Self-similar growth of Bose stars

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INR RAS

AD, D. Levkov, A. Panin, I. Tkachev, arXiv:2305.01005

Light bosonic (axion-like) dark matter

unknown mass!

Light DM

10^{-22} eV — $10^{-(3\div5)}$ eV — 10^{2} eV → m

String axions (Fuzzy DM)

- Predicted by string theory
- Any mass m

Arvanitaki et al '10

- Fuzzy DM: $m \sim 10^{-22}$ eV

e.g. Schive et al '04

$\Rightarrow$ Quantum at galaxy scales!

- $\lambda_4 \sim 10^{-100}$ — only gravity!

QCD axions

- Solve strong CP problem

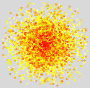
Peccei, Quinn '77

- Dark Matter:

$m \sim 10^{-5} - 10^{-3}$ eV

Klaer, Moore '17, Gorghetto et al '20

- Form miniclusters:



$M \sim 10^{-17} - 10^{-12}\, M_\odot$

Kolb, Tkachev '93; Pierobon et al '23

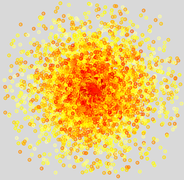
- $\lambda_4 \sim 10^{-50}$ — only gravity!

It's wave-like!

Any small structure

a dwarf galaxy

or axion minicluster



$\boxed{\boldsymbol{\rho}, \ \mathbf{v}}$ — known!

density, velocity

- **Large phase-space density!**

$$m \ll 10^2 \text{ eV} \quad \Rightarrow \quad f \sim \frac{\rho/m}{(m\mathbf{v})^3} \gg 1$$

$$\Rightarrow \quad \text{classical field } \boxed{\psi(t, \mathbf{x})}$$

or collective wave function

- **Nonrelativistic approximation**

$v \ll 1 \Rightarrow$

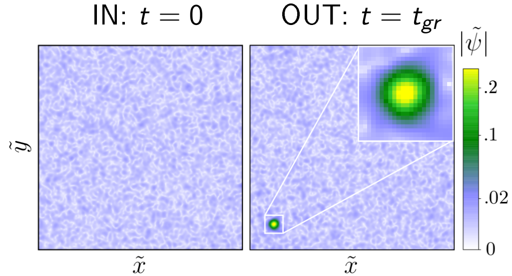
Schrödinger-Poisson (SP) eqs

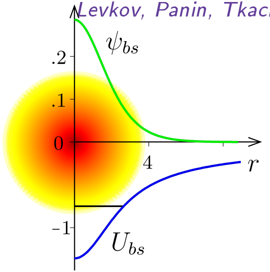
$$i\partial_t \psi = -\Delta\psi/2m + m\mathbf{U}\psi$$

$$\Delta \mathbf{U} = 4\pi G \ m|\psi|^2$$

grav. potential $U(t, x)$ field $\psi(t, x)$

Light DM Bose-condenses by gravitational scattering!



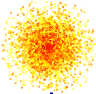


- **Bose star** = Bose-condensate on a single level of U_{bs}
- **Gravitational kinetic relaxation:**

$$t_{gr} = \frac{b\sqrt{2}}{12\pi^3} \frac{m^3 \mathbf{v}^6}{G^2 \Lambda \rho^2}, \quad b \approx 0.9$$

Coulomb logarithm:
$\Lambda \equiv \log(mvR)$

- **Fuzzy DM in dwarf galaxies:** $t_{gr} \gtrsim 10^3$ **yr**



- **QCD axions in miniclusters:** $t_{gr} \gtrsim$ **hr**
- DM is light $\Rightarrow$ **The Universe is packed with Bose stars!**

Problem

How do the Bose stars grow?

- A difficult problem
- Numerical simulations: conflicting results

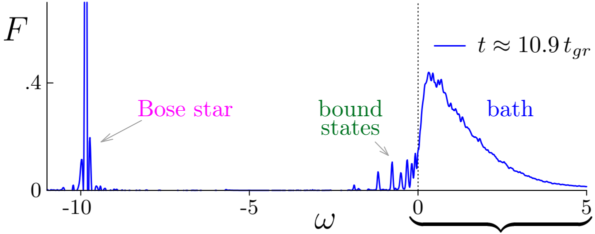
$$M_{bs} \propto \mathbf{t^{1/2}},\ t^{1/4},\ t^{1/8}$$

e.g. Levkov et al '18, Eggemeier et al '19, Chan et al '22

But the solution is simple and analytical!

Distribution of particles over energies

$$F(t,\,\omega) \equiv \frac{1}{N}\,\frac{dN}{d\omega} = \int \frac{dt_1\, d^3\mathbf{x}}{2\pi N}\;\psi(t,\mathbf{x})\,\psi^*(t + t_1,\,\mathbf{x})\,\mathrm{e}^{i\omega t_1 - t_1^2/\Delta t^2}$$

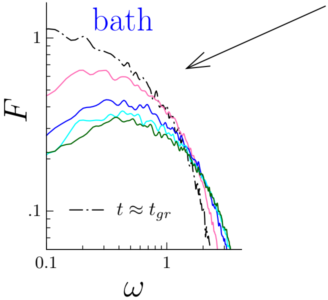


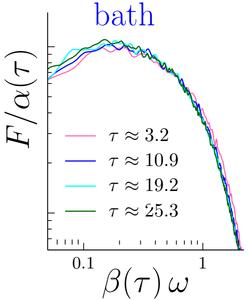
Scaling symmetry

$$F = \alpha F_s(\beta\omega)$$

$\alpha = \tau^{-1/D}$	$\beta = \tau^{2/D-1}$

$$\tau \equiv t/t_{gr}, \quad D = 2.8$$





Self-similar bath

- Consider the bath $\omega > 0$
- Ignore the Bose star potential ~~$U_{bs}(r)$~~

$\Rightarrow$ $\boxed{\partial_t F = \mathrm{St}\, F}$ bath kinetic eq.

Ansatz passes the equation!

$$F(\tau, \omega) = \alpha(\tau) \underbrace{F_s}_{\text{self-similar profile}} (\beta(\tau)\omega)$$

$$\alpha = \tau^{-1/D} \qquad \beta = \tau^{2/D-1} \qquad \tau \equiv t/t_{gr}$$

$\Rightarrow$ Profile Eq:

$$(2/D - 1)\omega_s \partial_{\omega_s} F_s - F_s/D = \mathrm{St}\, F_s$$

$\Rightarrow$ Power-law bath mass & energy

$$M_b \propto \tau^{1-3/D}, \quad E_b \propto \tau^{2-5/D} \qquad \boxed{E_b^3/M_b^5 \propto \tau}$$

non-autonomous system!

$\Rightarrow$ Condensation:

$$M_b \propto \tau^{k_M}, \quad k_M < 0: \qquad E_b \propto \tau^{k_E}, \quad k_E > 0$$

So: $D \in (2.5 : 3)$

Self-similar bath

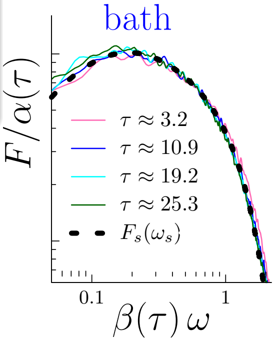
Two types of solutions

Energy Flux	Particle Flux
$J_{Es} < 0$ at $\omega = 0$ heating of the Bose-star	$J_{Ns} \neq 0$ at $\omega = 0$ evoporating?
$J_{Es} = 0$ at $\omega = 0$ $\Rightarrow$ Condensation:	$J_{Ns} \neq 0$ at $\omega = 0$
$J_{Es} = 0$ at $\omega = 0$	$J_{Ns} < 0$ at $\omega = 0$

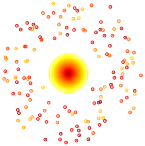
- Nontrivial BC: Particle Flux $\neq 0$ at $\omega = 0$

Then the solution exists
And it is a kinetic attractor

$\tau \equiv t/t_{gr}$



Assumptions:



- Quasi-stationary & self-similar bath:

$$D = D(t) \quad \text{but still} \quad \boxed{E_b^3/M_b^5 \propto \tau}$$

- Energy & mass conservation:

$$E_b = E - E_{bs} - E_e, \qquad M_b = M - M_{bs} - M_e$$

Bose star (E_{bs}, M_{bs}); ex. states (E_e, M_e)

- Bose star energy:

$$E_{bs} = -\gamma M_{bs}^3, \quad \gamma \approx 0.0542\, m^2 G^2$$

constant

- Low occupancies of bound states:

$x_e \equiv M_e/M \approx \text{const}$, $E_e \approx 0$ — confirmed by simulations

$$\frac{(1 + x^3/\epsilon^2)^3}{(1 - x_e - x)^5} = \frac{\tau - \tau_i}{\tau_*}$$

- $x(\tau) \equiv M_{bs}/M$
- $\tau \equiv t/t_{gr}$

A simple and predictive law!

Compare with simulations

Equation

$$\frac{(1+x^3/\epsilon^2)^3}{(1-x_e-x)^5} = \frac{\tau-\tau_i}{\tau_*}$$

Variables

- $x(\tau) \equiv M_{bs}/M$
- $\tau \equiv t/t_{gr}$

Parameters

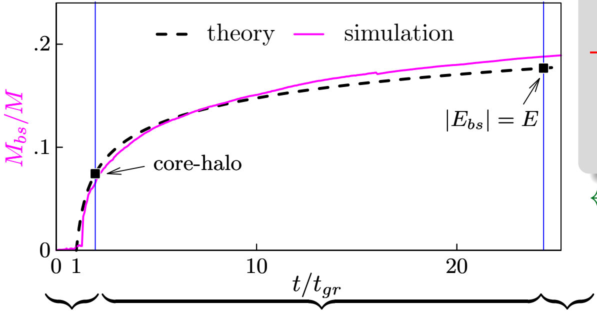
Known:

- $\epsilon^2 \equiv E/\gamma M^3$
- $\tau_* = \frac{1-\tau_i}{(1-x_e)^5}$ $\leftrightarrow$ $M_{bs}(1) = 0$

To fit:

- $x_e \equiv M_e/M$ — small
- $\tau_i \approx -0.1$ — universal

Comparison:



$\leftarrow$ $\epsilon = 0.074$, $x_e = 0.043$, $\tau_i = -0.1$

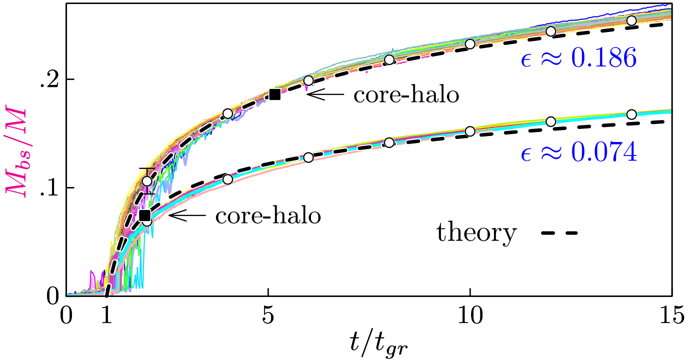
$M_{bs} \propto t^{1/2}$ $t^{1/4}$ $t^{1/8}$ — like in simulations!

cf. Levkov et al '18, Eggemeier et al '19, Chan et al '22

core-halo: $|E_{bs}/M_{bs}| = E/M$ — stars in simulations "stop growing"

Schive et al '14

- 33 simulations
- Essentially different t_{gr} & two values of ϵ



Other performed tests

- + Nonzero self-interaction $\lambda_4 \neq 0$
- + Condensation in miniclusters

Nonzero self-interaction $\lambda\psi^4$

Exactly the same!

- Gravitational kinetic relaxation only:
- But Bose star energy:

$$E_{bs} = -\gamma M_{bs}^3 \mathcal{E}(M_{bs}/M_\lambda),$$
$$\gamma \approx 0.0542\, m^2 G^2$$
$$\mathcal{E}(M_{bs}/M_\lambda) \approx 1;\ M_{bs} \text{ – small}$$
$$\mathcal{E}(M_{bs}/M_\lambda) \neq 1;\ M_{bs} \text{ – big}$$

Gross–Pitaevskii-Poisson (GPP) eqs

$$i\partial_t\psi = -\Delta\psi/2m + m\mathbf{U}\psi + +\lambda|\psi|^2\psi/(8m^2)$$
$$\Delta\mathbf{U} = 4\pi G\ m|\psi|^2$$

Equation

$$\frac{(1+x^3\mathcal{E}/\epsilon^2)^3}{(1-x_e-x)^5} = \frac{\tau-\tau_i}{\tau_*}$$

Variables

- $x(\tau) \equiv M_{bs}/M$
- $\tau \equiv t/t_{gr}$

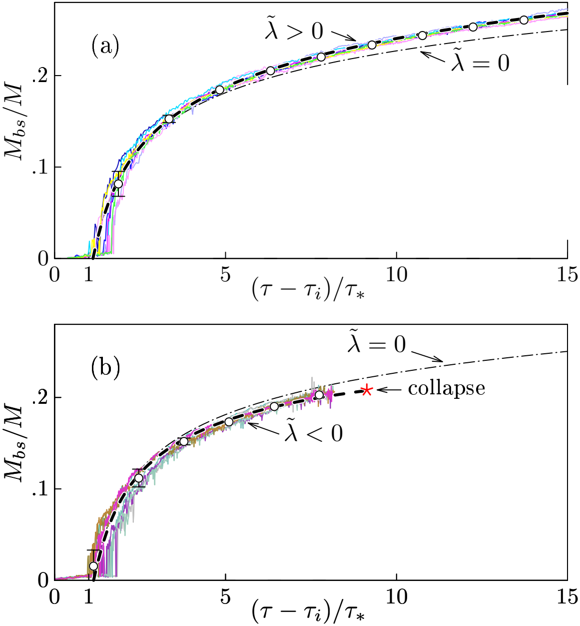
Parameters

To fit:

- $x_e \equiv M_e/M$ — small
- τ_i — universal

Known:

- $\epsilon^2 \equiv E/\gamma M^3$
- $\tau_* = \frac{1-\tau_i}{(1-x_e)^5} \leftrightarrow M_{bs}(1) = 0$
- $\mathcal{E} \equiv \mathcal{E}(xM/M_\lambda)$



Repulsion

$\tilde\lambda = \lambda v_0^2/(m^2 G) = 15$

From fit:

- $x_e \equiv M_e/M = 0.022$
- $\tau_i \approx -0.51$

Faster! *Chen et al '21*

Attraction

$\tilde\lambda = \lambda v_0^2/(m^2 G) = -6$

From fit:

- $x_e \equiv M_e/M = 0.025$
- $\tau_i \approx 0.14$

Slower! *Chen et al '21*

Collapse! *Chen et al '21*

Applications to cosmology: String axions

$$\frac{(1+x^3/\epsilon^2)^3}{(1-x)^5} \sim \frac{t}{t_{gr}}$$

$$x(t) \equiv \frac{M_{bs}}{M}$$

Core of Fornax dwarf

$M \sim 10^8\, M_\odot$

$v_{\rm vir} \sim 20$ km/s

- Parameters:

$$t_{gr} \sim 0.05\, \frac{m^3 (GM)^4}{\Lambda {v_{\rm vir}}^6}$$

$$\epsilon \sim 3\, \frac{v_{\rm vir}}{GmM}$$

- Time to "core-halo" slowdown:

$$t_{\rm c/h} \sim 9\epsilon t_{gr} \sim \Lambda^{-1} m^2 (GM)^3 / v_{\rm vir}^5$$

- Fuzzy DM in Fornax Dwarf ($m \sim 10^{-22}$ eV)
  $t_{\rm c/h} \sim 10^7\, yr$ — form & grow
- Experimental bound: $m \gtrsim 2 \cdot 10^{-20}$ eV
  $t_{\rm c/h} \gtrsim 10^{11}\, yr$ — do not grow! (in Fornax)
  Larger galaxies are worse!

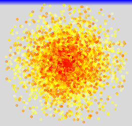
Only small Bose stars exist in the Universe!

Applications to cosmology: QCD axion, $m = 10^{-4}$ eV

$$\frac{(1 + x^3/\epsilon^2)^3}{(1-x)^5} \sim \frac{t}{t_{gr}}$$

$$x(t) \equiv \frac{M_{bs}}{M}$$

Axion minicluster



$$M \sim 10^{-17\div 12}\, M_\odot$$

$$\Phi = \delta\rho_a/\bar{\rho}_a\big|_{RD} = 0 \div 10^3$$

Hogan, Rees '88; Kolb, Tkachev '93

- Parameters:

$$t_{gr} \sim \frac{5\cdot 10^8\,\mathrm{yr}}{\Phi^4}\left(\frac{M}{10^{-14}\,M_\odot}\right)^2\left(\frac{m}{10^{-4}\,\mathrm{eV}}\right)^3$$

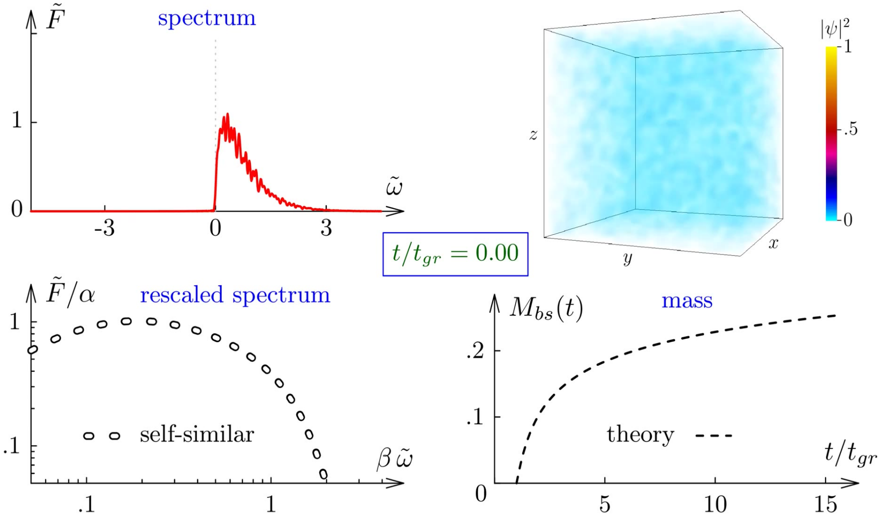
$$\epsilon \sim 0.02\,\Phi^{2/3}\left(\frac{M}{10^{-14}\,M_\odot}\right)^{-2/3}\left(\frac{m}{10^{-4}\,\mathrm{eV}}\right)^{-1}$$

- Time to eat 10% of minicluster:

$$t_{10} \sim t_{gr}\frac{(10\%)^9}{\epsilon^6} < 10^{10}\,\mathrm{yr} \qquad \text{if} \qquad \Phi \gtrsim 1$$

All denser miniclusters turn into

The Universe is packed with grown-up axion stars!


$\tilde{F}$
spectrum
1
0
$\tilde{\omega}$
-3
0
3
$t/t_{gr} = 0.00$
$|\psi|^2$
1
.5
0
z
y
x
$\tilde{F}/\alpha$
rescaled spectrum
1
.1
self-similar
$\beta\,\tilde{\omega}$
.1
1
$M_{bs}(t)$
mass
.2
.1
0
theory
t/t_{gr}
5
10
15

Conclusions II: Implications of Bose stars in axion cosmology

- Less diffuse DM $\Rightarrow$ weaker signals in DM detectorts
- Parametric resonance: radio explosions of heavy stars — explain FRB?

  Levkov, Panin, Tkachev '20; Chung-Jukko et al '22
- Bosenovas: heavy stars collapse & emit relativistic axions

  Levkov, Panin, Tkachev '17; Eby et al '22

THANK YOU FOR ATTENTION!

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