Stable Nonsingular cosmologies in Galileons with curvature and torsion

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Outline

1. Introduction: Galileons (Horndeski) on a curved spacetime (without Torsion)

2. Galileons on a spacetime with both, curvature and torsion

   - **why is this interesting? ->** besides NEC violation, **break the No-Go theorems**

   … The No-Go theorems are accidental to the assumptions (e.g. torsionless)

   -> Can obtain „Healthy“ modes (at linear order)

3. Perturbations

4. Status of the No-Go Theorems: How to break them

   a. Example, a healthy bounce (at least at linear order)

- S. Mironov and M. V-V, *“Stability of nonsingular cosmologies in galileon models with torsion: A no-go theorem for eternal subluminality,”* Phys. Rev. D, vol. 109, p. 044073, Feb 2024.
- S. Mironov and M. V-V, *“Quartic Horndeski-Cartan theories in a FLRW universe,”* Phys. Rev. D, vol. 108, no. 2, p. 024057, 2023.
- S. Mironov and M. V-V, *“Healthy Horndeski gravities with torsion”* 2405.08673

1. Introduction:

Galileons on a spacetime

without Torsion

1. Introduction: Galileons on a spacetime without Torsion

On top of GR, $\int \mathrm{d}^4x\, \sqrt{-g}\, R$

consider four general functions $G_2(\phi, X)\,,\ G_3(\phi, X)\,,\ G_4(\phi, X)\,,\ G_5(\phi, X)$

with $X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

Horndeski/ Generalized Galileons: Lorentz invariant combinations

of

$$R \qquad \Big(\nabla_\mu \nabla_\nu \phi\Big)^p \qquad p \leq 3$$

With coefficients $G_2(\phi, X)\,,\ G_3(\phi, X)\,,\ G_4(\phi, X)\,,\ G_5(\phi, X)$

1. Introduction: Galileons on a spacetime without Torsion

Horndeski/ Generalized Galileons:

$$\mathcal{S} = \int \mathrm{d}^4x \sqrt{-g} \Big(G_2 - G_3 \nabla_\mu \nabla^\mu \phi + \underline{G_4(\phi, X) R} + G_{4,X} \left((\nabla_\mu \nabla^\mu \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right)$$
$$+ G_5 G^{\mu\nu} \nabla_\mu \nabla_\nu \phi - \frac{G_{5X}}{6} \big((\nabla_\mu \nabla^\mu \phi)^3 - 3 (\nabla_\mu \nabla^\mu \phi) (\nabla_\nu \nabla_\rho \phi) \nabla^\nu \nabla^\rho \phi$$
$$+ 2 (\nabla_\mu \nabla_\nu \phi) (\nabla^\nu \nabla^\rho \phi) \nabla^\mu \nabla_\rho \phi \big) \Big)$$

Notation: $G_{4,X} = \partial G_4 / \partial X$, (-,+,+,+).

1. Introduction: Galileons on a curved spacetime <u>without</u> Torsion

Horndeski/ Generalized Galileons:

- No Ostrogradsky-Ghost (Horndeski, 1974)
- *(No)* NEC (See *e.g.* Rubakov, 2014) - (*No P-H. Non-singular cosmologies*)
- Generality
- Inspired by the low energy effective theory of DGP model (A. Nicolis, R. Rattazzi, and E. Trincherini, 2009).
- NO-GO (Libanov, Mironov and Rubakov, 2016): No nonsingular, non-ghosty, stable cosmological solution (provided the general case, with no specific asymptotics)

There are <u>**Global stability issues**</u>

2. Galileons on a spacetime

with Curvature and Torsion

Why? -> to obtain „Healthy“ modes (at linear order)

2. Galileons on a spacetime with Torsion

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right) \qquad \nabla \to \tilde{\nabla}\,, \qquad \tilde{\Gamma}^{\nu}_{\mu\lambda} \neq \tilde{\Gamma}^{\nu}_{\lambda\mu}$$

Horndeski/ Generalized Galileons with Torsion: Lorentz invariant combinations of

$$\tilde{R} \qquad (\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi)^p \qquad p \leq 3 \qquad \left[\tilde{\nabla}_\mu, \tilde{\nabla}_\nu\right] \phi \neq 0$$

With coefficients

$$G_2(\phi, X)\,,\ G_3(\phi, X)\,,\ G_4(\phi, X)\,,\ G_5(\phi, X)$$

2. Galileons on a spacetime <u>with</u> Torsion.

- Example: **Quartic Galileons with Torsion**

$$\mathcal{S}_4 = \int \mathrm{d}^4x \sqrt{-g} \left(G_4(\phi, X)\, R + G_{4,X} \left((\nabla_\mu \nabla^\mu \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right) \right)$$

$$\nabla \to \tilde{\nabla} \,,$$

$G_{4,X} \quad (\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi)^2$?

$$G_{4,X} \left(\left(\tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi \right)^2 + c \left(\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi \right) \tilde{\nabla}^\mu \tilde{\nabla}^\nu \phi + s \left(\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi \right) \tilde{\nabla}^\nu \tilde{\nabla}^\mu \phi \right) , \qquad c + s = -1$$

c parameterises a family of theories with <u>different dynamics</u>.

- S. Mironov and M. Valencia-Villegas, Phys. Rev. D **108**, 024057, 2023

2. Galileons on a spacetime with Torsion.

- **The Lagrangian for this talk**:

$$\mathcal{S} = \int \mathrm{d}^4x \, (\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5) \, ,$$

$$\mathcal{L}_2 = G_2 \, ,$$

$$\mathcal{L}_3 = -G_3 \, \tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi \, ,$$

$$\mathcal{L}_4 = G_4 \, \tilde{R} + G_{4,X} \left(\left(\tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi \right)^2 - \left(\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi \right) \tilde{\nabla}^\nu \tilde{\nabla}^\mu \phi \right)$$

$$\mathcal{L}_5 = G_5 \, \tilde{G}^{\mu\nu} \, \tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi - \frac{1}{6} G_{5,\,X} \Big((\tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi)^3 + (\tilde{\nabla}_\nu \tilde{\nabla}_\rho \phi) \Big(2 (\tilde{\nabla}^\mu \tilde{\nabla}^\nu \phi) \tilde{\nabla}^\rho \tilde{\nabla}_\mu \phi - 3 (\tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi) \tilde{\nabla}^\rho \tilde{\nabla}^\nu \phi \Big) \Big)$$

Why is this Action interesting?… besides NEC violation

1. **Break the No-Go: More Mixing with torsion perturbations breaks the link between the scalar and tensor sectors**

2. Galileons on a spacetime with Torsion.

- **Torsion in the metric (second order) formalism:**

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}_{\mu\nu} - \tilde{\Gamma}^{\rho}_{\nu\mu}\,, \qquad K^{\rho}{}_{\mu\nu} = -\frac{1}{2}\left(T_{\nu}{}^{\rho}{}_{\mu} + T_{\mu}{}^{\rho}{}_{\nu} + T^{\rho}{}_{\mu\nu}\right)\,,$$

- Assume: connection is not an independent field:

$$\tilde{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} - K^{\rho}{}_{\mu\nu} \qquad \Gamma^{\rho}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(\partial_{\mu}g_{\nu\sigma} + \partial_{\nu}g_{\mu\sigma} - \partial_{\sigma}g_{\mu\nu}\right)$$

$$\tilde{\nabla}_{\mu}V^{\nu} = \nabla_{\mu}V^{\nu} - K^{\nu}{}_{\mu\lambda}V^{\lambda} \qquad \nabla_{\rho}\,g_{\mu\nu} = 0$$

3. Torsionful Galileons about the FLRW background

3. Torsionful Galileons about the FLRW background

- **Linearization: <u>Spatially flat FLRW background</u> (conformal time)**

$$\eta_{\mu\nu}\mathrm{d}x^\mu\,\mathrm{d}x^\nu = a^2(\eta)\left(-\mathrm{d}\eta^2 + \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\right)$$

$$\begin{aligned}\delta g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = a^2(\eta)\,\big(&-2\,\alpha\,\mathrm{d}\eta^2\\ &+2\,(\partial_i B + S_i)\,\mathrm{d}\eta\,\mathrm{d}x^i + (-2\,\psi\,\delta_{ij}\\ &+2\,\partial_i\partial_j E + \partial_i F_j + \partial_j F_i + \boxed{2\,h_{ij}\,)}\,\mathrm{d}x^i\,\mathrm{d}x^j\big)\end{aligned}$$

- Perturbation of contortion:

24 independent components, 8 scalars, 6 vectors, 2 (2-component) tensors

$$\boxed{\begin{aligned}\delta K^{\text{tensor}}_{ij0} &= T^{(1)}_{ij}\\ \delta K^{\text{tensor}}_{ijk} &= \partial_i T^{(2)}_{jk} - \partial_k T^{(2)}_{ji}\end{aligned}}$$

3. Torsionful Galileons about the FLRW background

- Linearization: Notation

The perturbed Horndeski scalar: $\phi = \varphi(\eta) + \Pi$

The perturbed contortion tensor: $K_{\mu\nu\sigma} = {}^0K_{\mu\nu\sigma} + \delta K_{\mu\nu\sigma}$

- Background contortion: with $K_{\mu\nu\sigma} = -K_{\sigma\nu\mu}$ on an isotropic and homogeneous spacetime

$$ {}^0K_{0jk} = x(\eta)\delta_{jk} \qquad {}^0K_{ijk} = y(\eta)\epsilon_{ijk} $$

4. Stability:

A. No-Go theorem in up to quartic Theory (L4)

B. Breaking the No-Go with L5

4. Stability of L4

A. No-Go theorem in up to quartic Theory (L4)

$$\mathcal{S} = \int \mathrm{d}^4x \, (\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5) \, ,$$

$$\mathcal{L}_2 = G_2 \, ,$$

$$\mathcal{L}_3 = -G_3 \, \tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi \, ,$$

$$\mathcal{L}_4 = G_4 \, \tilde{R} + G_{4,X} \left(\left(\tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi \right)^2 - \left(\tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi \right) \tilde{\nabla}^\nu \tilde{\nabla}^\mu \phi \right)$$

$$\mathcal{L}_5 = G_5 \, \tilde{G}^{\mu\nu} \, \tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi - \frac{1}{6} G_{5,X} \left((\tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi)^3 + (\tilde{\nabla}_\nu \tilde{\nabla}_\rho \phi) \left(2 (\tilde{\nabla}^\mu \tilde{\nabla}^\nu \phi) \tilde{\nabla}^\rho \tilde{\nabla}_\mu \phi - 3 (\tilde{\nabla}_\mu \tilde{\nabla}^\mu \phi) \tilde{\nabla}^\rho \tilde{\nabla}^\nu \phi \right) \right)$$

4. Torsionful Galileons about the FLRW background

- **Quadratic Action for tensor sector:** Torsionless (c=0) vs Torsionful

$$\mathcal{S}_{\mathcal{T}} = \int \mathrm{d}\eta\, \mathrm{d}^3p\, \Big(b_1\, (\dot{h}_{ij})^2 + b_2\, \vec{p}^{\,2}(h_{ij})^2 + b_3(h_{ij})^2$$

$$+ \Big(c_1\, \vec{p}^{\,2}(T^{(2)}_{ij})^2 + c_2\, h_{ij}\, T^{(1)}_{ij} + c_3\, \dot{h}_{ij}\, T^{(1)}_{ij} + c_4\, (T^{(1)}_{ij})^2 \Big)\Big)$$

$$c_1\, T^{(2)}_{ij} \equiv 0$$

4. Torsionful Galileons about the FLRW background

- **Final Quadratic Action:**

$$\mathcal{S}_\tau = \int \mathrm{d}\eta\, \mathrm{d}^3x\, a^4 \left[\frac{1}{2\,a^2}\left(\mathcal{G}_\tau \left(\dot{h}_{ij}\right)^2 - \mathcal{F}_\tau (\partial_k\, h_{ij})^2\right)\right]$$

$$\mathcal{S}_s = \int \mathrm{d}\eta\, \mathrm{d}^3x\, a^4 \left(\frac{1}{a^2}\,\mathcal{G}_\mathcal{S}\,\dot{\psi}^2 - \frac{1}{a^2}\,\mathcal{F}_\mathcal{S}\,(\partial_i\psi)^2\right)$$

- One tensor perturbation
- No dynamical vector perturbation
- One scalar perturbation

$$\mathcal{G}_\tau = 2\,\frac{G_4^2}{G_4 + 2\,X\,G_{4,X}}\,,$$
$$\mathcal{F}_\tau = 2\,G_4\,,$$

$$\mathcal{G}_\mathcal{S} = 3\,\mathcal{G}_\tau + \frac{\mathcal{G}_\tau^2\,\Sigma}{\Theta^2}\,, \quad \mathcal{F}_\mathcal{S} = \frac{1}{a^2}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{a\,\mathcal{G}_\tau\,T}{\Theta}\right) - \mathcal{F}_\tau$$

$$T = \mathcal{F}_\tau\,(c_g^2 - 2)$$

$$c_g^2 = \mathcal{F}_\tau/\mathcal{G}_\tau$$

Key part,

Follow a similar reasoning as initially proved for a subclass of generalized Galileons in (Libanov, Mironov and Rubakov, 2016) and then extended to the full Horndeski action in (Kobayashi, 2016)…

4. **No-Go theorem:**

For L4 Galileons on a spacetime with torsion the following assumptions for a first order perturbative expansion about FLRW are mutually inconsistent:

I) Nonsingular cosmology: namely, there is a lower bound on the scale factor $a(\eta) > b_1 > 0.$

II) The graviton and the scalar mode are not ghosts and they suffer no gradient instabilities: $\mathcal{G}_\mathcal{T} > 0\,, \mathcal{F}_\mathcal{T} > 0\,, \mathcal{F}_\mathcal{S} > 0\,, \mathcal{G}_\mathcal{S} > 0.$

III) ***The graviton is always sub/ luminal:*** $\boxed{(c_g)^2 \leq 1}$

IV) ***...***

4. **No-Go theorem:**

…

IV) There is a lower bound $\mathcal{F}_\tau(\eta) > b_2 > 0$ *as* $\eta \to \pm\infty$ *(no „Strong gravity" at linear order (Ageeva, Petrov and Rubakov, 2021)).*

V) Θ *Vanishes at most a finite amount of times (To cover generic theories not defined by the equation* $\Theta \equiv 0$ *(Mironov and Shtennikova, 2023. Also, talk))*

4. Stability:

The argument: take $\mathcal{F}_{\mathcal{S}} = \frac{1}{a^2}\frac{dN}{d\eta} - \mathcal{F}_\tau > 0$

$$N = \frac{a\,\mathcal{G}_\tau\,T}{\Theta}$$

With Torsion, L4 / (without Torsion)

$$T = \mathcal{F}_\tau\,\left(c_g^2 - 2\right) < 0 \qquad\qquad T = \mathcal{G}_\tau > 0$$

Argument With Torsion: (I)-(III) imply

$$N \neq 0$$

Because Θ is a regular function of *H and* ϕ

4. Stability:

$$\boxed{N \neq 0}$$

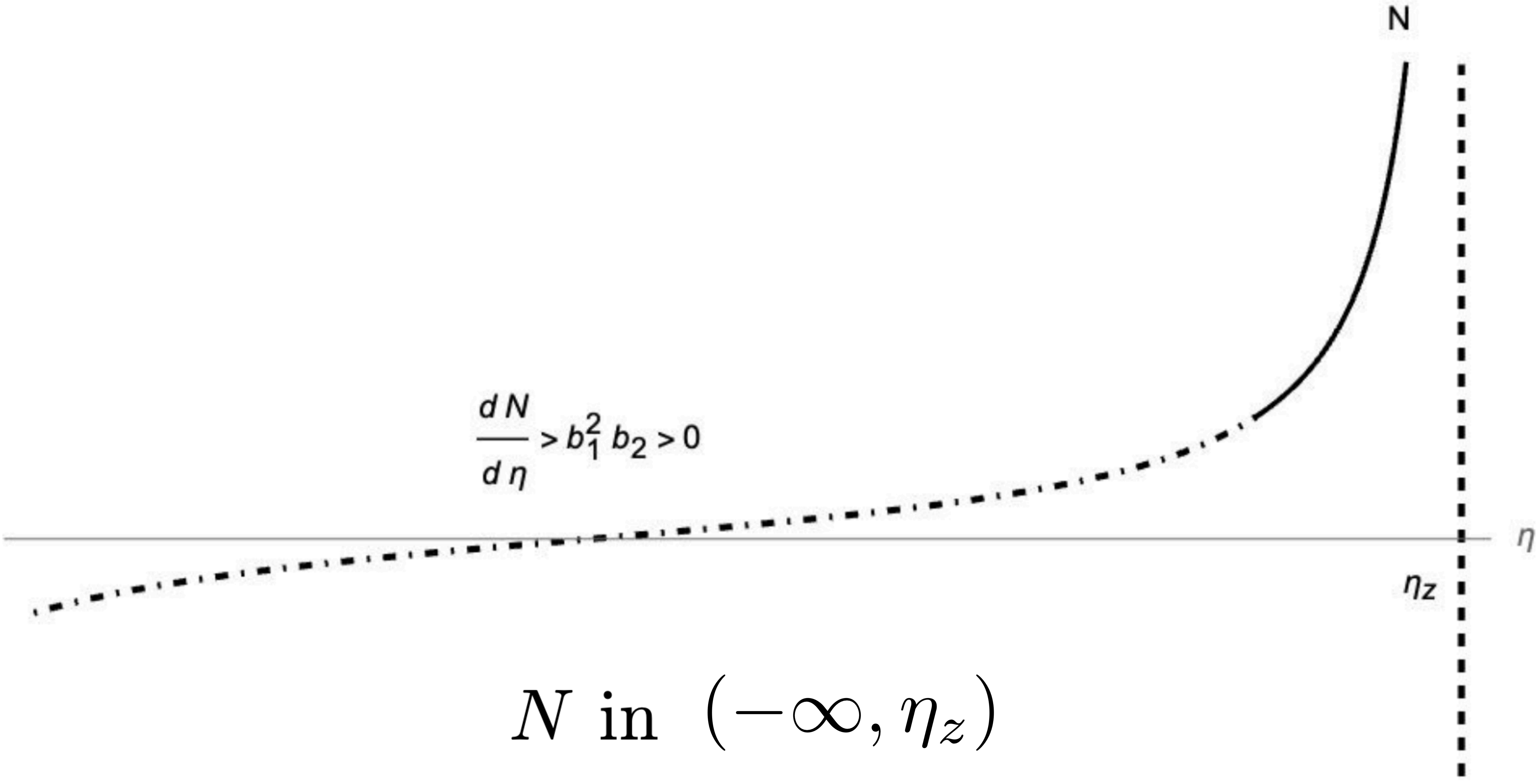
On the other hand $\mathcal{F}_\tau > 0$ $\qquad \mathcal{F}_\mathcal{S} = \frac{1}{a^2}\frac{dN}{d\eta} - \mathcal{F}_\tau > 0$

$$\boxed{N = \frac{a\,\mathcal{G}_\tau\,T}{\Theta}} \quad \begin{cases} \dfrac{\mathrm{d}N}{\mathrm{d}\eta} > a^2\,\mathcal{F}_\tau > 0 \\[2ex] \boxed{\dfrac{\mathrm{d}N}{\mathrm{d}\eta} > b_1^2\,b_2 > 0} \\ \boxed{\eta \to \pm\infty} \end{cases}$$

4. Stability:

$$\frac{\mathrm{d}N}{\mathrm{d}\eta} > b_1^2\, b_2 > 0$$

$$\eta \to \pm\infty$$



$$N \neq 0\,?$$

$$N \text{ in } (-\infty, \eta_z)$$

4. Stability of L4

No-Go for nonsingular, all-time stable and <u>sub/ luminal</u> solutions

<u>(In torsionful L4 G. stability -sub/luminality)</u>

S. Mironov and M. V-V,

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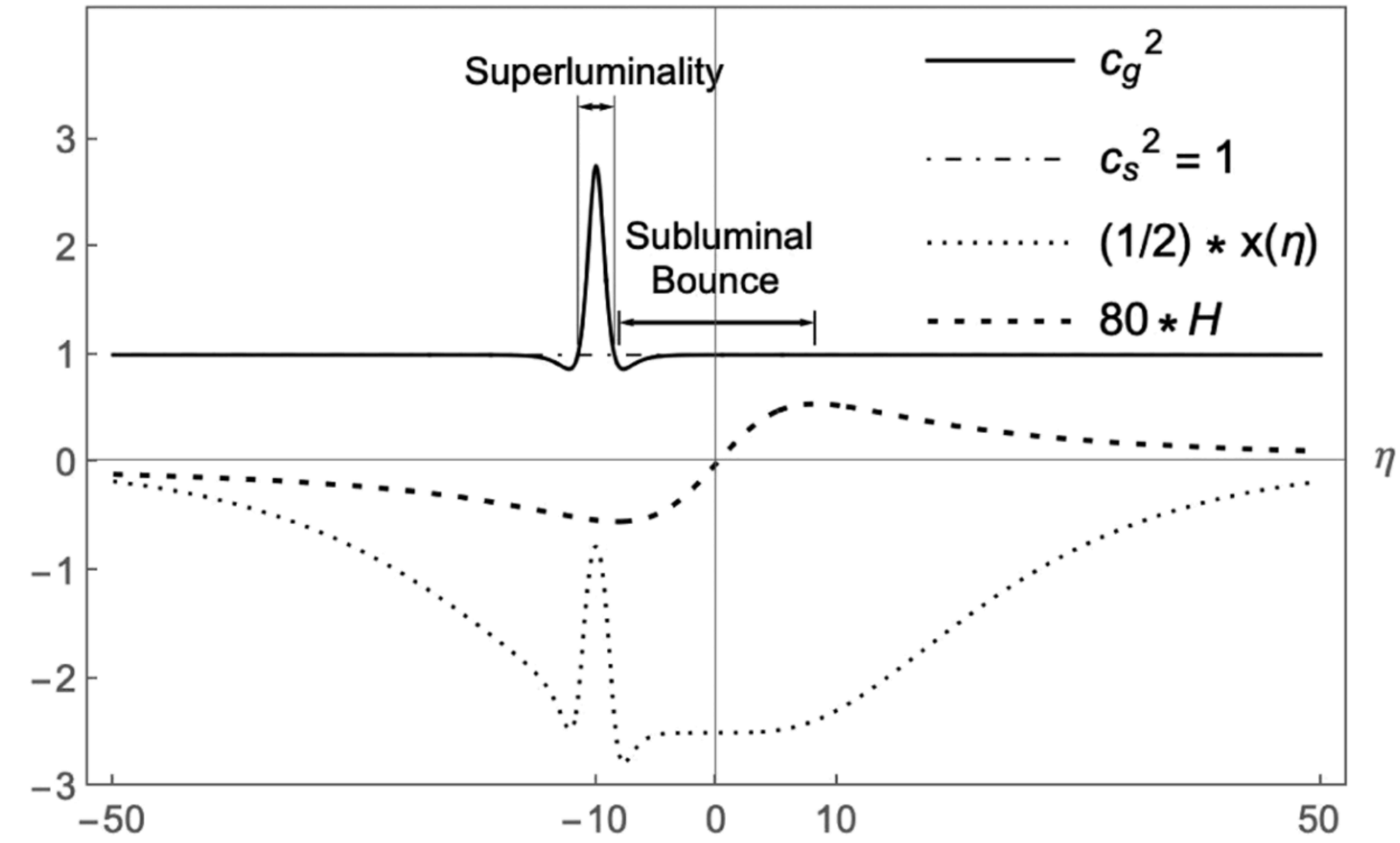


4. Stability of L4

Why a No-Go?

There is a tight relation between the

action for the graviton <-> action for the scalar

$$\mathcal{G}_{\mathcal{S}} = 3\,\mathcal{G}_{\tau} + \frac{\mathcal{G}_{\tau}^{2}\,\Sigma}{\Theta^{2}}\,,\quad \mathcal{F}_{\mathcal{S}} = \frac{1}{a^{2}}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{a\,\mathcal{G}_{\tau}\,T}{\Theta}\right) - \mathcal{F}_{\tau}$$

$$T = \mathcal{F}_{\tau}\,(c_g^2 - 2) \qquad c_g^2 = \mathcal{F}_{\tau}/\mathcal{G}_{\tau}$$

$$\mathcal{G}_{\tau} > 0\,, \mathcal{F}_{\tau} > 0\,, \mathcal{F}_{\mathcal{S}} > 0\,, \mathcal{G}_{\mathcal{S}} > 0.$$

4. Stability:

B. Breaking the No-Go with L5

- Modified graviton

4. Stability of L5

- **The Lagrangian to break the No-Go:**

$$\mathcal{S} = \int \mathrm{d}^4x \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_2 = G_2\,,$$

$$\mathcal{L}_3 = -G_3\,\tilde{\nabla}_\mu\tilde{\nabla}^\mu\phi\,,$$

$$\mathcal{L}_4 = G_4\,\tilde{R} + G_{4,X}\left(\left(\tilde{\nabla}_\mu\tilde{\nabla}^\mu\phi\right)^2 - \left(\tilde{\nabla}_\mu\tilde{\nabla}_\nu\phi\right)\tilde{\nabla}^\nu\tilde{\nabla}^\mu\phi\right)$$

$$\mathcal{L}_5 = G_5\,\tilde{G}^{\mu\nu}\,\tilde{\nabla}_\mu\tilde{\nabla}_\nu\phi - \frac{1}{6}G_{5,X}\Big((\tilde{\nabla}_\mu\tilde{\nabla}^\mu\phi)^3 + (\tilde{\nabla}_\nu\tilde{\nabla}_\rho\phi)\Big(2(\tilde{\nabla}^\mu\tilde{\nabla}^\nu\phi)\tilde{\nabla}^\rho\tilde{\nabla}_\mu\phi - 3(\tilde{\nabla}_\mu\tilde{\nabla}^\mu\phi)\tilde{\nabla}^\rho\tilde{\nabla}^\nu\phi\Big)\Big)$$

4. Stability of L5

- **Quadratic action for the Tensor sector with L5:**

more mixing of perturbations of torsion with perturbations of the metric

$$
\begin{aligned}
\mathcal{S}_\tau = \int \mathrm{d}\eta\, \mathrm{d}^3p\, \Big(& b_1\, (\dot{h}_{ij})^2 + b_2\, \vec{p}^{\,2} (h_{ij})^2 + b_3 (h_{ij})^2 \\
& + \Big(c_1\, \vec{p}^{\,2} (T^{(2)}_{ij})^2 + c_2\, h_{ij}\, T^{(1)}_{ij} + c_3\, \dot{h}_{ij}\, T^{(1)}_{ij} + c_4\, (T^{(1)}_{ij})^2 \Big) \\
& + \vec{p}^{\,2} \Big(d_1\, T^{(1)}_{ij} + d_2\, \dot{h}_{ij} + d_3\, h_{ij} \Big) T^{(2)}_{ij} + d_4\, \vec{p}^{\,2} h_{ij}\, T^{(1)}_{ij} \Big)
\end{aligned}
$$

4. Stability of L5

- **Quadratic action for the Tensor sector with L5:**

$$\mathcal{S}_\tau = \int \mathrm{d}\eta\, \mathrm{d}^3 p \,\Big(b_1\,(\dot h_{ij})^2 + b_2\,\vec p^{\,2}(h_{ij})^2 + b_3(h_{ij})^2$$
$$+\Big(c_1\,\vec p^{\,2}(T^{(2)}_{ij})^2 + c_2\,h_{ij}\,T^{(1)}_{ij} + c_3\,\dot h_{ij}\,T^{(1)}_{ij} + c_4\,(T^{(1)}_{ij})^2\Big)$$
$$+\vec p^{\,2}\Big(d_1\,T^{(1)}_{ij} + d_2\,\dot h_{ij} + d_3\,h_{ij}\Big)\,T^{(2)}_{ij} + d_4\,\vec p^{\,2} h_{ij}\,T^{(1)}_{ij}\Big)$$

$$c_1\,T^{(2)}_{ij} \equiv 0$$

$$T^{(2)}_{ij} = -\frac{1}{2\,c_1}\Big(d_1\,T^{(1)}_{ij} + d_2\,\dot h_{ij} + d_3\,h_{ij}\Big) \rightarrow -\frac{1}{4\,c_1}\big(-4\,c_1\,c_4 + \vec p^{\,2}\,d_1^2\big)\,(T^{(1)}_{ij})^2$$

$$T^{(1)}_{ij} = \frac{1}{f_2 + \vec p^{\,2}\,f_3}\Big(\big(2\,c_1\,c_3 - \vec p^{\,2}\,d_1\,d_2\big)\,\dot h_{ij} + \big(2\,c_1\,c_2 - \vec p^{\,2}\,(d_1\,d_3 - 2\,c_1\,d_4)\big)\,h_{ij}\Big)$$

4. Stability of L5

- **Modified graviton:**

$$\mathcal{G}_\tau = \frac{\bar{\mathcal{G}}_\tau}{f_2 + \vec{p}^{\,2} f_3} \qquad \mathcal{F}_\tau = \mathcal{F}_\tau(p^2)$$

- **Modified Scalar Sector:**

$$N = \frac{a\,\mathcal{G}_\tau\,T}{\Theta} \quad \longrightarrow \quad N =: \frac{a\,\bar{\mathcal{G}}_S\,T}{\Theta}$$

$$\mathcal{F}_S = \frac{1}{a^2}\frac{dN}{d\eta} - \mathcal{F}_\tau > 0 \quad \longrightarrow \quad \mathcal{F}_S = \frac{1}{a^2}\frac{\mathrm{d}N}{\mathrm{d}\eta} - \bar{\mathcal{F}}_S$$

$$T = \mathcal{F}_\tau\,(c_g^2 - 2) < 0$$

$$\bar{\mathcal{G}}_S \neq \mathcal{G}_\tau(p^2) \qquad \bar{\mathcal{F}}_S \neq \mathcal{F}_\tau(p^2)$$

$$\mathcal{G}_\tau(\vec{p}^{\,2} = 0) = \bar{\mathcal{G}}_S\,, \quad \mathcal{F}_\tau(\vec{p}^{\,2} = 0) = \bar{\mathcal{F}}_S$$

4. Stability:

$$N =: \frac{a\,\bar{\mathcal{G}}_{\mathcal{S}}\,T}{\Theta}$$

With Torsion, L4 / **With Torsion, L5**

$$\bar{\mathcal{G}}_{\mathcal{S}} = \mathcal{G}_{\tau} > 0$$

$$T = \mathcal{F}_{\tau}\,\left(c_g^2 - 2\right) < 0$$

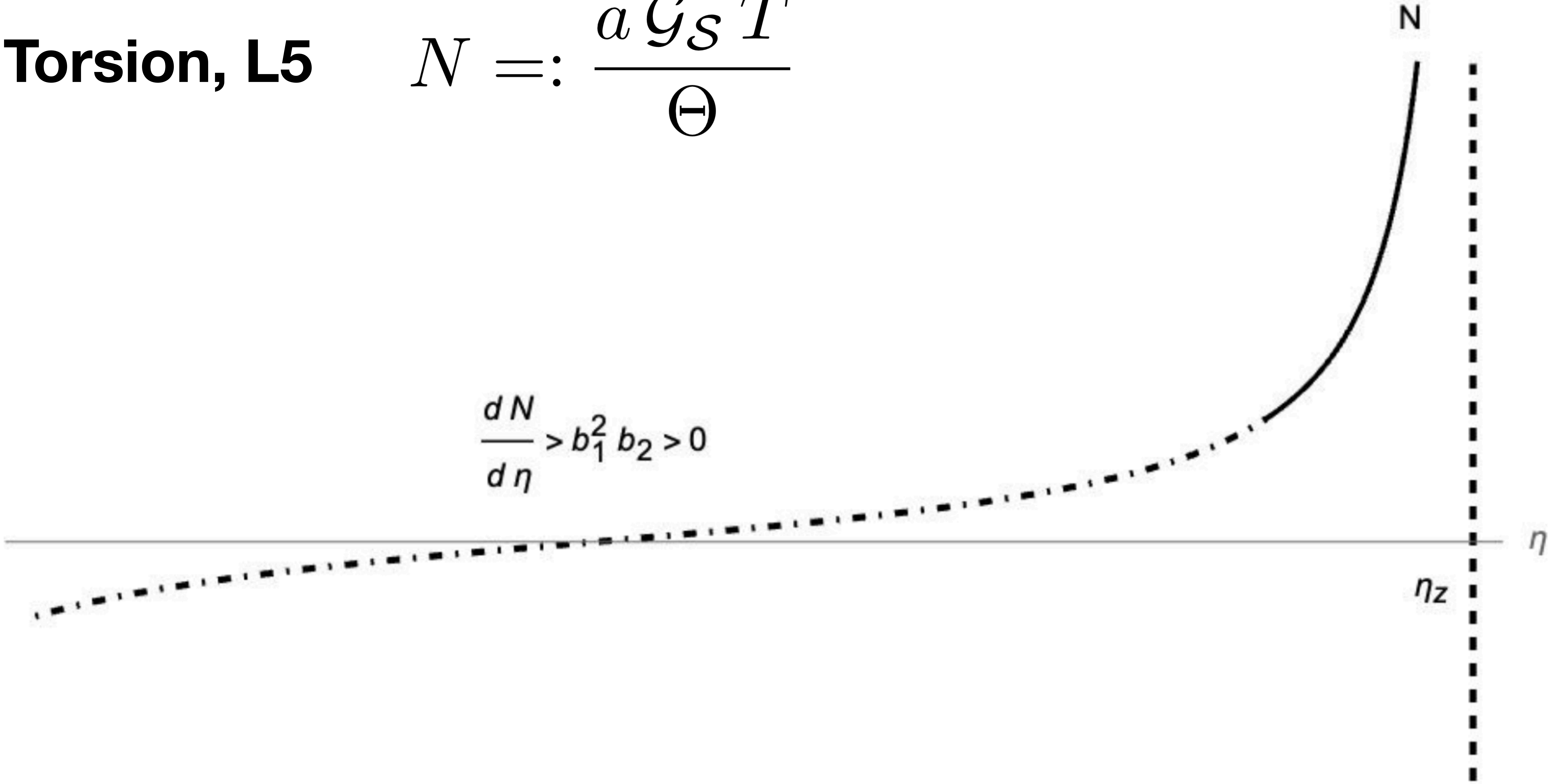
$$\boxed{N \neq 0}$$

$$\boxed{\bar{\mathcal{G}}_{\mathcal{S}} \neq \mathcal{G}_{\tau}(\vec{p}^{\,2})}$$

$$\boxed{T = \ldots}$$

4. Stability: $\mathcal{G}_\tau > 0\,, \mathcal{F}_\tau > 0\,, \mathcal{F}_\mathcal{S} > 0\,, \mathcal{G}_\mathcal{S} > 0.$

With Torsion, L5 $$N =: \frac{a\,\bar{\mathcal{G}}_\mathcal{S}\,T}{\Theta}$$



4. Stability of L5

Mixing with Torsion perturbations has broken the link between the

action for the graviton <-|-> action for the scalar

$$\mathcal{G}_{\mathcal{T}} > 0\,, \mathcal{F}_{\mathcal{T}} > 0\,, \mathcal{F}_{\mathcal{S}} > 0\,, \mathcal{G}_{\mathcal{S}} > 0.$$

(And other "healthy criteria“) Do not meet contradictions

Which theories avoid the No-Go?

With Torsion L5, that at some time satisfy

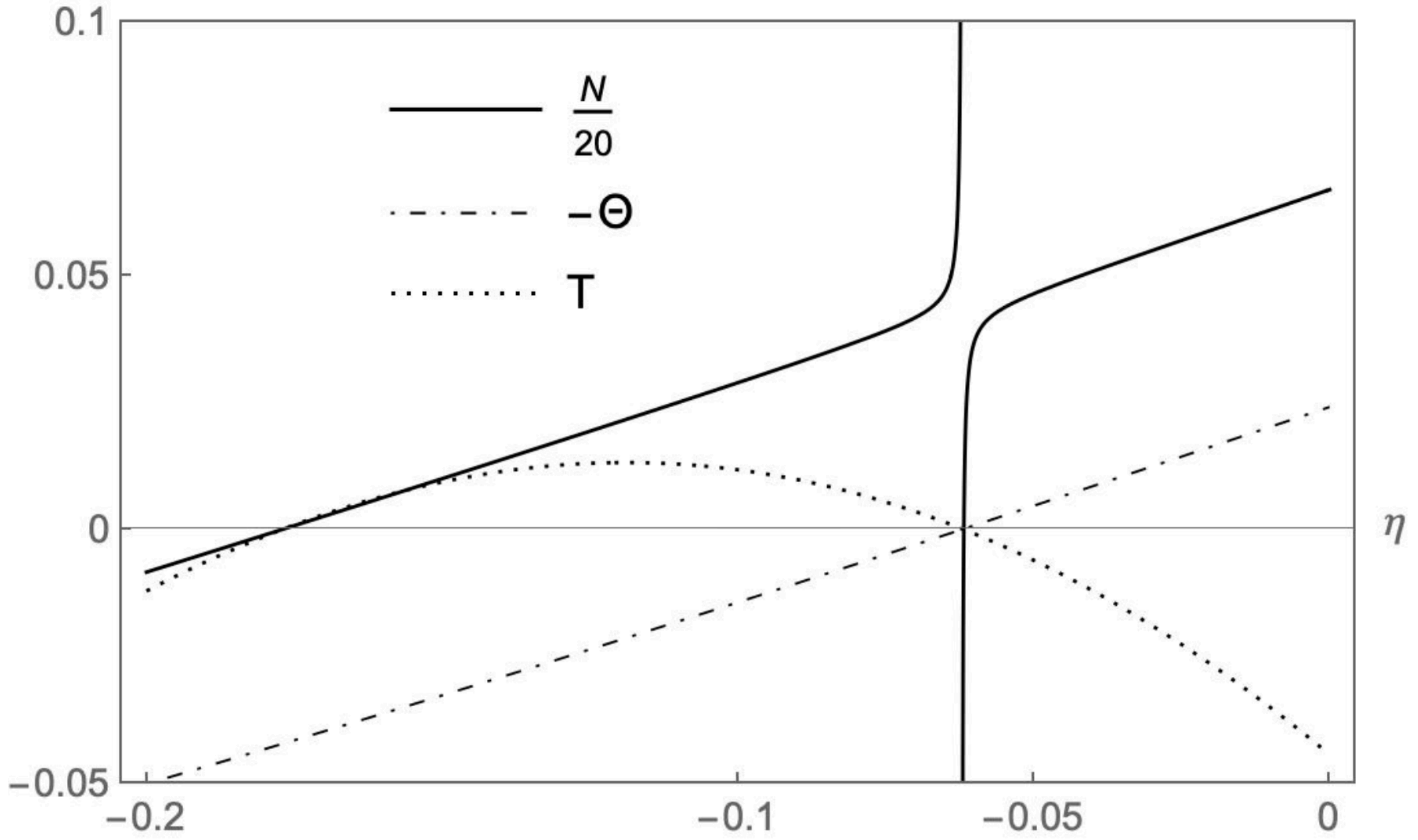
$$\boxed{T(\eta^*) = 0}$$

4. Stability: There are healthy cosmologies with L5

- example. A proof of principle

4. Stability:

By-pass the no-go? L5 and $\boxed{T(\eta^*) = 0}$



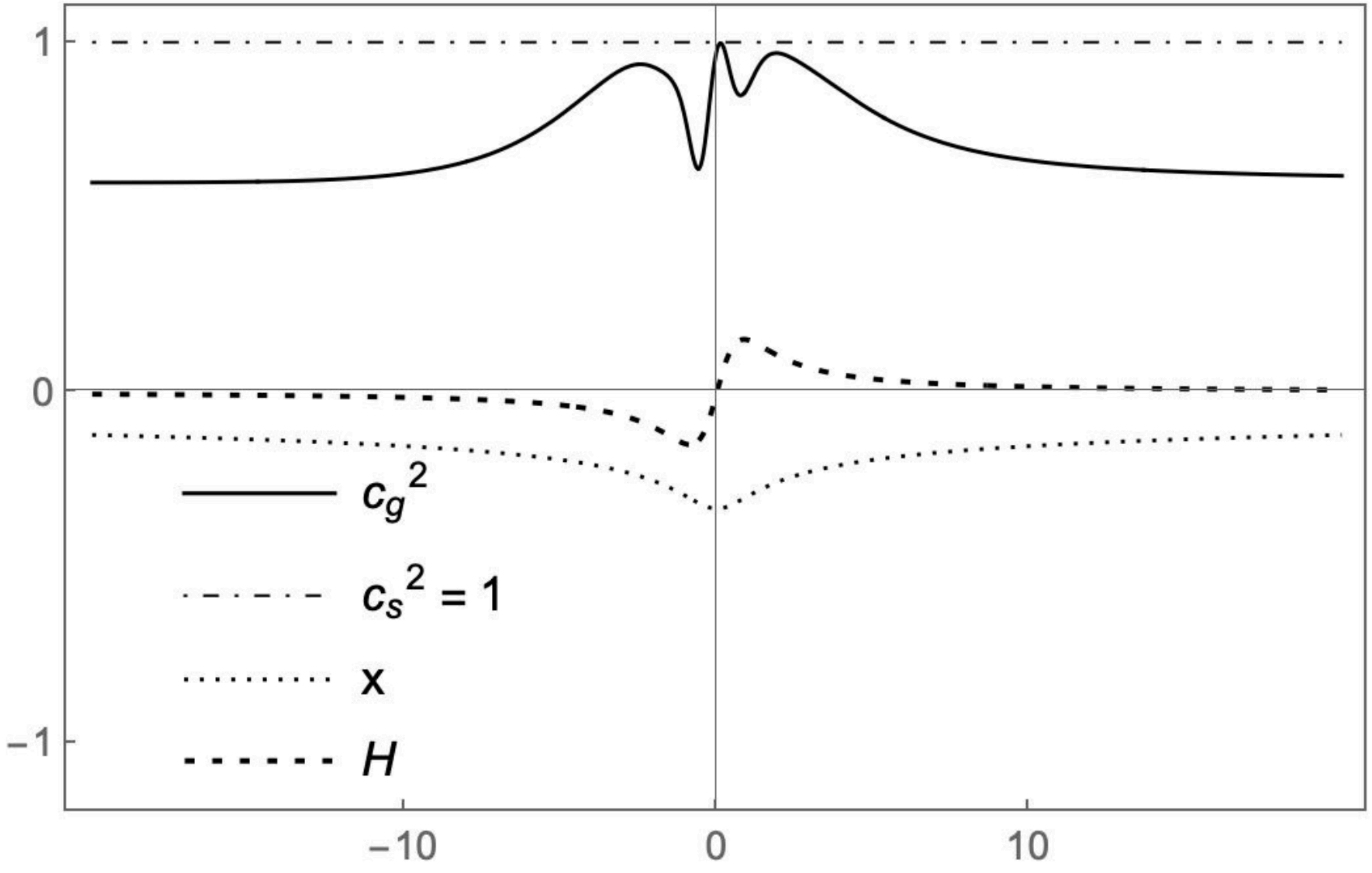
- S. Mironov and M. V-V, arXiv:2405.08673

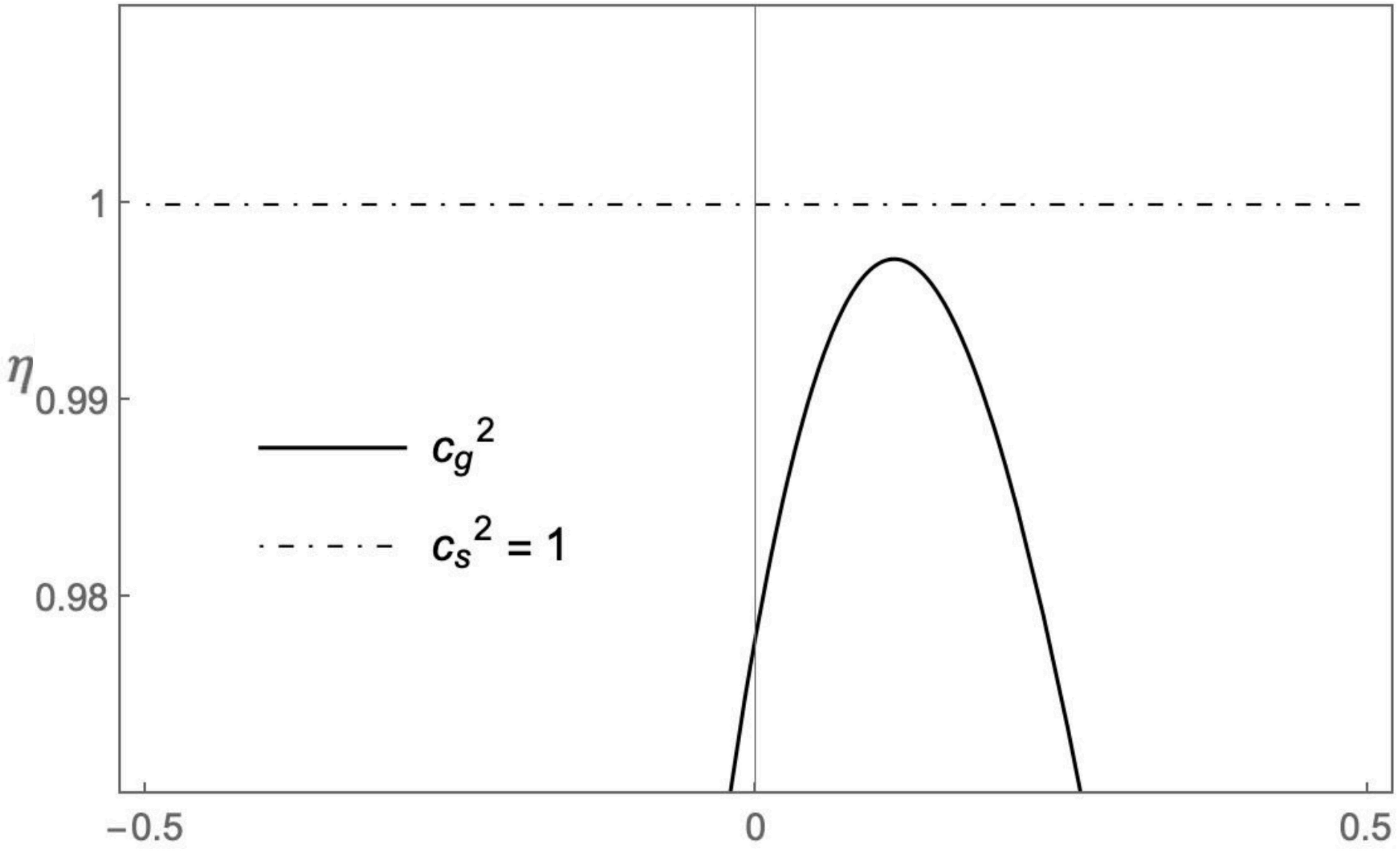
4. Stability:

With L5

$$\mathcal{G}_\tau > 0\,, \mathcal{F}_\tau > 0\,, \mathcal{F}_\mathcal{S} > 0\,, \mathcal{G}_\mathcal{S} > 0.$$

$$a = (\tau^2 + \eta^2)^{\frac{1}{6}}\,,\;\; H = \frac{\dot{a}}{a^2} = \frac{\eta}{3\,(\tau^2+\eta^2)^{\frac{7}{6}}}\,,\;\; \phi = \eta\,,\;\; x = -\frac{1}{3(1+\eta^2)^{\frac{1}{6}}}\,,$$





- S. Mironov and M. V-V, arXiv:2405.08673

Conclusions

- The mathematically and physically unjustified assumption of a torsionless spacetime leads to accidental relations at linear order, which restrict the healthiness of the solutions. **Simplifications enable the global instability issues.**

- **<u>The full Horndeski theory (with L5) with both curvature and torsion can support nonsingular, stable and subluminal cosmological solutions at all times.</u>** <u>The usual No-Go theorem that holds in a curved spacetime is avoided.</u>

- No-Go theorem in L4 Galileons with torsion.

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Additional Material

Background Equations

3. Torsionful Galileons about the FLRW background

- **Linearization: structure of the background equations**

- Example: In up to Quartic S4, We can solve for *H*, $\ddot{\varphi}(\eta)$ and *y*, *x*.

e.g.

$$\mathcal{E}_{K_{ijk}} = \epsilon_{ijk} \, \frac{2}{a^6} G_4 \, y = 0 \,, \qquad y(\eta) \equiv 0$$

$$\mathcal{E}_{K_{ij0}} = 0 \,, \qquad x(\eta) = -\frac{a^3 \, \mathcal{G}_\tau \, (8 \, H \, X \, G_{4,X} + \, a \, \dot{\phi} \, (G_3 - 2 \, G_{4,\phi}))}{8 \, G_4^2}$$

Additional Material

Linearization

3. Torsionful Galileons about the FLRW background

- **Linearization: Notation. The perturbed metric**

$$\mathrm{d}s^2 = (\eta_{\mu\nu} + \delta g_{\mu\nu})\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

Spatially flat FLRW background in conformal time

$$\eta_{\mu\nu}\mathrm{d}x^\mu\,\mathrm{d}x^\nu = a^2(\eta)\left(-\mathrm{d}\eta^2 + \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\right)$$

4 scalars, 2 (2-component) vectors and a (2-component) tensor

perturbation (graviton)

$$\begin{aligned}\delta g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = a^2(\eta)\big(&-2\,\alpha\,\mathrm{d}\eta^2\\ &+2\,(\partial_i B + S_i)\,\mathrm{d}\eta\,\mathrm{d}x^i + (-2\,\psi\,\delta_{ij}\\ &+2\,\partial_i\partial_j E + \partial_i F_j + \partial_j F_i + \underline{2\,h_{ij}})\,\mathrm{d}x^i\,\mathrm{d}x^j\big)\end{aligned}$$

3. Torsionful Galileons about the FLRW background

- **Linearization: Notation**

- Perturbation of contortion: with $K_{\mu\nu\sigma} = -K_{\sigma\nu\mu}$,

24 independent components

8 scalars,

$$\begin{aligned}
\delta K^{\text{scalar}}_{i00} &= \partial_i C^{(1)} \\
\delta K^{\text{scalar}}_{ij0} &= \partial_i \partial_j C^{(2)} + \delta_{ij} C^{(3)} + \epsilon_{ijk} \partial_k C^{(4)} \\
\delta K^{\text{scalar}}_{i0k} &= \epsilon_{ikj} \partial_j C^{(5)} \\
\delta K^{\text{scalar}}_{ijk} &= \left(\delta_{ij}\partial_k - \delta_{kj}\partial_i\right) C^{(6)} + \epsilon_{ikl}\partial_l\partial_j C^{(7)} + \left(\epsilon_{ijl}\partial_l\partial_k - \epsilon_{kjl}\partial_l\partial_i\right) C^{(8)}
\end{aligned}$$

3. Torsionful Galileons about the FLRW background

- **Linearization: Notation**

6 (2-component) vectors

$$\begin{aligned}
\delta K^{\text{vector}}_{i00} &= V^{(1)}_i \\
\delta K^{\text{vector}}_{ij0} &= \partial_i V^{(2)}_j + \partial_j V^{(3)}_i \\
\delta K^{\text{vector}}_{i0k} &= \partial_i V^{(4)}_k - \partial_k V^{(4)}_i \\
\delta K^{\text{vector}}_{ijk} &= \delta_{ij} V^{(5)}_k - \delta_{kj} V^{(5)}_i + \partial_j \partial_i V^{(6)}_k - \partial_j \partial_k V^{(6)}_i
\end{aligned}$$

and 2 (2-component) tensors

$$\begin{aligned}
\delta K^{\text{tensor}}_{ij0} &= T^{(1)}_{ij} \\
\delta K^{\text{tensor}}_{ijk} &= \partial_i T^{(2)}_{jk} - \partial_k T^{(2)}_{ji}
\end{aligned}$$

Additional Material

Details No-Go

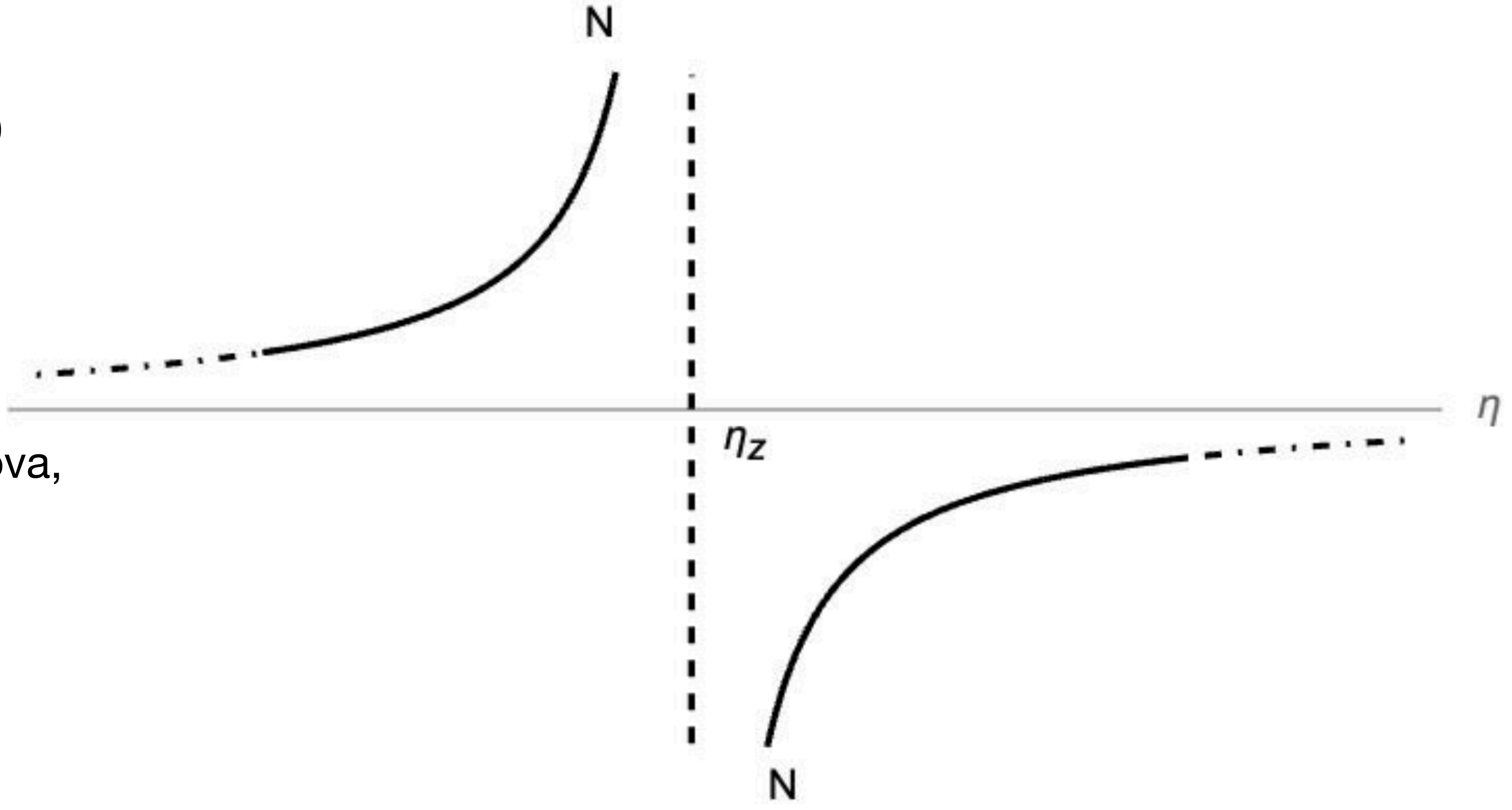
4. Stability: $N = \frac{a\,\mathcal{G}_\tau\,T}{\Theta}$

Even with Zeros of Θ

$$\boxed{\frac{\mathrm{d}N}{\mathrm{d}\eta} > a^2\,\mathcal{F}_\tau > 0}$$

S. Mironov, V. Rubakov, and V. Volkova, (2018).

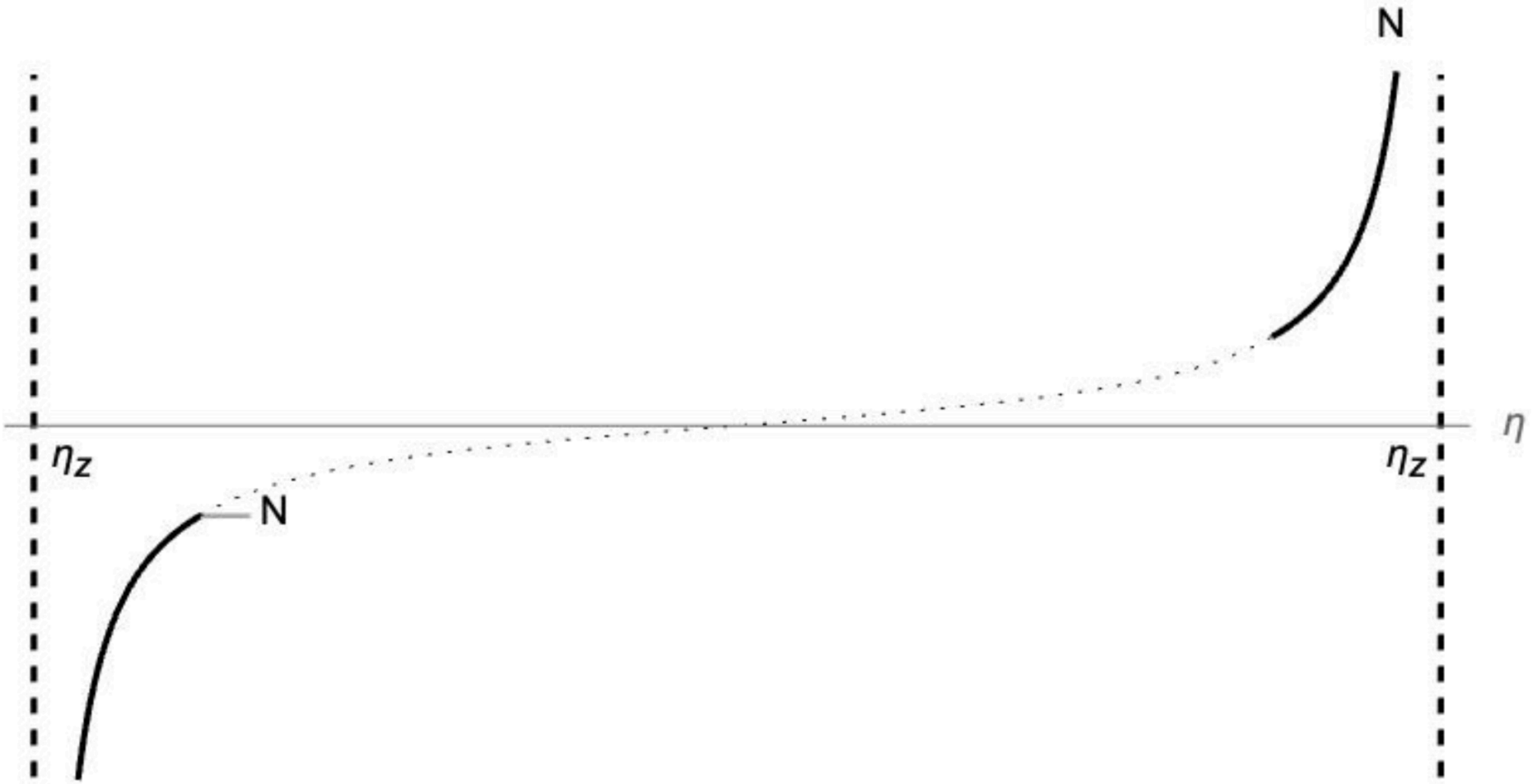
S. Mironov, (2019).



Behavior of $N(\eta)$ around zeros of Θ (denoted as η_z)

4. Stability: $\boxed{\dfrac{\mathrm{d}N}{\mathrm{d}\eta} > a^2\,\mathcal{F}_\tau > 0}$

Thus



$$\boxed{N \neq 0\,?}$$

N in between any two zeros η_z