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EXTRA SYMMETRIES IN AFFINE-METRIC GRAVITATIONAL THEORIES

XXII International Seminar "QUARKS-2024"
(19-24 May 2024)

Affine-Metric Theory of Gravity (MAG)

- Gauge Theory of $GA(4,R) = GL(4R) \triangleleft T(4)$ **group**
- Dynamical variables
$\Omega_{ab\mu}$ **- local Lorenz connection**
h^a_μ **- vierbain**
or in coordinate map
$\Gamma^\sigma_{\mu\nu} \neq \Gamma^\sigma{}_{\nu\mu}$ **- affine connection**
$g_{\mu\nu}$ **- metric**
Main geometrical values

$$R^\sigma{}_{\lambda\mu\nu} = \partial_\mu \Gamma^\sigma{}_{\lambda\nu} - \partial_\nu \Gamma^\sigma{}_{\lambda\mu} + \Gamma^\sigma{}_{\alpha\mu}\Gamma^\alpha{}_{\lambda\nu} - \Gamma^\sigma{}_{\alpha\nu}\Gamma^\alpha{}_{\lambda\mu} \text{ - \textbf{curvature tensor}},$$

$$R_{\mu\nu} = R^\sigma{}_{\mu\sigma\nu} \text{ - \textbf{Richi tensor}},$$

$$R = R_{\mu\nu} g^{\mu\nu} \text{ - \textbf{scalar of curvature}},$$

$$Q^\lambda{}_{\mu\nu} = \frac{1}{2}\left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}\right) \text{ - \textbf{tosion tensor}},$$

$$W_{\sigma\mu\nu} = \nabla_\sigma g_{\mu\nu} \equiv \partial_\sigma g_{\mu\nu} - \Gamma^\lambda_{\mu\sigma} g_{\lambda\nu} - \Gamma^\lambda_{\nu\sigma} g_{\mu\lambda} \text{ - \textbf{tensor of nonmetrisity}}.$$

The Lagrangian of MAG is the solution of jet equation

$$\Big[g^{\mu\nu}\frac{\delta}{\delta\delta g^{\mu\nu}} + Q^{\alpha}_{\mu\nu}\frac{\delta}{\delta Q^{\alpha}_{\mu\nu}} + 2W^{\alpha}_{\mu\nu}\frac{\delta}{\delta W^{\alpha}_{\mu\nu}} + R^{\alpha}_{\mu\beta\nu}\frac{\delta}{\delta R^{\alpha}_{\mu\beta\nu}} + R^{\mu\nu}\frac{\delta}{\delta R^{\mu\nu}}\Big] L(g, \partial g, \Gamma, \partial\Gamma) = 0$$

Theor+Dim	**4**	**3**	**2**
Metric-Affine Theory (g, Γ)	**234**	**75**	**15**
Einstein-Cartan Theory $(\nabla_\sigma g_{\mu\nu} = 0)$	**194**	**57**	**9**
Einstein Theory $(\Gamma^{\sigma}_{\mu\nu} = \left\{ {\sigma \atop \mu\nu} \right\})$	**10**	**3**	**1**

So

$L(R, Q, W) = R(g, \Gamma) - 2\Lambda + L(R^2) + L_{QQ} + L_{WW} + L_{WQ} +$ **other invariants**
At the first step we consider only two types of the MAG Lagrangians

The action of gravitational field is the sum of 13 invariants that has the canonical dimension $[M^{-2}]$

$$S_2 = -\frac{1}{\varkappa^2}\int d^4x\sqrt{-g}\left\{2\Lambda - R + L_{(QQ)} + L_{(WW)} + L_{(WQ)}\right\},$$

where

$$L_{(QQ)}(\Gamma) = Q^{\sigma}_{\mu\nu}Q^{\lambda}{}_{\alpha\beta}I^{\mu\nu\alpha\beta}_{\sigma\lambda(QQ)},$$

$$L_{(WW)}(\Gamma) = W_{\sigma\mu\nu}W_{\lambda\alpha\beta}I^{\sigma\mu\nu\lambda\alpha\beta}_{(WW)},$$

$$L_{(WQ)}(\Gamma) = W_{\sigma\mu\nu}Q^{\lambda}{}_{\alpha\beta}I^{\sigma\mu\nu\alpha\beta}_{\lambda(WQ)},$$

$$I^{\mu\nu\alpha\beta}_{\sigma\lambda(QQ)} = a_1 g_{\lambda\sigma}g^{\mu\alpha}g^{\nu\beta} + a_2\delta^{\nu}_{\sigma}\delta^{\beta}_{\lambda}g^{\mu\alpha} + a_3\delta^{\nu}_{\lambda}\delta^{\beta}_{\sigma}g^{\mu\alpha},$$

$$I^{\sigma\mu\nu\lambda\alpha\beta}_{(WW)} = b_1 g^{\lambda\sigma}g^{\mu\alpha}g^{\nu\beta} + b_2 g^{\sigma\alpha}g^{\mu\lambda}g^{\nu\beta} + b_3 g^{\sigma\lambda}g^{\mu\nu}g^{\alpha\beta} + b_4 g^{\sigma\nu}g^{\mu\alpha}g^{\lambda\beta} + b_5 g^{\sigma\alpha}g^{\mu\nu}g^{\lambda\beta},$$

$$I^{\sigma\mu\nu\alpha\beta}_{\lambda(WQ)} = c_1\delta^{\beta}_{\lambda}g^{\mu\nu}g^{\sigma\alpha} + c_2\delta^{\beta}_{\lambda}g^{\sigma\nu}g^{\mu\alpha} + c_3\delta^{\mu}_{\lambda}g^{\nu\beta}g^{\sigma\alpha}.$$

$$L_{(QQ)} = a_1 Q_{\lambda\mu\nu} Q^{\lambda\mu\nu} + a_2 Q_\sigma Q^\sigma + a_3 Q_{\lambda\mu\nu} Q^{\nu\mu\lambda}$$

$$L_{(WW)} = b_1 W^{\sigma\mu\nu} W_{\sigma\mu\nu} + b_2 W^{\sigma\mu\nu} W_{\nu\mu\sigma} + b_3 W^{(1)\sigma} W^{(1)}_\sigma + + b_4 W^{(2)}_\sigma W^{(2)}_\sigma + b_5 W^{(2)\sigma} W^{(1)}_\sigma,$$

$$L_{(WQ)} = c_1 W^{\sigma\mu\nu} Q_{\nu\mu\sigma} + c_2 W^{(1)\sigma} Q_\sigma + c_3 W^{(2)\sigma} Q_\sigma$$

where

$W^{(1)}_\mu = W^\sigma{}_{\mu\sigma}$**- the first type trace of nonmetricity field**

$W^{(2)}_\sigma = W_{\sigma\mu}{}^\mu$ **- the second type trace of nonmetricity field**

$Q_\sigma = Q^\nu_{\sigma\nu}$ **- trace of torsion field**

$L(R^2)$ **will be considered in the end of my talk**

Symmetries of action

$$S_2 = \int d^4x\sqrt{-g}\ \{2\Lambda - R + L_{(QQ)} + L_{(WW)} + L_{(WQ)}\}$$

$$x_\mu \to x'_\mu(x)$$

$$g'_{\mu\nu}(x') = g_{\alpha\beta}\frac{\partial x_\alpha}{\partial x'_\mu}\frac{\partial x_\beta}{\partial x'_\nu}$$

$$\Gamma'^{\sigma}_{\mu\nu} = \Gamma^{\lambda}_{\alpha\beta}\frac{\partial x'^{\sigma}}{\partial x_\lambda}\frac{\partial x_\alpha}{\partial x'_\mu}\frac{\partial x_\beta}{\partial x'_\nu} + \frac{\partial^2 x^\lambda}{\partial x'_\mu \partial x'_\nu}\frac{\partial x'^{\sigma}}{\partial x_\lambda}$$

Generalized projective symmetries

$$\Gamma'^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\mu\nu} + D^{\sigma}_{\mu\nu}$$

Idea

$$\left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\}' \to \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} + \delta^{\alpha}_{\mu} A_\nu$$

$$D^{\sigma}{}_{\mu\nu} = \delta^{\sigma}_{\mu} A_\nu + \delta^{\sigma}_{\nu} B_\mu + g_{\mu\nu} F^\sigma + E_{\lambda\mu\nu} g^{\lambda\sigma} + U_{\lambda\mu\nu} g^{\lambda\sigma}$$

Special types of transformation (so called forth type of transformation)

$$D^{\sigma}{}_{\mu\nu} = g^{\sigma\lambda} E_{\lambda\mu\nu}$$

,

where $E_{\lambda\mu\nu}$ - antisymmetrical tensor
Fifth type of transformations

$$D^{\sigma}_{\mu\nu} = g^{\sigma\lambda} U_{\lambda\mu\nu}$$

,

where $U_{\lambda\mu\nu}$ - traceless tensor

$$U^{\lambda}{}_{\mu\lambda} = U^{\lambda}{}_{\lambda\mu} = U_{\mu\lambda}{}^{\lambda} \equiv 0$$

$$U_{\sigma\mu\nu} + U_{\nu\sigma\mu} + U_{\mu\nu\sigma} - U_{\sigma\nu\mu} - U_{\nu\mu\sigma} - U_{\mu\sigma\nu} = 0$$

Restrictions on a_i, b_j, c_n

$$S_2(g,\Gamma) \Rightarrow S_2(g,\Gamma') = S_2(g,\Gamma) + \Delta_1(Q,D) + \Delta_2(W,D) + \Delta_3(D^2)$$

Invariants of $S_2(g,\Gamma)$ is equivalent to $\Delta_1(Q,D) = 0, \Delta_2(W,D) = 0, \Delta_3(D^2) = 0$

1. $D^{\sigma}_{\mu\nu} = \delta^{\sigma}_{\mu} A_{\nu}$

$$-(2a_1 + 3a_2 + a_3) - 2(4c_1 + c_2 + c_3) = 0$$

$$-2(2b_1 + 8b_3 + b_5) + \frac{1}{2}(3c_1 + c_3) = 0$$

$$(2b_2 + 2b_4 + 4b_5) + \frac{1}{2}(c_3 - 3c_2) = 0$$

$$2a_1 + 3a_2 + a_3 + 4(4b_1 + b_2 + 16b_3 + b_4 + 4b_5) - 4(4c_1 + c_2 + c_3) = 0$$

4. $D^{\lambda}{}_{\mu\nu} = g^{\lambda\sigma} E_{\sigma\mu\nu}$

$$(a_1 - a_3 + 1) = 0$$

5. $D^{\lambda}{}_{\mu\nu} = g^{\lambda\sigma} U_{\sigma\mu\nu}$

$$c_3 + 2(1 + 2a_1 + a_3) = 0$$

$$c_3 + 2b_1 + b_2 = 0$$

$$1 + 2a_1 + a_3 + 4b_1 + b_2 + c_3 = 0$$

2. $D^{\sigma}{}_{\mu\nu} = \delta^{\sigma}_{\nu} B_{\mu}$

$$2a_1 + 3a_2 + a_3 + c_3 - 2(c_1 + c_2) + 4 = 0$$

$$2b_2 + 4b_3 + 5b_5 + \frac{1}{2}(3c_1 + c_3) + 3 = 0$$

$$4b_1 + 2b_2 + 10b_4 + 2b_5 + \frac{1}{2}(3c_2 - c_3) + 1 = 0$$

$$3 + \frac{3}{4}(2a_1 + 3a_2 + a_3) + 10b_1 + 7b_2 + 4b_3 + 28b_4 + 10b_5 + \frac{3}{2}(c_3 + 2(c_1 + c_2)) = 0$$

3. $D^{\lambda}{}_{\mu\nu} = g_{\mu\nu} F^{\lambda}$

$$12 + c_3 - 5c_2 - 2c_1 = 0$$

$$2 - 4b_1 - 2b_2 - 10b_4 + 2b_5 = 0$$

$$4 + 2b_2 + 4b_3 + 5b_5 = 0$$

$$3 + 10b_1 + 7b_2 + 4b_3 + 25b_4 + 10b_5 = 0$$

Equation for "defect"of connection

Equation of motion for Γ^r_{pq} will be

$$\frac{\delta S_2}{\delta\Gamma^r_{pq}} = 0 \Rightarrow$$

$$\nabla_r g^{pq} - \delta^q_r \nabla_n g^{pn} - g^{pq} d^r - \delta^q_r d^p + d^{qp}_r - d^{pq}_r + d^l_{mn}(I^{pqmn}_{rl(\Sigma)} + I^{mnpq}_{lr\Sigma}) = 0$$

where d^r_{pq} is "defect"of connection

$$d^r_{pq} = \Gamma^r_{pq} - \left\{ \begin{matrix} r \\ pq \end{matrix} \right\}$$

After some algebra we will get the algebraic equation for "defect"of connection

$$\begin{aligned} - \delta^q_r(2d^p - \frac{1}{3}d^l_{mn}(I^{ptmn}_{tl(\Sigma)} + I^{mnpt}_{lt(\Sigma)}))+ \\ + -\delta^p_r(2d^q - \frac{1}{3}d^l_{mn}(I^{qtmn}_{tl(\Sigma)} + I^{mnqt}_{lt(\Sigma)}))+ \\ + d^{pq}_r - d^{pq}_r + d^l_{mn}(I^{pqmn}_{rl(\Sigma)} + I^{mnpq}_{lr(\Sigma)})- \\ - d^{qp}_r + d^{qp}_r - d^l_{mn}(I^{qpmn}_{rl(\Sigma)} + I^{mnqp}_{lr(\Sigma)}) = 0 \end{aligned} \quad (1)$$

and $I^{qpmn}_{rl(\Sigma)}$ is the linear combination of $I_{(QQ)}$, $I_{(WW)}$ and $I_{(WQ)}$

$$I^{abmn}_{sl(\Sigma)} = I^{[ab][mn]}_{sl(QQ)} + I^{(ab)(mn)}_{sl(WW)} + I^{[ab](mn)}_{sl(QW)} \quad (2)$$

Solution

d^k_{mn} **is tensor,so**

$$d^k{}_{mn} = \delta^k_m a_n + \delta^k_n b_m + g_{mn} f^k + e_{lmn} g^{lk} + u_{lmn} g^{lk}$$

$$\Downarrow$$

$$-\delta^q_r(2d^p - \frac{1}{3}d^l_{mn}(I^{ptmn}_{tl(\Sigma)} + I^{mnpt}_{lt(\Sigma)})) + \delta^p_r(2d^q - \frac{1}{3}d^l_{mn}(I^{qtmn}_{tl(\Sigma)} + I^{mnqt}_{lt(\Sigma)}))$$
$$+d^{pq}_r - d^{pq}_r + d^l_{mn}(I^{pqmn}_{rl(\Sigma)} + I^{mnpq}_{lr(\Sigma)}) - d^{qp}_r + d^q_r{}^p - d^l_{mn}(I^{qpmn}_{rl(\Sigma)} + I^{mnqp}_{lr(\Sigma)}) = 0$$

we will get the restrictions on a_i, b_j **and** c_k

$$D = d$$

Extra symmetries and quantization of MAG models

$$L(g,\Gamma) = R(g,\Gamma) - 2\Lambda + a_1 Q_{\lambda\mu\nu}Q^{\lambda\mu\nu} + a_2 Q_\sigma Q^\sigma + a_3 Q_{\lambda\mu\nu}Q^{\nu\mu\lambda}$$

Denote

$$\varepsilon_1 = 2a_1 + 3a_2 + a_3$$

$$\varepsilon_2 = a_1 - a_3 + 1$$

$$\varepsilon_3 = 1 + 2a_1 + a_3$$

$\varepsilon_i = 0$, $i = 1,2,3 \mapsto L(g,\Gamma)$ **is invariant under 1,4,5 types of transformation of** Γ

Quantization

Let's now move focus on the quantum properties of the affine-metric gravity with torsion terms. The effective action is determined by the following general formula:

$$\Gamma^{(1)}[\Phi] = \frac{i}{2}\ln\left(\det\frac{\delta^2 S}{\delta\Phi^2}\right). \tag{3}$$

Performing the background field expansion:

$$\begin{aligned} g_{\mu\nu}(x) &= \hat{g}_{\mu\nu}(x) + h_{\mu\nu}(x), \\ \Gamma^\lambda{}_{\mu\nu}(x) &= \hat{\Gamma}^\lambda{}_{\mu\nu}(x) + \gamma^\lambda{}_{\mu\nu}(x), \end{aligned}$$

where fields $(\hat{g},\hat{\Gamma})$ **define classical part and** (h,γ) − **quantum corrections, the effective action can be written using the functional integral form:**

$$\Gamma^{(1)} = -i\ln\int D\gamma Dh\,\exp\left\{-\frac{i}{2k^2}\int d^4x\sqrt{-g}\,(h,\gamma)\begin{pmatrix}\hat{\mathcal{O}}_{hh} & \hat{\mathcal{O}}_{h\gamma}\\ \hat{\mathcal{O}}_{\gamma h} & \hat{\mathcal{O}}_{\gamma\gamma}\end{pmatrix}\begin{pmatrix}h\\ \gamma\end{pmatrix}\right\}. \tag{4}$$

One-loop counterterms

$$\left(\begin{array}{cc} \hat{\mathcal{O}}_{hh} & \hat{\mathcal{O}}_{h\gamma} \\ \hat{\mathcal{O}}_{\gamma h} & \hat{\mathcal{O}}_{\gamma\gamma} \end{array}\right) \Rightarrow \left(\begin{array}{cc} \hat{\mathcal{D}}_{hh} & 0 \\ 0 & \hat{\mathcal{D}}_{\gamma\gamma} \end{array}\right)$$

Effective metric theory

$$\Gamma^{(1)} = -i\ln\int D\phi \exp\left\{-\frac{i}{2k^2}\int\sqrt{-g}\left(\Lambda\phi_{\mu\nu}J^{\mu\nu,\alpha\beta}\phi_{\alpha\beta} - (\hat{T}_{\lambda}^{\mu\nu,\alpha\beta}\phi_{\alpha\beta})(F^{-1})^{\lambda}{}_{\mu\nu}{}^{\sigma}{}_{\alpha\beta}(\hat{T}_{\sigma}{}^{\rho\tau,\gamma\delta}\phi_{\gamma\delta})\right)\right\},$$

Effective connection theory

$$\Gamma^{(1)} = -i\ln\int d^4x\sqrt{-g}\hat{\mathcal{D}}_2(\hat{\mathcal{D}}_1)^{-1/2}\exp\left\{-\frac{i}{2k^2}\int\sqrt{-g}\left(\Lambda H_{\mu\nu}J^{\mu\nu,\alpha\beta}H_{\alpha\beta}-\right.\right.$$

$$\left.\left.(\hat{T}_{\lambda}{}^{\mu\nu,\alpha\beta}H_{\alpha\beta})(F^{-1})^{\lambda}{}_{\mu\nu}{}^{\sigma}{}_{\rho\tau}(\hat{T}_{\sigma}{}^{\rho\tau,\gamma\delta}H_{\gamma\delta})\right)\right\},$$

where

$$\hat{T}^{\sigma}{}_{\mu\nu,\alpha\beta} = \frac{1}{2}\delta^{\sigma}_{\mu}\left(\hat{g}_{\beta\nu}\hat{\nabla}_{\alpha} + \hat{g}_{\alpha\nu}\hat{\nabla}_{\beta}\right) + \delta_{\mu\nu,\alpha\beta}\hat{\nabla}^{\sigma}$$

and the inverse tensor F^{-1} is determined by the equation:

$$F_{\lambda}{}^{\mu\nu}{}_{\sigma}{}^{\rho\tau}\left(F^{-1}\right)^{\sigma}{}_{\rho\tau}{}^{\gamma}{}_{\alpha\beta} = \delta^{\gamma}_{\lambda}\delta^{\mu}_{\alpha}\delta^{\nu}_{\beta} - \frac{1}{4}\delta^{\gamma}_{\beta}\delta^{\mu}_{\alpha}\delta^{\nu}_{\lambda}.$$

$F_{p}{}^{mn}{}_{q}{}^{rs}$ **is the linear combination of** $I^{mnrs}_{(QQ)pq}$

$$F_{p}{}^{mn}{}_{q}{}^{rs} = \delta^{s}_{q}\delta^{r}_{p}g^{mn} + \delta^{m}_{q}\delta^{n}_{p}g^{rs} - \delta^{r}_{p}\delta^{n}_{q}g^{ms} - \delta^{s}_{p}\delta^{m}_{q}g^{rn} +$$
$$+a_1 g_{pq}g^{mr}g^{ns} - a_1 g_{pq}g^{ms}g^{nr} +$$
$$+\frac{1}{2}a_2 g^{mr}\delta^{s}_{q}\delta^{n}_{p} + \frac{1}{2}a_2 g^{sn}\delta^{r}_{q}\delta^{m}_{p} - \frac{1}{2}a_2 g^{nr}\delta^{s}_{q}\delta^{m}_{p} - \frac{1}{2}a_2 g^{ms}\delta^{r}_{q}\delta^{n}_{p} +$$
$$+\frac{1}{2}a_3 g^{mr}\delta^{s}_{p}\delta^{n}_{q} + \frac{1}{2}a_3 g^{sn}\delta^{r}_{p}\delta^{n}_{q} - \frac{1}{2}a_3 g^{nr}\delta^{s}_{p}\delta^{n}_{q} - \frac{1}{2}a_3 g^{ms}\delta^{r}_{p}\delta^{n}_{q}$$

Role of symmetries

Kalmykov M.Yu., Pronin P.I., Poslavsky S.B. and Shatov M.F. (2000-2024)

$$(F^{(-1)})^{qc}_{rsab} = c_1 g^{qc} g_{rs} g_{ab} + c_2 g^{qc} g_{ra} g_{sb} + c_3 g^{qc} g_{rb} g_{sa} + c_4 \delta^q_r \delta^c_a g_{sb}$$

$$+ c_5 \delta^q_r \delta^c_b g_{sa} + + c_6 \delta^q_r \delta^c_s g_{ab} + c_7 \delta^q_s \delta^c_a g_{rb}$$

$$+ c_8 \delta^q_s \delta^c_b g_{ra} + c_9 \delta^q_s \delta^c_r g_{ab} + c_{10} \delta^q_a \delta^c_b q g_{rs} +$$

$$+ c_{11} \delta^q_a \delta^c_r g_{sb} + c_{12} \delta^q_a \delta^c_s g_{rb} + c_{13} \delta^q_b \delta^c_r g_{sa} + c_{14} \delta^q_b \delta^c_s g_{ra} + c_{15} \delta^q_b \delta^c_a g_{rs}$$

where

$$c_1 = -\frac{1}{4}, \quad c_6 = c_9 = c_{10} = c_{15} = \frac{1}{4}$$

$$c_2 = \frac{1}{3\varepsilon_3} - \frac{1}{12\varepsilon_2} + \frac{1}{4}, \quad c_3 = \frac{1}{12\varepsilon_2} - \frac{1}{3\varepsilon_3} + \frac{1}{4}$$

$$c_4 = \frac{3(2a_1 - a_2 + a_3) + 4}{6\varepsilon_1\varepsilon_3} - \frac{1}{12}, \; c_5 = \frac{1}{6\varepsilon_3} - \frac{1}{12}, \; c_7 = c_6, \; c_8 = -\frac{1}{6\varepsilon_3} - \frac{1}{12}$$

$$c_{11} = c_{14} = \frac{1}{6\varepsilon_3} + \frac{1}{12\varepsilon_2} - \frac{1}{4}, \quad c_{12} = c_{13} = -\frac{1}{6\varepsilon_3} - \frac{1}{12\varepsilon_2} - \frac{1}{4}$$

$$\varepsilon_i = 0, i = 1, 2, 3 \rightrightarrows (F^{(-1)})^{qc}_{rsab} \text{ will be singular}$$

Gauge fixing

It is need to fix not only the general coordinate invariance but it is need to fix the extra symmetries through adding the additional terms in the Lagrangian of MAG

$$L_{gf}^{(1)} = -\chi_1 \left(Q_\mu\right)^2$$

$$L_{gf}^{(2)} = -\chi_2 \left(\epsilon^{\sigma\mu\nu\alpha}\Gamma_{\mu\nu\alpha}\right)^2$$

$$L_{gf}^{(3)} = -\chi_3 \left(Q_{\mu\nu\alpha}Q^{\mu\nu\alpha}\right)^2$$

The adding of these terms is equal to redefinition of ε_i

$$\varepsilon_1 \to \varepsilon_1 + \chi_1 + 2\chi_3$$

$$\varepsilon_2 \to \varepsilon_2 - \chi_2 - \chi_3$$

$$\varepsilon_3 \to \varepsilon_3 + 2\chi_3$$

Fixing of general coordinate invariance may be made in two ways

$$\nabla_\sigma h^{\alpha\sigma} = 0$$

or

$$\nabla^\sigma \nabla^\lambda \gamma^\nu_{\sigma\lambda} = 0$$

Minimal MAG models with quadratic nonmetricity terms

There is only one model

$$S_2 = -\frac{1}{\varkappa^2}\int d^4x\sqrt{-g}\left\{2\Lambda - R + L_{(QQ)} + L_{(WW)} + L_{(WQ)}\right\},$$

Projective transformations $D^{\sigma}_{\mu\nu} = \delta^{\sigma}_{\mu}A_{\nu}$

$$-(2a_1 + 3a_2 + a_3) - 2(4c_1 + c_2 + c_3) = \zeta_1$$
$$-2(2b_1 + 8b_3 + b_5) + \frac{1}{2}(3c_1 + c_3) = \zeta_2$$
$$(2b_2 + 2b_4 + 4b_5) + \frac{1}{2}(c_3 - 3c_2) = \zeta_3$$
$$2a_1 + 3a_2 + a_3 + 4(4b_1 + b_2 + 16b_3 + b_4 + 4b_5) - 4(4c_1 + c_2 + c_3) = \zeta_4$$

Then

$$(F^{(-1)})(\zeta_i = 0, i = 1,2,3,4) \to \infty$$

Gauge fixing term

$$L_{gf} = -\frac{1}{2}\eta_1 Q_\alpha Q^\alpha - \frac{1}{2}\eta_2 W^{(1)}_\alpha W^{(1)}_\beta g^{\alpha\beta} - \frac{1}{2}\eta_3 W^{(2)}_\alpha W^{(2)}_\beta g^{\alpha\beta} - \frac{1}{2}\eta_4 W^{(2)}_\alpha W^{(1)}_\beta g^{\alpha\beta}$$

MAG models with quadratic curvature terms

$$S(g,\Gamma) = R(g,\Gamma) - 2\Lambda + a_1 Q_{\lambda\mu\nu}Q^{\lambda\mu\nu} + a_2 Q_\sigma Q^\sigma + a_3 Q_{\lambda\mu\nu}Q^{\nu\mu\lambda}$$
$$+\xi_0 R^2(g,\Gamma) + \xi_1 R_{\nu\mu\alpha\beta}R^{\nu\mu\alpha\beta} + \xi_3 R_{\alpha\beta}R^{\alpha\beta}$$

Symmetries

1. $\xi_2 = \xi_3 = 0, \zeta_1 \equiv 2a_1 + 3a_2 + a_3 = 0$

$$\Gamma'^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\mu\nu} + \delta^\sigma_\mu A_\nu$$

2. $\xi_2 = 0, \xi_3 \neq 0, \zeta_2 \equiv 1 + 4a_3 - 4a_1 = 0$

$$\Gamma'^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\mu\nu} + g^{\sigma\lambda}\epsilon_{\lambda\mu\nu\alpha}\partial^\alpha f(x)$$

3. $\xi_2 \neq 0, \xi_3 = 0, \zeta_3 \equiv 1 + 4a_3 + 8a_1 = 0$

$$\Gamma'^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\mu\nu} + g^{\sigma\lambda}V_{\lambda\mu\nu\alpha}\partial^\alpha f(x)$$

where

$$V_{\lambda\mu\nu\alpha} = V_{\lambda\nu\mu\alpha}$$

and

$$V_{\mu\nu\alpha\lambda} + V_{\nu\alpha\mu\lambda} + V_{\alpha\mu\nu\lambda} = 0$$

If anyone $\zeta_k = 0, k = 1,2,3,$ *propagators for* h *and* γ *do not exists!!!*
We are to fix this new projective invariance!!

THANK YOU ON YOUR ATTENTION!!!!!