Quantum corrections to the Hawking radiation

E.T.Akhmedov

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Setup

- The background metric ($d\Omega^2 = d\theta^2 + \cos^2\theta d\varphi^2$)

$$ds^2 = \begin{cases} dt_-{}^2 - dr^2 - r^2 d\Omega^2, & r \leq R(t) \\ \left(1 - \frac{r_g}{r}\right) dt^2 - \frac{dr^2}{1-\frac{r_g}{r}} - r^2 d\Omega^2, & r \geq R(t) \end{cases} ,$$

- The radius of the spherical shell

$$R(t) = \begin{cases} R_0, & t \leq 0 \\ r_g \left(1 + \frac{R_0 - r_g}{r_g} e^{-\frac{t}{r_g}}\right), & t \to +\infty \end{cases} ,$$

- The time

$$t_- = \begin{cases} \sqrt{1 - \frac{r_g}{R_0}}\, t, & t \leq 0 \\ \frac{R_0 - r_g}{\nu} \left(1 - e^{-\frac{t}{r_g}}\right), & t \to +\infty \end{cases} \quad ; \quad \nu = \left| \frac{dR(t_-)}{dt_-} \right|$$

Setup

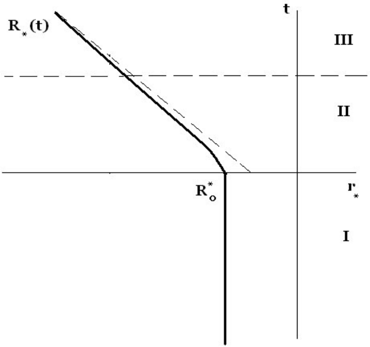

Figure: The collapse of a massive, thin shell in tortoise coordinates, $R_*(t) = R(t) + r_g \log\left(\frac{R(t)}{r_g} - 1\right)$. We divide the collapse into three phases: I, II and III

- We consider massive scalar field theory on such a background

$$S=\int d^4x\sqrt{|g|}\left[(\partial_\mu\phi)^2-m^2\phi^2-\frac{\lambda}{4!}\phi^4\right].$$

- EOM for the free modes in the collapse background

$$\begin{cases}\left[\partial_{t_-}^2-\partial_r^2+m^2+\frac{l(l+1)}{r^2}\right](r h_l)=0, & r\le R(t)\\ \left[\partial_t^2-\partial_{r^*}^2+\left(1-\frac{r_g}{r}\right)\left(m^2+\frac{l(l+1)}{r^2}+\frac{r_g}{r^3}\right)\right](r h_l)=0, & r\ge R(t)\end{cases},$$

- Spherical decomposition and the boundary conditions

$$\phi(t,r,\theta,\varphi)=\sum_{l,n}Y_{l,n}(\theta,\varphi)\,h_l(t,r),$$

$$h_l\Big[R(t)-0\Big]=h_l\Big[R(t)+0\Big],\quad \partial_{\rm n}h_l\Big[R(t)-0\Big]=\partial_{\rm n}h_l\Big[R(t)+0\Big],$$

Setup

- The free Hamiltonian depends on time

$$H_0(t) = \sum_l (2l+1) \int_0^\infty dr \, \frac{\sqrt{|g|}}{\sin(\theta)} \times$$

$$\times \left[g^{tt} \left(\partial_t h_l\right)^2 - \frac{1}{\sqrt{|g|}} h_l \partial_t \left(\sqrt{|g|} g^{tt} \partial_t h_l \right) \right].$$

  But the Hamiltonian is time independent before the start of the collapse and becomes time independent at the very final stage of the collapse.

- Hence, the free Hamiltonian can be diagonalized at past and future infinity by different modes. Before the start of the collapse it can be diagonalized by the in-modes, while at the future infinity — by the out-modes. That is the hint for the presence of the particle creation.

The problem to address

- We want to calculate the stress-energy flux at future infinity in the vicinity of the horizon for the in-ground initial state

$$J = \int_{S_2} d^2\Omega \left\langle in \right| : T^r_t \ \ (r \approx r_g, t \to +\infty) : \left| in \right\rangle$$

- The in-modes and the corresponding Fock space ground state are defined as

$$\phi\left(\underline{x}\right) = \sum_{l,n} Y_{l,n}(\theta,\varphi) \int_m^\infty \frac{d\omega}{2\pi} \left[a_{\omega,l,n} \bar{h}_{\omega,l}(r,t) + \mathrm{h.c.} \right],$$

$$a_{\omega,l,n} \left| in \right\rangle = 0$$

The behaviour of the in-modes

- Inside the shell ($r \leq R_0$) before the collapse ($t \leq 0$)

$$\bar{h}_{\omega,l}(t,r) = \frac{\mathcal{A}_\omega}{\sqrt{r}} J_{l+\frac{1}{2}}\left(\sqrt{\omega_-{}^2 - m^2}\; r\right) e^{-i\,\omega_-\,t_-},\; \omega_- = \frac{\omega}{\sqrt{1-\frac{r_g}{R_0}}};$$

- Outside the shell, before the collapse ($k = \sqrt{\omega^2 - m^2}$)

$$\bar{h}_{\omega,l}(t,r) = \frac{e^{-i\,\omega\,t}}{r}\begin{cases} A_\omega e^{-i\omega r_*} + B_\omega e^{i\omega r_*}, & |r - R_0| \ll r_g, \\ C_\omega\, e^{-ikr_*} + D_\omega\, e^{ikr_*}, & r \gg R_0, \end{cases} ,$$

- Just outside the shell ($|r - R_0| \ll r_g$) at the future infinity ($t \to +\infty$) ($u = t - r_*$, $v = t + r_*$)

$$r\,\bar{h}_{\omega,l}(r,t) \approx \sqrt{\pi\,R(u)} J_{l+\frac{1}{2}}\left[\sqrt{\omega_-{}^2 - m^2}\; R(u)\right] \times$$
$$\times \exp\left\{-i\omega_- \frac{(R_0 - r_g)}{\nu}\left(1 - e^{-\frac{u + R_0^* + r_g - R_0}{2r_g}}\right)\right\} + A_\omega\, e^{-i\,\omega\,v}$$

The tree-level flux

- The flux at future infinity near the shell is defined as

$$J\left(r \approx r_g, t \to +\infty\right) = \sum_{l=0}^{+\infty} (2l+1)\left(J_u^{(l)} - J_v^{(l)}\right),$$

- Due to the amplification of the zero point fluctuations one obtains the seminal Hawking radiation

$$J_u^{(l)} - J_v^{(l)} \approx \left(1 - \frac{r_g}{R_0}\right)^{\frac{1}{2}} \int_m^\infty \frac{d\omega}{2\pi}\omega\, n(\omega), \;\; n(\omega) = \frac{1}{e^{4\pi r_g \omega} - 1}.$$

Interestingly enough, if one cuts out UV modes ($\omega \to \infty$), then the Hawking flux abruptly drops off after sime time.

- The flux was found from the Keldysh propagator

$$T_t^r = \partial^{r_1}\partial_{t_2}\, D^K(1,2)\Big|_{1=2} = \frac{1}{2}\,\partial^{r_1}\partial_{t_2}\,\Big\langle\Big\{\phi(\underline{x}_1)\ ,\ \phi(\underline{x}_2)\Big\}\Big\rangle\Big|_{x_1=x_2}$$

- The Keldysh propagator for a generic state is

$$D^K(1,2) \approx \sum_{l_1,n_1,l_2,n_2} Y_{l_1,n_1}(\Omega_1)\,Y_{l_2,n_2}(\Omega_2)\int \frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\times$$

$$\times\Bigg\{\Big[N_{12} + \delta_{l_1 l_2}\,\delta_{n_1 n_2}\,\delta(\omega_1-\omega_2)\Big]\,\bar{h}^*_{\omega_1,l_1}(t_1,r_1)\,\bar{h}_{\omega_2,l_2}(t_2,r_2) +$$

$$+K_{12}\,\bar{h}_{\omega_1,l_1}(t_1,r_1)\,\bar{h}_{\omega_2,l_2}(t_2,r_2) + \mathrm{h.c.}\Bigg\}$$

For the in-state at tree level $N_{12} = \langle a_1^+\, a_2\rangle = 0$, $K_{12} = \langle a_1\, a_2\rangle = 0$. The flux was due to zero point fluctuations.

The effect on the flux due to loop corrections

- In the interaction picture at tree-level the "level-populations", N, and "anomalous averages", K, always remain zero:

$$N_{12} = \langle a^{+}_{\omega_1, l_1, n_1} \, a_{\omega_2, l_2, n_2} \rangle = 0, \quad K_{12} = \langle a_{\omega_1, l_1, n_1} \, a_{\omega_2, l_2, n_2} \rangle = 0$$

- In the loops they start to depend on time. E.g.:

$$N_{12} = \langle in | S^{+}(t_2, t_0) a^{+}_{\omega_1, l_1, n_1} \, S(t_2, t_1) \, a_{\omega_2, l_2, n_2} S(t_1, t_0) | in \rangle$$

In the high energy physics (describing scattering of particles in the vacuum) "level-population" and "anomalous average" are always zero. That is in accordance with the Poincare symmetry. There is no any Poincare symmetry in strong background fields.

Secular growth

- E.g. the leading contribution to the level population in the future infinity, when $t = (t_1 + t_2)/2 \gg |t_1 - t_2|$

$$N_{\omega,l,n|\omega',l',n'}(t) \sim \lambda^2\, t\, \frac{F(l,n|l',n')}{\sqrt{\omega\,\omega'}} \times$$

$$\times \int_{-\infty}^{\infty} d\tau \int_{r_g}^{\infty} r_3^2 dr_3 \int_{r_g}^{\infty} r_4^2 dr_4 \prod_{j=1}^{3} \int_{\omega_j > m} \frac{d\omega_j}{4\pi\omega_j} \times$$

$$\times \left\{ \left[n(-\omega_j)\, e^{-i\omega_j(\tau - \Delta r)} + n(\omega_j)\, e^{i\omega_j(\tau-\Delta r)} \right] + e^{-i\omega_j(\tau+\Delta r)} \right\},$$

  where $\Delta r = r_3 - r_4$ and factor $F(l,n|l',n')$ is constructed from spherical harmonics and appears instead of delta-functions establishing momentum conservation. Similar expression appears for anomalous average K.
- The right hand side here is essentially nothing but the collision integral in the situation when there is no energy conservation due to the time dependence of the background metric.

Loop corrections to the Hawking radiation

The presence of N and K means the modification of the Hawking flux. In fact, in the case when $K = 0$ and $N \neq 0$ the equation for the flux is as follows:

$$J_u \approx \sum_{l,n} \iint_m^\infty \frac{d\omega_1\, d\omega_2}{(2\pi)^2} \int_{|\omega'|>m} \frac{d\omega'}{2\pi} \int_{|\omega''|>m} \frac{d\omega''}{2\pi} \sqrt{|\omega'\, \omega''|}$$
$$\alpha(\omega_1, \omega')\, \alpha^*(\omega_2, \omega'')\, \left\langle \left\{ a^\dagger_{\omega_1,l,n},\, a_{\omega_2,l,n} \right\} \right\rangle e^{-i\,(\omega' - \omega'')\,u}.$$

The standard factor appearing in the calculations of the Hawking radiation $|\alpha\,(\omega, \omega')\,| \sim r_g \sqrt{\frac{|\omega'|}{\omega}} \cos\left[\frac{\pi\,(l+1)}{2} - \omega_- r_g\right]\, |\Gamma(-2i\omega' r_g)|$.

It is not hard to also restore the contribution of K. Now because of the loop corrections we have that

$$\left\langle \left\{ a^\dagger_{\omega_1,l,n},\, a_{\omega_2,l,n} \right\} \right\rangle = \delta(\omega_1 - \omega_2) + 2\, N_{\omega_1,l,n|\omega_2,l,n}(t).$$

Physical consequences

- After long enough time loop corrections become comparable to the tree-level contribution: $N, K \sim \lambda^2\, t\, e^{-4\,\pi\, r_g\, m} \sim 1$;
  Of course this effect, as well the Hawking effect, is visible for microscopic black holes;
- If the effect is there one has to resum leading corrections from all loops. The result of the resumation would depends on the initial conditions;
- To do the resummation one has to solve an analog of the kinetic equations for N and K;
- Even if the result of the resumation leads to a stationary state, still N and K carry all information about the state of QFT. That has something to do with the information paradox;
- Possibility for the violation of the Ehrenfest theorems. Namely after the resumation N and K can in principle blow up in finite proper time. Quantum corrections become stronger than classical contributions.

Other non-stationary backgrounds

We observe similar secular effects in the stress-energy flux and electric currents in other strong background fields. E.g.:

- FRW expanding and contracting universes, including de Sitter space-time;
- Strong electric fields. Both constant field and electric pulse;
- Dynamical Casimir effect due to the moving mirrors;
- Strong scalar field background

Strong background fields are more similar to the condensed matter physics rather than to high energy particle physics.

Conclusions

In accelerators one pumps out vacuum and scatters single particles. That is the reason to consider stationary Poincare invariant correlation functions to construct amplitudes and cross-sections. In the very early universe and in the background of collapsing microscopic black holes there is no reason to assume the state to be vacuum or even thermal.

THANKS!