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Paradoxicality of the weak equivalence principle

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OUTLINE

- Experimental data
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Experimental data

The weak equivalence principle (WEP) being an important part of general relativity (GR) states that "All test particles at the alike spacetime point, in a given gravitational field, will undergo the same acceleration, independent of their properties, including their rest mass"
P. Touboul *et al.*, *Phys. Rev. Lett.* **119**, 231101 (2017); *Phys. Rev. Lett.* **129**, 121102 (2022).

PHYSICAL REVIEW LETTERS **129,** 121102 (2022)

Editors' Suggestion Featured in Physics

MICROSCOPE Mission: Final Results of the Test of the Equivalence Principle

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(MICROSCOPE Collaboration)

We found no violation of the WEP, with the Eötvös parameter of the titanium and platinum pair constrained to $\eta(\mathrm{Ti, Pt})=[-1.5 \pm 2.3(\mathrm{stat}) \pm 1.5(\mathrm{syst})] \times 10^{-15}$ at 1σ in statistical errors.

Previous studies and a paradoxicality of the weak equivalence principle

Particle Dynamics in Static Gravitational Fields and Noninertial Frames

$$g_{00}^{(C)} = 1 - \frac{r_g}{r}, \quad g_{0i}^{(C)} = 0, \quad g_{ij}^{(C)} = -\left(\delta_{ij} - \frac{r_g x_i x_j}{r^3}\right)$$

and

$$g_{00}^{(i)} = 1 - \frac{r_g}{r}, \quad g_{0i}^{(i)} = 0, \quad g_{ij}^{(i)} = -\left(1 - \frac{r_g}{r}\right)\delta_{ij}$$

in the Cartesian and isotropic coordinates, respectively. Here $r_g = 2GM/c^2$

$$g_{00}^{(\mathrm{acc})} = \left(1 + \frac{\boldsymbol{a}\cdot\boldsymbol{r}}{c^2}\right)^2, \quad g_{ij}^{(\mathrm{acc})} = -\delta_{ij}.$$

The conventional general equations lead to the following known equations of the particle motion in the Cartesian coordinates:

$$\frac{du_i}{ds} = \frac{(u^0)^2 r_g}{2r^3}\left\{x^i\left[1 + \frac{3(\boldsymbol{\beta}\cdot\boldsymbol{r})^2}{r^2}\right] - 2\beta^i(\boldsymbol{\beta}\cdot\boldsymbol{r})\right\}$$

$$= -\frac{(u^0)^2}{c^2}\left\{g^i\left[1 + \frac{3(\boldsymbol{\beta}\cdot\boldsymbol{r})^2}{r^2}\right] - 2\beta^i(\boldsymbol{\beta}\cdot\boldsymbol{g})\right\},$$

$$\frac{du^i}{ds} = -\frac{(u^0)^2 r_g}{2r^3} x^i \left[1 + 2\boldsymbol{\beta}^2 - \frac{3(\boldsymbol{\beta}\cdot\boldsymbol{r})^2}{r^2}\right]$$

$$= \frac{(u^0)^2}{c^2} g^i \left[1 + 2\boldsymbol{\beta}^2 - \frac{3(\boldsymbol{\beta}\cdot\boldsymbol{r})^2}{r^2}\right], \qquad \frac{du_0}{ds} = 0,$$

$$\frac{du^0}{ds} = -\frac{(u^0)^2 r_g(\boldsymbol{\beta}\cdot\boldsymbol{r})}{r^3} = 2\frac{(u^0)^2}{c^2}(\boldsymbol{\beta}\cdot\boldsymbol{g}),$$

where $\boldsymbol{\beta} = \boldsymbol{V}/c$, $ds = cd\tau$ is the interval, and $du^i/(ds) = (u^0/c)du^i/(dt) = u^0 w^i/c$.

These equations of motion can be compared with the related equations for the uniformly accelerated frame:

$$\frac{du_i}{ds} = \frac{(u^0)^2 a^i}{c^2}, \qquad \frac{du^i}{ds} = -\frac{(u^0)^2 a^i}{c^2}, \qquad \frac{du_0}{ds} = 0, \qquad \frac{du^0}{ds} = -2\frac{(u^0)^2}{c^2}(\boldsymbol{\beta}\cdot\boldsymbol{a}).$$

In the two cases, forces are different.

We suppose that g=-a.

The passive gravitational mass of a system of *N* interacting and confined nonrelativistic particles is given by

$$m_g^{(p)} = \frac{R + 3T + 2U}{c^2}, \quad R = \sum_{i=1}^{N} m_i c^2, \quad T = \sum_{i=1}^{N} \frac{m_i v_i^2}{2};$$

where *R*, *T*, and *U* denote the rest, kinetic, and potential energies, respectively. Only electromagnetic interactions with the Coulomb potential have been previously considered. For the Coulomb interaction energy,

$$\langle T \rangle = \sum_{i=1}^{N} \frac{m_i}{2} \langle v_i^2 \rangle = -\frac{1}{2} \sum_{i=1}^{N} \langle U_i \rangle = -\frac{1}{2} \langle U \rangle.$$

The passive gravitational mass of a compound particle is equal to its rest mass in the nonrelativistic case:

$$\langle m_g^{(p)} \rangle = \frac{R + \langle T + U \rangle}{c^2}$$

- **There are other reasons which prevent the use of the virial theorem for the proof of the WEP. An evident reason is the strong interaction. This interaction makes a maximum contribution to a relativistic motion inside of compound objects. At the same time, the strong interaction of quarks and gluons has a string-like potential and its contribution to gravitational masses of compound objects cannot be explained by the virial theorem. Another problem is a necessity for relativistic corrections because velocities of constituent parts of compound objects are rather large. Taking these corrections into account can change even the result obtained with the virial theorem for electromagnetic interactions with the Coulomb potential. The relativistic virial theorem is given by [W. Lucha and F. F. Schoberl, Phys. Rev. Lett. 64,** 2733 (1990); **W. Lucha, Mod. Phys. Lett. A 30,** 2473 (1990)**] (for the Coulomb interaction)**

$$\left\langle \frac{\boldsymbol{p}^2}{\sqrt{m^2+\boldsymbol{p}^2}} \right\rangle = \left\langle \boldsymbol{r}\cdot\frac{\partial U}{\partial \boldsymbol{r}} \right\rangle = -\langle U\rangle .$$

$$T = \sqrt{m^2+\boldsymbol{p}^2} - m = \frac{\boldsymbol{p}^2}{2m}\left(1-\frac{\boldsymbol{p}^2}{4m^2}\right),$$

$$\left\langle \frac{\boldsymbol{p}^2}{\sqrt{m^2+\boldsymbol{p}^2}} \right\rangle = \left\langle \frac{\boldsymbol{p}^2}{m}\left(1-\frac{\boldsymbol{p}^2}{2m^2}\right) \right\rangle = 2\left\langle T\left(1-\frac{T}{m}\right) \right\rangle = -\langle U\rangle$$

An agreement between the WEP and the virial theorem disappears.

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Connection between gravitational and inertial masses of compound objects and the weak equivalence principle

However, this approach fails for a box of photons, despite claims in S. Carlip, Am. J. Phys. 66, 409 (1998); **M. Zych, L. Rudnicki and I. Pikovski, Am. J. Phys. 66,** 409 (1998). **It has been obtained that the Newtonian potential is coupled to the sum** $3T + 2U^{light}$**, where** T **is the kinetic energy of the box and** U^{light} **is the light energy. The virial theorem** ($\langle 2T + U^{light}\rangle = 0$) **has been used and the box energy has been obtained in the form** $E = \langle 3T + 2U^{light}\rangle = \langle T + U^{light}\rangle$. **However, this is evidently incorrect because** $\langle T\rangle > 0$, $\langle U^{light}\rangle > 0$, **and therefore** $\langle 2T + U^{light}\rangle > 0$. **In addition, previous studies do not take into account the elastic energy conditioned by the pressure of light on the box walls.**

Gravity for steady waves

Standing Waves into a Box

$$\Psi = 2A_0 \cos kx \cos \omega t$$

The momentum density is equal to zero for standing waves and is nonzero for running ones. The energy density is nonzero in the both cases. However, the free running plane wave has the momentum $p = E/c$ (E **is the kinetic energy) and the average momentum of the confined standing wave is equal to zero. If we consider standing waves with zero average momenta instead of a presentation of particles like bullets, the WEP is always conserved.**

Rigorous proof

Let us consider the gravitational interaction between the immobile box and the Earth. Let the box be at a great distance from a structureless particle. The sum of momenta acquired by the box and the particle is equal to zero ($\mathbf{p}_b=-\mathbf{p}_p$). The momenta are defined by the Newtonian force. We should compare the passive gravitational mass of the box in a gravitational field and the active gravitational mass of the same box creating its own gravitational field. In SR, the connection between the kinematic mass and energy of any body at rest is defined by the Einstein formula $E=Mc^2$. The active mass creating the gravitational field should have the same connection with the energy because GR and SR should not disagree. It is also important that the average momentum density is equal to zero in any spatial point of the box.

Summary

- Previous studies based on the virial theorem fail to explain the weak equivalence principle which seems like a paradox
- We need to take into account wave properties of particles being constituent parts of a compound particle. Such parts can be considered as steady waves. For these waves, the weak equivalence principle is valid
- The weak equivalence principle can also be proven in the general form by another method


Thank you for your attention