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Conundrum of higher derivative quantum gravity: Hořava model, renormalization group and asymptotic freedom

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Plan

Horava gravity:

1) Renormalizable Hořava gravity: projectable models

2) Asymptotic freedom in (2+1)-dimensional model

3) Beta functions and RG fixed points in (3+1)-dimensions

4) RG flows and AF in (3+1)-dimensions

5) Riddles of higher derivative gravity models:

complexity of operator dimensions;

renormalizability and AF of nonprojectable HG?;

tadpoles, IR divergences and modification of beta functions in quadratic gravity (Donoghue et al);

no running of G and Λ *--- metamorphosis of the running scale*

Renormalization of Horava gravity

Saving unitarity in renormalizable QG

Einstein GR

$$S_{EH} = \frac{M_P^2}{2} \int dt d^d x \; R$$

nonrenormalizable

$$\Rightarrow \frac{M_P^2}{2} \int dt d^d x \; (h_{ij} \Box h_{ij} + h^2 \Box h + \dots)$$

Higher derivative gravity

Stelle (1977)

$$\int (M_P^2 R + R_{\mu\nu} R^{\mu\nu} + R^2)$$

$$\Rightarrow \int (M_P^2 h_{ij} \Box h_{ij} + h_{ij} \Box^2 h_{ij} + \dots)$$

dominates at $k \gg M_P$

The theory is renormalizable and asymptotically free !

Fradkin, Tseytlin (1981)
Avramidy & A.B. (1985)

But has ghost poles $\Rightarrow$ no unitary interpretation

Horava (2009)

$$\int \underbrace{dt\, d^d x}_{\propto\, b^{-(z+d)}} (\dot{h}_{ij}\dot{h}_{ij} - h_{ij}(-\Delta)^z h_{ij} + \ldots)$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}\,, \qquad t \mapsto b^{-z} t \qquad \Rightarrow \qquad h_{ij} \mapsto b^{(d-z)/2} h_{ij}$$

Critical theory in $z = d$

LI is necessarily broken. We want to preserve as many symmetries, as possible

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \qquad \Rightarrow \qquad \gamma_{ij} \quad N^i, \qquad i = 1, \ldots, d$$

$$t \mapsto \tilde{t}(t) \qquad \Rightarrow \qquad N$$

Foliation preserving diffeomorphisms $x^i \mapsto \tilde{x}^i(\mathbf{x},t)\ ,\quad t \mapsto \tilde{t}(t)$

ADM metric decomposition

$$ds^2 = N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\ ,\quad i,j = 1,\ldots,d$$

space dimensionality

Anisotropic scaling transformations and scaling dimensions

$$x^i \to \lambda^{-1} x^i, \quad t \to \lambda^{-z} t, \quad N^i \to \lambda^{z-1} N^i, \quad \gamma_{ij} \to \gamma_{ij},$$

$$[x] = -1,\ [t] = -z,\ [N^i] = z-1,\ [\gamma_{ij}] = 0, \qquad [K_{ij}] = z.$$

extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

Basic versions of Horava gravity: ``projectable" theory* $(N = \mathrm{const} = 1)$ *vs ``non-projectable" theory $(N(x,t) \neq \mathrm{const})$

Projectable Horava gravity action

kinetic term -- unitarity

$$S = \frac{1}{2G}\int dt\, d^d x \sqrt{\gamma} N \Big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}(\gamma) \Big)$$

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

Potential term in (3+1) dimensions

relevant

$$\mathcal{V}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}R^{ij} + \nu_1 R^3 + \nu_2 R R_{ij} R^{ij}$$

$$+\nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \ldots$$

Extra structures in non-projectable theory

$$N(x,t) \neq \mathrm{const} \;\; \Rightarrow \;\; a_i = \nabla_i \ln N, \ldots$$

Physical spectrum in d+1=4: TT-graviton and scalar

Unitarity domain (no ghosts) $\frac{1-\lambda}{1-3\lambda} > 0$

$$\omega_{TT}^2 = \eta k^2 + \mu_2 k^4 + \nu_5 k^6 \, ,$$
$$\omega_s^2 = \frac{1-\lambda}{1-3\lambda}\Big(- \eta k^2 + (8\mu_1 + 3\mu_2) k^4 + (8\nu_4 + 3\nu_5) k^6 \Big)$$

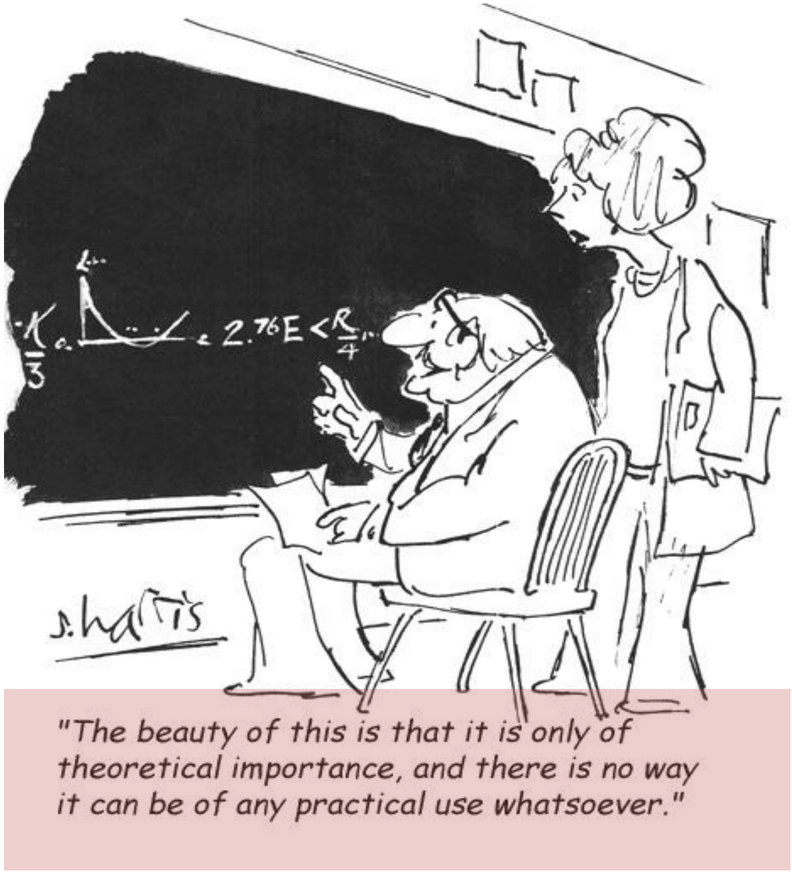
↑

tachyon in IR (whichever sign of η)

No general relativistic IR limit!

Phenomenologically useless model in contrast to nonprojectable HG which has a healthy IR limit fitting GR [D. Blas, O. Pujolas, S.Sibiryakov, JHEP04(2011)018].

GR: active lapse, $\lambda = 1, \eta = 1, \{\nu, \mu\} = 0$



Single example of local, unitary, renormalizable and asymptotically free (consistent in UV limit) quantum gravity – projectable Hořava gravity

Consider UV limit dominated by marginal operators, disregard relevant cosmological and Einstein terms and check AF.

Long list of problems to be solved that have been solved:

Renormalizability -- projectable HG is renormalizable in any d (nonprojectable?)	D. Blas, M. Herrero-Valea, S. Sibiryakov C. & A.B., PRD 93, 064022 (2016), arXiv:1512.02250
Gauge invariance of counterterms: preserving BRST structure of renormalization	D. Blas, M. Herrero-Valea, S. Sibiryakov C. Steinwachs & A.B., JHEP07(2018)035, arXiv:1705.03480,
Asymptotic freedom of (2+1)-dimensional model	D. Blas, M. Herrero-Valea, S. Sibiryakov C. Steinwachs & A.B., PRL 119, 211301 (2017), arXiv:1706.06809
Beta-functions of (3+1)-dimensional model	A.Kurov, S.Sibiryakov & A.B., PRD 105 (2022) 4, 044009 arXiv: 2110.14688
RG flows of (3+1)-dimensional model and asymptotic freedom	A.Kurov, S.Sibiryakov & A.B. PRD 108 (2023) 12, L121503, arXiv:2310.07841

Asymptotic freedom in (2+1)-dimensions

$$S = \frac{1}{2G} \int dt\, d^2x\; N\sqrt{\gamma}\; \left(K_{ij}K^{ij} - \lambda K^2 + \mu R^2\right)$$

Off-shell extension is not unique:

$$\Gamma_{1-\mathrm{loop}} \to \Gamma_{1-\mathrm{loop}} + \int dt\, d^dx\, \Omega_{ij} \frac{\delta S}{\delta \gamma_{ij}}$$

Essential coupling constants:

$$\lambda, \quad \mathcal{G} \equiv \frac{G}{\sqrt{\mu}}$$

background covariant
gauge-fixing term
σ, ξ *– free parameters*

$$S_{\mathrm{gf}} = \frac{\sigma}{2G} \int dt\, d^2x\, \sqrt{\gamma}\, F_i\, \mathcal{O}^{ij} F_i$$

$$F_i = \partial_t n_i + \frac{1}{2\sigma} \mathcal{O}^{-1}_{ij} (\nabla^k h^j_k - \lambda \nabla^j h)$$

$$\mathcal{O}^{ij} = -[\gamma_{ij} \Delta + \xi \nabla_i \nabla_j]^{-1}$$

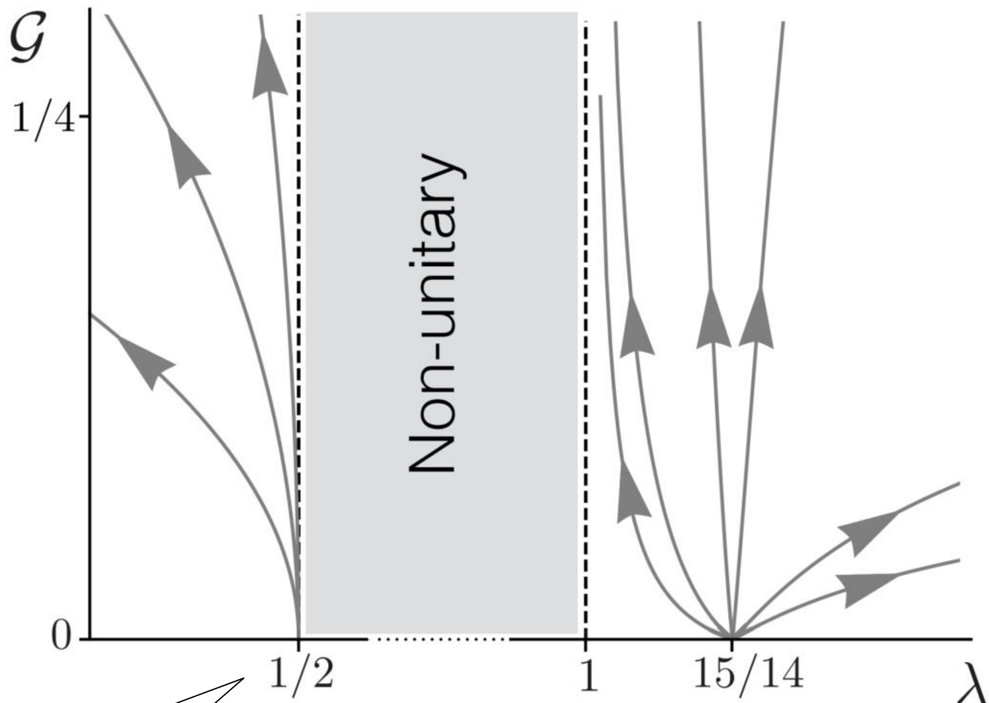
⇓

Mathematica package xAct

$$\beta_\lambda = \frac{15 - 14\lambda}{64\pi} \sqrt{\frac{1 - 2\lambda}{1 - \lambda}}\, \mathcal{G}$$

$$\beta_{\mathcal{G}} = -\frac{(16 - 33\lambda + 18\lambda^2)}{64\pi(1 - \lambda)^2} \sqrt{\frac{1 - \lambda}{1 - 2\lambda}}\, \mathcal{G}^2$$

Renormalization flows:



strongly coupled fixed point

$$\mathcal{G} \to \tilde{\mathcal{G}} = \frac{G}{\sqrt{1-2\lambda}}$$

$$\beta_{\tilde{\mathcal{G}}} = -\frac{(1-2\lambda)^2}{64\pi(1-\lambda)^{3/2}}\,\tilde{\mathcal{G}}^2$$

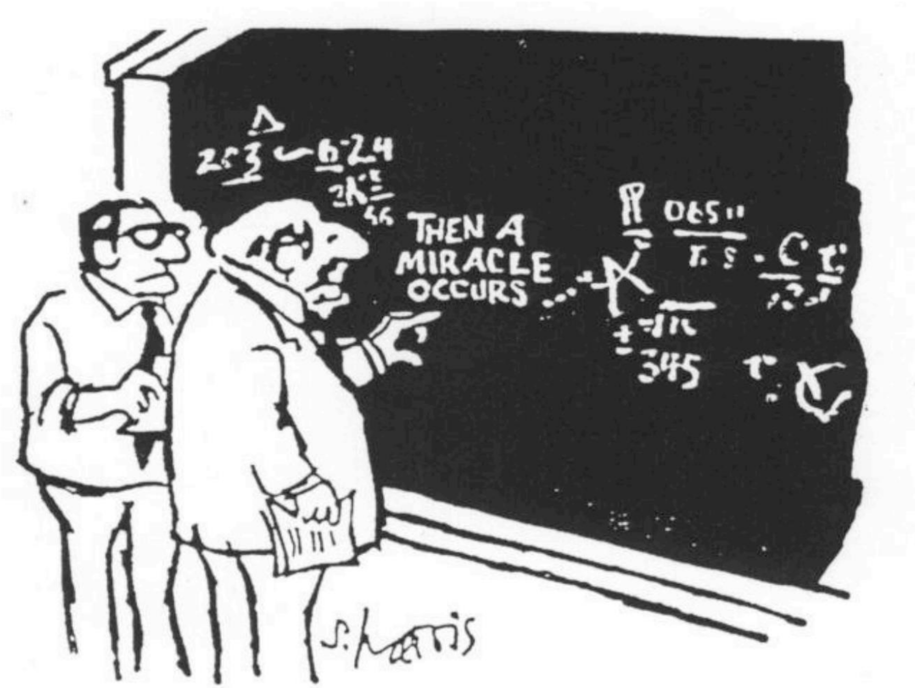
AF UV fixed point

(3+1)-dimensional Horava gravity

$$S = \frac{1}{2G} \int dt\, d^d x \sqrt{\gamma} \Big(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V}(\gamma) \Big)$$

$$\mathcal{V}(\gamma) = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \dots$$

Six essential coupling constants $\mathcal{G},\ \lambda$ and $\chi = (u_s, v_1, v_2, v_3)$



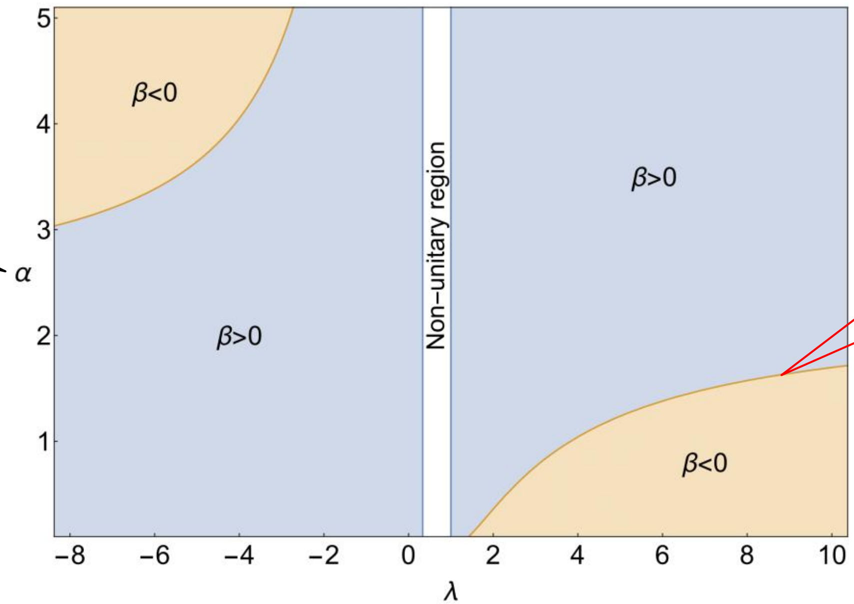
I think you should be a little more specific, here in Step 2

$$\beta_G, \quad \beta_\lambda$$

obtained by usual Feynman diagrams

M. Herrero-Valea, S. Sibiryakov & A.B., PRD100 (2019) 026012

$$\alpha \equiv \frac{1}{\nu_5}\frac{(1-\lambda)(8\nu_4+3\nu_5)}{1-3\lambda}$$



$$\lambda(\alpha) = \frac{9+7\sqrt{\alpha}-2\alpha+2\sqrt{10(\alpha+\alpha^{3/2})}}{3(3+\sqrt{\alpha}-2\alpha)}$$

The curves correspond to potential location of fixed points of the full RG flow. The region $\lambda \in [1/3,1]$ is excluded by the requirement of unitarity.

Background field method + heat kernel method + dimensional reduction

One-loop effective action

$$\Gamma_{\rm one-loop} = \frac{1}{2}{\rm Tr}_4 \ln \hat{F}(\nabla) = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\,{\rm Tr}_4\, e^{-s\hat{F}(\nabla)}$$

Action Hessian $\hat{F}(\nabla) = F^A_B(\nabla)$ *acting in the space of fields* $\varphi = \varphi^A(x)$

Heat kernel (Schwinger-DeWitt) expansion for minimal second order operators

$$\hat{F}(\nabla) = \Box + \hat{P} - \frac{\hat{1}}{6}R, \qquad \Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$$

$$e^{-s\hat{F}(\nabla)}\delta(x,y) = \frac{\mathcal{D}^{1/2}(x,y)}{(4\pi s)^{d/2}}\,g^{1/2}(y)\,e^{-\frac{\sigma(x,y)}{2s}}\sum_{n=0}^{\infty} s^n\,\hat{a}_n(x,y)$$

Schwinger-DeWitt (Gilkey-Seely) coefficients

$$\hat{a}_0\Big|_{y=x} = \hat{1}, \quad \hat{a}_1\Big|_{y=x} = \hat{P},$$

$$\hat{a}_2\Big|_{y=x} = \frac{1}{180}\left(R^2_{\alpha\beta\gamma\delta} - R^2_{\mu\nu} + \Box R\right)\hat{1} + \frac{1}{12}\hat{\boldsymbol{R}}^2_{\mu\nu} + \frac{1}{2}\hat{P}^2 + \frac{1}{6}\Box\hat{P}, \ldots$$

One-loop divergences $\Gamma^{\rm div}_{\rm one-loop} = -\frac{1}{32\pi^2\varepsilon}\int dx\, g^{1/2}{\rm tr}\,\hat{a}_2(x,x), \quad \varepsilon = 2 - \frac{d}{2} \to 0$

However, in Horava gravity operators are nonminimal

Set of quantum fields $\varphi(x) = h_{ij}(x), n^i(x) \;+\; FP\;\; ghosts$

Structure of operators on a static 3-metric background with generic 3-metric $\gamma_{ij}(\mathbf{x})$

$$\hat{F}(\nabla) = -\hat{1}\,\partial_\tau^2 + \hat{\mathbb{F}}(\nabla) \qquad\qquad \hat{\mathbb{F}} = \left\{\mathbb{F}_{ij}{}^{kl}, \mathbb{F}^k{}_i\right\} \sim \nabla^6 + R\nabla^4 + R^2\nabla^2 + R^3$$

space parts of metric and vector (shifts and ghosts) operators:

Example – for the ghost operator in σ, ξ -family of gauges:

$$\mathbb{F}^i{}_j(\nabla) = -\frac{1}{2\sigma}\delta^i_j\Delta^3 - \frac{1}{2\sigma}\Delta^2\nabla_j\nabla^i - \frac{\xi}{2\sigma}\nabla^i\Delta\nabla^k\nabla_j\nabla_k$$
$$-\frac{\xi}{2\sigma}\nabla^i\Delta\nabla_j\Delta + \frac{\lambda}{\sigma}\Delta^2\nabla^i\nabla_j + \frac{\lambda\xi}{\sigma}\nabla^i\Delta^2\nabla_j, \qquad \Delta = \gamma^{ij}\nabla_i\nabla_j$$

*Extension to **non-minimal and higher-derivative operators** -- the method of universal functional traces (I. Jack and H. Osborn (1984), G.A.Vilkovisky & A.B., Phys. Rept. 119 (1985) 1)*

Idea:

$$\mathrm{Tr}\,\ln\big(\Box^N + P(\nabla)\big) = N\,\mathrm{Tr}\,\ln\Box + \mathrm{Tr}\,\ln\Big(1 + P(\nabla)\frac{1}{\Box^N}\Big)$$
$$= N\,\mathrm{Tr}\,\ln\Box + \mathrm{Tr}\,P(\nabla)\frac{1}{\Box^N} + \cdots$$

⇓

$$\Gamma^{\mathrm{div}} = \sum_{m,n} \int d^4x\, \mathcal{R}_n^{\mu_1\ldots\mu_m} \nabla_{\mu_1}\ldots\nabla_{\mu_m}\frac{\hat{1}}{\Box^n}\delta(x,y)\Big|_{y=x}^{\mathrm{div}}$$

↑ ***universal functional traces***

$$\nabla\ldots\nabla\frac{\hat{1}}{\Box^n}\delta(x,y)\Big|_{y=x}^{\mathrm{div}} = \frac{(-1)^n}{\Gamma(n)}\nabla\ldots\nabla\int_0^\infty ds\, s^{n-1}\, e^{s\Box}\,\hat{\delta}(x,y)\Big|_{y=x}^{\mathrm{div}}$$

↑ *Schwinger-DeWitt expansion*

Dimensional reduction *method on a static background with generic 3-metric*

$$\mathrm{Tr}_4 \ln(-\partial_\tau^2 + \mathbb{F}) = -\int_0^\infty \frac{ds}{s}\, \mathrm{Tr}_4 e^{-s(-\partial_\tau^2+\mathbb{F})} = -\int d\tau\; \mathrm{Tr}_3 \sqrt{\mathbb{F}}$$

square root

How to proceed with the square root of the 6-th order differential operator?

$$\mathbb{F} = \sum_{a=0}^{6} \mathcal{R}_{(a)} \sum_{6\geq 2k\geq a} \alpha_{a,k} \nabla_1 ... \nabla_{2k-a} (-\Delta)^{3-k}, \quad \mathcal{R}_{(a)} = O\Big(\frac{1}{l^a}\Big)$$

⇩

Pseudodifferential operator – infinite series in curvature invariants $\mathcal{R}_{(a)}$

$$\sqrt{\mathbb{F}} = \sum_{a=0}^{\infty} \mathcal{R}_{(a)} \sum_{k\geq a/2}^{K_a} \tilde{\alpha}_{a,k} \nabla_1 ... \nabla_{2k-a} \frac{1}{(-\Delta)^{k-3/2}}$$

⇩

3D universal Functional traces

$$\mathrm{Tr}_3 \sqrt{\mathbb{F}}\,\Big|^{\mathrm{div}} = \sum_{a=2}^{6} \sum_k \tilde{\alpha}_{a,k} \int d^3x\, \mathcal{R}_{(a)}(\mathbf{x}) \nabla_1 ... \nabla_{2k-a} \frac{1}{(-\Delta)^{k-3/2}} \delta(\mathbf{x},\mathbf{x}')\,\Big|^{\mathrm{div}}_{\mathbf{x}=\mathbf{x}'}$$

Results for beta functions of (3+1)-dimensional Horava gravity

$$S = \frac{1}{2G}\int dt\, d^dx \sqrt{\gamma} N \Big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}(\gamma) \Big)$$

$$\mathcal{V}(\gamma) = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk}$$

Six essential coupling constants $\mathcal{G}$, λ and $\chi = (u_s, v_1, v_2, v_3)$

$$\mathcal{G} = \frac{G}{\sqrt{\nu_5}}, \quad \lambda, \quad u_s = \sqrt{\frac{(1-\lambda)(8\nu_4 + 3\nu_5)}{(1-3\lambda)\nu_5}}, \quad v_a = \frac{\nu_a}{\nu_5}, \quad a = 1, 2, 3$$

$$\beta_{\mathcal{G}} = \frac{\mathcal{G}^2}{26880\pi^2(1-\lambda)^2(1-3\lambda)^2(1+u_s)^3 u_s^3} \sum_{n=0}^{7} u_s^n \mathcal{P}_n^{\mathcal{G}}[l, v_1, v_2, v_3]$$

$$\beta_\lambda = \frac{\mathcal{G}}{120\pi^2(1-\lambda)(1+u_s)u_s}\Big[27(1-\lambda)^2 + 3u_s(11-3\lambda)(1-\lambda) - 2u_s^2(1-3\lambda)^2\Big]$$

$$\beta_\chi = \frac{A_\chi \mathcal{G}}{26880\pi^2(1-\lambda)^3(1-3\lambda)^3(1+u_s)^3 u_s^5} \sum_{n=0}^{9} u_s^n \mathcal{P}_n^{\chi}[l, v_1, v_2, v_3]$$

$$A_{u_s} = u_s(1-\lambda), \quad A_{v_1} = 1, \quad A_{v_2} = A_{v_3} = 2$$

$\mathcal{P}_n^{\chi}[l, v_1, v_2, v_3,]$ *are polynomials in* λ *and* v_a,

Use of Mathematica package xAct

Example (one of the longest ones):

$$\mathcal{P}_5^{v_1}=-2(1-\lambda)^2(1-3\lambda)\Big\{168v_2^3(51\lambda^3-149\lambda^2+125\lambda-27)-108v_3^3(9\lambda^3+9\lambda^2$$
$$-25\lambda+7)-4v_2^2(1-\lambda)\big[18v_3(117\lambda^2-366\lambda+109)-284\lambda^2-7265\lambda+5425\big]$$
$$+40320v_1^2(1-\lambda)^2(\lambda+1)-9v_3^2(3467\lambda^3-8839\lambda^2+6237\lambda-865)$$
$$+v_1\big[64v_2^2(1-\lambda)^2(1717\lambda-581)-16v_2(1-\lambda)\big(3v_3(2741\lambda^2-3690\lambda+949)$$
$$+25940\lambda^2-40662\lambda+12022\big)+27v_3^2(961\lambda^3-2395\lambda^2+1835\lambda-401)$$
$$+6v_3(52267\lambda^3-148963\lambda^2+129881\lambda-33185)-288353\lambda^3+542255\lambda^2$$
$$-333355\lambda+83485\big]-2v_2\big[162v_3^2(3\lambda^3+35\lambda^2-51\lambda+13)+24v_3(1265\lambda^3$$
$$-2191\lambda^2+691\lambda+235)+30971\lambda^3-40323\lambda^2+13167\lambda-4451\big]-12v_3(6551\lambda^3$$
$$-11593\lambda^2+6124\lambda-1112)+109519\lambda^3-252396\lambda^2+177357\lambda-34396\Big\}$$

Check of the results: independence of essential beta functions on the choice of gauge (σ, ξ - family of gauge conditions) and spectral sum method in dimensional and zeta-functional regularization.

$\mathcal{G} \to 0$ *asymptotic freedom*

$\mathcal{G} \to \infty$ *Landau pole*

Fixed points equations:

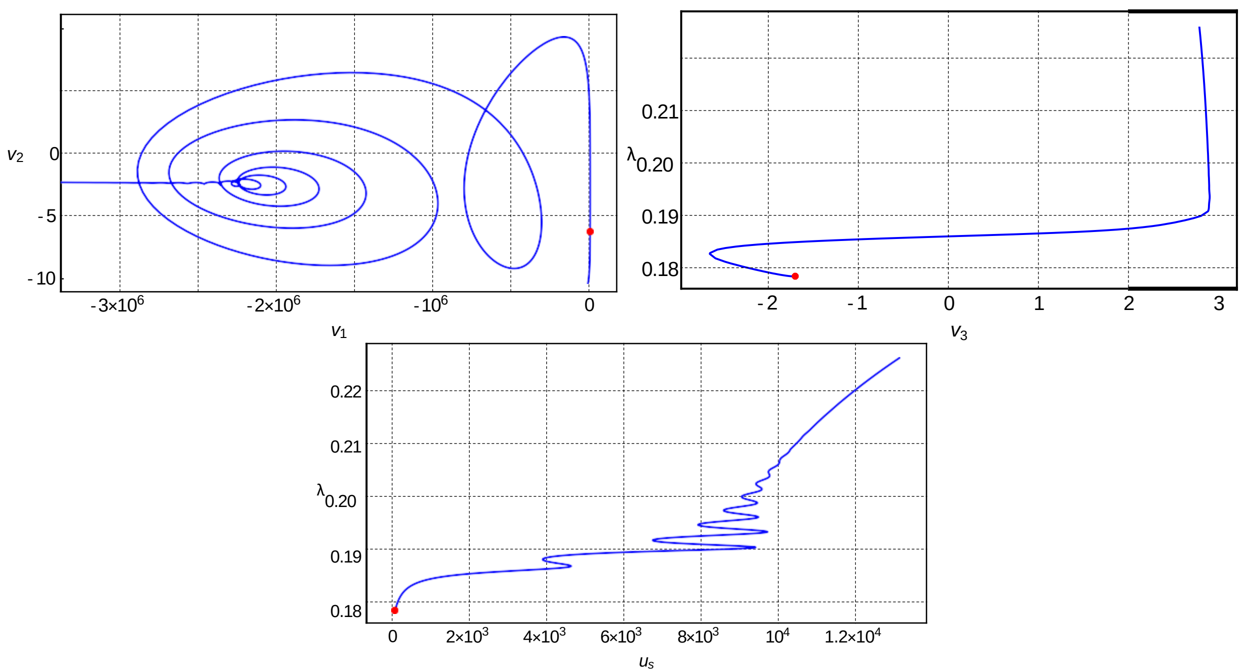
$$\beta_\lambda / \mathcal{G} = 0 \,,$$

$$\beta_\chi / \mathcal{G} = 0 \,, \qquad \chi = u_s, v_1, v_2, v_3$$

λ	u_s	v_1	v_2	v_3	$\beta_\mathcal{G}/\mathcal{G}^2$	AF?	UV attractive along λ?
0.1787	60.57	-928.4	-6.206	-1.711	-0.1416	yes	no
0.2773	390.6	-19.88	-12.45	2.341	-0.2180	yes	no
0.3288	54533	3.798×10^8	-48.66	4.736	-0.8484	yes	no
0.3289	57317	-4.125×10^8	-49.17	4.734	-0.8784	yes	no

i

Fixed points at finite λ . All of them are asymptotically free but they RG run into Landau poles.



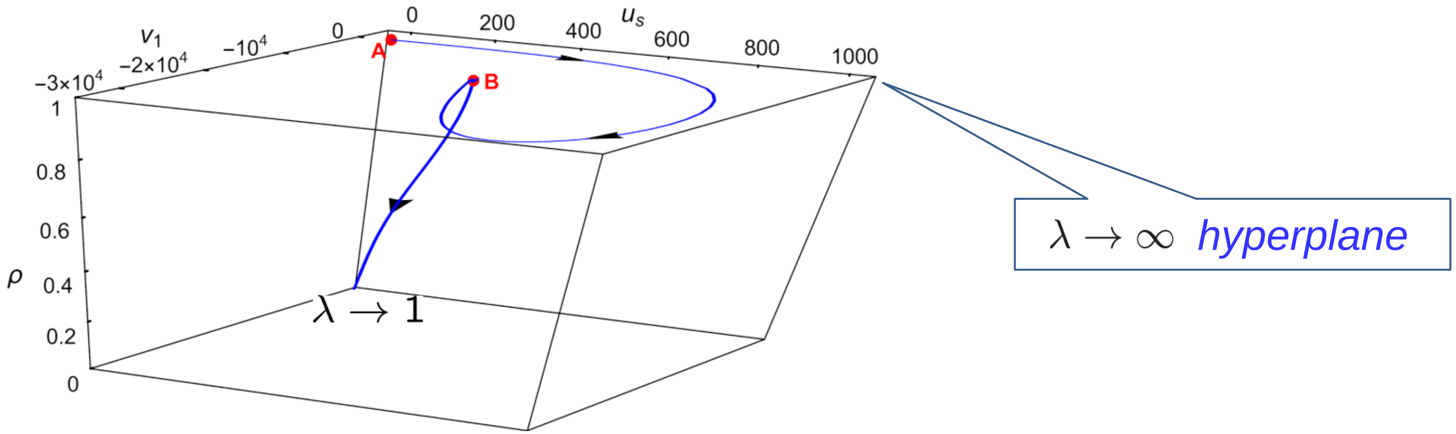
Special limit: $\lambda \to \infty$ *(cosmology implication, A.E. Gumrukcuoglu, S. Mukohyama, 1104.2087)*

Fixed points at $\lambda \to \infty$

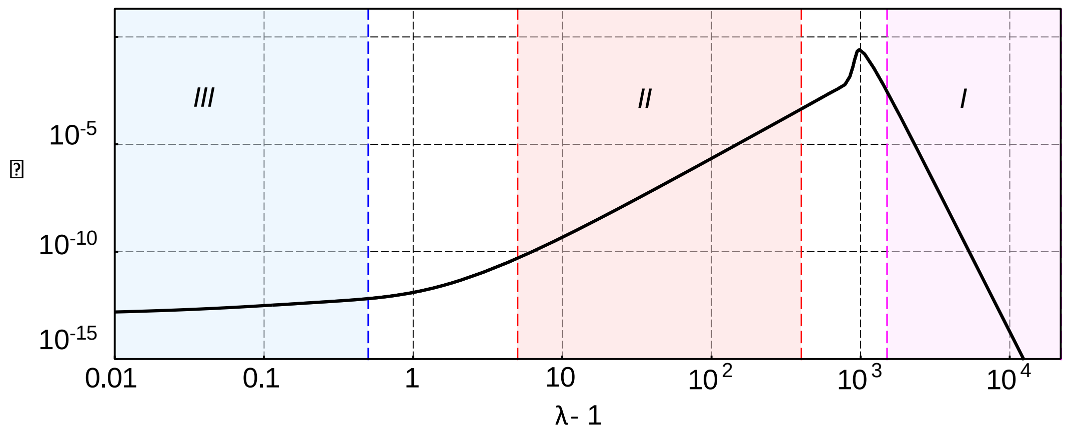
	u_s	v_1	v_2	v_3	$\beta_{\mathcal{G}}/\mathcal{G}^2$	asymptotically free?	UV attractive along λ?
	0.01950	0.4994	-2.498	2.999	-0.2004	yes	no
	0.04180	-0.01237	-0.4204	1.321	-1.144	yes	no
	0.05530	-0.2266	0.4136	0.7177	-1.079	yes	no
	12.28	-215.1	-6.007	-2.210	-0.1267	yes	yes
A	21.60	-17.22	-11.43	1.855	-0.1936	yes	yes
B	440.4	-13566	-2.467	2.967	0.05822	no	yes
	571.9	-9.401	13.50	-18.25	-0.07454	yes	yes
	950.6	-61.35	11.86	3.064	0.4237	no	yes

Special fixed points: **A** *is AF,* **B** *is not AF*

*Fixed point **B** as a transient of RG flow from AF UV point **A** to IR limit with* $\lambda \to 1$



Behavior of G on the flow: $G \to 0$ *in UV and IR*



Behavior of G as a function of $(\lambda - 1)$ along an RG trajectory connecting the point A to $\lambda \to 1^+$.

In regions I, II and III the dependence is well described by the power law $G \propto (\lambda - 1)^k$ with $k_I = -13.69$, $k_{II} = 3.84$, $k_{III} \approx 0.37$.

Riddles of higher derivative gravity models

1. Complexity of operator dimensions – eigenvalues of stability matrices.

№	θ^1	θ^2	θ^3	θ^4	θ^5
1	1.154	-1.235	0.9825	-0.2734 ± 0.2828 i	
2	0.5302	12.35	-0.3207	-71.95 ± 5.134 i	
3	0.3970	10.77	0.3012	-64.72 ± 0.6149 i	
4	-0.01334	-0.3436	-0.09353	0.2200 ± 0.1806 i	
5 (A)	-0.01414	-0.06998	0.2565	0.3204	0.06569
6 (B)	-0.01515	0.6032	0.3079	0.0924 ± 0.2890 i	
7	-0.01516	-1.722	0.1328	-0.3324 ± 0.3289 i	
8	-0.01517	-0.3657	1.326	0.4340 ± 0.4849 i	

Violation of LI or lack of gauge invariant operators ?

2. Renormalizability and AF of nonprojectable HG?

It has healthy IR limit fitting GR [D. Blas, O. Pujolas, S.Sibiryakov, JHEP04(2011)018].There are indications that it is renormalizable.

J. Bellorın, C. Borquez, B. Droguett,
Phys. Rev D 106, 044055 (2022), 2207.08938
arXiv:2405.04708

3. Tadpoles, IR divergences and modification of beta functions in quadratic gravity (Donoghue et al)

The problem of running Λ and G

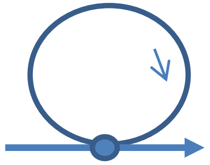
Running coupling constant = nonlocal form factor in effective action

$$g \Rightarrow g(\mu), \quad \mu\frac{d}{d\mu}g(\mu) = \beta\big(g(\mu)\big),$$

$$-\frac{1}{4g^2(\mu)}\int d^4x\, F_{\mu\nu}^2 \Rightarrow -\int d^4x\, F_{\mu\nu}(x)\frac{1}{4g^2\big(\sqrt{-\Box}\big)}F^{\mu\nu}(x) = -\int d^4p\, \hat{F}_{\mu\nu}(-p)\frac{1}{4g^2\big(p\big)}\hat{F}^{\mu\nu}(p)$$

running scale

Tadpole (total derivative) nature problem for running Λ and G



$$-\frac{\Lambda}{16\pi G}\int d^4x\, g^{1/2} \Rightarrow -\frac{1}{16\pi}\int d^4x\, g^{1/2}\frac{\Lambda(\Box)}{G(\Box)}1 = -\frac{1}{16\pi}\int d^4x\, g^{1/2}\frac{\Lambda(0)}{G(0)}$$

$$-\frac{1}{16\pi G}\int d^4x\, g^{1/2}R \Rightarrow -\frac{1}{16\pi}\int d^4x\, g^{1/2}\frac{1}{G(\Box)}R = -\frac{1}{16\pi}\int d^4x\, g^{1/2}\frac{1}{G(0)}R$$

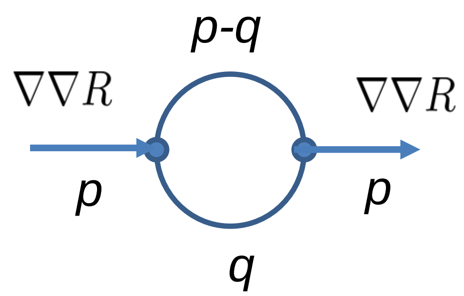
no running

Quadratic gravity action and propagator

$$S \sim \int d^4x \sqrt{g}(-M^2 R + R^2) \sim \int d^4q \, h(q)(M^2q^2 + q^4)h(-q) + \ldots \Rightarrow \ \textit{propagator} \sim \frac{1}{M^2q^2 + q^4}$$

UV finite and IR divergent at M=0

$$\int \frac{d^4q}{(q^4 + M^2q^2)((p-q)^4 + M^2(p-q)^2)} \sim \frac{1}{p^4} \log \frac{p^2}{M^2}$$



⇩

$$\int d^4p \, \nabla\nabla R(p) \frac{1}{p^4} \log \frac{p^2}{M^2} \nabla\nabla R(-p) \sim \int d^4x \, R \log\left(\frac{-\Box}{M^2}\right) R$$

running constant of R^2-- action

⇩

Modification of beta-functions by UV finite terms!

D.Buccio, J.Donoghue, G.Menezes, R.Percacci, arXiv:2403.02397

4. No running of G and Λ --- metamorphosis of the running scale

Renormalization group in covariant curvature expansion

$$S[g_{\mu\nu}] = \sum_{m,N} \Lambda_N^{(m)} \int d^dx \sqrt{g}\, \Re_N^{(d+m)} = 0$$

$$\Re_N^{(m)} = \underbrace{\nabla\ldots\nabla}_{m-2N} \overbrace{\Re\ldots\Re}^{N}, , \quad \dim \Re_N^{(m)} = m$$

Invariant curvature monomes of order N and dimension m

Covariant perturbation theory $\quad g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}, \quad \tilde{R}^{\mu}{}_{\nu\alpha\beta} = 0$

G.A.Vilkovisky & A.B. (1986-1990)

Monomes in metric perturbations

$$I_M^{(m)}(h) \propto \underbrace{\tilde{\nabla}\ldots\tilde{\nabla}}_{m} \overbrace{h(x)\ldots h(x)}^{M} \equiv I_M^{(m)}(h_1, h_2, \ldots h_M)\Big|_{x_1=x_2=\ldots x_M=x}$$

RG scale μ

$$\Lambda_N^{(m)} = \mu^{d-m} \lambda_N^{(m)}(\mu)$$

dimensionless couplings $\lambda_N^{(m)}(\mu) = \{\lambda(\mu)\}$

Effective action

$$\Gamma[g_{\mu\nu}] = \sum_{(m)} \mu^{d-m} \sum_{M=0}^{\infty} \int d^dx \sqrt{\tilde{g}}$$

$$\times \gamma_M^{(m)}\Big(\{\lambda(\mu)\}, \frac{\tilde{\nabla}_1}{\mu}, \ldots \frac{\tilde{\nabla}_M}{\mu}\Big) I_M^{(m)}(h_1, \ldots h_M)\Big|_{\{x\}=x},$$

nonlocal form factors

Employing RG: $\mu\frac{d\Gamma}{d\mu}=0 \;\rightarrow\; \mu\frac{d}{d\mu}\lambda(\mu)=\beta(\mu)\Big(\{\lambda(\mu)\}\Big)$ **for all** $\lambda_N^{(m)}(\mu)=\{\lambda(\mu)\}$

Choice of scale $\mu \rightarrow ?$

$$\tilde{D}\equiv\Big(-\sum_{N=1}^{\infty}\tilde{\Box}_N\Big)^{1/2},\qquad \tilde{\Box}_N\equiv\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu^N\tilde{\nabla}_\nu^N=0,$$

$$\tilde{D}I_N=\tilde{D}_N I_N,\qquad \tilde{D}_N\equiv\Big(-\sum_{M=1}^{N}\tilde{\Box}_M\Big)^{1/2},\qquad \tilde{D}_0=0.$$

Replacement $\mu\rightarrow\tilde{D}$ **and UV limit** $\tilde{\nabla}\rightarrow\infty,\quad \frac{\tilde{\nabla}}{\tilde{D}_N}\rightarrow O(1)$

$$\mu^{4-m}\gamma\Big(\lambda(\mu)\,\Big|\,\frac{\tilde{\nabla}_1}{\mu},\ldots\frac{\tilde{\nabla}_N}{\mu}\Big)\Big|_{\mu\rightarrow\tilde{D}_N}\;\Longrightarrow\;(\tilde{D}_N)^{4-m}\gamma\Big(\lambda(\tilde{D}_N)\,\Big|\,O(1)\Big)\equiv(\tilde{D}_N)^{4-m}\lambda\Big(\tilde{D}_N\Big)$$

Covariantization:

$$h_{\mu\nu}=-\frac{2}{\Box}R_{\mu\nu}+O[\Re^2],\qquad \tilde{\nabla}_\mu=\nabla_\mu+O[\Re],$$

$$I_N^{(m)}(h_1,h_2,\ldots h_N)\rightarrow\frac{1}{\Box_1\ldots\Box_N}\underbrace{\nabla\ldots\nabla}_{m}\Re_1\ldots\Re_N+O[\Re^{N+1}]$$

RG form factor

$$\Gamma[g_{\mu\nu}] \to \int d^4x\,\sqrt{g}\sum_{m,N}^{\infty}\frac{(D_N)^{4-m}}{\Box_1\ldots\Box_N}\,\lambda_N^{(m)}(D_N)\underbrace{\nabla\ldots\nabla}_{m}\Re_1\ldots\Re_N\Big|_{\{x\}=x}$$

dimensionful factors

For a cosmological constant term: $\Lambda_0^{(4)} = \frac{\Lambda}{16\pi G}$

$N = 2$ *any* $m \geq 0$ $\quad \int d^4x\,\sqrt{g}\,\lambda_2^{(m)}\left(\sqrt{-\Box_1 - \Box_2}\right)\Re(x_1)\,\Re(x_2)\Big|_{x_1=x_2}$

all factors in $\frac{(D_2)^{4-m}}{\Box_1\Box_2}\underbrace{\nabla\ldots\nabla}_{m}$ ***completely cancel out due to integration by parts !***

$$= \int d^4x\sqrt{g}\Big(R_{\mu\nu}F_1(\Box)R^{\mu\nu} + RF_2(\Box)R\Big) + O[\Re^3]$$

dimensionless
RG form factors

Metamorphosis to high-energy partners of the cosmological constant
[J. F. Donoghue, Phys. Rev. D 105, 105025 (2022), 2201.12217]

Conclusions

Single, known now, example of local, unitary, renormalizable and asymptotically free (consistent in UV limit) quantum gravity –
projectable Hořava gravity

Salvation of unitarity in local renormalizable QG via LI violation

Asymptotic freedom in (2+1)-dimensional theory

Beta functions and AF RG flows of (3+1)-dimensional HG

Enigma of higher-derivative gravity models

THANK YOU!