Large solitons flattened by small quantum corrections

Eduard Kim[1], Emin Nugaev [2], Yakov Shnir [3,4,5]

[1] Moscow Institute of Physics and Technology

[2] Institute for Nuclear Research

[3] Joint Institute for Nuclear Research

[4] Hanse-Wissenschaftskolleg

[5] University Oldenburg

based on e-Print: 2405.09262

Quarks-2024 seminar, 24 May 2024, Pereslavl

Outline

Q-balls

Q-ball is a single-field non-topological soliton, with standard ansatz $\phi(t,\vec{x}) = e^{-i\omega t} f(\vec{x})$.

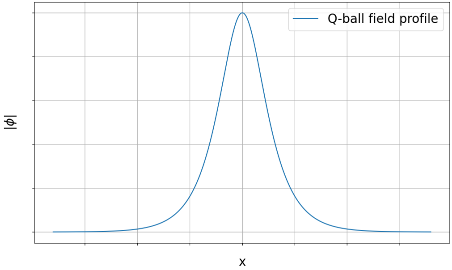

Figure 1: An illustration of typical Q-ball profile.

In order for Q-balls to exist, it is necessary that the potential satisfy the relation

$$V''(0) > \min_{\phi} \left[2V(|\phi|)/|\phi|^2 \right] . \quad (1)$$

The hint of Q-ball's existence can be seen from the stability analysis of the condensate

$$V''(f) - \frac{V'(f)}{f} < 0 . \quad (2)$$

A simplest choice of the potential would be the renormalizable non-linear quartic self-interaction, which supplements the mass term, i.e.

$$V(|\phi|) = m^2|\phi|^2 - \frac{\lambda_\phi}{4}|\phi|^4 \,. \tag{3}$$

- Negatively defined energy functional;
- Does not support any stable classical solutions.

However, classical Q-balls may exist in a model with a single complex scalar field and a suitable nonrenormalizable self-interaction potential.

FLS solitons

Friedberg-Lee-Sirlin (FLS) model (1976) provides non-topological solitons in $(3+1)$ dimensions. No mass parameters in $(3+1)$.

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - V(|\phi|, \chi),$$

$$V(|\phi|, \chi) = h^2 |\phi|^2 \chi^2 + \frac{\varkappa^2}{2}(\chi^2 - v^2)^2 \tag{4}$$

Symmetries:
global $U(1)$ for complex field ϕ, $\phi \to \mathrm{e}^{i\alpha}\phi$,
discrete $\mathcal{Z}_2$ for real χ, $\chi \to -\chi$.

Motivation and Goals

1. FLS model as a QFT;
2. Quantum corrections (mass hierarchy);
3. EFT for Q-balls.

UV-completed FLS

The general renormalizable UV-completed FLS model Lagrangian has the form

$$\begin{aligned}\mathcal{L} &= \partial_\mu\phi^*\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(|\phi|,\chi),\\ V(|\phi|,\chi) &= m_\phi^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4 + h^2|\phi|^2\chi^2 + \frac{\varkappa^2}{2}(\chi^2 - v^2)^2.\end{aligned} \tag{5}$$

For two-field theories, condensate stability condition (2) should be modified as

$$V''_{\chi\chi}(f,\chi)\left[V''_{ff}(f,\chi) - \frac{1}{f}V'_f(f,\chi)\right] + \left[V''_{f\chi}(f,\chi)\right]^2 < 0. \tag{6}$$

It is convenient to introduce the potential

$$U(|\phi|) = \omega^2 |\phi|^2 - V_{eff}(|\phi|), \qquad (7)$$

where

$$V_{eff}(|\phi|) = V(|\phi|, \chi)\Big|_{\frac{\partial V(|\phi|,\chi)}{\partial \chi}=0}. \qquad (8)$$

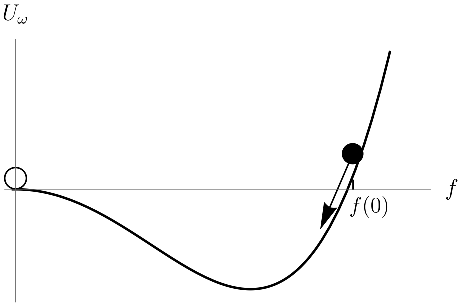

Figure 2: Mechanical potential $U(|\phi|)$.

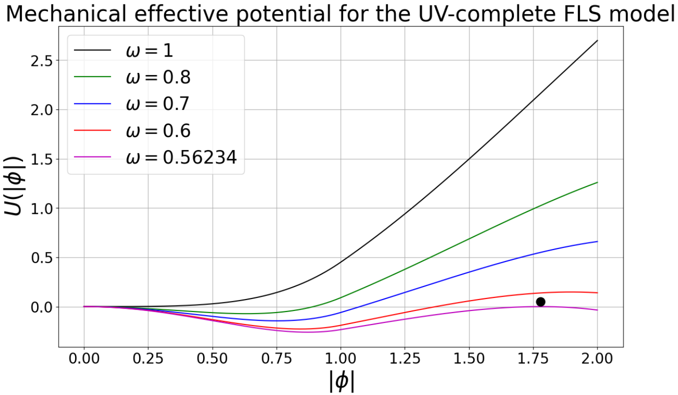

Figure 3: The mechanical potential $U(|\phi|)$ of the model (5) plotted for different values of parameters ω. The couplings of the model are set to $h = 1$, $\varkappa = 1$ and $\lambda = 0.1$, while the nondimensionalization of the Lagrangian (5) is done by the substraction of the v^4 multiplier. Without loss of generality, we set $v = 1$. The black dot represents a condensate.

As can be seen from Eq.(7), the quartic self-interaction λ-term induces a lower bound of parameter ω (see Fig.3)

$$\omega_- = v(\varkappa^2\lambda)^{\frac{1}{4}}, \text{ while } \lambda \leq \frac{h^4}{4\varkappa^2}. \tag{9}$$

At $\omega = \omega_-$, the only possible initial value of $|\phi|$ is

$$|\phi|_{in} = C = \frac{\omega_-}{\sqrt{\lambda}}, \tag{10}$$

which also provides a local extremum, so that $U(C) = 0$ and $U^{'}(C) = 0$. Thus, a $\phi = Ce^{-i\omega_- t}$ is a homogeneous solution that represents a condensate.

Coleman-Weinberg potential

We start by shifting fields by their classical values

$$\begin{aligned} \phi^{(*)} &= \hat{\phi}^{(*)} + \phi_q^{(*)}, \\ \chi &= \hat{\chi} + \chi_q. \end{aligned} \tag{11}$$

Now, let us introduce the mass matrix W as $W_{ij} = \frac{\partial^2 V(\varphi)}{\partial \varphi_{q,i} \partial \varphi_{q,j}}$ for a theory with a set of $\{\varphi_i\}$ fields.

$$W = \begin{pmatrix} m_\phi^2 + \lambda|\hat{\phi}|^2 + h^2\hat{\chi}^2 & 2h^2\hat{\phi}\hat{\chi} \\ 2h^2\hat{\phi}^*\hat{\chi} & 2h^2|\hat{\phi}|^2 + \varkappa^2(6\hat{\chi}^2 - 2v^2) \end{pmatrix} \tag{12}$$

We assume the following mass hierarchy: $W_{\phi^*\phi} \ll W_{\chi\chi}$. In this case, we set the mass matrix W's power counting by assuming that couplings $h \sim z$ and $\varkappa \sim 1$, while $z \ll 1$.

In the leading order of perturbations, $\phi_q^{(*)} = 0$ and χ_q is integrated-out as

$$V_{1-loop} = \frac{1}{64\pi^2}\left[W_{\chi\chi}^2\left(\log\frac{W_{\chi\chi}}{\mu_H^2} - \frac{3}{2}\right)\right], \tag{13}$$

where μ_H is an energy scale of order $\mu_H^2 \sim W_{\chi\chi}$, and the 1-loop contribution is determined specifically at non-negative $W_{\chi\chi}$.

$$V_{CW}(|\hat{\phi}|, \hat{\chi}) = h^2|\hat{\phi}|^2\hat{\chi}^2 + \frac{\varkappa^2}{2}(\hat{\chi}^2 - v^2)^2 + V_{1-loop} \tag{14}$$

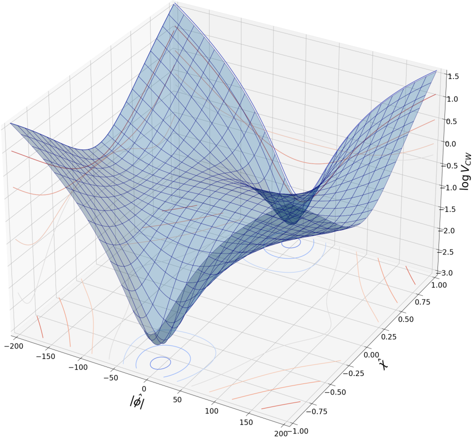

Figure 4: The 1-loop Coleman-Weinberg potential profile of the UV-completed FLS model. The parameters of the model are set to $h = 0.01$ and $\varkappa = 1$.

Quantum corrected Q-balls

Firstly, we integrate-out heavy field χ on the classical level by using equations of motion

$$\frac{\partial V_{CW}(|\hat{\phi}|, \hat{\chi})}{\partial \hat{\chi}} = 0. \tag{15}$$

The effective potential is of the form

$$\begin{aligned} V_{eff}(|\hat{\phi}|) = \left[m_L^2|\hat{\phi}|^2 - \frac{\lambda_L}{2}|\hat{\phi}|^4 + \hat{\Lambda}_L\right]\Theta\left(\frac{m_L}{\sqrt{\lambda_L}} - |\hat{\phi}|\right) + \\ + \left[-m_R^2|\hat{\phi}|^2 + \frac{\lambda_R}{2}|\hat{\phi}|^4 + \hat{\Lambda}_R\right]\Theta\left(|\hat{\phi}| - \frac{m_L}{\sqrt{\lambda_L}}\right), \end{aligned} \tag{16}$$

where $\hat{\Lambda}_{L,R}$ is a constant shift, and $\Theta(x)$ is a Heaviside step function.

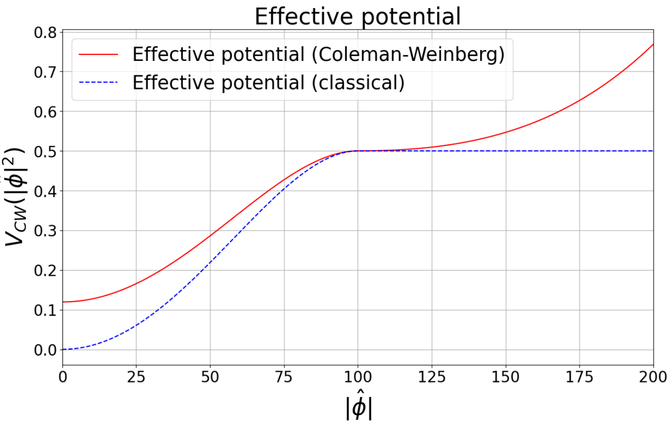

Figure 5: The effective potential $V_{CW}(|\phi|^2)$ is shown. The parameters are set to $h = 0.01$ and $\varkappa = 1$.

After constructing an effective potential, we use redefined Eq.(7)

$$U(|\hat{\phi}|) = \omega^2|\hat{\phi}|^2 - V_{eff}(|\hat{\phi}|) + \hat{\Lambda}_L, \tag{17}$$

so that $U(0) = 0$. Q-ball solutions possess the same type of behaviour that was shown for the UV-completed FLS model. The lower bound on parameter ω is

$$\omega_-^2 = \sqrt{2\lambda_R(\hat{\Lambda}_R - \hat{\Lambda}_L)} - m_R^2. \tag{18}$$

In the limit $\omega \to \omega_-$, the complex field ϕ solution profile resembles a thin-wall regime solution, and then finally settles at the condensate solution

$$\hat{\phi} = Ce^{-i\omega_- t}, \text{ where } C = \sqrt{\frac{2(\hat{\Lambda}_R - \hat{\Lambda}_L)}{\lambda_R}}. \tag{19}$$

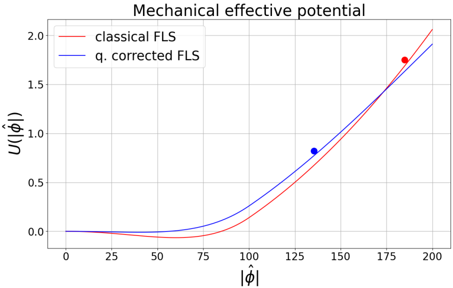

Figure 6: The comparison of the initial values $f(0)$ (presented as color-filled dots on the plot) of the Q-ball solutions for the classical/quantum corrected FLS models effective potential at $h = 0.01$, $\varkappa = 1$ and $\omega = 0.8$.

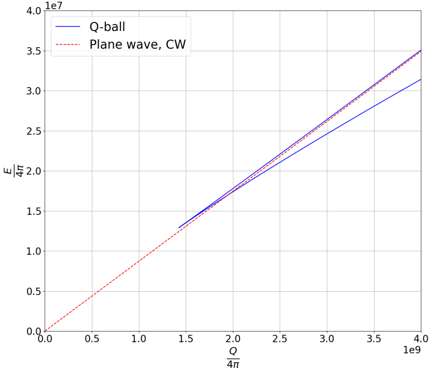

Figure 7: The Q-ball energy $\frac{E}{v}$ (regularized by subtracting $\hat{\Lambda}_L$ from the effective potential to set $V_{eff}(0) = 0$) is plotted as a function of $U(1)$ charge Q for theory with effective potential. Parameters are set to $h = 0.01$ and $\varkappa = 1$.

Results and outlook

- UV-completion of the FLS model;
- Thin-wall approximation;
- Coleman-Weinberg mechanism induces a new mass scale ω_-.

THANK YOU FOR ATTENTION!

This work was supported by the grant RSF 22-12-00215