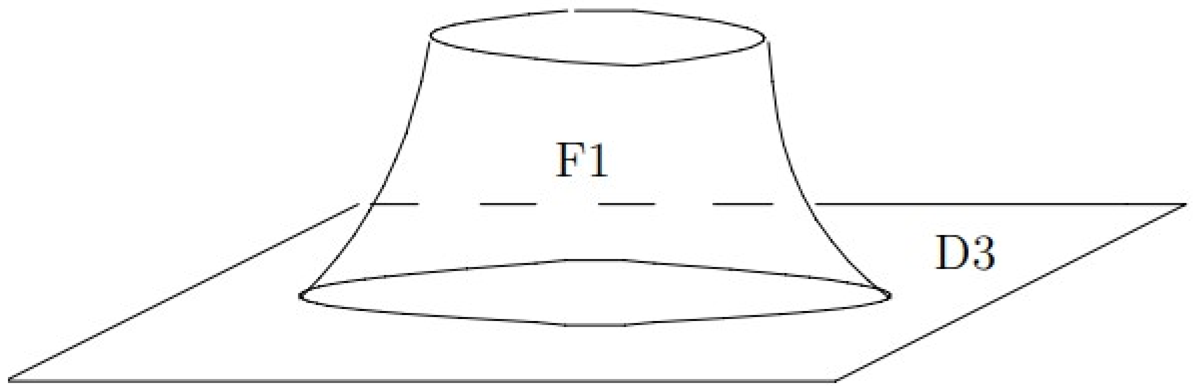


Fig.8: AdS/CFT correspondence at work: (a) Type IIB brane picture of the W-boson pair creation induced by the U(1) part of the gauge strength in $\mathcal{N} = 4$ theory with the U(N)×U(1) gauge group. (b) Gravity dual picture: "cap"-like surface in the AdS_5 space.

Holographic Schwinger pair production



Creation of states in fundamenal representation. In the confined state
The open strings representing quarks are extended between the
Cut-off scale and the corresponding D8 fravor brane

Many papers in the different versions of confined phase Yoshida, Hashimoto 13,14....
Effects of entanglement between the confined pair
Kharzeev,Zahed,Gruninger 2023

Baryon-pair production. Skyrmion picture and corrections

$$L = \frac{f_\pi^2}{4} Tr\left(\partial_\mu U \partial_\mu U^\dagger\right) + \frac{1}{32e^2} Tr\left(\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2\right).$$

$$B^\mu = \frac{i}{24\pi^2}\epsilon^{\mu\nu\sigma\rho} Tr(L_\nu L_\sigma L_\rho) + \frac{\epsilon_{\mu\nu\alpha\beta}}{24\pi^2}\partial_\nu[3ieA_\alpha TrQ(U^{-1}\partial_\beta U + \partial_\beta U U^{-1}]$$

$$J_\mu^Q = J_\mu^3 + \frac{1}{16\pi^2}\epsilon_{\mu\nu\alpha\beta} Tr[Q\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U U^{-1} + U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U]$$

$$+\frac{ie}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu A^\alpha Tr[Q^2\partial_\beta U U^{-1} U^{-1}\partial_\beta U + Q\partial_\beta U Q U^{-1} - \frac{1}{2} QUQ\partial_\beta U^{-1}]$$

$$U_0 = e^{if(r)\hat{x}\sigma},\ f(0) = \pi,\ f(\infty) = 0.$$

$$L_a = \frac{e}{16\pi^2} A_0 \varepsilon^{0\nu\alpha\beta} \operatorname{Tr} Q \left(\partial_\nu U U^{-1} \partial_\alpha U U^{-1} \partial_\beta U U^{-1} + U^{-1} \partial_\nu U U^{-1} \partial_\alpha U U^{-1} \partial_\beta U\right),$$

Relevant interaction term in the Lagrangian

$$\int d^4x L_a = \int dt \int 4\pi r^2 dr \frac{e}{16\pi^2} \frac{4}{r^2} (\sin^2 f) f' A_0 = \int dt \frac{en}{2} A_0.$$

$$S = ML + \Lambda \int dz (a_0')^2 + (a')^2,\ a_0^2 + a^2 = 1.$$

$$|p\uparrow\rangle = \frac{1}{\pi}(a_1 + ia_2), |p\downarrow\rangle = -\frac{i}{\pi}(a_0 - ia_3);$$

$$|n\uparrow\rangle = \frac{i}{\pi}(a_0 + ia_3), |n\downarrow\rangle = -\frac{i}{\pi}(a_1 - ia_2).$$

$$w \propto \exp\left(-\frac{M_p^2}{eE}\right)$$

Baryon-pair production in holographic QCD

$$S = \sigma \int d^4x dz (h(z) \operatorname{Tr} F_{\mu\nu}^2 + g(z) \operatorname{Tr} F_{\mu z}^2) + S_{CS}$$

Baryon- instanton in the
5d theory on the flavor branes

$$A_\mu = -i f(\eta) g_{inst}(x,z) \partial_\mu g_{inst}, \qquad A_0(x,z) = 0, \qquad f(\eta) = \frac{\eta^2}{\eta^2 + \rho^2},$$

$$g_{inst} = \frac{(z - z_0) - i(\vec{x} - \vec{x_0})\vec{\tau}}{\sqrt{(z - z_0)^2 + |\vec{x} - \vec{x_0}|^2}},$$

$$B_i(x,z) = 0, \qquad B_0(x,z) = -\frac{1}{8\pi^2 \lambda \eta^2}[1 - \frac{\rho^4}{(\eta^2 + \rho^2)^2}],$$

$$\eta = \sqrt{(z - z_0)^2 + |\vec{x} - \vec{x_0}|^2},$$

Son,Stephanov, 04
Sakai-Sugimoto 06
…………

$$B = \int d^3x dr (\operatorname{Tr} F_L \tilde{F}_L - \operatorname{Tr} F_R \tilde{F}_R)$$

Baryonic charge

Baryon-pair production in holographic QCD

- In the brane terms the baryon is the D4 brane wrapped around the internal S^4 sphere in Witten-Sakai-Sugimoto geometry
R_{3,1} x S^4 x cygar

Wrapped brane ->Massive particle — Witten 98.

Due to the 5d CS term there are N_C strings attached to the wrapped Brane → baryonic vertex   Witten 98

If we take into account the chiral condensate the holographic baryon Is identified with the dyonic instanton in 5d gauge-scalar theory

Krikun,Gustavvson, A.G. 2012

Baryon-pair production in holographic QCD

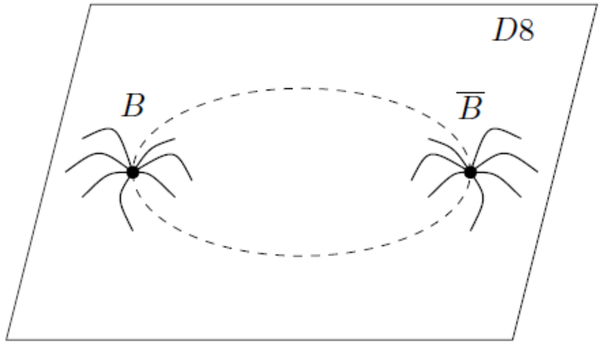
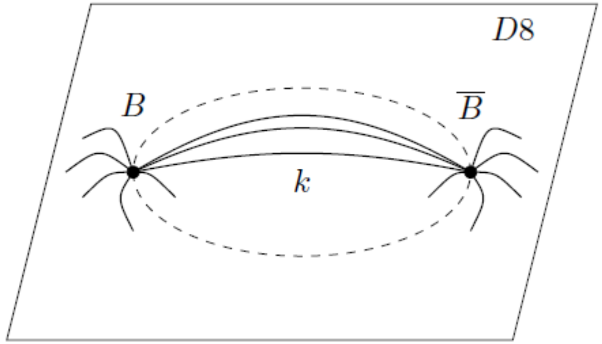


Figure 1: All strings are connected to D8 branes



$$S = 2\pi RM - qE_{eff}\pi R^2.$$

$$E_{eff} = E - kT_{srt}, \qquad T_{str} \propto \Lambda^2$$

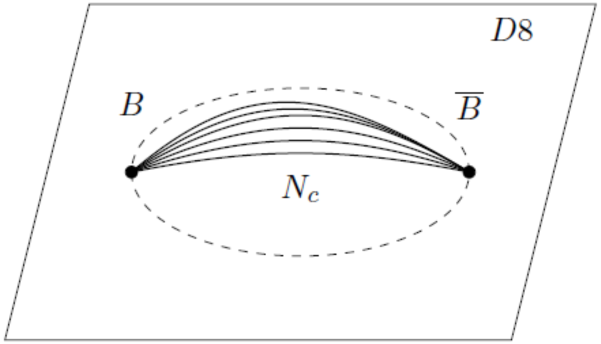
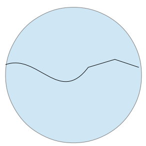


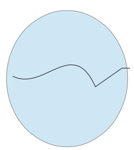
Figure 3: All strings are between vertex and anti-vertex

$$E_{eff} = E - NT_{srt}$$

Corrections to the probability rate

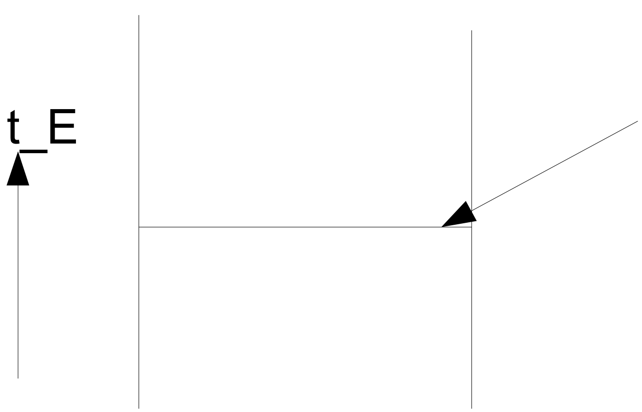


Photon exchange. Small correction due to the coupling



Meson exchange is suppressed by the pion mass if
the radius of the critical bounce is large enough

Non-perturbative correction due to the extended instanton S-Skyrmion



Extended S-Skyrmion , instant baryonic current

Grekov, A.G. 19'

Account of such non-perturbative contribution
yields additional suppression of the rate

Account of the chiral condensate.Dyonic instanton in 5d

$$F_{\mu\nu} = *F_{\mu\nu}, \quad E_\mu = D_\mu \varphi.$$

$$S = \int d^5x \, \mathrm{Tr} \left(\frac{1}{2} F_{\mu\nu}^2 + E_\mu^2 + (D_\mu \varphi)^2 + (D_5 \varphi)^2 \right)$$

$$A_\mu = \frac{2}{g} \frac{\rho^2}{r^2(\rho^2 + r^2)} \bar{\eta}^a_{\mu\nu} x_\nu \frac{\sigma^a}{2}.$$

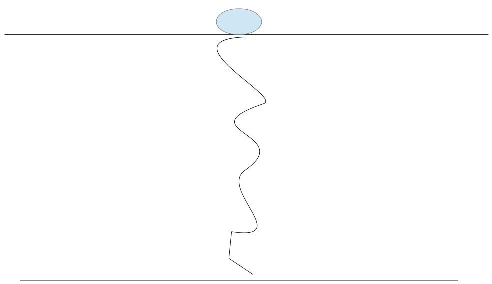
$$\varphi = v \frac{r^2}{r^2 + \rho^2} \frac{\sigma^3}{2}.$$

If we take into account the nontrivial profile of the chiral condensate in the baryon solution it is represented by the dyonic instanton in the 5d theory on the flavor branes Gustavson,Krikun,A.G. 12'

The vev of the scalar — chiral condensate. The account of the nontrivial profile of the bifundamental scalar does not modify the probability rate significantly(numerics)

Baryons with heavy quarks

$$S = 2\pi M r_0 - qEr_0^2 + S_{NG},$$



$$ds^2 = \left(\frac{u}{R}\right)^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)d\tau^2) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \quad f(u) = 1 - \frac{u_k^3}{u^3}.$$

$$S = T_s \int d^2x\sqrt{g} = T_S \int d\varphi du\, r \left(\frac{u}{R}\right)^{3/2}\sqrt{r'^2 + \frac{R^3}{u^3 - u_k^3}},$$

We take into account the contribution of the baryonic vertex plus the contribution from the string extended till the brane corresponding to the heavy flavor

Baryons with heavy quarks

$$\frac{rr''}{1+r'^2(u^3-u_k^3)/R^3}+\frac{3rr'}{2u}+\left[\frac{3rr'u^2/(2R^3)}{1+r'^2(u^3-u_k^3)/R^3}-1\right]\frac{R^3}{u^3-u_k^3}=0.$$

$$S=2\pi M_1 r_0-\pi qEr_0^2-\frac{A}{r_0},\quad A=\pi T_s R^3\ln\left(\frac{\alpha u_q}{u_k}\right).$$

$$w\sim e^{-S}=\exp\left(-\frac{\pi M_1^2}{qE}+A\frac{qE}{M_1}\right),$$

$$M_1=M+T_s(u_q-u_k)$$

The mass of the baryon involving the
contribution from the string extended
In radial coordinate

The decay of neutron in external electric field

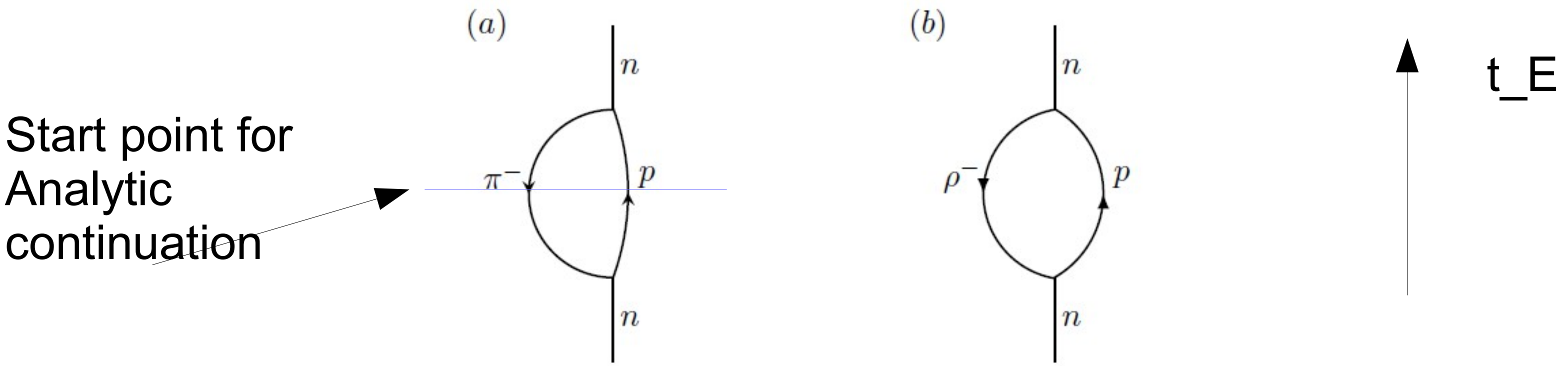


Figure 4: Bounce for the neutron decay: (a) via π^- channel; (b) via ρ^- channel

$$S_{inst} = m_\pi L_\pi + M_p L_p - eE(Area) - M_n H$$

$$Im\delta M_n \propto e^{-S_{inst}}$$

Neutron decay probability

$$S_{inst} = \frac{m_\rho^2}{eE} \arccos \frac{M_n^2 + m_\rho^2 - M_p^2}{2m_\rho M_n} + \frac{M_p^2}{eE} \arccos \frac{M_n^2 - m_\rho^2 + M_p^2}{2M_p M_n} - \frac{m_\rho M_n}{eE} \sqrt{1 - \left(\frac{M_n^2 + m_\rho^2 - M_p^2}{2m_\rho M_n}\right)^2} \tag{35}$$

Non-perturbative creation of specific states in electric fields

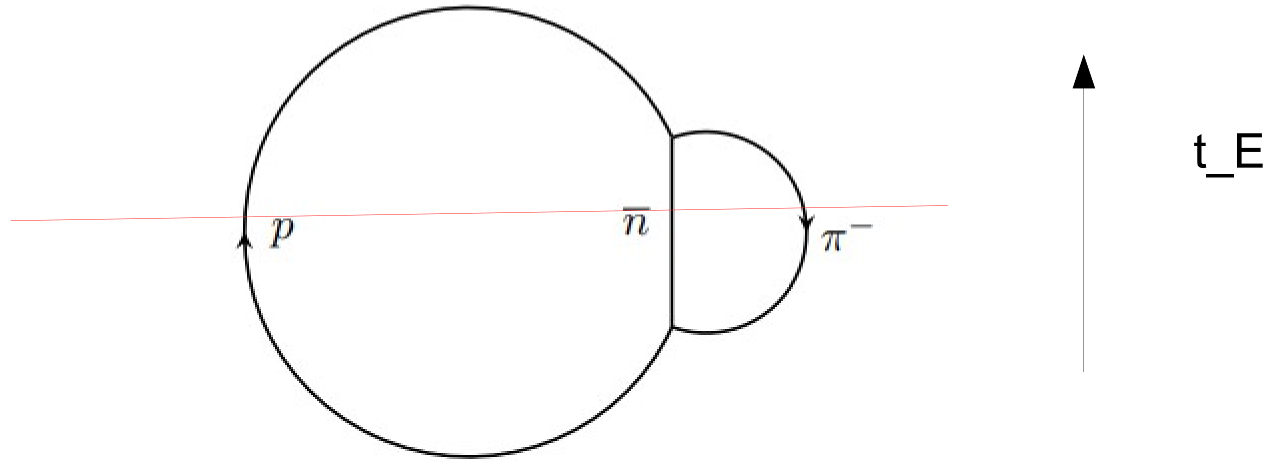


Figure 5: The Euclidean trajectory for $p\bar{n}\pi^-$ creation

$$\omega \propto \exp\left(-\frac{\pi(M_p^2+m_\rho^2)}{eE} - \frac{M_pM_n}{eE}\sqrt{1-\left(\frac{M_p^2-m_\rho^2+M_n^2}{2M_pM_n}\right)^2} + \right.$$

$$\left. + \frac{M_p^2}{eE}\arccos\frac{M_p^2-m_\rho^2+M_n^2}{2M_pM_n} + \frac{m_\rho^2}{eE}\arccos\frac{m_\rho^2-M_p^2+M_n^2}{2m_\rho M_n}\right).$$

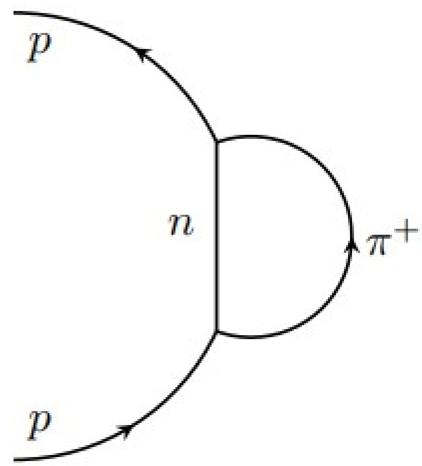


Figure 6: Conjectual Euclidean trajectory yielding the proton decay in electric field

Accelerated particle at the initial state. Some subtle points concerning transition Minkowski-> Euclid->Minkowski

Probability rates and possible marks of the processes

- Certainly the probabilities are exponentially small
- Probably the highly energetic external particles could enhance the rate enough. The example of the Schwinger processes assisted by the laser beams
- Some specific marks. For instance, for the production of the (pn+meson) state we get

  the neutron at rest + asymmetric pair in the final state

Conclusion and open questions

- The composite worldline instantons are effective for many non-perturbative processes
- To prove the proton decay in electric field(in progress)
- Find the composite worldline and worldsheet(string creation) instantons at finite temperature
- Marginal stability curves for BPS states in external field for SUSY YM theories