Emission of gravitational waves by cosmic domain walls

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Introduction

- Pulsar timing arrays: NANOGrav, EPTA with InPTA, PPTA, and Chinese PTA ⟶
- stochastic gravitational waves background
- supermassive black hole binaries are the most likely source
- primordial sources are also possible
- we focus on domain walls

- CosmoLattice computer code – Figueroa et al., 2021

Introduction

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2 \qquad \langle \phi \rangle = \pm\eta$$

$$\frac{d^2\phi}{dz^2} - \lambda\left(\phi^2 - \eta^2\right)\phi = 0 \qquad \phi(z \to \pm\infty) = \pm\eta$$

$$\phi(z) = \eta \tanh\left(\frac{z}{\Delta}\right) \qquad \Delta^2 = \frac{2}{\lambda\eta^2}$$

- domain wall width

Scaling regime

- One domain wall per horizon

- Distance between walls = Hubble radius

$$\rho_{wall} \sim \frac{H^{-2}\sigma_{wall}}{H^{-3}} \sim \sigma_{wall}/t$$

$$\xi_{dw} = \frac{\rho_{wall}}{\sigma_{wall}} t$$ - scaling parameter

$$\sigma_{dw} = \frac{2\sqrt{2\lambda}\eta^3}{3}$$ - tension of domain walls

Scaling regime

$$P \sim \dddot{Q}^2_{ij} / \left(40\pi M^2_{Pl}\right) \quad Q_{ij} \sim M_{wall}/H^2 \qquad M_{wall} \sim \sigma_{wall}/H^2$$

$$\rho_{gw} \sim PtH^3 \sim \frac{\sigma^2_{wall}}{40\pi M^2_{Pl}}$$

Hiramatsu et al., 2013

$$k\tau \ll 1$$

Caprini et al., 2009

$$\frac{d\rho_{gw}}{d\ln k} \propto k^3$$

Initial conditions

$$\langle \phi(\mathbf{k})\phi(\mathbf{q})\rangle = A(k)\delta(\mathbf{k}+\mathbf{q})$$

$$\langle \dot{\phi}(\mathbf{k})\dot{\phi}(\mathbf{q})\rangle = B(k)\delta(\mathbf{k}+\mathbf{q})$$

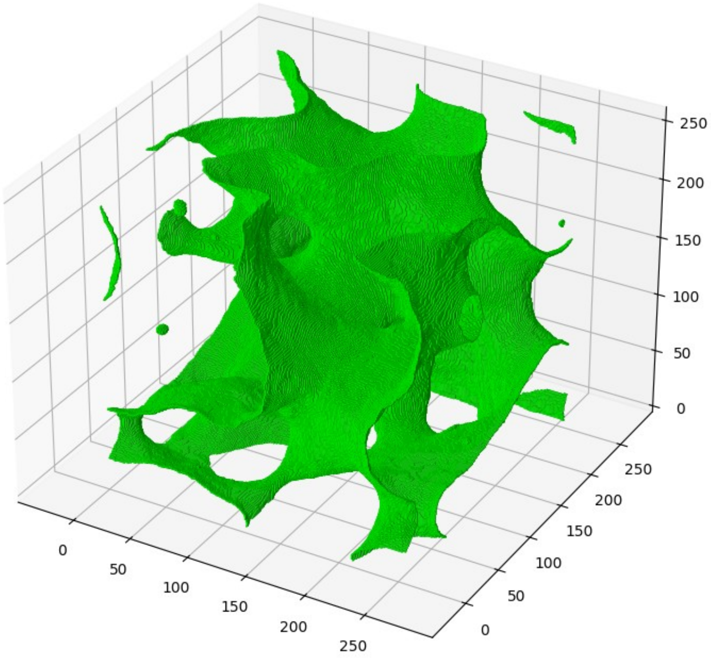
Vacuum:

$$A(k) = \frac{1}{2k} \qquad B(k) = \frac{k}{2}$$

Thermal:

$$A(k) = \frac{1}{k\cdot\left(e^{\frac{k}{T}}-1\right)} \qquad B(k) = \frac{k}{e^{\frac{k}{T}}-1}$$

Computation of scaling parameter

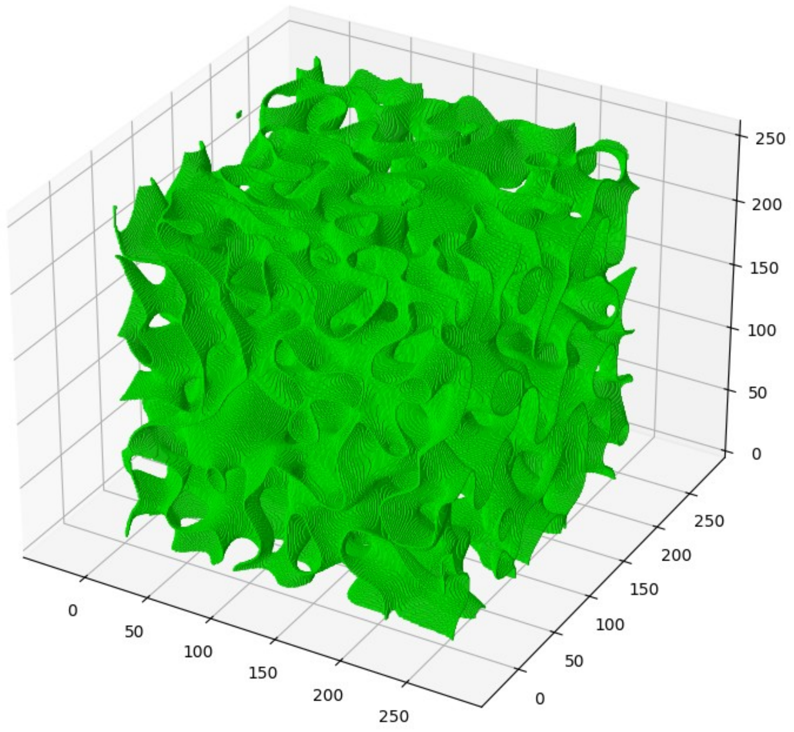


$$\xi = \frac{At}{a(t)V} = \frac{\rho_{wall}}{\sigma_{wall}} t$$

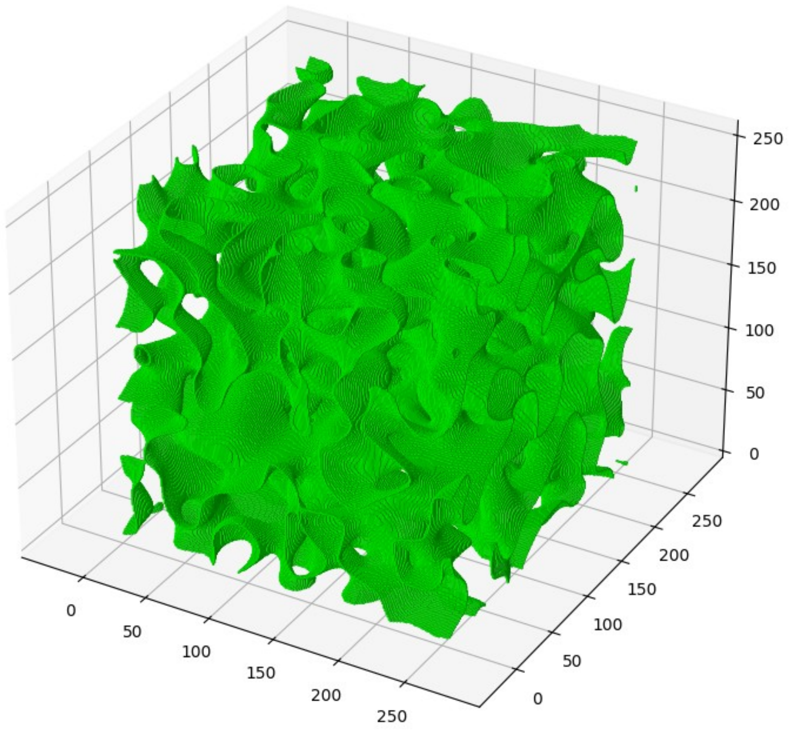
$$A = \Delta A \cdot \sum_{\text{links}} \delta \cdot \frac{|\nabla\phi|}{|\phi_{,x}| + |\phi_{,y}| + |\phi_{,z}|}$$

Press, Ryden and Spergel, 1989

Evolution of DW network

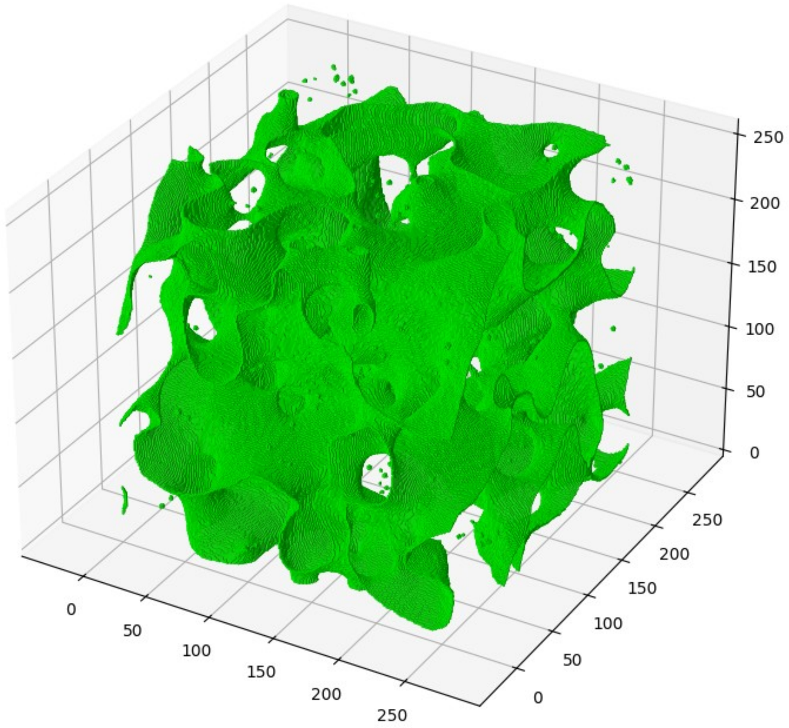


τ = 1

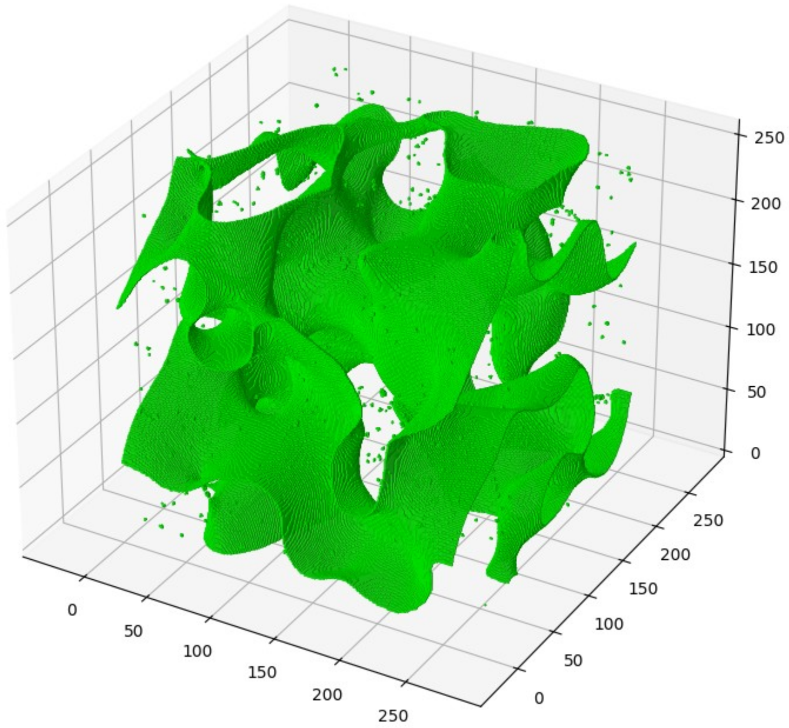


τ = 5

Evolution of DW network

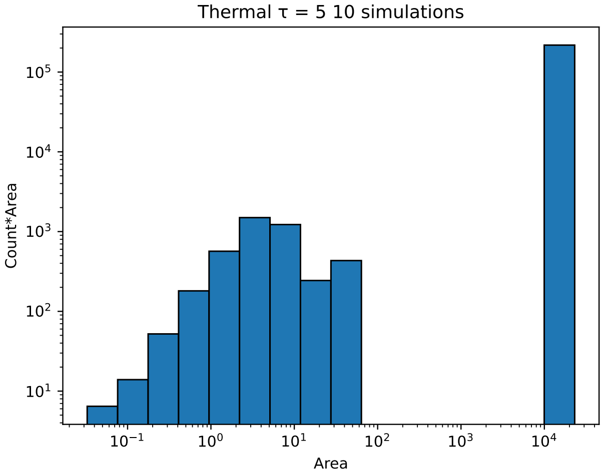


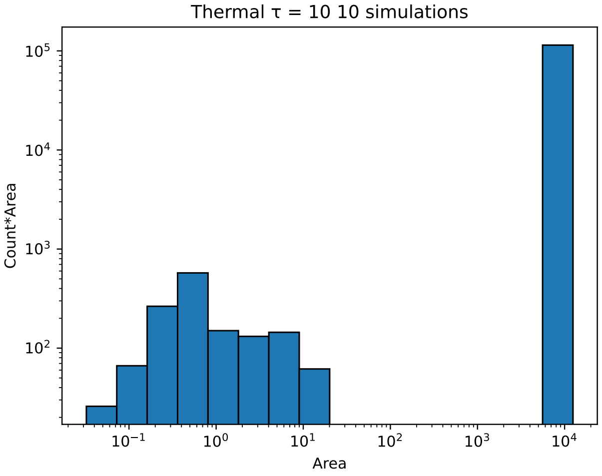
τ = 10

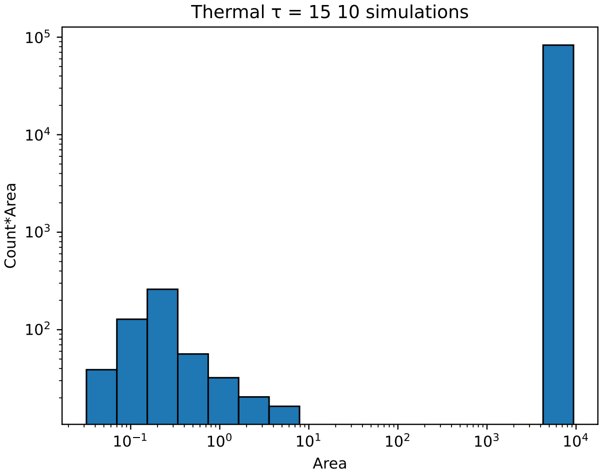


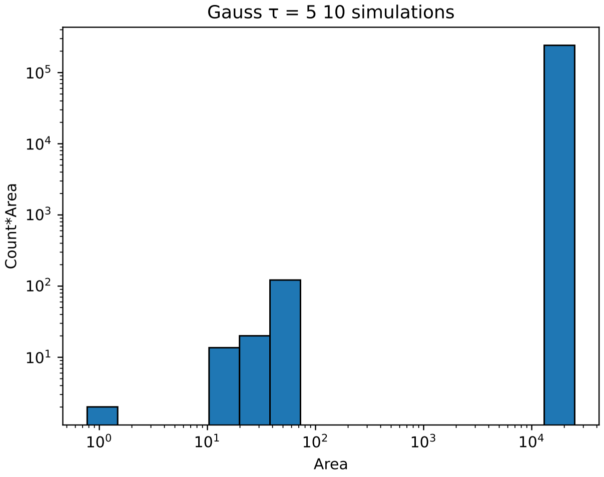
τ = 15

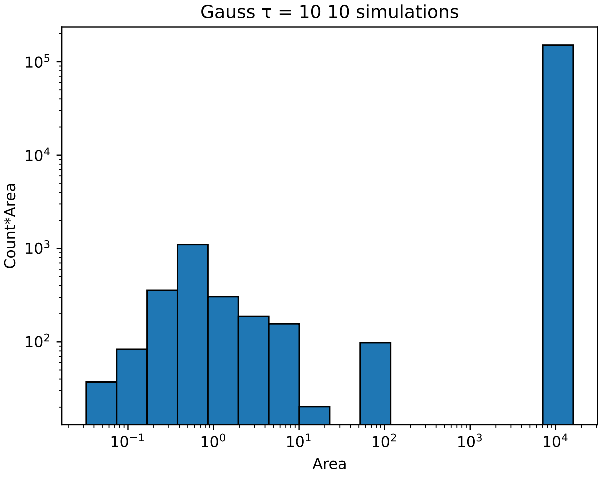
Distribution over areas

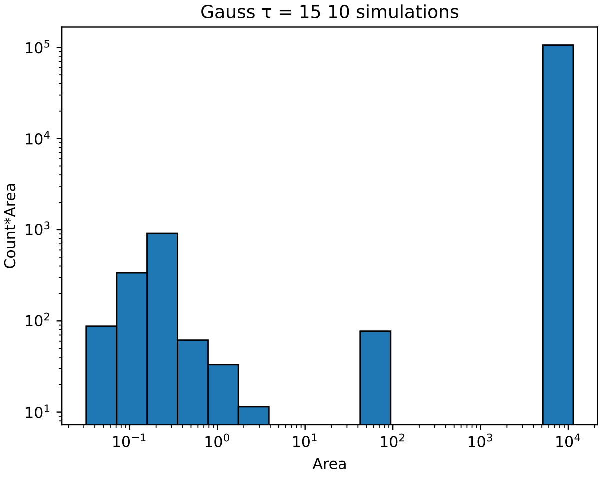




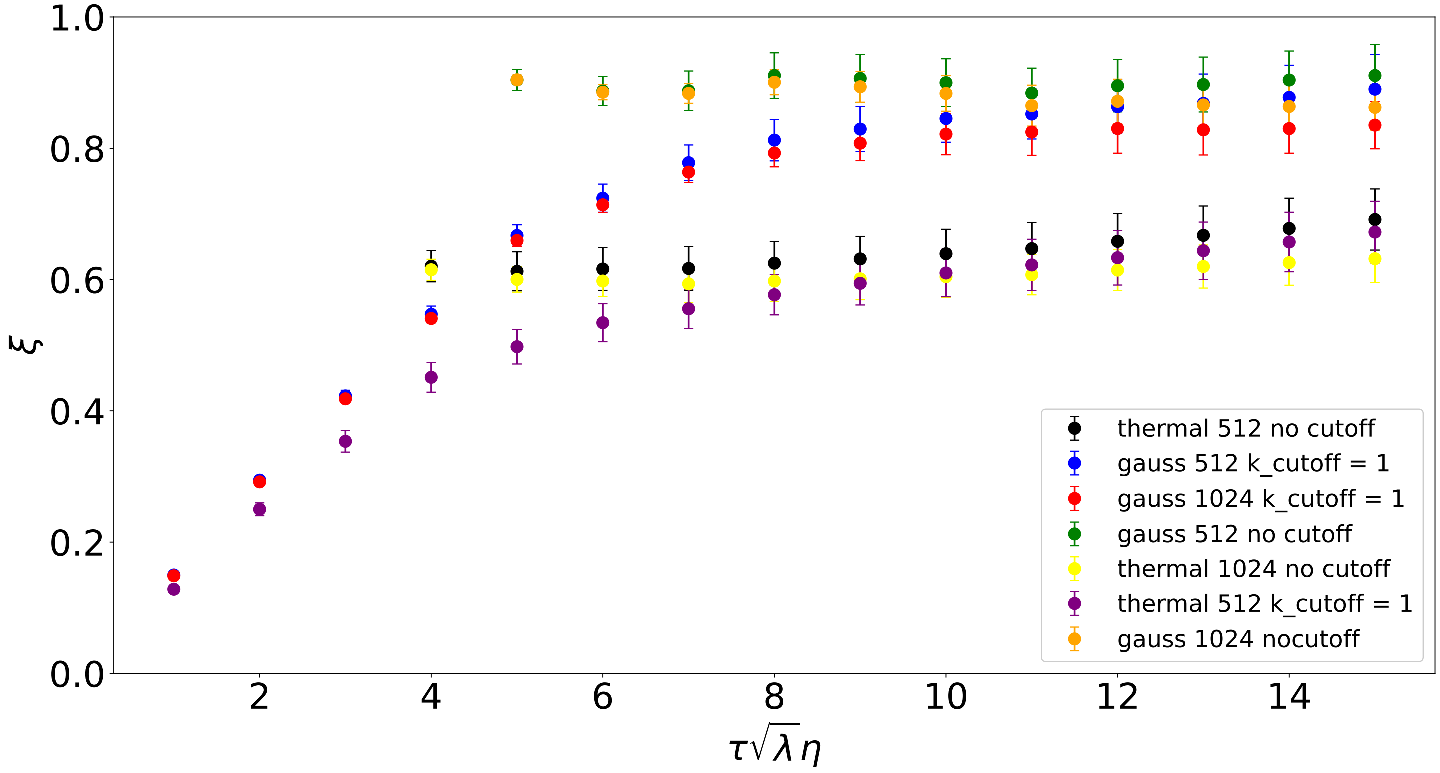








Scaling parameter



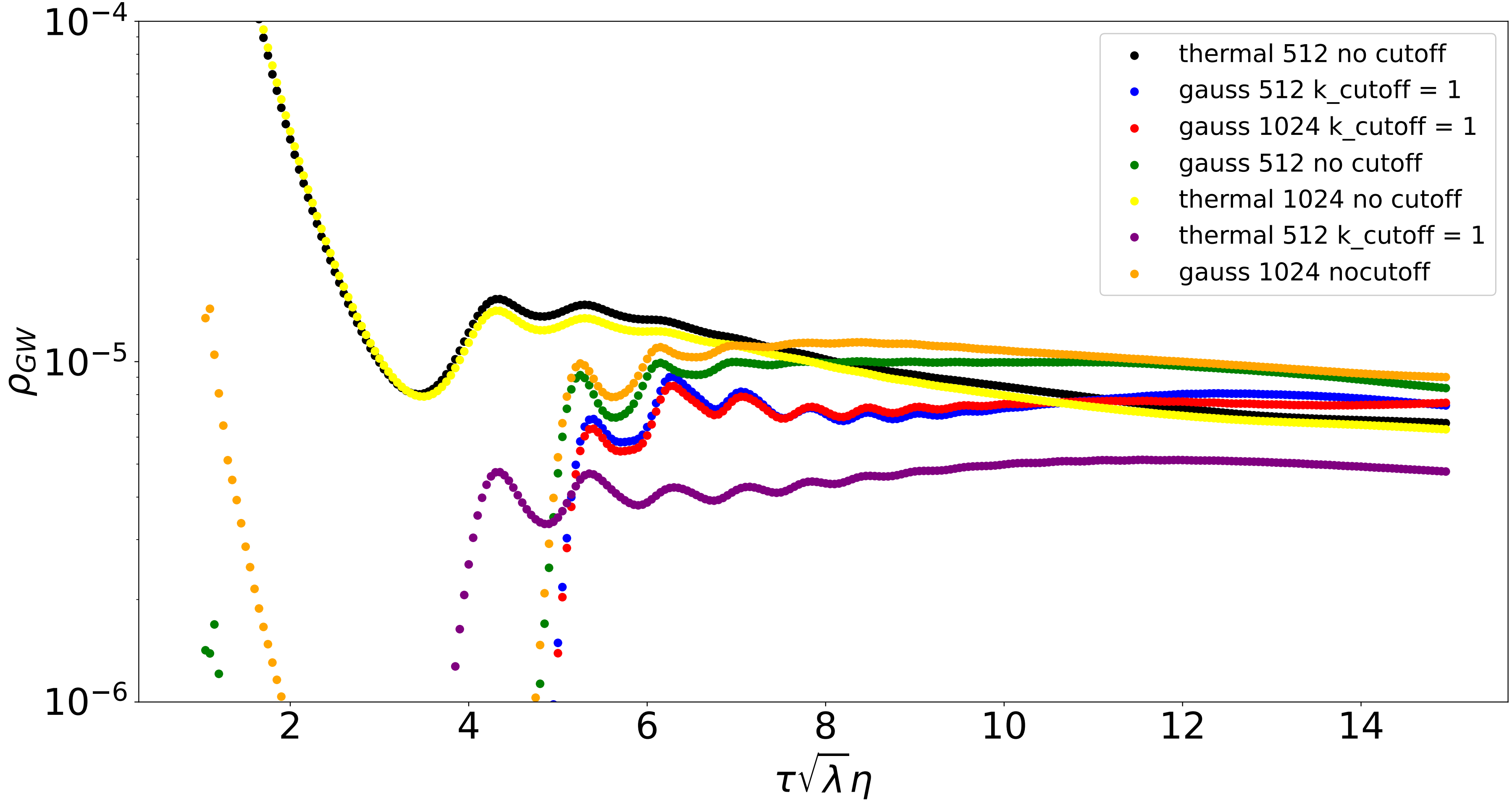
$$\tau_f \simeq \frac{5}{\sqrt{\lambda}\eta}$$

$$\delta_w \simeq 0.06 H^{-1}$$

$\xi \approx 0.8$ - vacuum

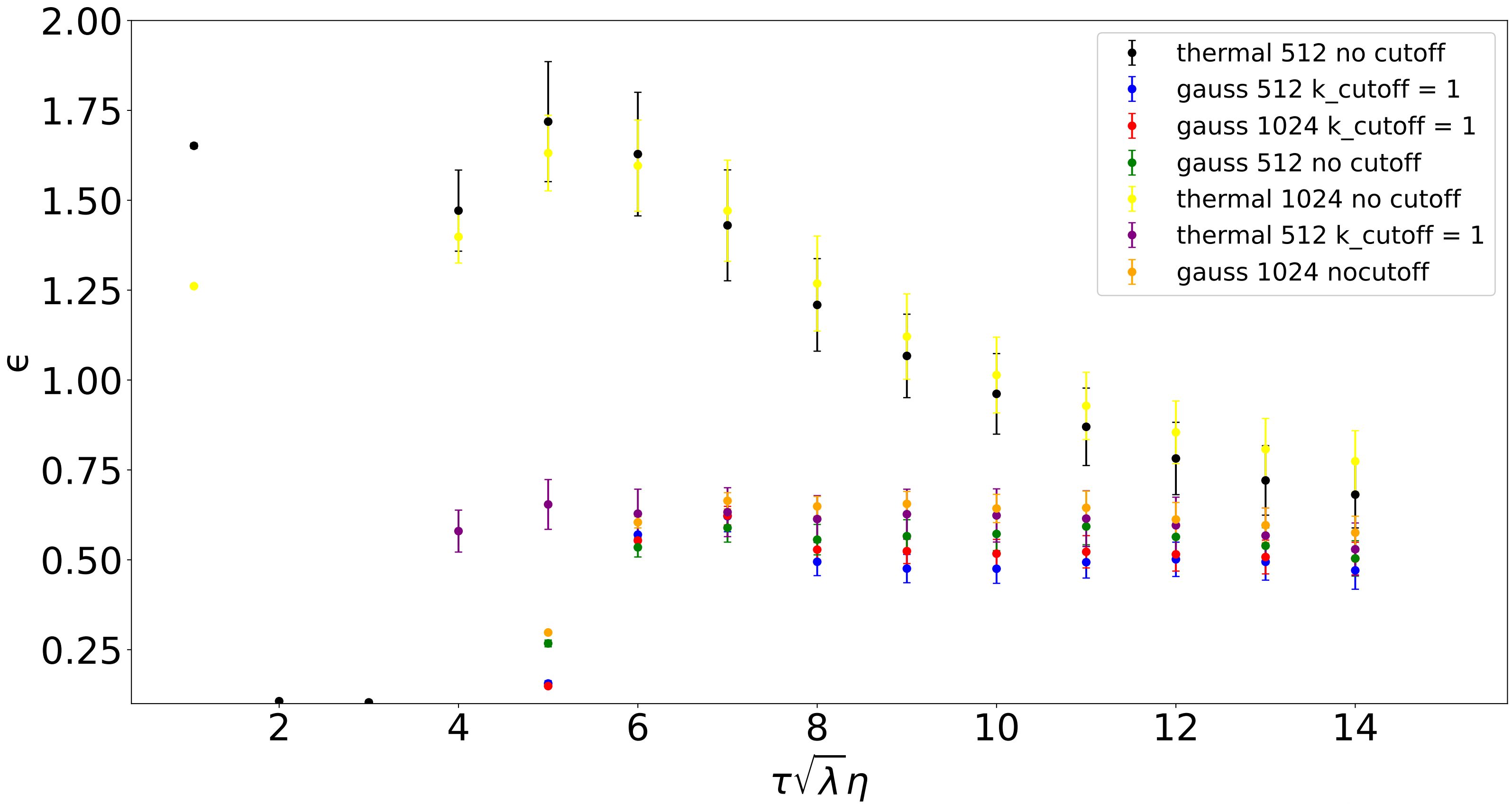
$\xi \approx 0.6$ - thermal

Energy density of GW



$$\epsilon_{gw} = \frac{\rho_{gw}}{G\xi^2\sigma_{gw}^2}$$

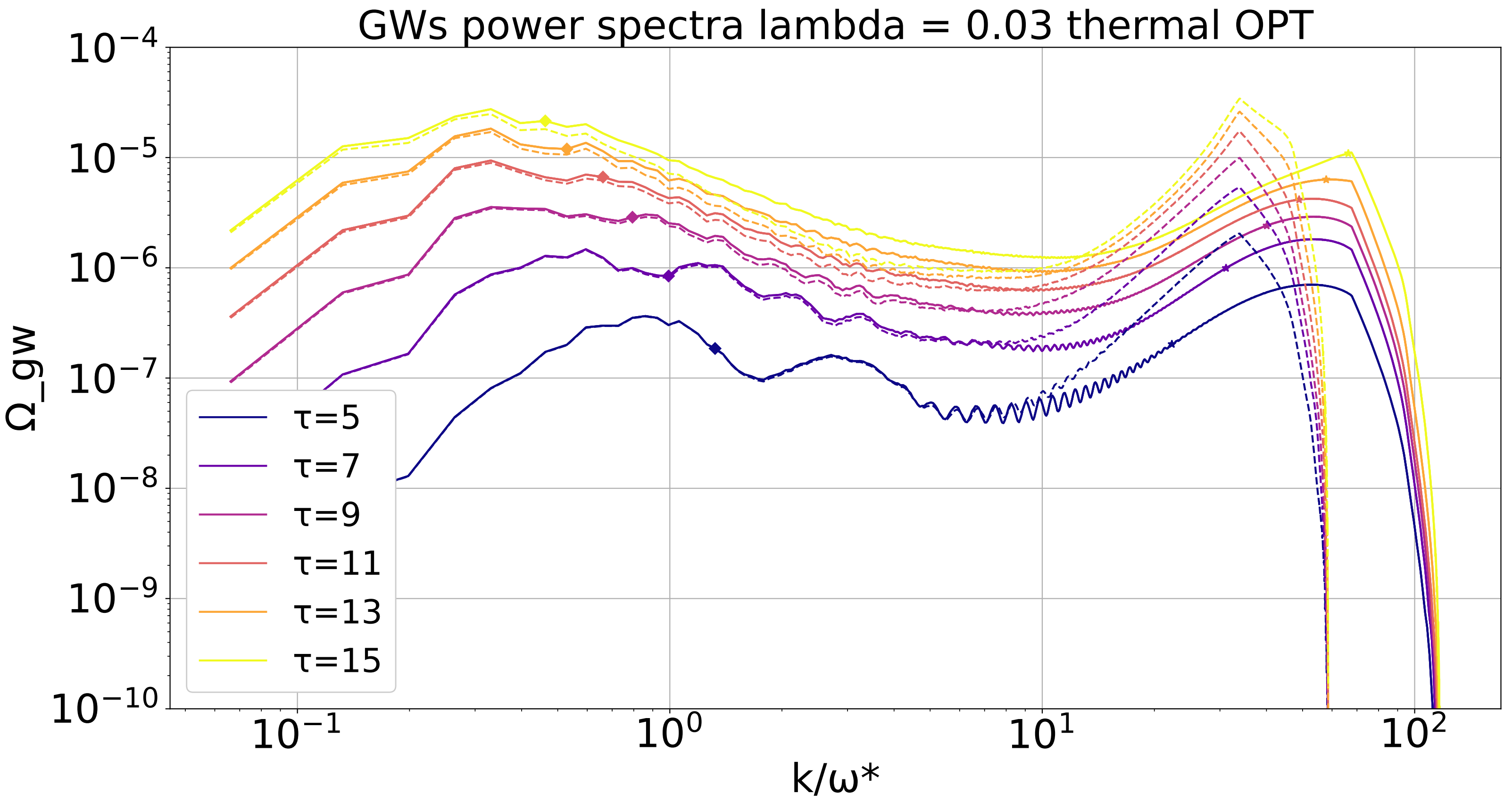
Efficiency of GW production



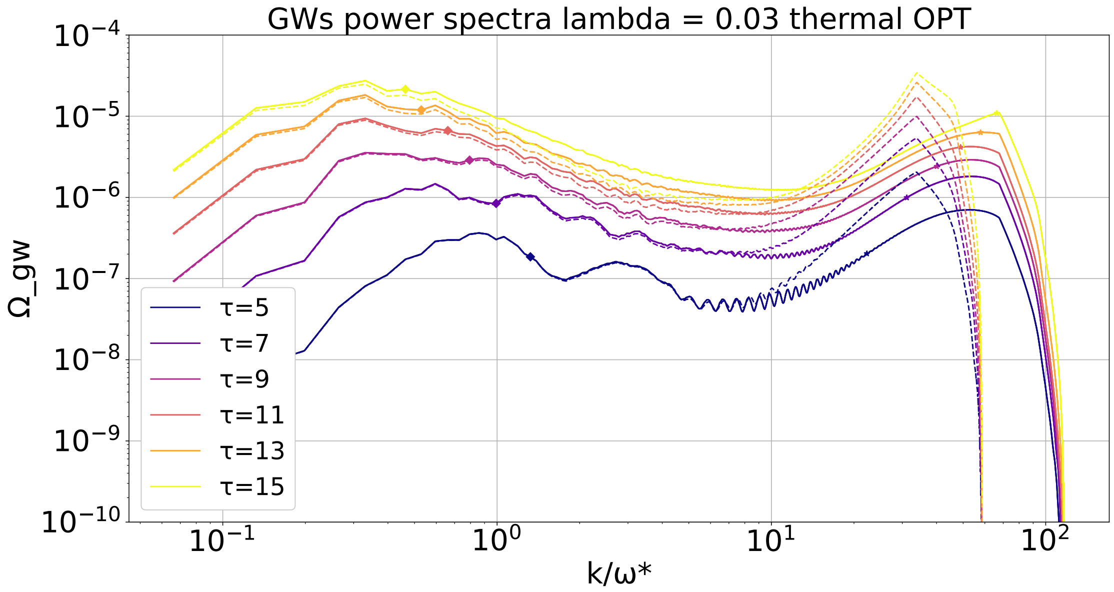
$\epsilon_{gw} \approx 0.5$ (vacuum) $\epsilon_{gw} \approx 0.9$ (thermal)

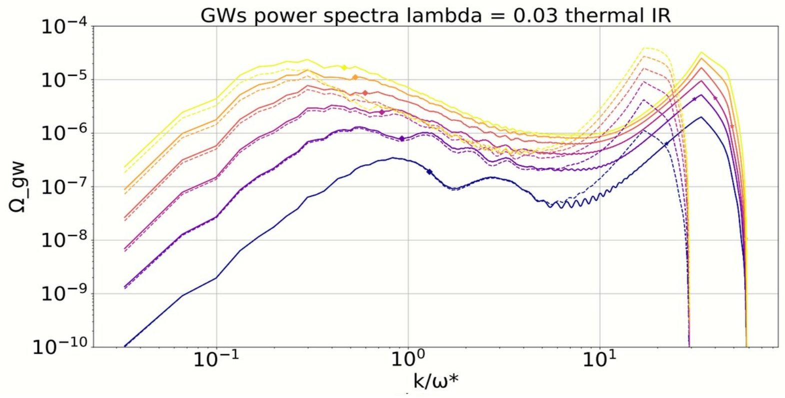
Numerical artifact

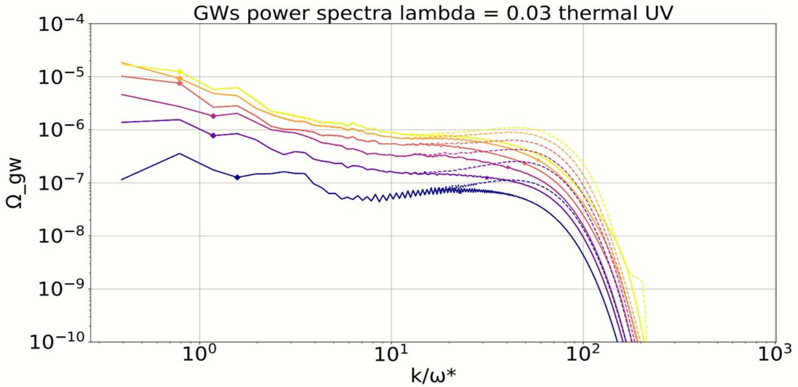
$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm GW}}{{\rm d ln}k}$$



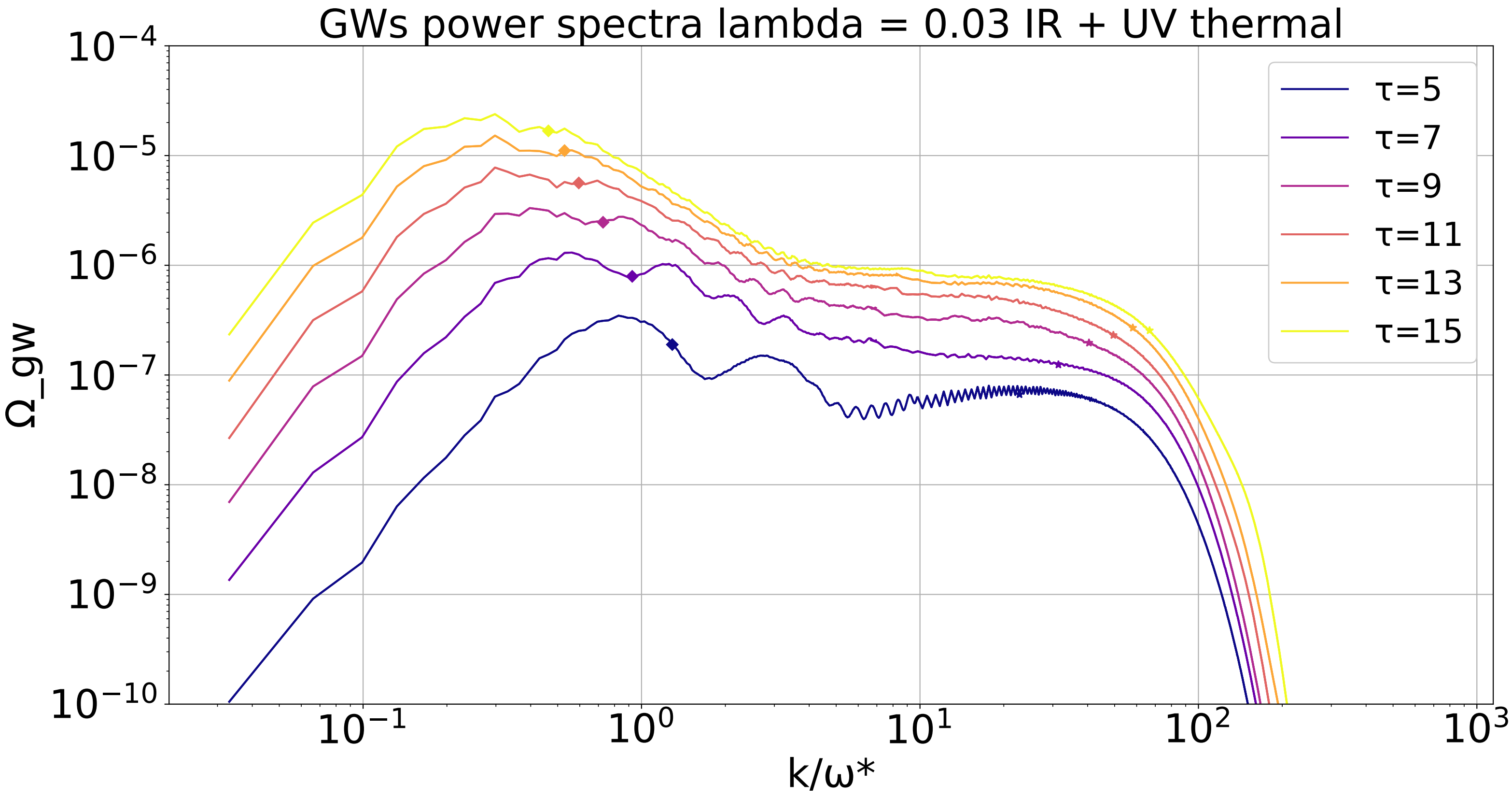
GW spectrum: thermal conditions



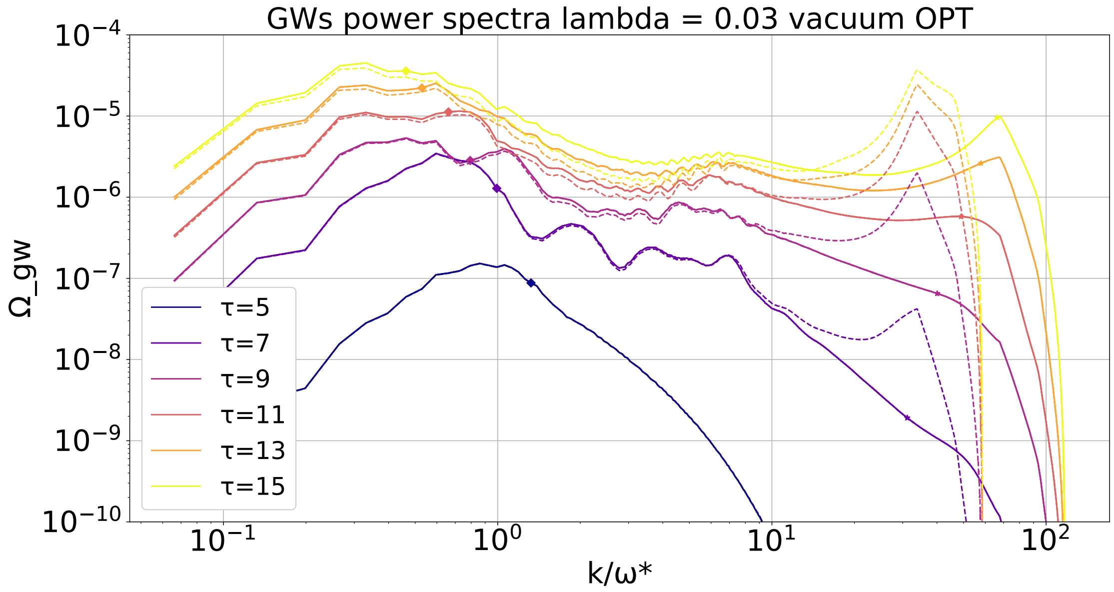


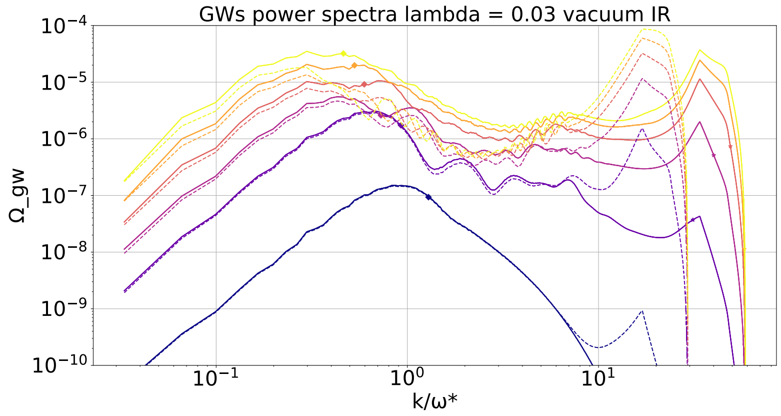


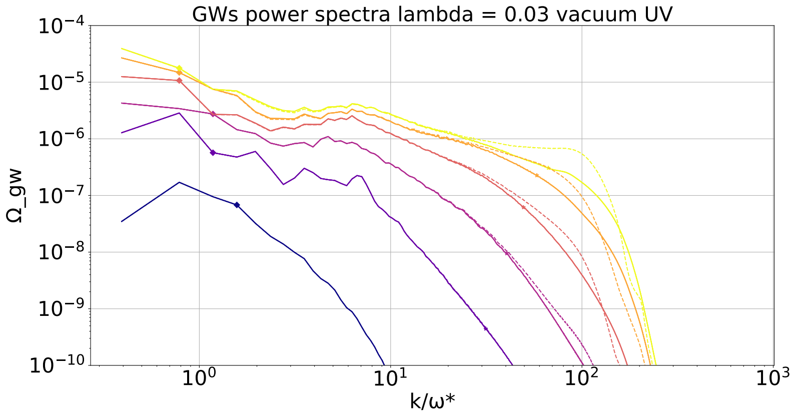
GW spectrum: thermal conditions



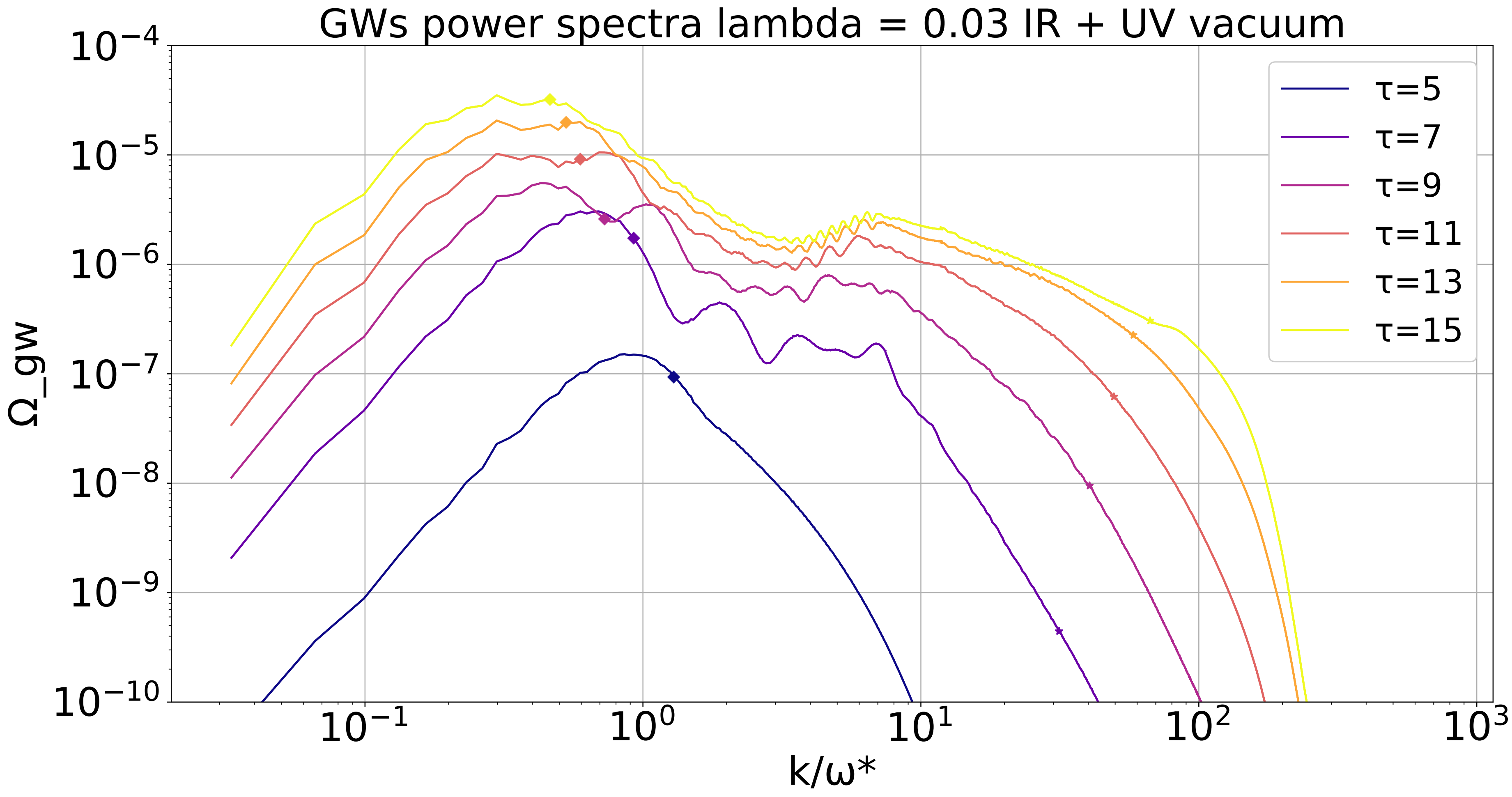
GW spectrum: vacuum conditions







GW spectrum: vacuum conditions



Results

$$\Omega_{gw,peak}(\tau) \approx 4.6\ (7.7) \times 10^{-10} \cdot \left(\frac{H_i}{H(\tau)}\right)^2 \cdot \left(\frac{\eta}{6 \cdot 10^{16}\ \text{GeV}}\right)^4 \qquad \text{thermal (vacuum)}$$

$$\Omega_{gw,peak} h_0^2 \simeq 0.6\ (1) \cdot 10^{-10} \cdot \left(\frac{100\ \text{MeV}}{T_{dec}}\right)^4 \cdot \frac{\sigma_{wall}^2}{(100\ \text{TeV})^6} \cdot \left(\frac{10}{g_*(T_{dec})}\right)^{4/3} \qquad \text{thermal (vacuum)}$$

$$\Omega_{gw} h_0^2 \propto k^3 \qquad k \ll k_{peak}$$

$$\Omega_{gw} h_0^2 \propto k^{-1.3} \qquad k_{peak} < k < k_*$$

$$\Omega_{gw} h_0^2 \simeq \text{const} \qquad k_* < k < 2\pi a/\delta_w$$

$$k\tau \ll 1$$

$$\frac{d\rho_{gw}}{d\ln k} \propto k^3$$

$$f_{peak} = F_{peak} \cdot \frac{a_{dec}}{a_0} \simeq 0.7\ H_{dec} \cdot \frac{a_{dec}}{a_0} \simeq 0.4\ \text{nHz} \left(\frac{T_{dec}}{100\ \text{MeV}}\right) \cdot \left(\frac{10}{g_*(T_{dec})}\right)^{1/3}$$

Summary

- Domain wall network settles to scaling regime: ξ and energy density of GW are constant

- Different value of ξ in the cases of thermal and vacuum conditions may have physical origin

- In the IR part spectrum behaves as expected

- It is possible to get rid of numerical artifact in the UV part of the spectrum

Thank you for your attention!