Cosmological models with arbitrary spatial curvature in the theory of gravity with non-minimal derivative coupling

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Dedicated to the memory of Alexei Starobinsky

Based on

- S.V. Sushkov, R. Galeev, Phys. Rev. D **108**, 044028 (2023)
- R. Galeev, R.K. Muharlyamov, A.A. Starobinsky, S.V. Sushkov, M.S. Volkov, PRD **103**, 104015 (2021)
- A.A. Starobinsky, S.V. Sushkov, M.S. Volkov, PRD, **101**, 064039 (2020)
- A.A. Starobinsky, S.V. Sushkov, M.S. Volkov, JCAP **1606** (2016) no.06, 007
- J. Matsumoto, S.V. Sushkov, JCAP, **01**, 040 (2018)
- M.A. Skugoreva, S.V. Sushkov, A.V. Toporensky, PRD **88**, 083539 (2013)
- S.V. Sushkov, PRD **85**, 123520 (2012)
- E.N. Saridakis, S.V. Sushkov, PRD **81**, 083510 (2010)
- S.V. Sushkov, PRD **80**, 103505 (2009)

Motivation

- GR has successfully been exploited for a long time to describe celestial motion in Solar system, a bending of light rays, gravitational waves, the universe expansion (ΛCDM model)
- GR is unable to solve the number already existing problems and appearing new ones
  - cosmological and black hole singularities
  - dark energy (accelerating expansion of the Universe)
  - initial inflation
  - large scale structure of the universe
  - dark matter evidence
  - cosmological constant problem
  - etc. . .
- These amazing discoveries have set new serious challenges before theoretical cosmology faced the necessity of radical *modification* or *extension* of General Relativity

Scalar-tensor theories

$$S = \int d^4x\sqrt{-g}\left[F(\phi)R - Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2U(\phi)\right] + S_m\left[\psi_m, g_{\mu\nu}\right]$$

- generalizations of the Brans-Dicke theories
- the scalar field is
  - minimally coupled with ordinary matter (physical or Jordan frame)
  - non-minimally coupled with the scalar curvature by the term $F(\phi)R$

Notice: Non-minimal coupling of the scalar field with the scalar curvature is provided by the terms $F(\phi)R$

In 1974, *Gregory Walter Horndeski* derived the action of the most general scalar-tensor theories with second-order equations of motion [G.Horndeski, *Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space*, IJTP **10**, 363 (1974)]

Horndeski Lagrangian:[1]

$$L_{\rm H} = \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)$$

$$\mathcal{L}_2 = G_2(\phi, X)\ ,$$
$$\mathcal{L}_3 = G_3(\phi, X)\,\Box\phi\ ,$$
$$\mathcal{L}_4 = G_4(\phi, X)R - 2G_{4,X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})\ ,$$
$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{\mu\nu} + \tfrac{1}{3}G_{5,X}(\phi, X)(\Box\phi^3 - 3\,\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}{}_{\sigma})\,,$$

$G_a(\phi, X)$ are four arbitrary functions, and $X = -\frac{1}{2}(\nabla\phi)^2$

Notice: Non-minimal coupling of the scalar field with curvature is provided by two terms, $G_4(\phi, X)R$ and $G_5(\phi, X)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$

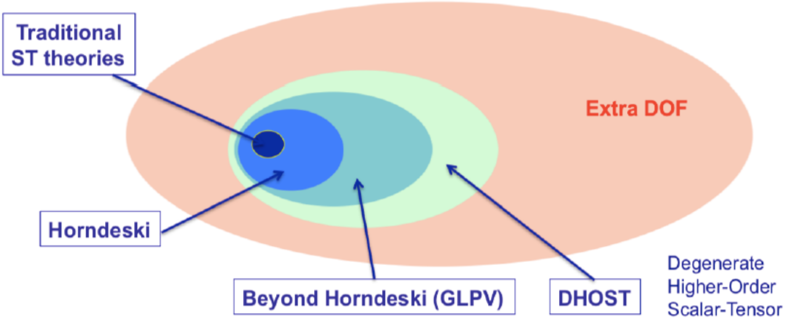
[1] T. Kobayashi, M. Yamaguchi, J. Yokoyama, Prog. Theor. Phys. **126**, 511 (2011).

Subclasses of the Horndeski theory

$$\mathcal{L}_H = \mathcal{L}\{G_2, G_3, G_4, G_5\}$$

- Hilbert-Einstein action (GR):
  $G_4(\phi, X) = \frac{1}{2}M_{Pl}^2 \quad \rightarrow \quad \mathcal{L}_H \sim \frac{1}{2}M_{Pl}^2 R$
- Nonminimal coupling: $G_4(\phi, X) = f(\phi) \quad \rightarrow \quad \mathcal{L}_H \sim f(\phi)R$
- GR with a scalar field: $G_2(\phi, X) = \epsilon X - V(\phi)$
- k-essence: $G_2 = K(\phi, X)$
- Kinetic gravity braiding (KGB):
  $G_3 = B(\phi, X) \quad \rightarrow \quad \mathcal{L}_H \sim B(\phi, X)\Box\phi$
- Nonminimal kinetic coupling:
  $G_5(\phi, X) = \eta\phi \quad \rightarrow \quad \mathcal{L}_H \sim \eta G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$
- Fab Four, Gallileons, etc.

Extended scalar-tensor theories



Landscape of scalar-tensor theories
D. Langlois, Dark energy and modified gravity in degenerate higher-order scalar-tensor (DHOST) theories: A review Int. J. Mod. Phys. D 28 (2019), no. 05 1942006

DHOST theories

$$S=\int d^4x\sqrt{-g}\Big[F_{(2)}(\phi,X)R+P(\phi,X)+Q(\phi,X)\Box\phi$$
$$+F_{(3)}(\phi,X)\,G_{\mu\nu}\phi^{\mu\nu}+\sum_{a=1}^{5}A_a(\phi,X)L_a^{(2)}+\sum_{a=1}^{10}B_a(\phi,X)L_a^{(3)}\Big]$$

$$L_1^{(2)}=\phi_{\mu\nu}\phi^{\mu\nu}\,,\qquad L_2^{(2)}=(\Box\phi)^2\,,\qquad L_3^{(2)}=(\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu\,,$$
$$L_4^{(2)}=\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu\,,\qquad L_5^{(2)}=(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2\,.$$

$$L_1^{(3)}=(\Box\phi)^3\,,\quad L_2^{(3)}=(\Box\phi)\,\phi_{\mu\nu}\phi^{\mu\nu}\,,\quad L_3^{(3)}=\phi_{\mu\nu}\phi^{\nu\rho}\phi^\mu_\rho\,,$$
$$L_4^{(3)}=(\Box\phi)^2\,\phi_\mu\phi^{\mu\nu}\phi_\nu\,,\quad L_5^{(3)}=\Box\phi\,\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\,,\quad L_6^{(3)}=\phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma\,,$$
$$L_7^{(3)}=\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^{\rho\sigma}\phi_\sigma\,,\quad L_8^{(3)}=\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\,\phi_\sigma\phi^{\sigma\lambda}\phi_\lambda\,,$$
$$L_9^{(3)}=\Box\phi\,(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2\,,\quad L_{10}^{(3)}=(\phi_\mu\phi^{\mu\nu}\phi_\nu)^3\,.$$

Notice: Non-minimal coupling of the scalar field with curvature is provided by two terms, $F_{(2)}(\phi,X)R$ and $F_{(3)}(\phi,X)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$

Non-minimal coupling of the scalar field with curvature

Notice: There are only two qualitatively different terms describing non-minimal coupling of the scalar field with curvature: $M(\phi, X)R$ and $N(\phi, X)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$.

- $M(\phi, X)R$ — Brans-Dicke-like theories
- $N(\phi, X)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ — theories with non-minimal derivative coupling

Theory with nonminimal derivative coupling. I

Focusing on non-minimal derivative coupling, we have

Action: $S = S^{(g)} + S^{(m)}$

$S^{(m)}$ — *the action for ordinary matter fields*

$$S^{(g)} = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2(R-\Lambda) - (\varepsilon\, g_{\mu\nu} + \eta\, G_{\mu\nu})\nabla^\mu\phi\nabla^\nu\phi - 2V(\phi)\right]$$

Λ — *cosmological constant*

$\varepsilon = 1$ *(ordinary scalar field)*;
$\varepsilon = -1$ *(phantom scalar field)*;
$\varepsilon = 0$ *(no standard kinetic term)*

η — *dimensional coupling parameter*; $[\eta] = (\textit{length})^2 \;\rightarrow\; \eta = \pm\ell^2$

ℓ — *characteristic scale of non-minimal coupling*

Theory with nonminimal derivative coupling. II

Field equations:

$$G_{\mu\nu} = -g_{\mu\nu}\Lambda + 8\pi \left[T^{(m)}_{\mu\nu} + T^{(\phi)}_{\mu\nu} + \eta\,\Theta_{\mu\nu}\right]$$

$$[\varepsilon g^{\mu\nu} + \eta G^{\mu\nu}]\nabla_\mu\nabla_\nu\phi = V'_\phi$$

$$\begin{aligned}
T^{(\phi)}_{\mu\nu} =& \varepsilon\left[\nabla_\mu\phi\nabla_\nu\phi - \tfrac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right] - g_{\mu\nu}V(\phi),\\
\Theta_{\mu\nu} =& -\tfrac{1}{2}\nabla_\mu\phi\,\nabla_\nu\phi\,R + 2\nabla_\alpha\phi\,\nabla_{(\mu}\phi R^\alpha_{\nu)} - \tfrac{1}{2}(\nabla\phi)^2 G_{\mu\nu} + \nabla^\alpha\phi\,\nabla^\beta\phi\,R_{\mu\alpha\nu\beta}\\
&+\nabla_\mu\nabla^\alpha\phi\,\nabla_\nu\nabla_\alpha\phi - \nabla_\mu\nabla_\nu\phi\,\Box\phi + g_{\mu\nu}\big[-\tfrac{1}{2}\nabla^\alpha\nabla^\beta\phi\,\nabla_\alpha\nabla_\beta\phi + \tfrac{1}{2}(\Box\phi)^2\\
&-\nabla_\alpha\phi\,\nabla_\beta\phi\,R^{\alpha\beta}\big]\\
T^{(m)}_{\mu\nu} =& (\rho + p)u_\mu u_\mu + p g_{\mu\nu}
\end{aligned}$$

Notice: *The field equations are of second order!*

Isotropic and homogeneous cosmological models

Ansatz: $V \equiv 0$ (no potential), $\varepsilon = +1$ (ordinary scalar)
$\phi = \phi(t)$, $T^{(m)}_{\mu\nu} = diag(\rho(t), p(t), p(t), p(t))$, and the FLRW metric

$$ds^2 = -dt^2 + \mathrm{a}^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]$$

$k = 0, \pm 1$, $\mathrm{a}(t)$ *cosmological factor*, $H(t) = \dot{\mathrm{a}}(t)/\mathrm{a}(t)$ *Hubble parameter*

Gravitational equations:

$$3\left(H^2 + \frac{k}{\mathrm{a}^2}\right) = \Lambda + 8\pi\rho + 4\pi\psi^2\left(1 - 9\eta\left(H^2 + \frac{k}{3\mathrm{a}^2}\right)\right),$$

$$2\dot{H} + 3H^2 + \frac{k}{\mathrm{a}^2} = \Lambda - 8\pi p - 4\pi\psi^2\left[1 + 2\eta\left(\dot{H} + \frac{3}{2}H^2 - \frac{k}{\mathrm{a}^2} + 2H\frac{\dot{\psi}}{\psi}\right)\right]$$

The scalar field equations:

$$\mathrm{a}^3\psi\left(1 - 3\eta\left(H^2 + \frac{k}{\mathrm{a}^2}\right)\right) = Q = const$$

where $\psi = \dot{\phi}$

Modified Friedmann equation (Master equation). I

Material content is a mixture of radiation and non-relativistic component:

$$\rho = \rho_m + \rho_r = \rho_{m0}\left(\frac{\mathrm{a}_0}{\mathrm{a}}\right)^3 + \rho_{r0}\left(\frac{\mathrm{a}_0}{\mathrm{a}}\right)^4$$

Introducing the dimensionless scales factor a, Hubble parameter h, and coupling parameter ζ:

$$a = \frac{\mathrm{a}}{\mathrm{a}_0}, \quad h = \frac{H}{H_0}, \quad \zeta = \eta H_0^2,$$

and the dimensionless density parameters:

$$\Omega_0 = \frac{\Lambda}{3H_0^2}, \quad \Omega_2 = \frac{k}{\mathrm{a}_0^2 H_0^2}, \quad \Omega_3 = \frac{\rho_{m0}}{\rho_{cr}}, \quad \Omega_4 = \frac{\rho_{r0}}{\rho_{cr}}, \quad \Omega_6 = \frac{4\pi Q^2}{3\mathrm{a}_0^6 H_0^2},$$

where $\rho_{cr} = 3H_0^2/8\pi$ is the critical density, one has

Modified Friedmann equation

$$h^2 = \Omega_0 - \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left(1 - 3\zeta(3h^2 + \frac{\Omega_2}{a^2})\right)}{a^6\left(1 - 3\zeta(h^2 + \frac{\Omega_2}{a^2})\right)^2}$$

Modified Friedmann equation (Master equation). II

Modified Friedmann equation

$$h^2 = \Omega_0 - \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left(1 - 3\zeta(3h^2 + \frac{\Omega_2}{a^2})\right)}{a^6\left(1 - 3\zeta(h^2 + \frac{\Omega_2}{a^2})\right)^2}$$

- Assuming $\Lambda \geq 0$, one has $\Omega_0 \geq 0$
- $\Omega_2 = k/\mathrm{a}_0^2 H_0^2$, hence
  $\Omega_2 = 0$, $\Omega_2 < 0$, $\Omega_2 > 0$ if $k = 0, -1, +1$, respectively
- $\zeta = \eta H_0^2 = \pm\left(\ell/\ell_H\right)^2$, where $\ell_H = 1/H_0$, hence
  ζ is proportional to the square of ratio of two characteristic scales,
  hence $\zeta \ll 1$???
- In case $\Omega_6 = 0$ (no scalar with non-minimal derivative coupling) one has the standard master equation of ΛCDM cosmological model
- In case $\Omega_6 \neq 0$ but $\zeta = 0$ (no non-minimal derivative coupling) one has a cosmological model with an ordinary scalar field

Denoting $y = h^2$ one can rewrite the master equation as a cubic in y algebraic equation

$$y^3 + c_2(a)y^2 + c_1(a)y + c_0(a) = 0$$

with the coefficients

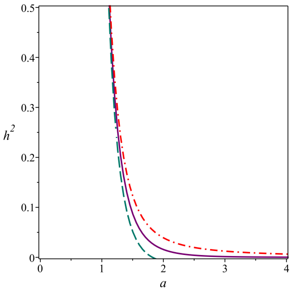
$$\begin{aligned}
c_2 &= -\Omega_0 + \frac{3\Omega_2}{a^2} - \frac{\Omega_3}{a^3} - \frac{\Omega_4}{a^4} - \frac{2}{3\zeta}, \\
c_1 &= -\frac{2\Omega_2}{a^2}\left(\Omega_0 - \frac{3}{2}\frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4}\right) \\
&\quad + \frac{1}{3\zeta}\left(2\Omega_0 - \frac{4\Omega_2}{a^2} + \frac{2\Omega_3}{a^3} + \frac{2\Omega_4}{a^4} + \frac{3\Omega_6}{a^6}\right) + \frac{1}{9\zeta^2}, \\
c_0 &= -\frac{\Omega_2^2}{a^4}\left(\Omega_0 - \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4}\right) \\
&\quad + \frac{\Omega_2}{3a^2\zeta}\left(2\Omega_0 - \frac{2\Omega_2}{a^2} + \frac{2\Omega_3}{a^3} + \frac{2\Omega_4}{a^4} + \frac{\Omega_6}{a^6}\right) \\
&\quad - \frac{1}{9\zeta^2}\left(\Omega_0 - \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6}{a^6}\right).
\end{aligned}$$

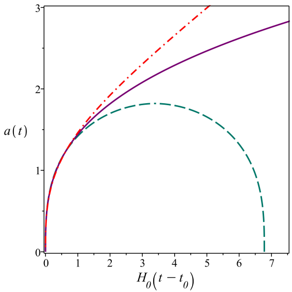
Notice: Roots $h = h(a)$ of the cubic polynomial (16) define a global cosmological behavior

Cosmological scenarios. I.
The case $\zeta = 0$ and $\Omega_0 = \Omega_3 = \Omega_4 = 0$

$$h^2 = -\frac{\Omega_2}{a^2} + \frac{\Omega_6}{a^6}$$

- At early times, when $a \to 0$, one has $h^2 \approx \Omega_6/a^{-6} \to \infty$, that is an initial cosmological singularity
- The later evolution essentially depends on the sign of Ω_2, i.e. on the spatial curvature of the universe





Cosmological scenarios. II.
The case $\zeta \neq 0$ and $\Omega_3 = \Omega_4 = 0$ (no matter)

Master equation:

$$h^2 = \Omega_0 - \frac{\Omega_2}{a^2} + \frac{\Omega_6\left(1 - 3\zeta(3h^2 + \frac{\Omega_2}{a^2})\right)}{a^6\left(1 - 3\zeta(h^2 + \frac{\Omega_2}{a^2})\right)^2}$$

The early time universe evolution (the limit $a \to 0$)

Asymptotics:

$$h^2 = -\frac{\Omega_2}{3a^2} + \left(\frac{1}{9\zeta} - \frac{8\zeta\Omega_2^3}{27\Omega_6}\right) + O(a^2)$$

- First two major terms in the asymptotic (18) do not contain the cosmological constant Ω_0!
- Following [2], we may say that the cosmological constant is *screened* at the early stage and makes no contribution to the universe evolution.

[2] A. A. Starobinsky, S. V. Sushkov, and M. S. Volkov, *The screening Horndeski cosmologies*, *JCAP* **1606** (2016), no. 06 007

Cosmological scenarios. II.
The case $\zeta \neq 0$ and $\Omega_3 = \Omega_4 = 0$ (no matter)

Zero spatial curvature ($k = 0$, $\Omega_2 = 0$):

$$h^2 = \frac{1}{9\zeta} + O(a^6)$$

- **Therefore** at early cosmological times one has an *eternal* ($t \to -\infty$) inflation with the quasi-De Sitter behavior of the scale factor: $a(t) \propto e^{H_\eta t}$, where $H_\eta = 1/\sqrt{9\eta}$.
- **Notice:** that the primary inflationary epoch is only driven by non-minimal derivative or *kinetic* coupling between the scalar field and curvature without introducing any fine-tuned potential, and so one can call this epoch as a *kinetic* inflation.

Cosmological scenarios. II.
The case $\zeta \neq 0$ and $\Omega_0 = \Omega_3 = \Omega_4 = 0$

Negative spatial curvature ($k = -1$, $\Omega_2 < 0$):

$$h^2 = \frac{|\Omega_2|}{3a^2} + \left(\frac{1}{9\zeta} + \frac{8\zeta|\Omega_2|^3}{27\Omega_6}\right) + O(a^2).$$

- The Hubble parameter h has a *singular* behavior at $a \to 0$, so that $h^2 \approx |\Omega_2|/3a^2 \to \infty$
- As a increases, the first term in the asymptotic (**??**) decreases and becomes negligible with respect to the second one. As the scale factor a grows further, the behavior of Hubble parameter is determined by the second term in (**??**), so that $h^2 \approx h_{dS}^2 = \frac{1}{9\zeta} + \frac{8\zeta|\Omega_2|^3}{27\Omega_6}$ and $a(t) \propto e^{h_{dS}(H_0 t)}$. This stage can be called as a *quasi-de Sitter era* with the de Sitter parameter h_{dS}.

Cosmological scenarios. II.
The case $\zeta \neq 0$ and $\Omega_0 = \Omega_3 = \Omega_4 = 0$

Positive spatial curvature ($k = +1$, $\Omega_2 > 0$):

$$h^2 = -\frac{\Omega_2}{3a^2} + \left(\frac{1}{9\zeta} - \frac{8\zeta\Omega_2^3}{27\Omega_6}\right) + O(a^2).$$

- There exists some small minimal value of $a = a_{min}$,

$$a_{min}^2 \approx 3\zeta\Omega_2 \left(1 - \frac{8\zeta^2\Omega_2^2}{3\Omega_6}\right)^{-1},$$

  such that the value of h^2 becomes to be zero!!!
- A moment t_B when the Hubble parameter h, or $\dot{a}$, equals to zero is a turning point in the universe evolution, or a *bounce*, when the stage of contraction is changing to expansion one.
- The minimal size of the universe can be estimated as follows

$$\mathrm{a}_{min} = \sqrt{3}\,\ell,$$

  where ℓ is the characteristic scale of nonminimal derivative coupling.

Cosmological scenarios. II.
The case $\zeta \neq 0$ and $\Omega_0 = \Omega_3 = \Omega_4 = 0$

Master equation:

$$h^2 = \Omega_0 - \frac{\Omega_2}{a^2} + \frac{\Omega_6\left(1 - 3\zeta(3h^2 + \frac{\Omega_2}{a^2})\right)}{a^6\left(1 - 3\zeta(h^2 + \frac{\Omega_2}{a^2})\right)^2}$$

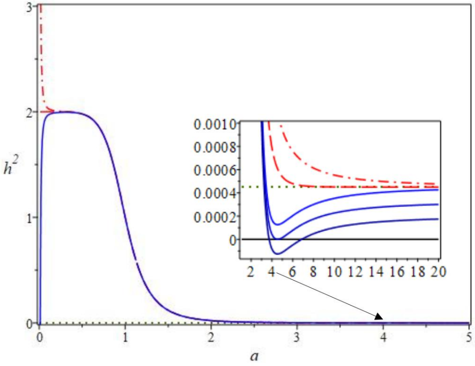
The late time universe evolution (the limit $a \to \infty$)

- In the case $\Omega_2 \leq 0$, at the late stage of evolution the universe enters a *secondary inflation epoch* with $h^2 = \Omega_0$, i.e. $H = H_\Lambda = \sqrt{\Lambda/3}$.
- In the case $\Omega_2 > 0$, the squared Hubble parameter has an extremal value h^2_{extr} such that $d(h^2)/da = 0$. In case $h^2_{extr} > 0$ one has the inflationary asymptotic $h^2 = \Omega_0$. In case $h^2_{extr} \leq 0$, there is a turning point in the universe evolution, when the expansion stage is changing to contraction one.
- In the last case one has a *cyclic scenario* of the universe evolution.

The case $\zeta \neq 0$ and $\Omega_0 = \Omega_3 = \Omega_4 = 0$

Graphical representation:

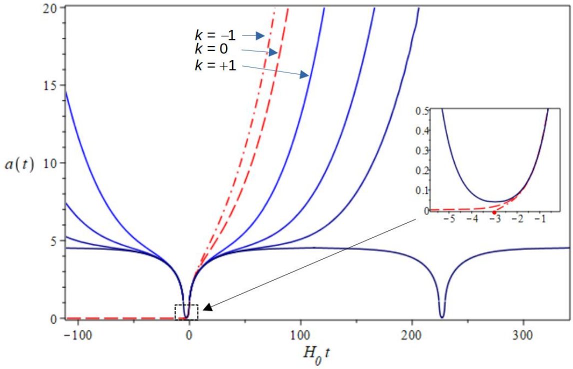
Plots of h^2 versus a



Cosmological scenarios. II.
The case $\zeta \neq 0$ and $\Omega_0 = \Omega_3 = \Omega_4 = 0$

Graphical representation:

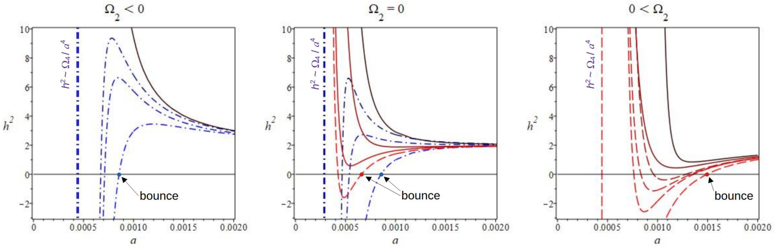
Plots of a versus t



Master equation:

$$h^2 = \Omega_0 - \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left(1 - 3\zeta\left(3h^2 + \frac{\Omega_2}{a^2}\right)\right)}{a^6\left(1 - 3\zeta\left(h^2 + \frac{\Omega_2}{a^2}\right)\right)^2}$$

Graphical representation:



Notice: For *all* types of spatial geometry of the homogeneous universe, $k = 0, \pm 1$, there exists a wide domain of parameters Ω_3 and Ω_4 such that one has a *bounce* !

Role of anisotropy

Notice: Small anisotropy of the universe observed today could be catastrophically large on early stages of the universe evolution. Therefore the results obtained for isotropic cosmological models may not be valid!

The Bianchi I metric

$$ds^2 = -dt^2 + a_1^2\,dx_1^2 + a_2^2\,dx_2^2 + a_3^2\,dx_3^2\,,$$

where $a_i = a_i(t)$ and $\phi = \phi(t)$

Let us use **the standard parametrization**:

$$a_1 = ae^{\beta_+ + \sqrt{3}\beta_-}, \quad a_2 = ae^{\beta_+ - \sqrt{3}\beta_-}, \quad a_3 = ae^{-2\beta_+}$$

$\sigma^2 = \dot\beta_+^2 + \dot\beta_-^2$ is the *anisotropy parameter*, and $H = \dot a/a$

Field equations:

$$\begin{aligned} 3M_{\rm Pl}^2(H^2-\sigma^2) &= \tfrac{1}{2}\left(1-9\eta\left(H^2-\sigma^2\right)\right)\dot\phi^2+\Lambda,\\ \frac{d}{dt}\left[\mathrm{a}^3\dot\beta_\pm(2M_{\rm Pl}^2+\eta\dot\phi^2)\right] &= 0,\\ \frac{d}{dt}\left[\mathrm{a}^3\left(3\eta\left(H^2-\sigma^2\right)-1\right)\dot\phi\right] &= 0. \end{aligned}$$

Anisotropy parameter:

$$\sigma^2 = \frac{C^2}{a^6(2M_{\rm Pl}^2 + \eta\dot{\phi}^2)^2}$$

Asymptotic behavior of anisotopy:

As expected, at late times anisotropy is *damping* in the usual way

$$a \to \infty \qquad \Longrightarrow \qquad \sigma^2 \sim a^{-6} \to 0$$

Suprisingly, unlike GR, anisotropy is *screened* at early times!

$$a \to 0, \; \dot{\phi}^2 \sim a^{-6} \qquad \Longrightarrow \qquad \sigma^2 \sim a^6 \to 0$$

Therefore, contrary to what one would normally expect, the early state of the Universe in the theory cannot be anisotropic!

The Bianchi IX metric

$$ds^2 = -dt^2 + \tfrac{1}{4}a_1^2\,\omega_1 \otimes \omega_1 + \tfrac{1}{4}a_2^2\,\omega_2 \otimes \omega_2 + \tfrac{1}{4}a_3^2\,\omega_3 \otimes \omega_3\,,$$

where ω_a are 1-forms, $d\omega_a = \varepsilon_{abc}\,\omega_b \wedge \omega_c$

Parameterization: $a_1 = ae^{\beta_+ + \sqrt{3}\beta_-},\ a_2 = ae^{\beta_+ - \sqrt{3}\beta_-},\ a_3 = ae^{-2\beta_+}$

$a^3 = a_1 a_2 a_3$ — a volume;
$H = \dot{a}/a$ — an 'average' Hubble parameter
$\beta_\pm$ parameterize deviation from isotropy
$\sigma^2 = \dot{\beta}_+^2 + \dot{\beta}_-^2$ — an anisotropy parameter
$\mathcal{H}^2 = H^2 - \sigma^2$ where $\mathcal{H}$ is an 'anisotropic' Hubble parameter

The effective spacial curvature $\mathcal{K}$: if $\beta_\pm = 0$, then $\mathcal{K} = 1$

$$\mathcal{K} = -\frac{1}{3}e^{-8\beta_+}\left(4e^{6\beta_+}\cosh^2(\sqrt{3}\beta_-) - 1\right)\left(4e^{6\beta_+}\sinh^2(\sqrt{3}\beta_-) - 1\right)$$

Field equations:

$$3M_{\rm Pl}^2\left(\mathcal{H}^2+\frac{\mathcal{K}}{a^2}\right)=\frac{1}{2}\dot{\phi}^2+\Lambda-\frac{9}{2}\eta\,\dot{\phi}^2\left(\mathcal{H}^2+\frac{\mathcal{K}}{3a^2}\right), \quad (1)$$

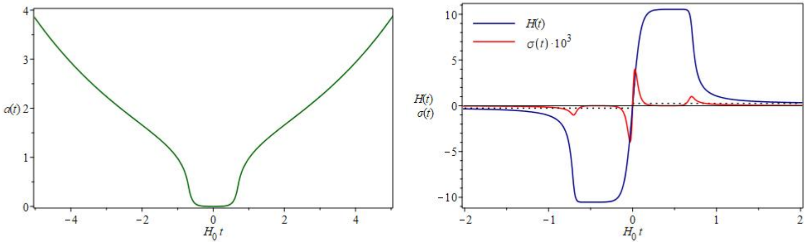
$$\frac{1}{a^2}\frac{d}{dt}\left[(2M_{\rm Pl}^2+\eta\dot{\phi}^2)a\dot{a}\right]=(2M_{\rm Pl}^2+\eta\dot{\phi}^2)(\tfrac{1}{2}\mathcal{H}^2-\sigma^2)$$
$$-(M_{\rm Pl}^2-\tfrac{1}{2}\eta\dot{\phi}^2)\frac{\mathcal{K}}{a^2}-\frac{1}{2}\dot{\phi}^2+\Lambda, \quad (2)$$

$$\frac{d}{dt}\left[a^3\dot{\beta}_\pm(2M_{\rm Pl}^2+\eta\dot{\phi}^2)\right]=a(M_{\rm Pl}^2-\tfrac{1}{2}\eta\dot{\phi}^2)\frac{\partial\mathcal{K}}{\partial\beta_\pm}, \quad (3)$$

$$\frac{d}{dt}\left[a^3\dot{\phi}\left(1-3\eta\left(\mathcal{H}^2+\frac{\mathcal{K}}{a^2}\right)\right)\right]=0. \quad (4)$$

Anisotropic cosmologies: Bianchi IX model. III

Numerical solution:



Notice: Contrary to the Belinskii-Khalatnikov-Lifshits mechanism of oscillatory approaching to the singularity, the anisotropy tends to zero at the moment of the bounce!

Conclusions

- The cosmological constant Λ (or Ω_0) turns out to be *screened* at early times and makes no contribution to the universe evolution
- Depending on model parameters, there are three qualitatively different initial state of the universe: an *eternal kinetic inflation*, an *initial singularity*, and a *bounce*. The bounce is possible for *all* types of spatial geometry of the homogeneous universe.
- For *all* types of spatial geometry, we found that the universe goes inevitably through the *primary quasi-de Sitter* (inflationary) epoch with the de Sitter parameter $h_{dS}^2 = \frac{1}{9\zeta} - \frac{8\zeta\Omega_2^3}{27\Omega_6}$.
- For $k = 0$ this epoch lasts eternally to the past, when $t \to -\infty$. When $k = -1$ or $+1$, the primary inflationary epoch starts soon after a birth of the universe from an initial singularity, or after a bounce, respectively.
- The mechanism of primary or *kinetic* inflation is provided by non-minimal derivative coupling and needs *NO* fine-tuned potential.

Conclusions (Continuation...)

- In the course of cosmological evolution the domination of η-terms is canceled, and this leads to a *change* of cosmological epochs.
- The late-time universe evolution depends both on k and Λ. In the case $k = 0$ (zero spatial curvature), or $k = -1$ (negative spatial curvature), at late times the universe enters an epoch of *accelerated expansion* or a secondary inflationary epoch with $H = H_\Lambda = \sqrt{\Lambda/3}$. In case $k = +1$ (positive spatial curvature), there is a *turning point* in the universe evolution, when the expansion stage is changing to contraction one.
- Depending on model parameters, there are *cyclic scenarios* of the universe evolution with the non-singular bounce at a minimal value of the scale factor, and a turning point at the maximal one.
- Contrary to what one would normally expect, anisotropy is *dumped* at early stages of the universe evolution!

THANKS FOR YOUR ATTENTION!