Infinite spin particle on gravity background

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based on

I.L. Buchbinder, SF, A.P. Isaev, V.A. Krykhtin , arXiv:2402.13879 [hep-th]; arXiv:2403.14446 [hep-th]

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Our results on describing the infinite (continuous) particle
on curve space-time will be presented.

In $4D$ space-time infinite spin representations are massless ($P^m P_m = 0$) unitary irreps of the Poincaré group which obey [E.Wigner, V.Bargmann]:

$$W^m W_m = -\mu^2 \neq 0\,, \qquad W_m = \tfrac{1}{2}\,\varepsilon_{mnkl} P^n M^{kl}\,.$$

Each of these representations contains infinite tower of states with all (integer or half-integer) helicities.

The same spectrum of states is used in higher spin theory that considers infinite tower of the helicity states ($W^m W_m = 0$, $W_m = \lambda P_m$).

There are some similarities higher spin theory and infinite spin theory.

But there are a number of differences:

- in highest spin theory all spins unite with each other through interaction, whereas in infinite spin theory all spins are combined together initially;
- in infinite spin theory there is dimensionfull parameter μ.

Infinite spin representations were researched in the 2000s in many studies (see, e.g., the papers by [L.Brink, A.Khan, P.Ramond, X.Xiong, J.Mund, B.Schroer, J.Yngvason, X.Bekaert, J.Mourad, N.Boulanger, P.Schuster, N.Toro, V.Rivelles, E.Skvortsov, M.Najafizadeh, M.Khabarov, Yu.Zinoviev, K.Alkalaev, M.Grigoriev, A.Chekmenev, A.Bengtsson, R.Metsaev, K.Zhou]).

But, in almost all paper, infinite spin particles were considered in flat space.

There are very few papers on infinite spin particle in curved space.

- In [Metsaev, 2017], the Lagrangian infinite spin formulation was constructed by specific deformation of the Fronsdal Lagrangian on AdS. This result was later confirmed in [Metsaev, 2019] within light-cone approach.
- In [Khabarov, Zinoviev, 2017], the Lagrangian construction for an infinite spin field in AdS space was realized on the basis of massive spin s field theory in the limit $m \to 0$, $s \to \infty$ and $sm \to \mu$.
- Infinite spin fields in AdS space were discussed also in the review [Bekaert, Skvortsov, 2017].
- In these papers, the Lagrangian formulation for infinite spin fields was constructed not in an arbitrary curved space-time, but only in AdS space.

This talk will present $1d$ and $4D$ models of infinite spin particles in curved space within the framework of our description of such representations.

$4D$ infinite (continuous) spin fields

Since infinite spin representations are infinite-dimensional (in a fixed space-time coordinate), it is necessary

- to consider an infinite tower of space-time fields
- or consider generalized fields that depend on an additional coordinate.

To take into account symmetries, the second choice is preferred.

One of the possible choices is to use commuting vector y^m as additional coordinate (see, for example, [E.Wigner, V.Bargmann] and [P.Schuster, N.Toro, V.Rivelles, X.Bekaert, J.Mourad, E.Skvortsov, M.Najafizadeh, M.Khabarov, Yu.Zinoviev, K.Alkalaev, M.Grigoriev, A.Chekmenev, R.Metsaev, K.Zhou]).

In such formulation, infinite spin field $\Phi(x^m, y^m)$ depends on the position vector x^m and vector coordinate y^m.

To describe irreducible infinite spin representations, the field $\Phi(x^m, y^m)$ obeys the Wigner-Bargmann equations of motion.

We use different formulation in describing infinite (continuous) spin representations.

$4D$ infinite spin fields with additional spinor coordinate

In our formulation, the additional variable is

commuting Weyl spinor ξ^{α}, $\bar{\xi}^{\dot{\alpha}} = (\xi^{\alpha})^*$, $\alpha = 1, 2$.

The field $\Phi(x;\xi,\bar{\xi})$ describing infinite integer-spin representation obeys the following equations of motion [I.Buchbinder, SF, A.Isaev, V.Krykhtin, 2018]

$$
\begin{aligned}
\partial^m \partial_m \, \Phi(x;\xi,\bar{\xi}) &= 0\,, \\
\Big[i \left(\xi \sigma^m \bar{\xi} \right) \partial_m \, + \, \boldsymbol{\mu} \Big] \Phi(x;\xi,\bar{\xi}) &= 0\,, \\
\left[i \left(\frac{\partial}{\partial \xi} \sigma^m \frac{\partial}{\partial \bar{\xi}} \right) \partial_m \, - \, \boldsymbol{\mu} \right] \Phi(x;\xi,\bar{\xi}) &= 0\,, \\
\left[\xi \frac{\partial}{\partial \xi} \, - \, \bar{\xi} \frac{\partial}{\partial \bar{\xi}} \right] \Phi(x;\xi,\bar{\xi}) &= 0\,.
\end{aligned}
$$

Due to the first three equations, the field $\Phi(x;\xi,\bar{\xi})$ satisfies

$$
W^2 \Phi(x;\xi,\bar{\xi}) \, = \, -\boldsymbol{\mu}^2 \Phi(x;\xi,\bar{\xi})\,.
$$

and describes irreducible massless infinite spin representation.

$4D$ infinite spin particle in flat space

In $1d$ description, the dynamics of infinite spin particle is determined by the set of the first class constraints: [I.Buchbinder, SF, A.Isaev, V.Krykhtin, 2018]

$$\begin{aligned} p^m p_m &\approx 0\,, \\ \left(\xi\sigma^m\bar{\xi}\right)p_m - \mu &\approx 0\,, \\ (\bar{\pi}\tilde{\sigma}^m\pi)\,p_m - \mu &\approx 0\,, \\ \xi\pi - \bar{\pi}\bar{\xi} &\approx 0 \end{aligned}$$

in the phase space with canonical Poisson brackets

$$\{x^m, p_n\}_{_{PB}} = \delta^m_n\,, \qquad \{\xi^\alpha, \pi_\beta\}_{_{PB}} = \delta^\alpha_\beta\,, \qquad \{\bar{\xi}^{\dot{\alpha}}, \bar{\pi}_{\dot{\beta}}\}_{_{PB}} = \delta^{\dot{\alpha}}_{\dot{\beta}}\,.$$

It is flat-space formulation of infinite spin particle.

The main task is to generalize this model to the case of curved space-time.

The basic requirements for such a construction are:

- General covariant generalization of constraints with a given flat limit.
- Closure of the algebra of new constraints.

These conditions lead restrictions on space-time geometry.

Generalization to curved space-time

Now $x^\mu(\tau)$ are local coordinates in curved space, ξ^α, $\bar{\xi}^{\dot{\alpha}}$, π_α, $\bar{\pi}^{\dot{\alpha}}$ are two-component spinors in tangent space. Their Poisson brackets are

$$\{x^\mu, p_\nu\}_{PB} = \delta^\mu_\nu\,, \qquad \{\xi^\alpha, \pi_\beta\}_{PB} = \delta^\alpha_\beta\,, \qquad \{\bar{\xi}^{\dot{\alpha}}, \bar{\pi}_{\dot{\beta}}\}_{PB} = \delta^{\dot{\alpha}}_{\dot{\beta}}\,.$$

Space-time geometry is described by metric $g_{\mu\nu}(x)$, vierbein $e^m_\mu(x)$ and spin connection $\omega_\mu{}^{mn}(x)$.

Transition from flat space-time to curved space-time involves the replacement

$$p_\mu \quad \to \quad \mathcal{P}_\mu = p_\mu + \frac{1}{2}\omega_\mu{}^{mn} M_{mn}\,, \qquad \mathcal{P}_m = e^\mu_m \mathcal{P}_\mu\,,$$

where $M_{mn} = \xi\sigma_{mn}\pi - \bar{\pi}\tilde{\sigma}_{mn}\bar{\xi}$ – generators of tangent Lorentz transformations

Components of "covariant momentum" satisfy the Poisson brackets algebra

$$\{\mathcal{P}_\mu, \mathcal{P}_\nu\}_{PB} = -\frac{1}{2} R_{\mu\nu}{}^{mn} M_{mn}\,,$$

where $R_{\mu\nu}{}^m{}_n$ is the curvature tensor.

Restriction on space-time geometry

Covariant generalization of the flat space constraints should be of the form:

$$\mathcal{P}^m\mathcal{P}_m + \cdots \approx 0\,, \qquad \left(\xi\sigma^m\bar{\xi}\right)\mathcal{P}_m - \boldsymbol{\mu} + \cdots \approx 0\,, \qquad \left(\bar{\pi}\tilde{\sigma}^m\pi\right)\mathcal{P}_m - \boldsymbol{\mu} + \cdots \approx 0$$

were the dots mean terms that disapper at vanishing the gravity background.

Closing the algebra of constraints leads to the following important consequences:

- Curved space must be the Riemannian space with zero torsion: $T_{\mu\nu}{}^m = 0$.
- Curvature tensor of the background geometry should be of the form:

$$R_{mn}{}^{k\ell} \;=\; \kappa\left(\delta^k_m\delta^\ell_n - \delta^\ell_m\delta^k_n\right),$$

  where κ is a constant. That is, only spaces of constant curvature are allowed as background: the Minkowski space ($\kappa = 0$), de Sitter space ($\kappa > 0$) and anti de Sitter space ($\kappa < 0$).
- All additional terms (the dots in the formulas above) in the constraints are fully defined.

Curve space-time constraints

In the case of AdS space the final constraints have the form

$$\begin{aligned}
\mathcal{F}_0 &= \mathcal{P}^m\mathcal{P}_m - \frac{1}{4}R_{mn}{}^{k\ell}M^{mn}M_{k\ell} - \frac{1}{2}\kappa\mathcal{K}^2 - 2\boldsymbol{\mu}|\kappa|^{1/2} \approx 0\,,\\
\mathcal{F} &= \left(\xi\sigma^m\bar{\xi}\right)\mathcal{P}_m - \boldsymbol{\mu} + \frac{1}{4}|\kappa|^{1/2}\mathcal{K}^2 \approx 0\,,\\
\tilde{\mathcal{F}} &= \left(\bar{\pi}\tilde{\sigma}^m\pi\right)\mathcal{P}_m - \boldsymbol{\mu} + \frac{1}{4}|\kappa|^{1/2}\mathcal{K}^2 \approx 0\,,\\
\mathcal{U} &= N - \bar{N} \approx 0\,,
\end{aligned}$$

where $\mathcal{K} = N + \bar{N}$, $N = \xi^\alpha\pi_\alpha$, $\bar{N} = \bar{\pi}_{\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}$.

The only non-zero Poisson bracket of constraints has the form

$$\left\{\mathcal{F},\tilde{\mathcal{F}}\right\}_{PB} = -K\mathcal{F}_0 + |\kappa|^{1/2}K\left(\mathcal{F}+\tilde{\mathcal{F}}\right)\,.$$

In dS space the constraints have the form

$$\begin{aligned}
\mathcal{F}_0 &= \mathcal{P}^m\mathcal{P}_m - \frac{1}{4}R_{mn}{}^{k\ell}M^{mn}M_{k\ell} - \frac{1}{2}\kappa\mathcal{K}^2 \approx 0\,, \\
\mathcal{F} &= \left(\xi\sigma^m\bar{\xi}\right)\mathcal{P}_m - \boldsymbol{\mu} + \frac{1}{4}\kappa^{1/2}\mathcal{K}^2 \approx 0\,, \\
\tilde{\mathcal{F}} &= \left(\bar{\pi}\tilde{\sigma}^m\pi\right)\mathcal{P}_m - \boldsymbol{\mu} - \frac{1}{4}\kappa^{1/2}\mathcal{K}^2 \approx 0\,, \\
\mathcal{U} &= N - \bar{N} \approx 0\,,
\end{aligned}$$

The only non-zero Poisson bracket of constraints has the form

$$\left\{\mathcal{F},\tilde{\mathcal{F}}\right\}_{PB} = -K\mathcal{F}_0 + |\kappa|^{1/2}K\left(\mathcal{F}-\tilde{\mathcal{F}}\right)\,.$$

Replacing phase space coordinates with operators (taking into account their ordering) and considering constraints as equations of motion (first operator quantization), we obtain field theory of infinite spin particle on curved space.

Important requirement in this case is to obtain Lagrangian theory, when all conditions are obtained from the Lagrangian.

BRST Lagrangian formulation of infinite spin field on AdS_4

We have constructed field formulation of the infinite spin particle in AdS space by using BRST methods.

The main points in the construction:

- Spinor quantities ξ_α, $\bar{\xi}_{\dot\alpha}$, $\partial/\partial\xi_\alpha$, $\partial/\partial\bar{\xi}_{\dot\alpha}$ are realized by creation c^α, $\bar{c}_{\dot\alpha}$ and annihilation a_α, $\bar{a}^{\dot\alpha}$ operators in the Fock space:

$$[\bar{a}^{\dot\beta}, \bar{c}_{\dot\alpha}] = \delta^{\dot\beta}_{\dot\alpha}, \quad [a_\beta, c^\alpha] = \delta^\alpha_\beta\,, \qquad (a_\alpha)^\dagger = \bar{c}_{\dot\alpha}\,, \quad (\bar{a}_{\dot\alpha})^\dagger = c_\alpha\,,$$

acting on vacuum states $|0\rangle$ and $\langle 0|$, $\langle 0|0\rangle = 1$ according to

$$\langle 0|\bar{c}_{\dot\alpha} = \langle 0|c^\alpha = 0\,, \qquad \bar{a}^{\dot\alpha}|0\rangle = a_\alpha|0\rangle = 0\,.$$

- BRST charge is constructed by the operators of the first class constraints:

$$
\begin{aligned}
L_0 &= D^2 + \kappa \left(N\bar{N} + N + \bar{N} \right) + 2\boldsymbol{\mu}\, |\kappa|^{1/2} - \frac{1}{2}\,\kappa K^2\,, \\
L &= i(a\sigma^m \bar{a}) e^{\mu}_{m} D_\mu - \boldsymbol{\mu} - \frac{1}{4}\,|\kappa|^{1/2} K^2, \\
L^+ &= i(c\sigma^m \bar{c})\, e^{\mu}_{m} D_\mu - \boldsymbol{\mu} - \frac{1}{4}\,|\kappa|^{1/2} K^2,
\end{aligned}
$$

Here the D_μ are the covariant derivative operators defined by

$$D_\mu \;=\; \partial_\mu \;+\; \tfrac{1}{2}\,\omega_\mu{}^{mn}\,\mathcal{M}_{mn}\,, \qquad \mathcal{M}_{mn} \;=\; c^\alpha\,(\sigma_{mn})_\alpha{}^\beta\, a_\beta \;+\; \bar{c}_{\dot\alpha}\,(\tilde\sigma_{mn})^{\dot\alpha}{}_{\dot\beta}\,\bar{a}^{\dot\beta}$$

L_0, L, L^+, N, $\bar{N}$, K are operators corresponding to $\mathcal{F}_0$, $\mathcal{F}$, $\tilde{\mathcal{F}}$, $\mathcal{N}$, $\bar{\mathcal{N}}$, $\mathcal{K}$.

Commutator algebra of the constraint operators is written in the form

$$
\begin{aligned}
[L, L_0] &= 0\,, \qquad [L^+, L_0] \;=\; 0\,, \\
[L^+, L] &= K\, L_0 \;+\; |\kappa|^{1/2}(K+1)\, L \;+\; |\kappa|^{1/2}(K-1)\, L^+\,.
\end{aligned}
$$

- The conditions for Hermitianity ($Q = Q^\dagger$) and nilpotency ($Q^2 = 0$) of BRST charge require the use of suitable ordering of operators in constraints:

$$
\begin{aligned}
Q \;=\; & \eta_0(L_0 - \kappa) \;+\; \eta^+ L \;+\; \eta L^+ + K\eta^+\eta\mathcal{P}_0 + \\
& + \frac{1}{2}|\kappa|^{1/2}(K+1)\,\eta^+\eta\mathcal{P} \;+\; \frac{1}{2}|\kappa|^{1/2}(K-1)\,\eta^+\eta\mathcal{P}^+ + \\
& + \frac{1}{2}|\kappa|^{1/2}(K+1)\,\mathcal{P}^+\eta^+\eta \;+\; \frac{1}{2}|\kappa|^{1/2}(K-1)\,\mathcal{P}\eta^+\eta\,.
\end{aligned}
$$

Here we have extended the Fock space by introducing fermionic ghost "coordinates" η_0, η, η^+ and their canonically conjugated ghost "momenta" $\mathcal{P}_0$, $\mathcal{P}^+$, $\mathcal{P}$ which obey the anticommutation relations

$$\{\eta, \mathcal{P}^+\} \;=\; \{\mathcal{P}, \eta^+\} \;=\; \{\eta_0, \mathcal{P}_0\} \;=\; 1$$

and act on the vacuum state as follows

$$\eta|0\rangle \;=\; \mathcal{P}|0\rangle \;=\; \mathcal{P}_0|0\rangle \;=\; 0.$$

- Lagrangian

$$\mathcal{L} \sim \langle \bar{\Psi} | Q | \Psi \rangle$$

is formulated in terms of BRST charge Q acting in Fock space of the vectors

$$|\Phi\rangle \; = \; |\varphi\rangle \; + \; \eta_0 \mathcal{P}^+ |\varphi_1\rangle \; + \; \eta^+ \mathcal{P}^+ |\varphi_2\rangle \, .$$

- To obtain off-shell Lagrangian formulation, it is necessary to use triplet of infinite spin fields (ket-vectors) and their conjugate ones (bra-vectors):

$$|\varphi(x)\rangle \, , \quad |\varphi_1(x)\rangle \, , \quad |\varphi_2(x)\rangle \qquad \text{and} \qquad \langle \bar{\varphi}(x)| \, , \quad \langle \bar{\varphi}_1(x)| \, , \quad \langle \bar{\varphi}_2(x)| \, .$$

Each of these fields contains infinite number of states, that are obtained by the action of creation or annihilation operators on the vacuum state.

Off-shell Lagrangian formulation of infinite spin field on AdS_4

The resulting Lagrangian has the form (up to the multiplier $\sqrt{-g}$)

$$
\begin{aligned}
\mathcal{L} \;=\; & \langle\bar{\varphi}| \left\{(L_0-\kappa)|\varphi\rangle - \left[L^+ + \frac{1}{2}\,|\kappa|^{1/2}(K-1)\right]|\varphi_1\rangle\right\} \\
& - \langle\bar{\varphi}_1| \left\{\left[L + \frac{1}{2}\,|\kappa|^{1/2}(K-1)\right]|\varphi\rangle - \left[L^+ - \frac{1}{2}\,|\kappa|^{1/2}(K+1)\right]|\varphi_2\rangle + K|\varphi_1\rangle\right\} \\
& - \langle\bar{\varphi}_2| \left\{(L_0-\kappa)|\varphi_2\rangle - \left[L - \frac{1}{2}\,|\kappa|^{1/2}(K+1)\right]|\varphi_1\rangle\right\}.
\end{aligned}
$$

This Lagrangian is invariant under the gauge transformations

$$
\delta|\varphi\rangle \;=\; \left[L^+ + \frac{1}{2}\,|\kappa|^{1/2}(K-1)\right]|\lambda\rangle\,, \qquad \delta|\varphi_1\rangle \;=\; (L_0-\kappa)|\lambda\rangle\,,
$$

$$
\delta|\varphi_2\rangle \;=\; \left[L - \frac{1}{2}\,|\kappa|^{1/2}(K+1)\right]|\lambda\rangle\,.
$$

The Lagrangian and gauge transformations at $\kappa = 0$ turn to Lagrangian and gauge transformations for infinite spin fields in Minkowski space.

Conclusion

We obtained the following results:

- We present a new particle model that generalizes for curved space-time an infinite spin particle in flat space. The model is described by commuting Weyl spinor additional coordinates.
- It is proved that this model is consistent only in an external gravitational field corresponding to constant curvature spaces.
- A full set of first-class constraints in de Sitter and anti-de Sitter spaces is obtained.
- Using BRST construction, we have developed the Lagrangian description of infinite spin field in AdS_4 space.
- The Lagrangian is formulated in terms of the operator BRST charge acting in the Fock space corresponding to the creation and annihilation operators with two-component spinor indices.
- We construct the BRST charge and derive the Lagrangian and gauge transformations for infinite spin field theory in AdS_4 space.

Thank you very much for your attention !