Quarks-2024
May 19 – May 25

MicrOMEGAs - a tool for calculation in Dark Matter models.

https://lapth.in2p3.fr/micromegas

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Main publictions (CPC) and steps of development:

1.MicrOMEGAs: A Program for calculating the **relic density in the MSSM** *hep-ph/0112278*

2.MicrOMEGAs 2.0: A Program to calculate the **relic density of dark matter in a generic model** *hep-ph/0607059*

3.Dark matter **direct detection rate** in a generic model with micrOMEGAs 2.2 *arXiv:0803.2360*

4.**Indirect search** for dark matter with micrOMEGAs2.4 *arXiv:1004.1092*
with P.Brun, S.Rosier-Lees, P.Salati

5. micrOMEGAs_3: A program for calculating dark matter observables arXiv:*1305.0237*

6. Limits on dark matter proton scattering from **neutrino telescopes** using micrOMEGAs *arXiv:1507.07987*

7.**Collider limits** on new physics within micrOMEGAs4.3 *arXiv:1606.03834*
with D.Barducci, J.Bernon, S.Kraml,U.Laa

8.micrOMEGAs5.0 : Freeze-in. *arXiv:1801.03509*
with A.Goudelis, A. Pukhov, B.Zaldivar

9.micrOMEGAs 6.0: N-component dark matter *arXiv:2312.14894*

Cited > 5000

Similar packages: DarkSusy, MadDM

Included models

MSSM/	
NMSSM/	Next-to-Minimal SuSy Model
CPVMSSM/	MSSM with complex parameters
IDM/	Inert dublet model
LHM/	Little Higgs Model
Z3IDM/	Z^3 model
Z4IDMS/	Z^4 model (2 DM paticles)
Z7M	Z^7 (3 DM particles)
ZpPortal	Z`

./newProgect newModelName - *creates new model*

Model is described by 4 files :vars1.mdl, func1.mdl, prtcles1.mdl lgrng1.mdl disposed in newModelName/work/models subdirectory

Model Files: Inert Doublet Model

Inert Doublet model contains two SU(2)*U(1) doublets

$$H_1 = \begin{pmatrix} 0 \\ <v> + h/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} \tilde{H}+ \\ (\tilde{X} + i \cdot \tilde{H3})/\sqrt{2} \end{pmatrix}$$

The Lagrangian contains only *even* powers of H_2 doublet

$$L = (SM\ terms) + D^\mu H_2^* D_\mu H_2$$

$$-\mu^2 H_2^2 - \lambda_2 H_2^4 - \lambda_3 H_1^2 H_2^2 - \lambda_4 |H_1^* H_2|^2 - \lambda_5 Re[(H_1^* H_2)^2]$$

Because of symmetry $H_2 \rightarrow -H_2$ the lightest of $\tilde{H}^+, \tilde{X}, \tilde{H3}$ is stable

Parameters $\mu, \lambda_3, \lambda_4$ can be expressed in terms of masses

New couplings are λ_2 , $\lambda_L = \lambda_3 + \lambda_4 + \lambda_5$

See details **arXiv:1106.1719**

Model Files: **Free parameters of the model.**

```
Inert Doublet Model
 Variables
 Name | Value       |>  Comment                                    <|
EE    |0.31333      |Electromagnetic coupling constant
SW    |0.474        |sin of the Weinberg angle
MZ    |91.187       |Mass of Z
MHX   |111          |Mass of Inert Doublet Higgs
MH3   |222          |Mass of CP-odd Higgs
MHC   |333          |Mass of charged Higgs
LaL   |0.01         |Coupling in Inert Sector
................................
```

Model Files: Constrained parameter of the model.

```
Inert Doublet
 Constraints
 Name  |> Expression
CW     |sqrt(1-SW^2)
MW     |MZ*CW
Mb     |MbEff(Q)
Mc     |McEff(Q)
mu2    |MHX^2-laL*(2*MW/EE*SW)^2
la3    |2*(MHC^2-mu2)/(2*MW/EE*SW)^2
la5    |(MHX^2-MH3^2)/(2*MW/EE*SW)^2
```

Model Files: Particles of the model

Full Name	P	aP	number	spin2	mass	width	color	aux	> LaTeX(A)
photon	A	A	22	2	0	0	1	G	A
Z boson	Z	Z	23	2	MZ	!wZ	1	G	Z
gluon	G	G	21	2	0	0	8	G	G
W boson	W+	W-	24	2	MW	!wW	1	G	W^+
neutrino	n1	N1	12	1	0	0	1	L	\nu^e
electron	e1	E1	11	1	0	0	1		e
mu-neutrino	n2	N2	14	1	0	0	1	L	\nu^\mu
muon	e2	E2	13	1	Mm	0	1		\mu
tau-neutrino	n3	N3	16	1	0	0	1	L	\nu^\tau
tau-lepton	e3	E3	15	1	Mt	0	1		\tau
u-quark	u	U	2	1	0	0	3		u
d-quark	d	D	1	1	0	0	3		d
c-quark	c	C	4	1	Mc	0	3		c
s-quark	s	S	3	1	Ms	0	3		s
t-quark	t	T	6	1	Mtop	wtop	3		t
b-quark	b	B	5	1	Mb	0	3		b
Higgs	h	h	25	0	Mh	!wh	1		h
odd Higgs	~H3	~H3	36	0	MH3	!wH3	1		(H3)
Charged Higgs	~H+	~H-	37	0	MHC	!wHC	1		(H+)
second Higgs	~X	~X	35	0	MHX	!wHX	1		(X)

Names of particles of odd sector are started with tilde ~

Model Files: Feynman rules

```
Inert Dublet
 Lagrangian
P1 |P2 |P3 |P4 |>   Factor          <|> dLagrangian/ dA(p1) dA(p2)dA(p3)
A  |W+ |W- |    |-EE                 |m3.p2*m1.m2-m1.p2*m2.m3- …...
A  |~H+|~H-|    |EE                  |m1.p3-m1.p2
B  |b  |A  |    |EE/3                |G(m3)
B  |b  |G  |    |GG                  |G(m3)
B  |b  |Z  |    |-EE/(12*CW*SW)      |4*SW^2*G(m3)-3*G(m3)*(1-G5)
B  |b  |h  |    |-EE*Mb/(2*MW*SW)    |1
B  |t  |W- |    |-EE*Sqrt2/(4*SW)    |G(m3)*(1-G5)
W+ |W- |~X |~X |EE^2/(2*SW^2)        |m1.m2
h  |~X |~X |    |-2*MW*SW/EE         |la3+la4+la5
Z  |Z  |~X |~X |EE^2/(2*CW2*SW^2)    |m1.m2
   …...............
```

Generation of new models in micrOMEGAs

./newProgect newModelName - *creates new model*

Model is described by 4 files :vars1.mdl, func1.mdl, prtcles1.mdl lgrng1.mdl disposed in newModelName/work/models subdirectory

Model files can be created by mean of

LanHEP, FeynRules, Sarah

```
let B1= -SW*Z+CW*A, W3=CW*Z+SW*A, W1=('W+'+'W-')/Sqrt2,  W2 =
i*('W+'-'W-')/Sqrt2.
let WW = {W1,  W2 , W3}.
lterm -F**2/4  where F=deriv^mu*WW^nu^a-deriv^nu*WW^mu^a
g*eps^a^b^c*WW^mu^b*WW^nu^c.


Let  hi= {-i*W+.f,  (h+ vev(2*MW/EE*SW)+i*Z.f)/Sqrt2).
let hh = { -i*'~H+',  ('~X'+i*'~H3')/Sqrt2 }.
%                              Hi and HH – conjugated doublets
lterm -la2*(hh*HH)**2. -la3*(hi*H)*(hh*HH). -la4*(hi*HH)*(Hi*hh).
lterm  -la5/2*(hi*HH)**2 + AddHermConj.vv
```

Generation of matrix elements

numout *cc ; // numout – is a type for matrix element in micrOMEGAs.

cc = newProcess(char*Process); // call CalcHEP to calculate symbolically and compile matrix element for given process. For instance

cc = newProcess(“e,E->m,M”);

Matrix element is presented as a shared library and stored in directory

MODEL**/work/so_generated**

Name of library is related to names of particles in the process.
If model library already was generated and the model was not changed, then library is not recompiled.

For example, cross sections of 2->2 processes can be calculated by

cs= cs22(cc,L,Pcm,cos_min,cos_max,&err);

Pcm – momentum in Center of Mass reference frame
cos_min, cos_max - cuts for cosine of scattering angle in the same frame
L=1 in case you have generated codes only for one process. For general case L numerates subprocesses.

So, micrOMEGAs works with a matrix element which is compiled by CalcHEP for given model and passed to micrOMEGAs

Loop induced vertices

micrOMEGAs is able to get numerical coefficients at vertex implemented in Lagrangian and use them to construct loop induced vertexes.
It is implemented for construction of Higgs-gamma-gamma and Higgs-glue-glue vertices which are needed for interface with HIGGSBOUNDS and LILITH for applying LHC constrains on Higgs particle
Also it need for correct calculation of Higgs width.

MicrOMEGAs fuctions

double complex lAAhiggs(Q, HiggsName);
double complex lGGhiggs(Q, HiggsName);

$$\lambda F_{\mu\nu} F^{\mu\nu}$$

double complex lAA5higgs(Q,HiggsName);
double complex lGG5higgs(Q,HiggsName);

$$\lambda F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Q is reserved for the case of off-shell vertex.

For example in IDM Lagrangian

func1.mdl
LAAH |-cabs(lAAhiggs(Mh,"h"))

lgrgn1.mdl
A |A |h | |-4*LAAH |p1.p2*m1.m2-m2.p1*m1.p2

Problem I – breaking of gauge invariance.

Increase of cross sections in unitary gauge and appearance of negative cross sections in Feynman one.

a) Loop correction if mass spectrum in MSSM.

We add corrections to potential according to
M. Carena, M. Quiros and C.E.M. Wagner, "Effective potential methods and the Higgs mass spectrum in the MSSM", Nucl. Phys. B461 (1996) 407.
But it is not enough.

b) Implementation of particle widths breaks gauge invariance.

Calculation of DM relic density (Freeze out)

Ωh^2 = darkOmega(&Xf,fast,Beps,&err) *fast =1 for for fast* calculation

We assume that all decays in odd sectors are fast and,so, in odd sector

$$\frac{N_i}{N_j} = \frac{N_i^{eq}}{N_j^{eq}} \approx exp(-(M_i - M_j)/T) \qquad (1)$$

And solve equation for total abundance $Y=N_{odd}/s$, where s – entropy density,

$$\frac{dY}{ds} = \frac{< v\sigma >}{3H}(Y^2 - Y_{eq}^2)$$

H – Hubble rate

Beps excludes co-annihilation if $exp(\frac{2M_{cdm} - M_1 - M_2}{T}) < Beps$

Co-annihilation: (annihilation of non-DM odd particles)
Problem: double counting caused by t-channel pole: ~m,~χ → h → b,B

Ωh^2 = darkOmegaN(fast, Beps, &err) calculates relic density for N-component DM.
By default DM sectors defined by the number of “~” symbols, But also can be defined by the used. So, one can split a DM thermal sector on 2 and check Eq,(1) ! Here we take into account decay and processes of co-scattering :

Dm1, SM1 → Dm2,SM2

Co-scattering and decays

1) As a rule decay plus co-scattering restore thermal equilibrium inside of DM sector with fixed charge of group of symmetry.

2) Decays are responsible for low temperatures, co-scattering is responsible for thermal equilibrium at high temperatures.

3) Infrared divergent processes with photon and gluon

g, ~b $\rightarrow$ *b, ~χ* (*gluon* + *s-botom decay)*

does not contribute to co-scattering

Width calculation in plasma:

arXiv:1110.2171, 1607.03910, 1207.6082

Kinetic equilibrium.

We assume kinetic equilibrium between DM particles and SM bath.

Indeed massive DM cools faster than SM particles, but kinetic equilibrium is supported by

reactions ~χ,SM->~χ,SM

It can be important when double DM mass is a little bit smaller than (Higgs) resonance.

Not solved in micrOMEGAs. (See darkSUSY)

Feebly interacting Dark Matter. Freeze-in

For DM which has nether been in thermal equilibrium with SM bath.

We get explicitly integrable equation

$$\frac{dY}{ds} = \frac{< v\sigma >}{3H}(\qquad -Y_{eq}^2)$$

Problem with quantum statistics. A proper account of it breaks Lorentz invarience and requires multi-dimension integration.
But the corresponding correction is small (<= 20%) and there is a fast way of approximate solution.

Vev(T) dependence. One can implement dependence of Lagrangian from temperature via parameter “T”

Problem of implementation of thermal masses. At high temperatures particle density is about T^3, the distance between particles is about 1/T, Debye mass are about gT. Thermal masses are taken into account via cut for small (-t) contribution of matrix matrix element.

T-cut

$$\int_{t_{max}}^{0} \frac{dt}{m(T)^2 - t} \approx \int_{t_{max}}^{-m(T)^2 - m_0^2} \frac{dt}{m_0^2 - t}$$

Y = darkOmegaFi(TR, *feebleParticle*, &err);
calculates the DM abundance after summing over all 2 → 2 processes involving particles in the bath B in the initial state and at least one *feebleParticle*_ in the final state.

Low-Temperature Reheating

During cosmic reheating, the inflaton ϕ decays into SM radiation with a total decay width Γ_ϕ. The dynamics of the background is driven by the set of Boltzmann equations for the inflaton energy density ρ_ϕ and the SM entropy density s

$$\frac{d\rho_\phi}{dt} + 3H\rho_\phi = -\Gamma_\phi \rho_\phi$$

$$\frac{ds}{dt} + 3Hs = \frac{\Gamma_\phi}{\rho}{}_\phi T$$

$$H^2 = \frac{\rho_\phi + \rho_R}{3M_P}$$

The evolution of the DM number density n can be tracked by the Boltzmann equation

$$\frac{dn}{dt} + 3Hn = - < \sigma v > (n^2 - n_{eq}^2$$

darkOmegaInflDecay(HI ,Γ, Beps,&aend , &err)