QUARKS-2024 XXII INTERNATIONAL SEMINAR ON HIGH-ENERGY PHYSICS

Methods of Black Hole Shadow Modelling when Rotation is Taken into Account

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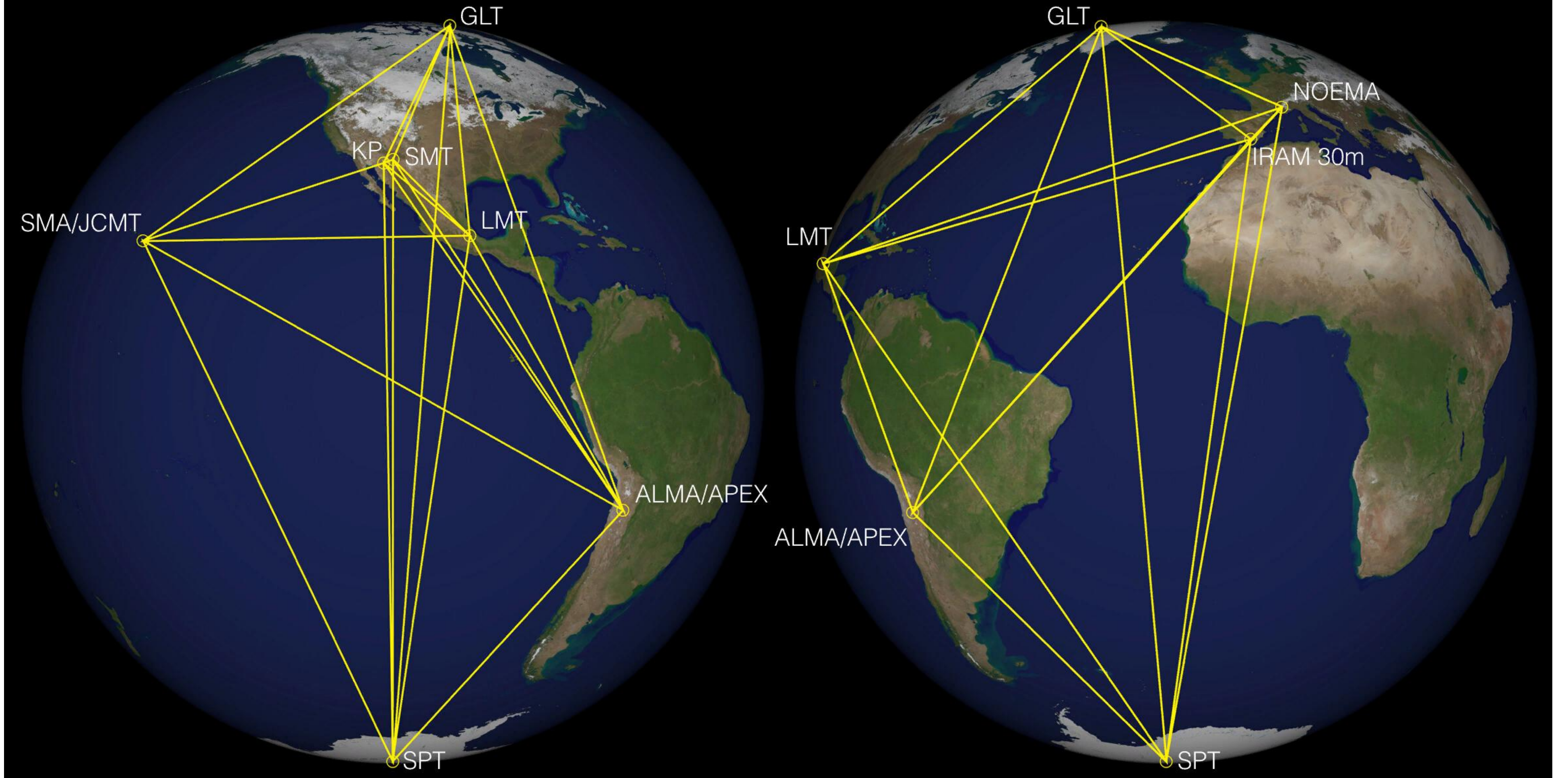
RSF Russian Science Foundation

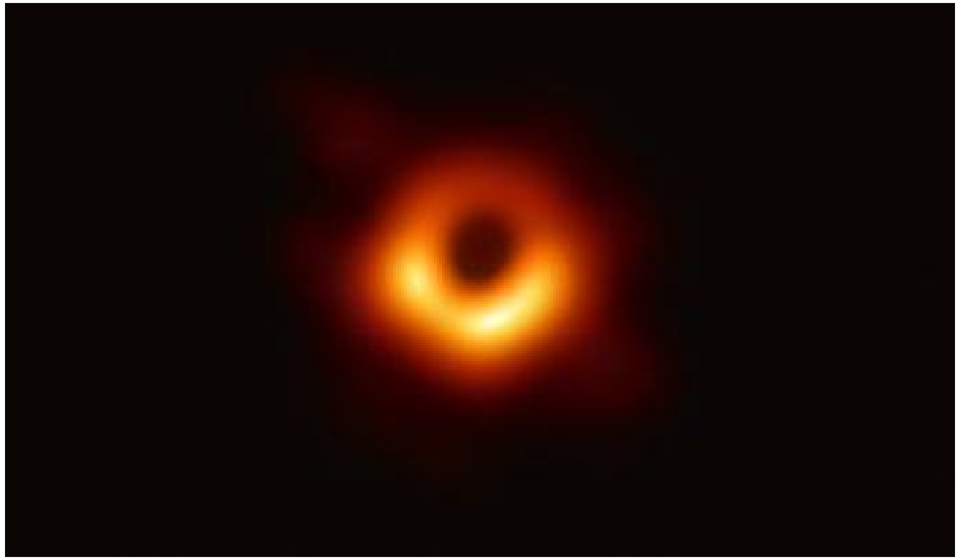
Supported by RSF via grant № 23-22-00073

Event Horizon Telescope

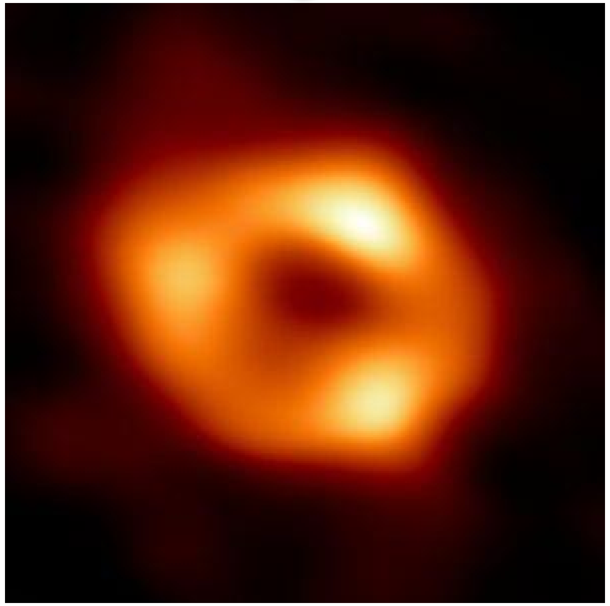


Black Hole Shadows (@EHT)

M 87*



Sgr A*



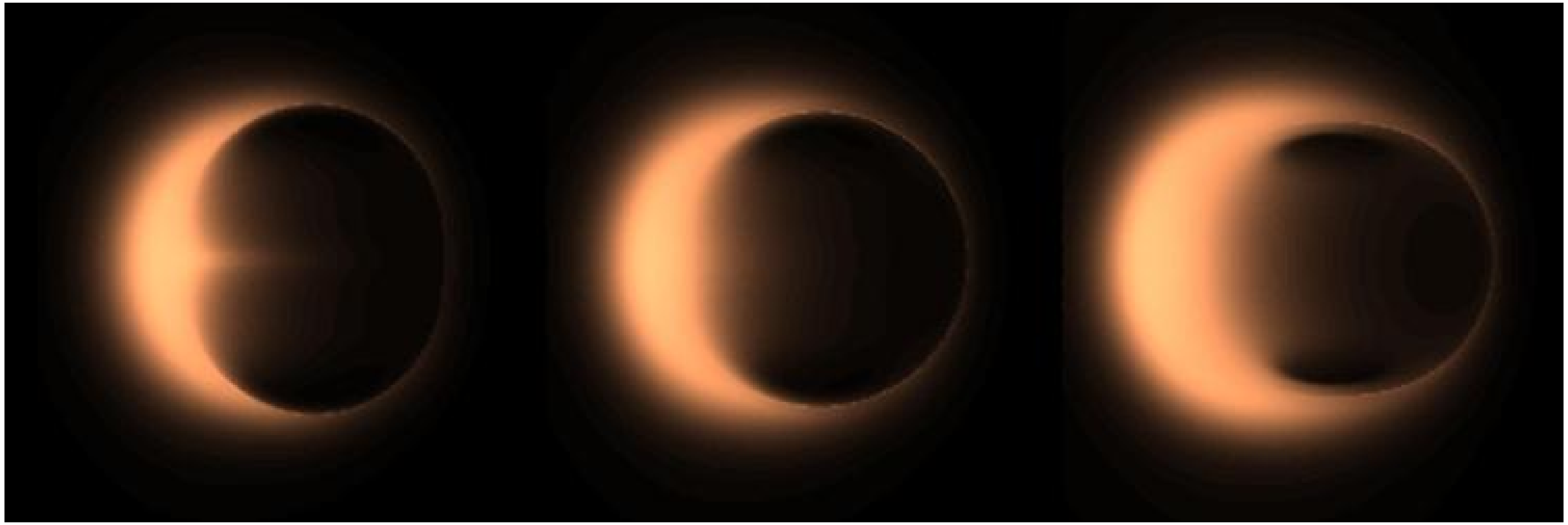
BH shadow size from 4.3M to 6.1M

K. Akiyama, et al., Astrophys. J. 875 (1) L5 (2019).

BH shadow size from 4.3M to 5.3M

The Event Horizon Telescope Collaboration, The Astrophysical Journal Letters 930 L17 (2022).

Constraints on gravity models from black hole shadows

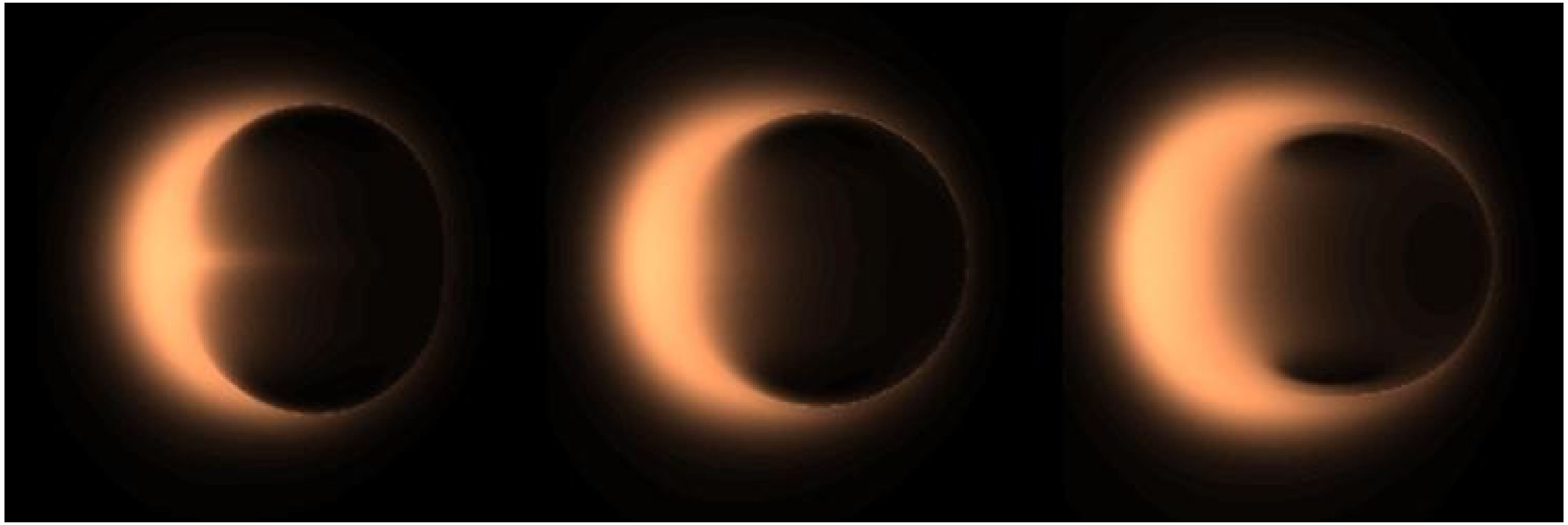


Pic is taken from https://www.eso.org/public/images/shadow-evt/

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

A.Zakharov, **Phys. Rev. D,** Vol.90, P.062007 (2014) ... V.Prokopov, SA, O.Zenin, **JETP**, Vol.135, p.842 (2022).

Constraints on gravity models from black hole shadows

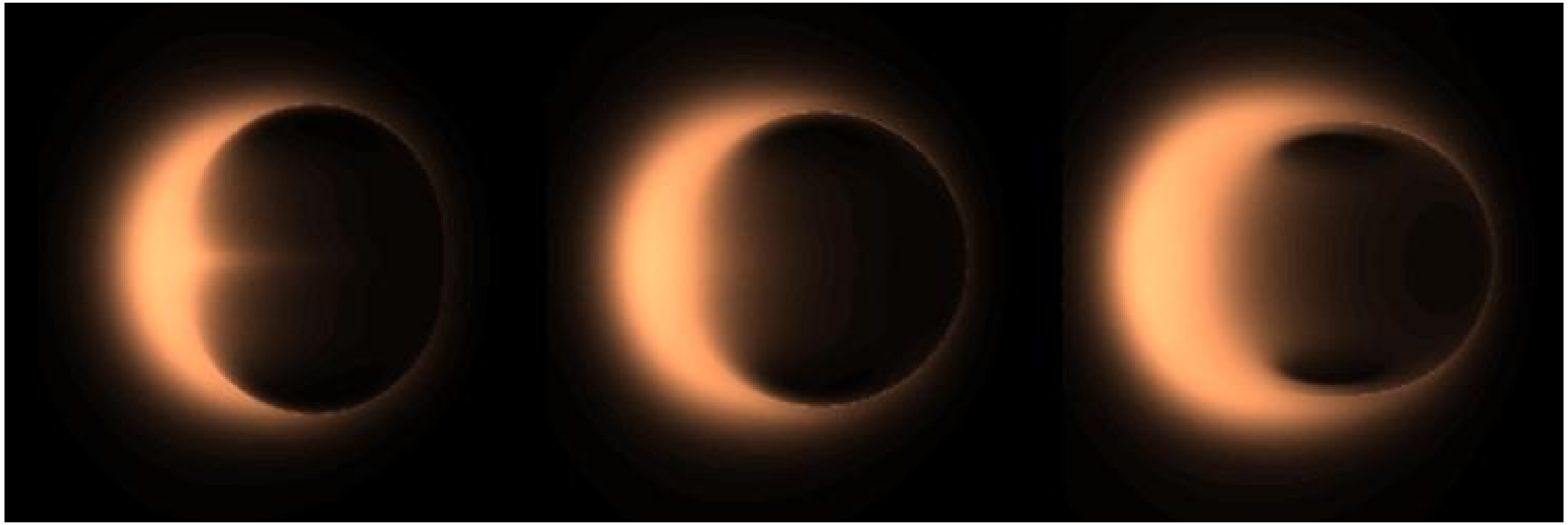


Pic is taken from https://www.eso.org/public/images/shadow-evt/

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

A.Zakharov, **Phys. Rev. D,** Vol.90, P.062007 (2014) ... V.Prokopov, SA, O.Zenin, **JETP**, Vol.135, p.842 (2022).

Constraints on gravity models from black hole shadows



Pic is taken from https://www.eso.org/public/images/shadow-evt/

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} - r^2 d\Omega^2$$

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\tilde{\varphi}\right)^2 - \frac{\sin^2\theta}{\rho^2}\left((r^2 + a^2)d\tilde{\varphi} - adt\right)^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2$$

A.Zakharov, **Phys. Rev. D,** Vol.90, P.062007 (2014) ... CA, A.A Байдерин, A.B. Немтинова, O.И. Зенин, ЖЭТФ 165, 508 (2024) arXiv 2404.16079

Idea:

The general form of spherically-symmetric metrics:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

Equation of motion: $\left(\frac{d\hat{r}}{d\tau}\right)^2 + \frac{L^2}{B(\hat{r})\hat{r}^2} = \frac{E^2}{A(\hat{r})B(\hat{r})}, \quad \frac{d\phi}{d\tau} = \frac{L}{\hat{r}^2}, \quad D = L/E$

Introduce: $u(r) = \left(\frac{d\hat{r}}{d\phi}\right)^2 = \frac{\hat{r}^4}{D^2 A(\hat{r})B(\hat{r})} - \frac{\hat{r}^2}{B(\hat{r})},$

To calculate the shadow size one has to find maximal root of

$$u(r) = 0, \quad \frac{du(r)}{dr} = 0, \quad \frac{d^2u(r)}{d^2r} > 0.$$

Results for some extended gravity models

Horndesky Model

$$A(r) = 1 - \frac{2M}{r} - \frac{2C_7}{7r^7}$$

$$B(r)^{-1} = 1 - \frac{2M}{r} - \frac{C_7}{r^7}$$

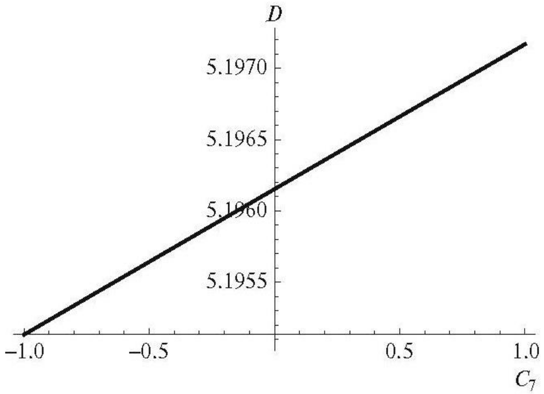


Fig. 3. The dependence of shadow size (D) versus the combination of model constants C_7 for Horndesky theory coupled with Gauss–Bonnet invariant (in the units of M, $M = 1$).

Loop quantum gravity

$$A(r) = \left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right)\left(1 - \frac{\alpha\beta M}{\alpha r^3 + \beta M}\right),$$

$$B(r)^{-1} = 1 - \frac{2Mr^2}{r^3 + 2Ml^2},$$

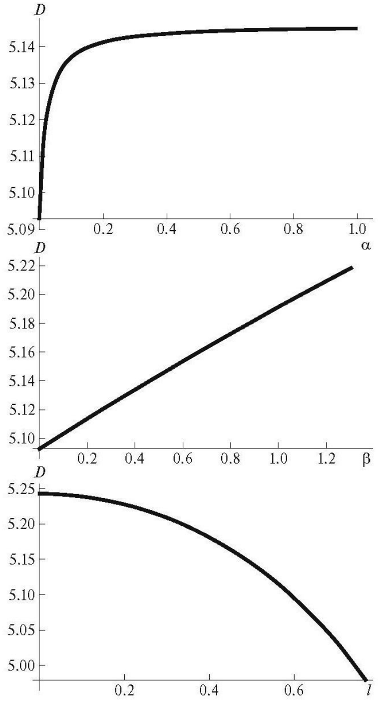


Fig. 4. The dependence of shadow size D upon the time delay α when $l = 0.5M$ and $\beta = 0.5$ (top image), upon the 1-loop quantum corrections β when $l = 0.5M$, $\alpha = 0.5$ (central image), upon the central energy density l when $\alpha = 0.5$, $\beta = 0.5$ (bottom image) for BH in modified Hayward metric in the units of M, $M = 1$.

V.Prokopov, SA, O.Zenin, **JETP**, Vol.135, P. 91 (2022), ***ibid***, p.842 (2022)

Results for some extended gravity models

Conformal gravity

$$A(r) = 1 - \frac{2M}{r} + \frac{Q_s^2}{r^2} + \frac{Q_s^2\left(-M^2 + Q_s^2 + \frac{6}{m_2^2}\right)}{3r^4} + \ldots,$$

$$B(r)^{-1} = 1 - \frac{2M}{r} + \frac{Q_s^2}{r^2} + \frac{2Q_s^2\left(-M^2 + Q_s^2 + \frac{6}{m_2^2}\right)}{3r^4} + \ldots,$$

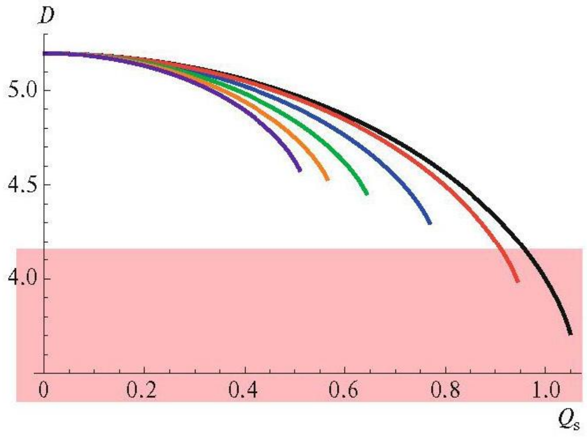


Fig. 5. The dependence of the shadow size D against the scalar charge Q_s for in new massive conformal gravity with different values of massive spin-2 mode m_2 (in the units of M, $M = 1$). Black line corresponds to $m_2 \to \infty$, red one corresponds to $m_2 = 2$, blue one corresponds to $m_2 = 1$, green one corresponds to $m_2 = 0.707$, orange one corresponds to $m_2 = 0.577$, purple one corresponds to $m_2 = 0.5$.

Bumblebee model

$$A(r) = \left(1 - \frac{2M}{r}\right),$$

$$B(r) = \frac{1 + l}{1 - \frac{2M}{r}},$$

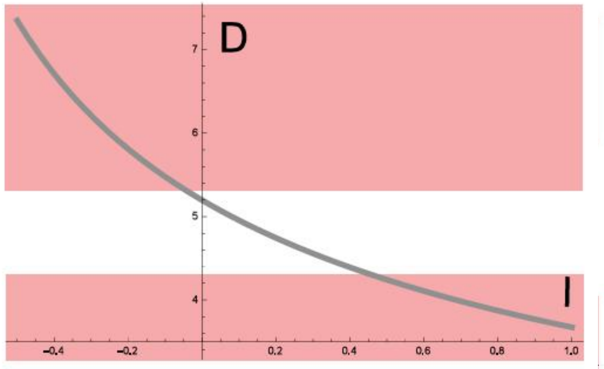


Рис. 1. The dependence of the shadow size D upon parameter l in alternative bumblebee generalization with Schwarzschild approximation (in the units of M, $M = 1$).

V.Prokopov, SA, O.Zenin, **JETP**, Vol.135, P. 91 (2022), ***ibid***, p.842 (2022)

Results for some extended gravity models

f(Q) gravity

$$A(r) = 1 - \frac{2M_{\rm ren}}{r} - \alpha\frac{32}{r^2},$$

$$B(r)^{-1} = 1 - \frac{2M_{\rm ren}}{r} - \alpha\frac{96}{r^2},$$

$$2M_{\rm ren} = 2M - \alpha\left(\frac{32}{3M} + c_1\right),$$

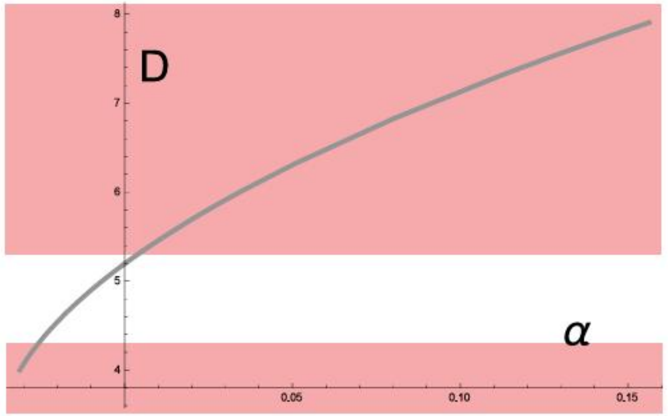


Рис. 2. The dependence of the shadow size D upon parameter α in $f(Q)$ gravity in M_{ren} units.

Scalar Gauss-Bonnet gravity

$$A = -f(r)\left[1 + \frac{\zeta}{3r^3 f(r)}h(r)\right],$$

$$B = \frac{1}{f(r)}\left[1 - \frac{\zeta}{r^3 f(r)}k(r)\right],$$

where

$$h(r) := 1 + \frac{26}{r} + \frac{66}{5r^2} + \frac{96}{5r^3} - \frac{80}{r^4},$$

$$k(r) := 1 + \frac{1}{r} + \frac{52}{3r^2} + \frac{2}{r^3} + \frac{16}{5r^4} - \frac{368}{3r^5},$$

$$f(r) := 1 - \frac{2}{r},$$

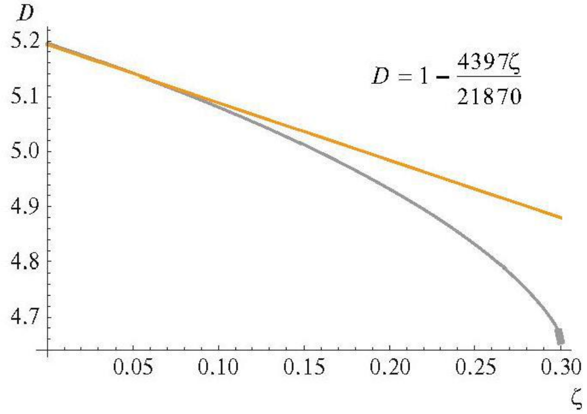


Fig. 8. The lower curve is the dependence of the shadow size D upon parameter ζ in scalar Gauss–Bonnet gravity (in the units of M, $M = 1$). The top line is the first order approximation.

V.Prokopov, SA, O.Zenin, **JETP**, Vol.135, P. 91 (2022), ***ibid***, p.842 (2022)

Constraints for these extended gravity models

The results in Horndesky with Gauss-Bonnet invariant, LQG, Bumbelby and Gauss-Bonnet scalar models are in complete agreement with the M87* observations. For most of considered examples the the model predictions are not pass the boundary established by the existing observational data.

In conformal gravity big values of m_2 and Q_s must be excluded (for example if $m_2 = 2$ then $Q_s < 0.9$).

In STEGR f(Q) gravity M87 observations constraint α as $-0.025 < \alpha < 0.04$.

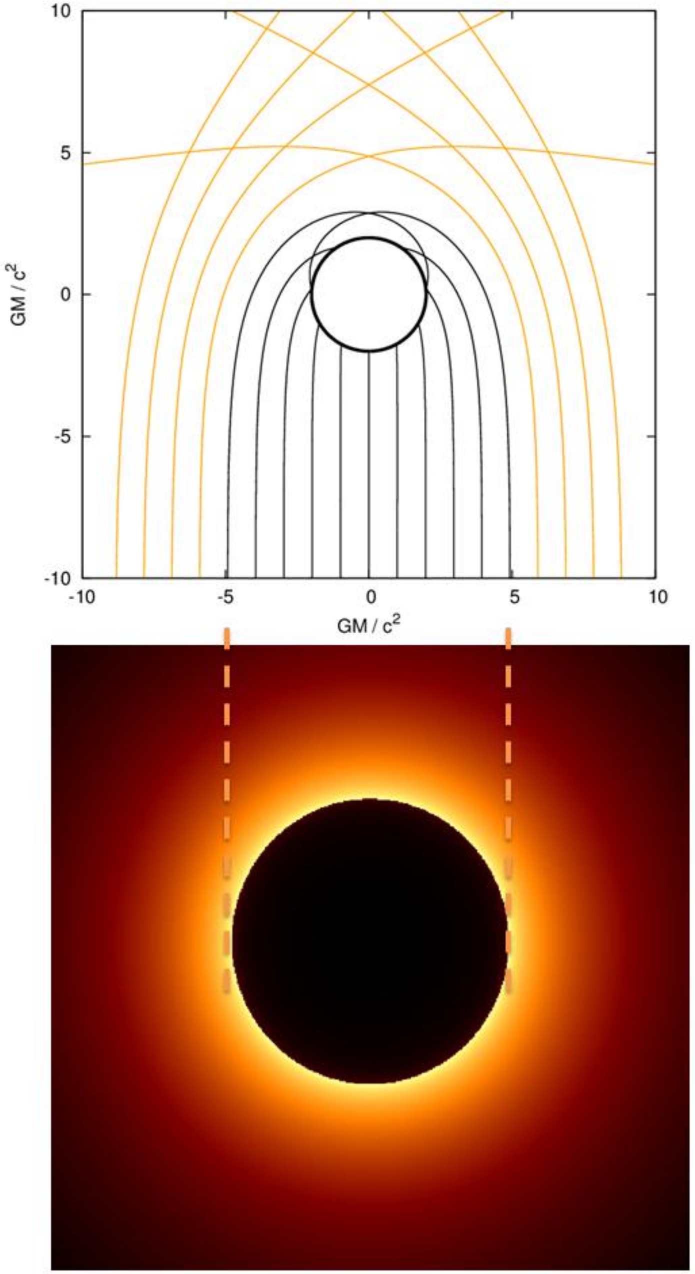
In alternative Bumblebee generalization with Schwarzschild approximation one obtains that $-0.3 < l < 0.45$.

These results demonstrates the maximum that could be distinguished when a BH rotation is not taken into account.

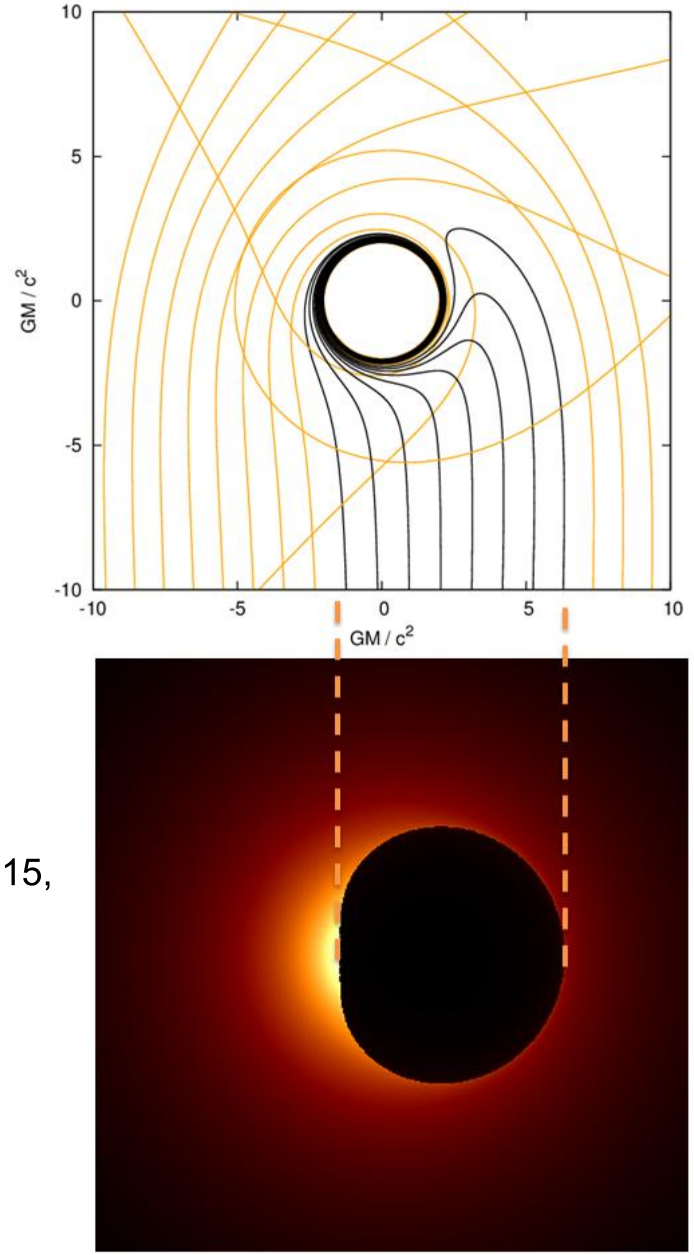
The upper bound on the size of the shadow for Sgr A* 5.3M appeared to be lower than for the case of M87* 6.1M, becoming comparable with the calculated size of the BH shadow in GR (about 5.2M). This fact makes possible to improve constraints on the alternative bumblebee metric ($-0.05 < l < 0.45$) from below and f(Q) gravity ($-0.025 < \alpha < 0.005$) from above.

V.Prokopov, SA, O.Zenin, **JETP**, Vol.135, P. 91 (2022), ***ibid***, p.842 (2022)

Schwarzschild BH shadow



Kerr BH shadow



The most probable values of the rotation parameter and inclination of the rotation plane in BH:

M87*:
$a = 0.9375$[1]
Sgr A*:
$a = 0.5$ or $a = 0.94$
and $\theta = \pi/6$[2]

[2] Cui, Y.; others, Nature 621, 711–715, (2023).

[1] Akiyama, K.; others, Astrophys. J. Lett. 930, L13, (2022).

Pic is taken from https://odysseyedu.wordpress.com/black-hole-shadow/

BH Shadow for Kerr black hole

Hamilton-Jacobi equation

$$g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu} = 0. \qquad \mathcal{R} = 0, \qquad \frac{d\mathcal{R}}{dr} = 0.$$

$$E = -p_t \quad L_z = p_\phi$$

$$S = -Et + L_z\phi + S_r(r) + S_\theta(\theta),$$

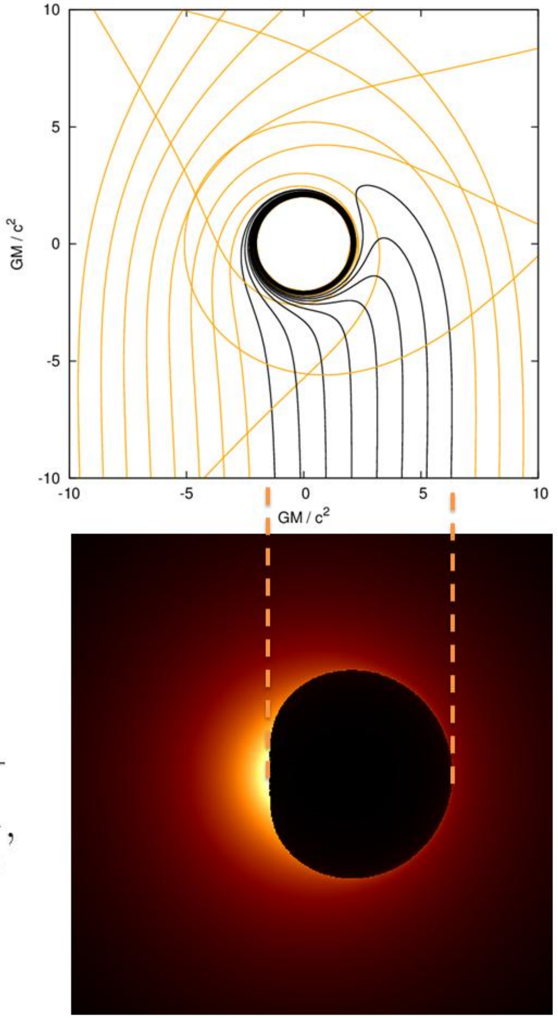
$$\mathcal{R}(r) = \left(\omega + a^2 - a\lambda\right)^2 - \left(f_r^{-1}r^2 + a^2\right)\left[\eta + (a-\lambda)^2\right]$$

$$\lambda = \frac{K + a^2}{a} - \frac{2K'}{a}\frac{(FH + a^2)}{(HF)'},$$

$$\eta = \frac{4(a^2 + FH)}{(HF)'^2}K'^2 - \frac{1}{a^2}\left[K - \frac{2(FH + a^2)}{(HF)'}K'\right].$$

$$x' = -\frac{\lambda}{\sin\theta_0},$$

$$y' = \pm\sqrt{\eta + a^2\cos^2\theta_0 - \frac{\lambda^2}{\tan^2\theta_0}},$$

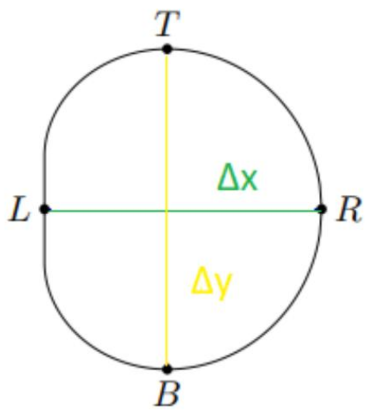


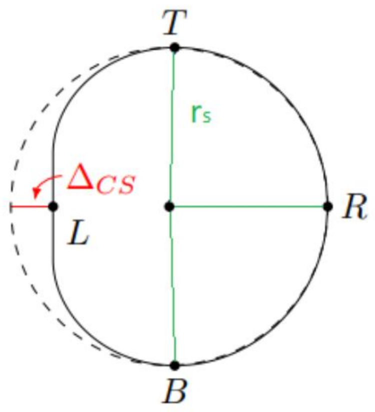
BH Shadow for Kerr black hole

Coordinates of the BH shadow on a plane perpendicular to the observer's line of sight

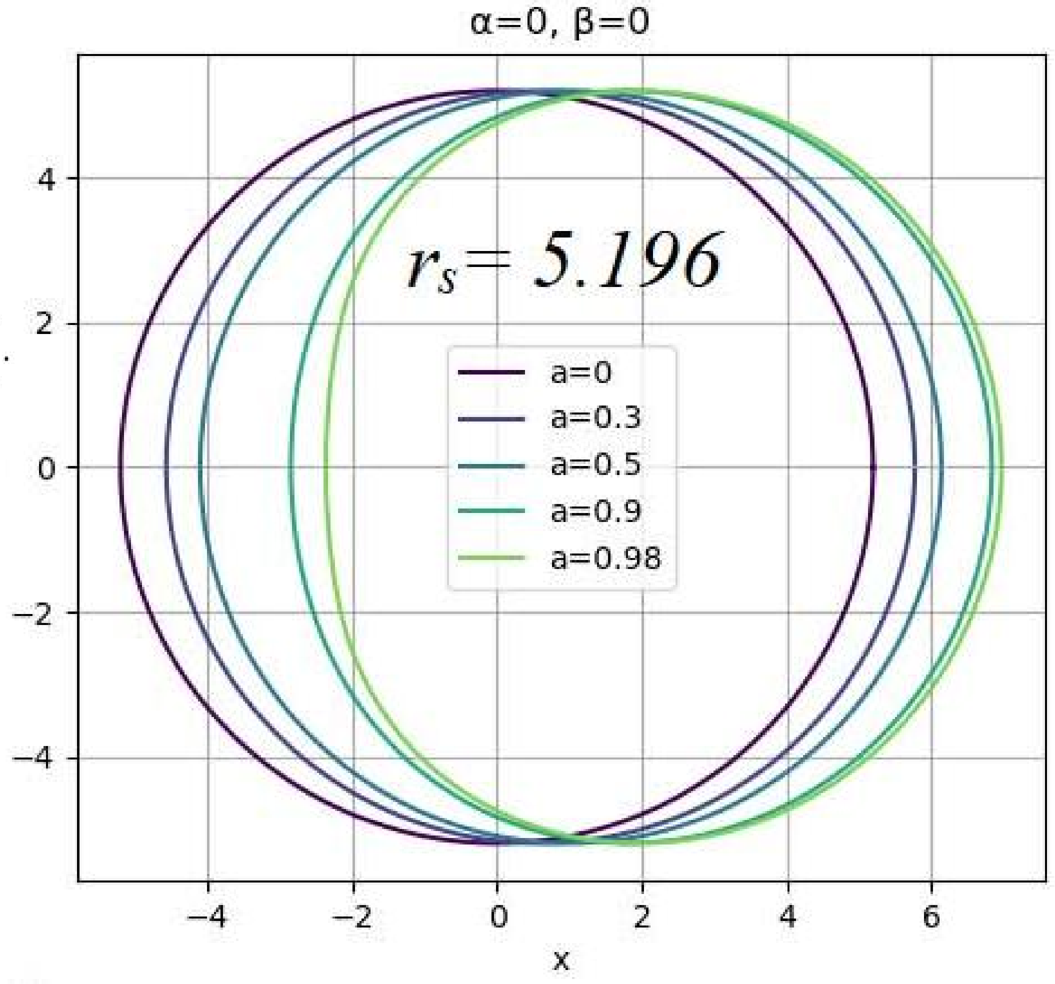
$$x' = -\frac{\lambda}{\sin\theta_0}, \qquad \lambda = \frac{K + a^2}{a} - \frac{2K'}{a}\frac{(FH + a^2)}{(HF)'},$$

$$y' = \pm\sqrt{\eta + a^2\cos^2\theta_0 - \frac{\lambda^2}{\tan^2\theta_0}}, \qquad \eta = \frac{4(a^2 + FH)}{(HF)'^2}K'^2 - \frac{1}{a^2}\left[K - \frac{2(FH + a^2)}{(HF)'}K'\right].$$





$$D = \frac{x_{min} + x_{max}}{2}, \qquad \delta_{cs} = \Delta_{cs}/r_s,$$



СА, А.А Байдерин, А.В. Немтинова, О.И. Зенин, ЖЭТФ 165, 508 (2024)

K. Hioki and Kei-ichi Maeda, Phys. Rev. D **80**, 024042 (2009).

Approved Newman-Janis method

$$ds^2 = -G(r)dt^2 + \frac{1}{F(r)}dr^2 + H(r)d\Omega^2$$

$$g_{tt} = -\frac{FH + a^2\cos^2\theta}{(K + a^2\cos^2\theta)^2}\Psi,$$

$$g_{t\phi} = -a\sin^2\theta\frac{K - FH}{(K + a^2\cos^2\theta)^2}\Psi,$$

$$g_{\theta\theta} = \Psi,$$

$$g_{rr} = \frac{\Psi}{FH + a^2},$$

$$g_{\phi\phi} = \Psi\sin^2\theta\left(1 + a^2\sin^2\theta\frac{2K - FH + a^2\cos^2\theta}{(K + a^2\cos^2\theta)^2}\right),$$

$$K = H(r)\sqrt{\frac{F(r)}{G(r)}}.$$

+ additional equations on Ψ

$$\lim_{a\to 0}\Psi(r, y^2, a) = H(r)$$

$$(K + a^2y^2)^2(3\Psi_{,r}\Psi_{,y^2} - 2\Psi\Psi_{,r,y^2}) = 3a^2K_{,r}\Psi^2,\ \Psi[K_{,r}^2 + K(2 - K_{,rr}) - a^2y^2(2 + K_{,rr})] + (K + a^2y^2)[(4y^2\Psi_{,y^2} - K_{,r}\Psi_{,r}] = 0.$$

E.T. Newman, A.I. Janis, *J. Math. Phys*. 6, 915 (1965) ... M. Azreg-Aïnou, , *Eur. Phys. J. C* 74, 2865 (2014)

Horndesky theory

$$ds^2 = -\quad (1 - \frac{2M}{r} - \frac{8\alpha_5\eta}{5r^3})dt^2 + \frac{1}{1 - \frac{2M}{r} - \frac{8\alpha_5\eta}{5r^3}}dr^2 + r^2 d\Omega^2.$$

$$G(r) = F(r) = 1 - \frac{2M}{r} - \frac{8\alpha_5\eta}{5r^3} + O(\frac{1}{r^4})$$

$$H(r) = K(r) = r^2, \qquad \Psi_n = r^2 + a^2 y^2.$$

$$g_{tt} = -\left(1 - \frac{2Mr}{\rho^2} - \frac{8\alpha_5\eta}{5r}\right),$$

$$g_{t\phi} = -\frac{2a\sin^2\theta}{5r\rho^2}\left(4\alpha_5\eta + 9Mr^2\right),$$

$$g_{rr} = \rho^2\left(-\frac{8\alpha_5\eta}{5r} + a^2 - 2Mr + r^2\right)^{-1},$$

$$g_{\theta\theta} = \rho^2,$$

$$g_{\phi\phi} = \frac{\sin^2\theta}{\rho^2}\left(r^4 + 2ar^2\cos^2\theta + a^4\cos^4\theta + \frac{8a^2\alpha_5\eta\sin^2\theta}{5r} + 2aMr\sin^2\theta + a^2r^2\sin^2\theta + a^4\cos^2\theta\sin^2\theta\right),$$

SA, A.Baiderin, O.Zenin, submitted to EPJ C

Bumblebee model

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{1+l}{1-\frac{2M}{r}}dr^2 + r^2 d\Omega^2,$$

$$G(r) = 1 - \frac{2M}{r}, \qquad F(r) = \frac{G(r)}{1+l},$$

$$H(r) = r^{\sqrt{1+l}+1}, \qquad K(r) = r^2\sqrt{1+l}^{-1},$$

$$\Psi_n = H(r) + a^2 y^2 r^{\sqrt{1+l}-1}\frac{8(1+l)^{1/2}-4}{8-2(1+(1+l)^{1/2})}.$$

$$g_{tt} = \frac{r^{-1+\sqrt{1+l}}AB}{\sqrt{1+l}CD},$$

$$g_{t\phi} = -\frac{ar^{-l+\sqrt{1+l}}EB\sin^2\theta}{(1+l)CD},$$

$$g_{rr} = -\frac{(1+l)r^{-l+\sqrt{1+l}}B}{CG},$$

$$g_{\theta\theta} = r^{1+\sqrt{1+l}} + \frac{a^2(-4+8\sqrt{1+l})r^{-l+\sqrt{1+l}}\cos^2\theta}{8-2(1+\sqrt{1+l})},$$

$$g_{\phi\phi} = \frac{r^{-l+\sqrt{1+l}}\sin^2\theta(B+5a^2\cos^2\theta)}{(1+l)CD} \times (D(1+l) - Ka^2\cos^2\theta),$$

$$A = (2Mr^{1+l} - r^{1+\sqrt{1+l}} - a^2\cos^2\theta - a^2 l\cos^2\theta),$$

$$B = -3r^2 + \sqrt{1+l}r^2 - 3a^2\cos^2\theta - 4a^2\sqrt{1+l}\cos^2\theta,$$

$$C = -3 + \sqrt{1+l},$$

$$D = r^2 + a^2\sqrt{1+l}\cos^2\theta,$$

$$E = -r^2 - lr^2 - 2\sqrt{1+l}Mr^{\sqrt{1+l}} + \sqrt{1+l}r^{1+\sqrt{1+l}},$$

$$G = a^2 + a^2 l - 2Mr^{1+l} + r^{1-\sqrt{1+l}},$$

$$F = -2Mr^{\sqrt{1+l}} + r^{1+\sqrt{1+l}} - a^2 l\cos^2\theta,$$

$$K = \sqrt{1+l}F - r - 2lr^2 - D.$$

Casana, R.; Cavalcante, A.; Poulis, F.P.; Santos, E.B. Phys. Rev. D 97, 104001, (2018)

S.Capozziello, S.Zare, H.Hassanabadi, e-Print: 2311.12896

SA, A.Baiderin, O.Zenin, submitted to EPJ C,

Scalar Gauss-Bonnet Gravity

$$ds^2 = -\ f_s(1+\frac{\xi}{3r^3 f_s})dt^2 + \frac{(1-\frac{\xi}{r^3 f_s})}{f_s}dr^2 + $$
$$+\ (r^2 + \frac{\xi}{3r} + \frac{2\xi M}{3r^2})d\Omega^2,$$

$$G(r) = f_s(1+\frac{\xi}{3r^3 f_s}) + o(\frac{1}{r^3}),$$
$$F(r) = \frac{f_s}{(1-\frac{\xi}{r^3 f_s})} + o(\frac{1}{r^3}),$$
$$H(r) = 2\frac{K}{K_r}r,$$
$$K(r) = r^2 + \frac{\xi}{3r} + \frac{2\xi M}{3r^2} + o(1/r^3),$$
$$\Psi_c = 2\frac{K}{K_r}r + a^2\frac{H^2(8K-K_r^2)y^2}{K^2(8H-H_r K_r)},$$
$$f_s = 1-\frac{2M}{r}.$$

$$\frac{1}{K_r} = \frac{1}{2r} + o(1/r^3).$$

$$g_{tt} = \frac{r^2(E+F\cos^2\theta)}{AB},$$
$$g_{t\phi} = -\frac{aCD\sin^2\theta}{AB},$$
$$g_{rr} = -\frac{AB}{r^2(E+F)},$$
$$g_{\theta\theta} = \frac{B}{3r^2},$$
$$g_{\phi\phi} = \frac{1+Q+9a^4r^4A\cos^4\theta+6a^2r^2G\sin^2\theta}{3r^2AB} + \frac{9a^4r^4A\cos^2\theta\sin^2\theta}{3r^2AB}.$$

$$A = \xi + 2Mr^2 - r^3,$$
$$B = 2\xi M + \xi r + 3r^4 + 3a^2r^2\cos^2\theta,$$
$$C = 2\xi M + \xi r + 3r^4,$$
$$D = A + 16M^2r^2 - 16Mr^4 + 4r^5,$$

$$E = 32\xi M^3 r - 16\xi M^2 r^2 - 8\xi M r^3 + 4\xi r^4 + 48M^2r^5 - 48Mr^6 + 12r^7,$$
$$F = -3a^2\xi - 6a^2Mr^2 + 3a^2r^3,$$
$$G = 16\xi M^3 r^5 + 2\xi r^6 + 24M^2r^7 - 24Mr^8 + 6r^9,$$
$$K = 2\xi^2 M + \xi^2 r + 4\xi M^2 r^2 + 2\xi r^4 + 6Mr^6 - 3r^7,$$
$$Q = 4\xi^3 M(M+r) + \xi^2 r^2(\xi + 2M^3 + 4M^2 r + 10Mr^2 + 5r^3) + 3\xi r^6(8M^2 + r^2) + 9r^{10}(2M-r).$$

SA, A.Baiderin, O.Zenin, submitted to EPJ C