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Weyl meson in the Weyl-Dirac theory as an extension of the Standard Model

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Content

1. Introduction. Background

Weyl geometry

Hermann Weyl introduced a generalization of Riemannian geometry in an attempt to unify gravity and electromagnetism in 1918. Although Weyl's initial version of gravity does not agree with experiments, Weyl's ideas about the use of additional vector and scalar fields (geometric, not electromagnetic in nature) were subsequently taken into account by a number of researchers Among them, we will highlight first of all those whose works we directly used (Dirac, Rosen, Drechsler). The main idea of Weyl, which was used by the listed researchers, is the assumed local scale (conformal in the sense of metric transformation) invariance of the equations of gravitation. If we try to classify approaches based on Weyl's ideas, we can distinguish the following options: a) the use of a Lagrangian linear in curvature $\breve{R}$, leading to second-order gravity equations with a non-minimal connection between the scalar field and gravity; b) use of the square of the Weyl curvature $\breve{R}^2$; c) use of the "square" of the Weyl tensor in the Lagrangian $C_{\alpha\beta\chi\delta}C^{\alpha\beta\chi\delta}$, leading to fourth-order gravity equations.

Let's consider Weyl (local scale) transformations:

$$g_{\mu\nu}(x) \rightarrow \tilde{g}_{\mu\nu}(x) = \Omega^2(x) \cdot g_{\mu\nu}(x) \ , \quad \Omega(x) = \exp(\sigma(\mathrm{x})) .$$

We will further denote Weyl quantities (analogs of Riemannian ones) by a convex bracket at the top of the mathematical symbol, and transformed quantities by a wavy line at the top of the mathematical symbol.

The corresponding theory of gravity is *Weyl invariant* if the action of the theory is invariant under Weyl transformations: $S \to \tilde{S} = S$. It should be noted that for the value: $g \equiv Det\ g_{\mu\nu}$:

$$\sqrt{-g} \to \sqrt{-\tilde{g}} = \Omega^4(x) \cdot \sqrt{-g} .$$

Thus, since $S = \int L\sqrt{-g} \cdot d^4x$, then for a theory with a conformally invariant action it is necessary that the Lagrangian be transformed as

$$L \to \tilde{L} = \Omega^{-4}(x) \cdot L \quad .$$

The exponent of a function $\Omega(x)$ in the corresponding Weyl transformation is called Weyl weight. The scalar function $\beta(x)$ is converted to:

$$\beta \to \tilde{\beta} = \Omega^{-1}(x) \cdot \beta .$$

We further assume $W(\beta)=-1$, $\frac{\partial\sigma}{\partial x^{\mu}}=\sigma_{,\mu}\equiv\sigma_{\mu}$, and $\frac{\partial\beta}{\partial x^{\mu}}=\beta_{,\mu}\equiv\beta_{\mu}$. To describe the Weyl manifold, in addition to the metric $g_{\mu\nu}(x)$, the geometric field of the Weyl vector $w_{\mu}(x)$ is introduced, which are transformed together with the metric transformations $g_{\mu\nu}\to\tilde{g}_{\mu\nu}$ as:

$w_{\mu}\to\tilde{w}_{\mu}=\tilde{w}_{\mu}-\frac{\partial\sigma}{\partial x^{\mu}}=\tilde{w}_{\mu}-\nabla_{\mu}\sigma,\quad \sigma=\ln\Omega(x)$. The Weyl derivative $\breve{\partial}$ is introduced for the scalar function: β : $\breve{\partial}_{\mu}\beta=\partial_{\mu}\beta-w_{\mu}\beta$. The Weyl derivative of a scalar β has a Weyl weight of -1, as does the function β itself:

$$\frac{\partial}{\partial x^{\mu}}\left(\Omega^{-1}\beta\right)=\Omega^{-1}\beta\cdot\left(\frac{\beta_{\mu}}{\beta}-\sigma_{\mu}\right),\ \breve{\partial}_{\mu}\beta\equiv\beta_{\mu}-w_{\mu}\cdot\beta\to\Omega^{-1}\left(\beta_{\mu}-w_{\mu}\cdot\beta\right)=\Omega^{-1}\cdot\breve{\partial}_{\mu}\beta.$$

Modified Weyl Christoffel symbols are defined using metric $g_{\alpha\beta}(x)$ and the Weyl vector $w_{\nu}(x)$; and look as foollows: $\breve{\Gamma}^{\lambda}_{\mu\nu}=\Gamma^{\lambda}_{\mu\nu}+\delta^{\lambda}_{\mu}w_{\nu}+\delta^{\lambda}_{\nu}w_{\mu}-g_{\mu\nu}w_{\lambda}$.

Here, $\Gamma^{\lambda}_{\mu\nu}$ are the usual Levi-Civita symbols.

In the case of a modified covariant derivative for an arbitrary tensor $B_{\mu\nu}$

$$\breve{\nabla}_{\lambda} B_{\mu\nu} = \frac{\partial}{\partial x^{\lambda}} B_{\mu\nu} - \breve{\Gamma}^{\alpha}_{\lambda\mu} B_{\alpha\nu} - \breve{\Gamma}^{\alpha}_{\lambda\nu} B_{\mu\alpha} ,$$

therefore for the metric tensor we get: $\breve{\nabla}_{\lambda} g_{\alpha\beta} = -2w_{\lambda} g_{\alpha\beta}$. This nonmetric relation characterizes Weyl manifolds, since for a Riemannian space the metricity condition $\nabla_{\lambda} g_{\alpha\beta} = 0$ holds.

We can define the Weyl covariant derivative: $\breve{D}_{\mu} \equiv \breve{\nabla}_{\mu} + 2w_{\mu}$, since the Weyl weight is $W(g_{\alpha\beta}) = 2$. Then $\breve{D}_{\mu} g_{\alpha\beta} = 0$. For scalar, vector and tensor:

$$\breve{D}_{\mu} h(x) = \breve{\partial}_{\mu} h(x) \equiv \left[\breve{\nabla}_{\mu} + W(h) w_{\mu} \right] h(x) ,$$

$$\breve{D}_{\mu} h_{\alpha} \equiv \left[\breve{\nabla}_{\mu} + W(h_{\alpha}) w_{\mu} \right] h_{\alpha} ,$$

$$\breve{D}_{\mu} h_{\alpha\beta} \equiv \left[\breve{\nabla}_{\mu} + W(h_{\alpha\beta}) w_{\mu} \right] h_{\alpha\beta} .$$

The modified curvature tensor is defined similarly to the case of Riemannian geometry:

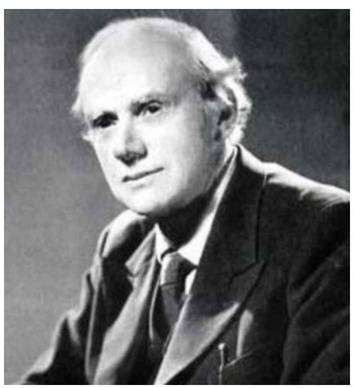
$$\breve{R}_{\mu\nu\rho}{}^{\alpha} = \partial_{\nu} \breve{\Gamma}^{\alpha}_{\mu\rho} - \partial_{\mu} \breve{\Gamma}^{\alpha}_{\nu\rho} + \breve{\Gamma}^{\alpha}_{\nu\lambda} \breve{\Gamma}^{\lambda}_{\mu\rho} - \breve{\Gamma}^{\alpha}_{\mu\lambda} \breve{\Gamma}^{\lambda}_{\nu\rho} .$$

$$\breve{R} = R - \nabla_{\alpha} w^{\alpha} - 6 w_{\alpha} w^{\alpha} .$$

Let's consider a number of authors who contributed to the development of Weyl gravity and the formation of the concept of the weylon (Weyl meson).

1) Dirac

In the 1970s, P.A.M. Dirac introduced Weyl geometry into a number of scalar-tensor theories. He was interested in the connection between some large numbers appearing in physics and proposed the “large number hypothesis” (Dirac, 1973). Weyl geometry was not widely known to the generation of physicists of the 1970s. Dirac then introduced the scalar-tensor theory of gravity combined with electromagnetism. Like Weyl in 1918, he identified the electromagnetic field potential with the Weyl vector.



$$L_\beta = \beta^2 R + 6\beta_\lambda \beta^\lambda + (\alpha - 6)(\beta_\mu - w_\mu \beta)\cdot(\beta^\mu - w^\mu \beta) + 2\lambda\beta^4 + \omega^2 E^{\mu\nu} E_{\mu\nu}$$

$$S_\beta = -\frac{M_P^2}{2}\int L_\beta \sqrt{|g|}\, d^4x = -\frac{1}{16\pi}\int L_\beta \sqrt{|g|}\, d^4x$$

P.Dirac. *Long range forces and broken symmetries.* Proceedings of Royal Society London A., 1973, № 333.P. 403–418.

2) Rosen and Israelit

Dirac's theory continued to be developed in subsequent decades by Nathan Rosen and Mark Israelit from the 1970s. In their development of the Dirac program, Rosen and Israelit initially adhered to the interpretation of the vector field as a field with a light massive (like Proca's field) photon. But in his paper (1982), Rosen discussed the possibility of interpreting the vector field as the potential of a new hypothetical massive heavy boson field. During 1990s, he and Israelit moved to the latter interpretation as the preferred physical interpretation of the Weyl vector in Weyl–Dirac theory. Rosen extended Dirac's approach in several ways. He added a scale-invariant mass term to the Lagrangian, studied dynamic equations, corresponding currents, Noether relations, and considered the question of various gauges. Although he recognized the importance of scale-covariant derivatives in giving the Lagrangian density a scale-invariant form, he did not write the dynamic equations in a scale-invariant form. The left side of the Einstein equation was written based on the Einstein tensor. Similarly, the right-hand expressions for the mass momentum energy and the scalar field separately were not Weyl invariant. All terms of the dynamic equations were reduced to Riemannian quantities. This was convenient from the point of view of physicists accustomed to the general theory of relativity.

Rosen also posed the question of how the Dirac "atomic gauge" in the sense of the Weyl gauge could be reconciled with the non-integrable geometric Weyl structure in order to solve the problem that Einstein formulated in 1918 as an objection to the application of the generalized Weyl gauge - namely, the problem of "second clocks". Studying the Lagrangian of the Weyl-Dirac theory, Rosen (1982) arrived at the generalized Proca equations for the Weyl vector:

$$\nabla_\nu F^{\mu\nu} + m^2 w^\mu = 0$$

with mass $m^2 = \frac{1}{2}(6-k)$, where k is a parameter of the Weyl-Dirac theory. Rosen concluded that in the case of k-6<0, two physical interpretations of the scale coupling are possible: the Weyl vector could be an electromagnetic field with massive photons of very low mass, or an extremely weak "mesonic" field interacting with ordinary matter. Rosen stated (1982), that these mesons could conceivably accumulate at the centers of galaxies and galaxy clusters and could provide the “missing mass” needed to create a closed universe.

Rosen N. *Weyl's Geometry and Physics.* Foundation of physics. 1982. V12, N3. P213-235.Israelit, Mark; Rosen, Nathan. 1992. *Weyl-Dirac geometry and dark matter.* Foundations of Physics. N22. P.555–568 (1992).

3) Utiyama

Unlike Dirac, Utiyama attempted to interpret Weyl's non-trivial Weylian scale coupling as a new fundamental field. In a series of papers he discussed its bosonic interpretation (Utiyama, 1973). He investigated the conditions under which the “Weyl gauge field” allows plane wave solutions, and came to the conclusion that it would be a “tachyon” field, that is, a field allowing superluminal propagation of disturbances. Therefore, according to Utiyama, this boson must be contained within particles of matter. However, he believed that this “unusual field may play some role in constructing a model of a stable elementary particle” (Utiyama, 1973). Smolin (1979) and Hong Cheng (1988) came to the conclusion that this field has a Planck mass.

Smolin, Lee. *Towards a theory of spacetime structure at very short distances*.Nuclear Physics B. N 160.P 253–268 (1979).

Cheng, Hung. *Possible existence of Weyl’s vector meson.* Physical Review Letters N 61.P 2182–2184 (1988).

H. Nishino and S. Rajpoot. *Standard Model and SU(5) GUT with Local Scale Invariance and the Weylon.* AIP Conf. Proc. P 82-93 (2007), Melville, New York (2006). ArXiv:0805.0613v1 [hep-th]. 5 May 2008.

4) Further development of the Weyl-Dirac theory

In general, the time until about 2000 was the first stage of exploration of a variety of approaches. For several years, direct successors of the Dirac line explored the astrophysical consequences of Dirac's distinction between “atomic gauge” and “Einstein gauge” (Canuto et al.), or refined and expanded the theory (Rosen). At first they adhered to the Dirac interpretation of large-scale coupling as an electromagnetic potential. In the 1980s, this interpretation of the Weyl-Dirac ideas lost its significance. Those who continued using the Dirac approach considered the scale connection as a kind of massive gauge field of the Proca type.

In later research in 1990s, Rosen and Israelit proposed hypotheses for applying the energy-momentum of a Weyl geometric gravitational-scalar field and a hypothetical “Weylon gas” to the accelerated expansion of the Universe and describe the emergence of dark matter. These studies were not related to specific astrophysical or astronomical observations.

E.Scholz: arXiv:1102.3478v2 [hep-th] 18 Apr 2011; arXiv:1206.1559v6 [gr-qc] 27 Jul 2015; arXiv:1703.03187v1 , 9 Mar 2017

2. What is a Weyl meson

The emergence of the concept of weylon

The concept of a vector meson arising in Weyl geometry was apparently first introduced by Rosen (1982) beyond the context of the electroweak model. This happened when we abandoned the interpretation of the Weyl vector as an electromagnetic potential within the framework of Weyl-Dirac gravity.

The fact that all fields of the Standard Model (SM), with the exception of the Higgs field potential, have conformally invariant Lagrangians on the basis of Minkowski space, was considered by Englert et. al in 1970s, and then by other authors. For some researchers (Smolin, Cheng; later Drechler and Tann), the emergence of the Standard Model (SM) made it possible to connect Weyl gravity with the standard fields of the model, in particular, with the Higgs field.

The Weyl vector was considered by Lee Smolin in 1979 as a new hypothetical field that, after quantization, results in a particle with a mass close to the Planck scale. Nine years later, this particle was examined again by Cheng and was named "weylon." The scale-covariant form for the dynamic gravity equations was studied in the works of Hong Cheng. Cheng's paper (1988) was the first description of the electroweak sector of the SM based on Weyl geometry. Note that Cheng's article contains references to Dirac's work.

A decade later, Drechler and Tann in 1998 continued to explore the connection between electroweak structure and Weyl geometry. They came up with the idea of considering the Higgs field as part of the gravitational structure. The question of "mass generation" by breaking local scale symmetry in Weyl's geometric approach to SM fields was studied by Drechler and Tann.

This question continued to attract the attention of researchers. Nishino and Rajput in the early 2000s studied how the symmetries of the Standard Model could be complemented by the interaction of a scalar field and a weylon.

Further, we can note the works in this direction by such authors as Erhard Scholz, Subhadib Mitra, Gopal Kashyap, Israel Quiros, Ichiro Oda, Dumitru Ghilenchea. The number of publications on this topic is constantly growing, so we simply did not mention many due to lack of time. There are works by some authors in similar fields, for example, Shimon Meir, Paul Steinhardt, Mikhail Shaposhnikov etc.

The question of how exactly classical local scale symmetry is related to the quantum level remains open.

Drechsler W., Tann H. *Broken Weyl invariance and the origin of mass*. Foundations of Physics. 1999. V29(7). P.1023–1064.

3. Features of our model

Our version of the development of the Weyl-Dirac theory assumes that the scalar Dirac function β is classical; the Weyl vector w^{α} included in the geometric connection is gradient. That vector B^{α}, which we further call the Weyl meson field or Weyl meson, or weylon, is included only in the Lagrangian. Therefore, unlike other authors, ***we distinguish between the concepts of Weyl vector*** w^{α} ***and weylon (Weyl meson)*** B^{α}**.**

It is possible to use a vector field B^{α} in the Lagrangian, similar to the Weyl vector w^{α}, and at the same time avoid the "second clock effect". To do this, we should modify Weyl-Dirac gravity by introducing two different vectors (one nonphysical, the other physical) instead of one Weyl vector. The Weyl vector w^{γ} in the definition of the Weyl space $(M, g_{\alpha\beta}, w^{\gamma})$, where M is a differentiable manifold, is set equal to: $w_{\nu} = \dfrac{\partial_{\nu}\beta(x)}{\beta} \equiv \dfrac{\beta_{\nu}(x)}{\beta} \equiv \partial_{\nu}(\ln\beta)$, where $\beta(x)$ is the real Dirac function. Our vector w_{ν} is non-physical. The Dirac function β under the Weyl transform changes as:

$$\beta \to \tilde{\beta} = \beta \cdot \Omega^{-1}.$$

Let us also introduce another vector $B_\mu(x)$, which, under the Weyl transformation $g_{\alpha\beta} \to \tilde{g}_{\alpha\beta} = \Omega^2 \cdot g_{\alpha\beta}$, changes as Weyl vector w_μ :

$$B_\mu \to \tilde{B}_\mu = B_\mu - \frac{\partial \ln \Omega(x)}{\partial x^\mu}$$

Here, $\Omega(x)$ is a positive smooth function (conformal factor), B_μ is a physical vector that cannot be reduced to a gradient. Then the curvature of Weyl space $\breve{R}$ is written as follows:

$$\beta^2 \breve{R} = \beta^2 R - 6\beta^2 w_\lambda w^\lambda + 12\beta\beta_\lambda w^\lambda = \beta^2 R + 6\beta_\lambda \beta^\lambda$$

and the Lagrangian describing gravity can take the form:

$$L_\beta = \beta^2 R + 6\beta_\lambda \beta^\lambda + \alpha\left(\beta_\mu - B_\mu \cdot \beta\right) \cdot \left(\beta^\mu - B^\mu \cdot \beta\right) + 2\lambda\beta^4 + \omega^2 E^{\mu\nu} E_{\mu\nu}$$

where α, λ, ω are parameters,

$$E_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,.$$

Let us emphasize that w^μ and B^μ are different vectors. The vector w_μ is used in the definition of the Weyl connection:

$$\breve{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + \delta^\lambda_\mu w_\nu + \delta^\lambda_\nu w_\mu - g_{\mu\nu} w^\lambda \,,$$

and it is related to the function gradient β.

The action for gravity looks like this:

$$S_\beta = -\frac{M_P^2}{2}\int L_\beta\sqrt{|g|}d^4x = -\frac{1}{16\pi}\int L_\beta\sqrt{|g|}d^4x\,.$$

If we consider the mass of a particle before local scale invariance is violated, then this mass should be variable:

$$m = m_0 \cdot \beta(x)$$

This is the case of integrable Weyl geometry. When fixed $\beta(x)=1$, the vector in the Weyl connection disappears and this connection becomes the Riemannian Levi-Civita connection $\Gamma^\lambda_{\mu\nu}$. The vector B_μ and its intensity $E_{\mu\nu}$ remain in the Lagrangian, but there is no "second clock" effect due to the use of integrable Weyl geometry. Let two electrons at different ends of the universe have different masses due to variability $\beta(x)$. When they are reduced to one point, they will have the same masses, in contrast to the case of non-integrable Weyl geometry, when there is one Weyl vector $B_\mu \equiv w_\mu$.

It is known that the Standard Model of elementary particles (SM) is scale invariant either for high energies or for zero mass of the Higgs boson. Therefore, guided by this consideration, a number of authors used Weyl geometry for a joint description of the SM and gravity with a non-minimal connection with the scalar field.

A common disadvantage of models of this kind is that the Weyl vector w^{μ} is used both in the geometric connection and in the Lagrangian, which, generally speaking, leads to a "second clock" effect [see, for example, Pauli's book]. We introduce the connection between Weyl-Dirac gravity and the Standard Model without such a problem. In our model, which combines gravity and the Standard Model, there is vector B_{μ} in the Lagrangian that is not reducible to the gradient of the scalar of the Dirac function β.

The derivatives of scalars, vectors, tensors, spinors with respect to coordinates are "extended" accordingly so that scale symmetry is maintained in curved space. In this case, for "extension" we can use either vector B_{μ} or vector w_{μ}. Thus, for fermions ψ the "extended" derivative has the form:

$$\partial_{\mu}\psi \to \tilde{D}_{\mu}\psi = \left(\partial_{\mu} + i\Gamma_{\mu}(x) - \frac{3}{2}\left\{w_{\mu}\ or\ B_{\mu}\right\}\right)\psi,$$

where $\Gamma_{\mu}(x)$ is the spin connection [Drechler]. The factor 3/2 comes from the Weyl weight of the fermion. Further extension of the derivative can be carried out taking into account gauge symmetry.

Thus, for a fermion doublet ψ_L in the Standard Model, the extended covariant gauge-invariant derivative has the form [Drechler, Rajput]:

$$D_\mu \psi_L \to \breve{D}_\mu \psi_L = \left(\partial_\mu + i\Gamma_\mu(x) - \frac{3}{2}\left\{ w_\mu \ or \ B_\mu \right\} + ig\tau W_\mu^{SM} + \frac{i}{2} g' Y_L B_\mu^{SM} \right) \psi_L ,$$

for a singlet of fermions ψ_R in the SM:

$$D_\mu \psi_R \to \breve{D}_\mu \psi_R = \left(\partial_\mu + i\Gamma_\mu(x) - \frac{3}{2}\left\{ w_\mu \ or \ B_\mu \right\} + \frac{i}{2} g' Y_R B_\mu^{SM} \right) \psi_R ,$$

for the Higgs field doublet H in the SM:

$$D_\mu H \to \breve{D}_\mu H = \left(\partial_\mu - \left\{ w_\mu \ or \ B_\mu \right\} + ig\tau W_\mu^{SM} + \frac{i}{2} g' B_\mu^{SM} \right) H .$$

Here, Y_L , Y_R - are hypercharges, W_μ^{SM} and B_μ^{SM} are gauge fields, τ are Pauli matrices, g and g' are charge parameters. In derivatives, some subscripts are omitted for brevity. Note that the Higgs field potential H must include function β [Rajput et al.] in order for it to have a suitable Weyl weight: $U(H,\beta) = \lambda_H \left(H^+ H \right)^2 - \mu_H \left(H^+ H \right) \beta^2 + \xi_H \beta^4$.

Thus, the action of the gravity model and the SM looks like this: $S = S_\beta + \int \breve{L}_{SM} \sqrt{|g|} d^4 x$,

where the modified SM Lagrangian $\breve{L}_{SM}$ includes the above derivatives.

4. Spinors and the Weyl meson

Let us consider a model version of the interaction of the Weyl meson B_μ and spinors using the example of neutrinos. Let us assume that the Weyl meson B_μ , as a dark analogue of a photon without an electric charge, is associated with the right singlet of a heavy neutrino. Next we use the See-Saw model. The right-handed Majorana neutrino mass arises from a hypothetical electrically neutral scalar singlet χ , which is the dark analogue of the Higgs field doublet.

So, the "elongated" derivatives have the form:

$$\breve{D}_\mu \nu_L = \left(\partial_\mu + i\Gamma_\mu(x) - \frac{3}{2}\partial_\mu \ln\beta + ig\tau W_\mu^{SM} + \frac{i}{2}g' Y_L B_\mu^{SM} \right)\nu_L ,$$

$$\breve{D}_\mu \nu_R = \left(\partial_\mu + i\Gamma_\mu(x) - \frac{3}{2}B_\mu + \frac{i}{2}g' Y_R B_\mu^{SM} \right)\nu_R .$$

Thus, by assumption, an ordinary active neutrino does not interact with the Weyl meson, but a sterile right-handed neutrino formally interacts with it.

The model Lagrangian can be written as follows:

$$L = -\frac{M_P^2}{2}L_\beta + g^{\mu\nu}\left(\breve{D}_\mu H\right)^+\left(\breve{D}_\nu H\right) + \frac{1}{2}g^{\mu\nu}\left(\breve{D}_\mu \chi\right)\left(\breve{D}_\nu \chi\right) - U(H,\chi,\beta) + L_{SM}(\beta)$$
$$+ \frac{i}{2}\left(\bar{\nu}_R\gamma^\mu \vec{\breve{D}}_\mu \nu_R - \bar{\nu}_R\gamma^\mu \overleftarrow{\breve{D}}_\mu \nu_R\right) + f_e \bar{e}_R H^+ l_L + f_\nu \bar{\nu}_R H^+ l_L + \frac{1}{2}\chi f_R \bar{\nu}_R^C \nu_R + h.c.$$

similar to the way it was done in [Rajput, arXiv:hep-th/0403039v1 2 Mar 2004].

After breaking the symmetry $SU(2)\times U(1)\times D(1)\to U(1)$ we get the Lagrangian for the right neutrino ν_R :

$$L\supset L_{LR}=(M_D\bar{\nu}_L\nu_R+M_M\bar{\nu}_R^C\nu_R+h.c.)+\frac{i}{2}\left(\bar{\nu}_R\gamma^\mu\vec{\bar{D}}_\mu\nu_R-\bar{\nu}_R\gamma^\mu\overleftarrow{\bar{D}}_\mu\nu_R\right)$$

Since the Dirac neutrino mass M_D is assumed to be much smaller than the right Majorana neutrino mass M_M , a small admixture of the right neutrino wave function ν_R is added to the wave function of the left neutrino ν_L .

The kinetic terms for the dark scalar χ and doublet H can provide indirect interaction with the Weyl meson, but there is still no direct interaction of spinors with the Weyl meson. The point is that the member $\frac{i}{2}\left(\bar{\nu}_R\gamma^\mu\vec{\bar{D}}_\mu\nu_R-\bar{\nu}_R\gamma^\mu\overleftarrow{\bar{D}}_\mu\nu_R\right)$ does not contain vector B_μ. This is how it should be for the Lagrangian for spinors to be Hermitian. This fact, common to spinors, was noted by various authors; for example let us refer to [Drechler, Tann]. The Weyl vector (and the Weyl meson) in the Lagrangian behaves in a sense as an imaginary electromagnetic vector. This analogy helps to understand why the Lagrangian of the spinor part should not contain an explicit Weyl vector. Otherwise there will be a violation of the Hermitianity of the Lagrangian. **Thus, the Weyl meson does not interact directly with spinors.**

5. Vectors and scalars

Vectors in the SM are of a phase nature. This is reflected in the gauge connection with matter fields. The Weyl vector and Weyl meson are related to the change in length scale, so they do not have a phase factor i. The coupling between the scalar and the Weyl meson B_ν is taken into account directly in the kinetic term. The standard expression for the kinetic term of an electrically charged scalar φ is:

$$g^{\mu\nu}\left(\partial_\mu\varphi-(B_\mu+iA_\mu)\cdot\varphi\right)^*\cdot\left(\partial_\nu\varphi-(B_\nu+iA_\nu)\cdot\varphi\right).$$

Here, A_μ is the electromagnetic potential. For convenience, we have introduced charges into the definition of vectors here, and do not consider the expression for the derivative taking into account other gauge vectors, which occurs for the Higgs boson.

The connection between the Weyl meson and the scalar is quite clear. The gauge vector A_μ compensates for the change in phase of the scalar φ, and the Weyl meson compensates for the change in the norm of the scalar B_μ .

Comment. If you need to set an arbitrary self-action potential for a scalar φ, then you should do this: $V(\varphi)\rightarrow\beta^4V\left(\dfrac{\varphi}{\beta}\right)$, so that the potential is Weyl invariant. The mass term of the scalar field $\sim m^2\varphi\varphi^*$ before Weyl symmetry is broken is written as follows $\sim\beta^2\varphi\varphi^*$.

Let's consider a model Lagrangian that includes a charged scalar field φ:

$$L = L_\beta + \xi \cdot g^{\mu\nu} \left(\partial_\mu \varphi - (B_\mu + iA_\mu) \cdot \varphi \right)^* \cdot \left(\partial_\nu \varphi - (B_\nu + iA_\nu) \cdot \varphi \right) + L_{AB}$$

In addition to the gravitational Lagrangian L_β, the Lagrangian includes a kinetic term for a charged scalar field φ. In the observable universe, the conformal symmetry is broken. Taking into account the violation of conformal symmetry, we set the scalar equal to $\beta = 1$, and, doing so, we obtain the effective Lagrangian L_{AB} to describe the interaction of the electromagnetic field A_μ and the analog of Weyl vector B_μ :

$$L_{AB} = \delta^2 F_{\mu\nu} F^{\mu\nu} + E_{\mu\nu} E^{\mu\nu} - 2\varepsilon\delta F_{\mu\nu} E^{\mu\nu} - 2m_B^2 B_\mu B^\mu - 4J_A^\mu A_\mu - 4J_B^\mu B_\mu \ .$$

Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $E_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $2m_B^2 = -\alpha$, J_A and J_B are electromagnetic and dilaton currents of the scalar field φ. These equations hold for the strengths of the vectors A_μ and B_μ:

$$E^{\mu\nu}{}_{;\mu} + m_B^2 B^\nu = \varepsilon\delta m_B^2 F^{\mu\nu}{}_{;\mu} - \xi \left[\partial^\mu \left(\varphi\varphi^* \right) - 2B^\mu \left(\varphi\varphi^* \right) \right] ,$$

$$\delta^2 F^{\mu\nu}{}_{;\mu} = \varepsilon\delta m_B^2 E^{\mu\nu}{}_{;\mu} - \xi \left[i\varphi^* \partial^\mu \varphi - i\varphi \cdot \partial^\mu \varphi^* - 2A^\mu \left(\varphi\varphi^* \right) \right] .$$

Let us give an interesting, but purely speculative example. Let us consider a model Lagrangian with a massless scalar φ , which is invariant under Weyl transformations

$$L = \xi \cdot \frac{\beta^2}{\varphi\varphi^*} g^{\mu\nu}\left(\partial_\mu\varphi - (B_\mu + iA_\mu)\cdot\varphi\right)^* \cdot \left(\partial_\nu\varphi - (B_\nu + iA_\nu)\cdot\varphi\right)$$
$$+\delta^2\left(E^{\mu\nu}E_{\mu\nu} + F^{\mu\nu}F_{\mu\nu}\right)$$

Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $E_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. Under Weyl transformations $B^\mu \to B^\mu - \frac{\beta^\mu}{\beta}$,

under gauge transformations $A^\mu \to A^\mu - \alpha^\mu$ (separately).

Now let there is a violation of Weyl symmetry and $\beta = 1$. The resulting Lagrangian is invariant under rotation transformations in the complex vector complex plane $C^\mu = B^\mu + iA^\mu$:

$$C'^\mu = B'^\mu + iA'^\mu = \left(B^\mu \cos\chi - A^\mu \sin\chi\right) + i\left(A^\mu \cos\chi + B^\mu \sin\chi\right),$$

if there is a joint change in the logarithm of a complex scalar: $\varphi = \rho\cdot\exp\left(i\theta\right)$, $\ln\varphi = \ln\rho + i\theta$:

$\ln\varphi' = \ln\rho' + i\theta' = \left(\ln\rho\cdot\cos\chi - \theta\sin\chi\right) + i\left(\theta\cos\chi + \ln\rho\cdot\sin\chi\right)$. Here, χ is an arbitrary angle in the complex plane (B, A). Let us denote $\varphi = \exp(\psi + i\theta)$, $\psi = \ln|\varphi|$, then

$$g^{\mu\nu}\left(\partial_\mu \ln\varphi - C_\mu\right)^* \cdot \left(\partial_\nu \ln\varphi - C_\mu\right) = g^{\mu\nu}\cdot\left[\left(\partial_\mu\psi - B_\mu\right)^2 + \left(\partial_\mu\theta - A_\mu\right)^2\right]$$

6. Vector meson and Higgs boson

Note that the Higgs field doublet can be related to gravity by adding the corresponding term:

$$L \supset \lambda_R H^+ H \cdot \breve{R} + g^{\mu\nu}\left(\breve{D}_\mu^b H\right)^+\left(\breve{D}_\nu^b H\right) - U(H,\beta) \ .$$

Here, $\breve{D}_\mu^b = \partial_\mu - \{(1-b)w_\mu + bB_\mu\} + igW_\mu^{SM} + \frac{i}{2} g' B_\mu^{SM}$, $0 \le b \le 1$. If $\lambda_R \ne 0$, then the value $\beta = 1$ must then be redefined, so that there is a transition to general relativity with an observable gravitational constant G_N. So, Weyl meson interacts only with the field of the Higgs doublet. After the breaking Weyl symmetry and after breaking the symmetry $SU(2) \times U(1) \times D(1) \to U(1)$ here are Higgs boson h and interaction terms:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}, \qquad L \supset b^2 vh \cdot B^\mu B_\mu + \frac{b^2}{2} h^2 B^\mu B_\mu \ .$$

1.Decay of Weyl meson B^μ. Let us consider in more detail the process of decay of the Weyl meson. See the preprint of Yong Tang and Yue-Liang Wu. *Weyl Symmetry Inspired Inflation and Dark Matter.* //arXiv:1904.04493v1 [hep-th] 9 Apr 2019. The interaction $\sim bh \cdot B^\mu \partial_\mu h$ cannot directly define the decay of Weyl meson, but there are an off-shell cannels of this decay. For $m_B >> m_h$ decay width Γ_B is

$$\Gamma_B \sim \frac{b^6 m_B}{32\pi^5} \approx 1.8 \cdot 10^{-25} s^{-1} \cdot \left(\frac{b}{9 \cdot 10^{-10}}\right)^6 \left(\frac{m_B}{2.2 \cdot 10^9 GeV}\right).$$

Here, m_B - mass of Weyl meson, m_h- mass of Higgs boson.

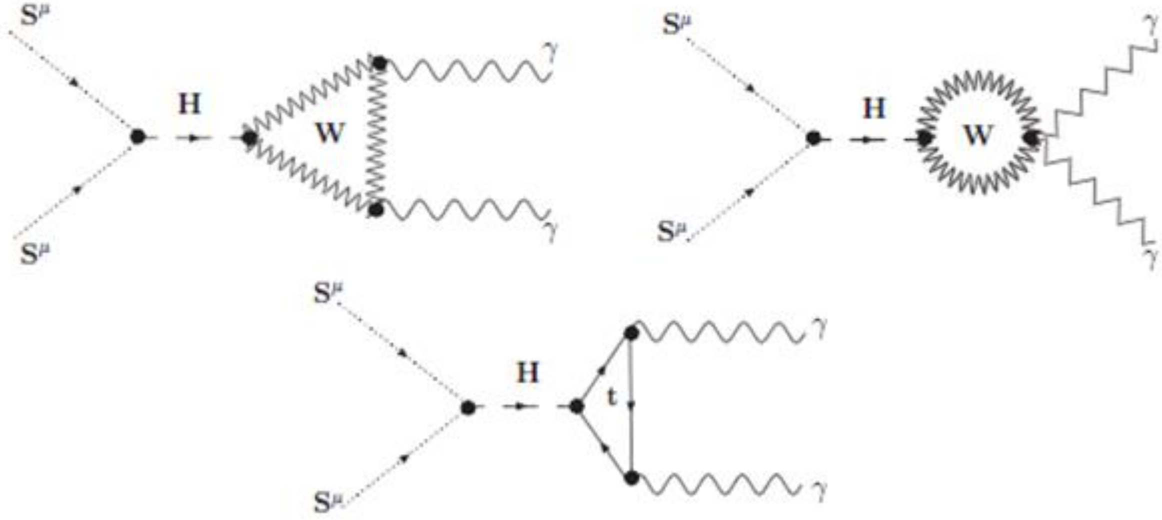
Another estimate of the decay rate of the Weyl meson by Gopal Kashyap ("Weyl meson and its implications in collider physics and cosmology", Physical review D: Particles and fields; July 2012 (arXiv:1207.6195v2 [hep-ph] 12 Jan 2013)) gives:

$$\Gamma_B \leq 4.6 \cdot \left(\frac{m_B}{100\ GeV} \right) \cdot 10^{-27} c \quad \text{for mass of Weyl meson} \quad m_B > 386\ GeV .$$

If Γ_B is sufficiently small, then Weyl meson may be a possible DM candidate; else there are different channels of decay, such as $B_\mu \to 3b + 3\bar{b}$ (bottom quark) and $B_\mu \to 3\tau^+ + 3\tau^-$.

2. Weyl meson annihilation to photon pair.

From the preprint by author, Gopal Kashyap, "Constraints on Weyl meson parameters imposed by Fermi-LAT gamma-ray observations," arXiv:1405.0679v2 [hep-ph] August 13, 2014, we present Weyl meson annihilation diagrams with the detection channel at two gamma quanta. Here, S^μ is Weyl meson.



The cross section of this process (annihilation to photon pair) was estimated by Gopal Kashyap as

$$< \sigma v >= \frac{3}{64\pi \cdot m_B^2} b^2 v^2 \frac{\left| M_{h\to\gamma\gamma} \right|^2}{\left(4m_B^2 - m_h^2\right)^2 + m_h^2 \Gamma_h^2}.$$

The cited works by Yong Tang, Yue-Liang Wu and Gopal Kashyap are the works on the possibility of testing the specific connection between Weyl gravity and particle physics. So, the Weyl meson is a naturally occurring particle in the combined phenomenological connection between the Weyl theory of gravity and the Standard Model.

7. Weylons as analogues of dark photons

The choice of a certain gauge of the function β=1 means the violation of conformal symmetry and the transition from Weyl gravity to general relativity. In our simple model, the remainder of this symmetry is the very light Weyl meson B_μ. The shape of the Higgs field potential at a sufficiently low temperature leads to spontaneous breaking of the SM gauge symmetry. Breaking the gauge symmetry of the SM leads to the appearance of a massless electromagnetic field A_μ. Let's consider the lagrangian:

$$L_{AB} = \delta^2 F_{\mu\nu}F^{\mu\nu} + E_{\mu\nu}E^{\mu\nu} - 2\varepsilon\delta F_{\mu\nu}E^{\mu\nu} - 2m_B^2 B_\mu B^\mu - 4J_A^\mu A_\mu - 4J_B^\mu B_\mu .$$

Let's rename the variables: $\tilde{A}^\mu = \delta \cdot A^\mu$, $\varepsilon = \sin\chi_0$, $m_B^2 = \cos^2\chi_0 \cdot m_{DF}^2$. Then

$$L_{AB} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}E_{\mu\nu}E^{\mu\nu} + \frac{\sin\chi_0}{2}\tilde{F}_{\mu\nu}E^{\mu\nu} + \frac{\cos^2\chi_0}{2}m_{DF}^2 B_\mu B^\mu + J_A^\mu \tilde{A}_\mu + J_B^\mu B_\mu .$$

If we neglect the dilaton current J_B, then the Lagrangian L_{AB} formally coincides with the Lagrangian from [1], which describes the interaction of photons $\tilde{A}^\mu$ and dark photons B^μ. Therefore, let's use the results of [1]. Basing on transformations

$$\{A^\mu, B^\mu\} \to \left\{\tilde{A}^\mu = \delta \cdot A^\mu, \tilde{B}^\mu = B^\mu\right\} \to \left\{\tilde{\tilde{A}}^\mu = \cos\chi_0 \cdot \tilde{A}^\mu, \tilde{\tilde{B}}^\mu = B^\mu - \sin\chi_0 \cdot \tilde{A}^\mu\right\},$$

we make a transition to the basis of interaction, in which oscillations of quanta of vector fields take place $\tilde{\tilde{A}}^{\mu}$ and $\tilde{\tilde{B}}^{\mu}$: $\tilde{\tilde{\gamma}}_A \leftrightarrow \tilde{\tilde{\gamma}}_B$.

There is a formula for the transition probability from [1]:

$$P(\tilde{\tilde{\gamma}}_A \to \tilde{\tilde{\gamma}}_B) = \sin^2 \chi_0 \cdot \sin^2\left(\frac{m_{DF}^2 L_{AB}}{4\omega}\right),$$

where L_{AB} is the mixing length, ω is the angular frequency of photon $\tilde{\tilde{\gamma}}_A$. Thus, there is a transition of the quanta of the Weyl-gauge vector B^{μ} into the quanta of the electromagnetic field A_{μ}. In this case, the analog of the dark photon is the vector B^{μ} , transforming according to the Weyl vector transformation rule, but not included in the affine connection.

1 . Alessandro Mirizzi, Gunter Sigl. *Microwave Background Constraints on Mixing of Photons with Hidden Photons*. Journal of Cosmology and Astroparticle Physics. 2009 (03).
2. James S. Bolton, Andrea Caputo, Hongwan Liu and Matteo Viel. *Comparison of Low-Redshift Lyman-α Forest Observations to Hydrodynamical Simulations with Dark Photon Dark Matter*. Phys.Rev.Letters, V.29, P.211102(1-8).2022.

The astrophysical consequences of the transition of ordinary photons to dark photons have been studied in various papers. In the recent publication [2], a variant was considered when dark photons, turning into ordinary ones, heat the interstellar gas to the observed temperature. The mass of dark photons, according to estimates in [2], is $m_{DF} \sim 8 \cdot 10^{-14} eV$, the mixing parameter is $\varepsilon = \sin\chi_0 \sim 5 \cdot 10^{-15}$. The probability P of the conversion $\tilde{\tilde{\gamma}}_B \to \tilde{\tilde{\gamma}}_A$ in a form convenient for comparison with observations is given in [2]:

$$P(\tilde{\tilde{\gamma}}_B \to \tilde{\tilde{\gamma}}_A) \cong \pi\varepsilon^2 \frac{m_{DF}^2 c^2}{\hbar^2} \left| \frac{d \ln m_\gamma^2}{dt} \right|_{t=t_R}^{-1} .$$

Here, $m_\gamma(\vec{r},t)$ is the effective plasma mass of an ordinary photon in interstellar gas, t_R is the time spent by the system $\tilde{\tilde{A}}^\mu$ and $\tilde{\tilde{B}}^\mu$ in resonance.

Thus, the existence of the interaction effect of the analogs of Weyl mesons and ordinary photons can be tested under astrophysical conditions.

The origin of kinetic mixing is a separate rather complex issue. If we have only phenomenology in mind, then we can simply use an addition $-2\varepsilon\delta F_{\alpha\beta}E^{\alpha\beta}$ to the Lagrangian L_{AB} , which corresponds to the symmetry $U(1)\times D(1)$.

8. Problems of non-integrable Weyl geometry

Let us consider nonintegrable Weyl geometry. The Weyl vector, when conformal invariance is violated, describes particles with spin 1 and mass $m_W = \frac{k\hbar}{c}$. At the same time, even when the gauge is fixed $\beta = 1$, the connectivity coefficients differ from Riemannian ones:

$$\breve{\Gamma}^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}\left(\breve{\partial}_{\mu} g_{\alpha\nu} + \breve{\partial}_{\nu} g_{\alpha\mu} - \breve{\partial}_{\alpha} g_{\mu\nu}\right) = \left\{ \begin{matrix} \lambda \\ \mu\ \nu \end{matrix} \right\} + \delta^{\lambda}_{\mu} w_{\nu} + \delta^{\lambda}_{\nu} w_{\mu} - g_{\mu\nu} w^{\lambda}$$

There remain two possibilities to satisfy the experimental observations: either to accept $w_{\mu} \equiv 0$, which leads to the integrable Weyl vector (IWG) model $w_{\alpha} = \nabla_{\alpha} \ln \beta$, or accept that, on average $\langle\ \rangle$ over some characteristic 4-volume:

$$\breve{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + \left\langle \delta^{\lambda}_{\mu} w_{\nu} + \delta^{\lambda}_{\nu} w_{\mu} - g_{\mu\nu} w^{\lambda} \right\rangle = \Gamma^{\lambda}_{\mu\nu}, \qquad \left\langle \delta^{\lambda}_{\mu} w_{\nu} + \delta^{\lambda}_{\nu} w_{\mu} - g_{\mu\nu} w^{\lambda} \right\rangle = 0.$$

Effective energy-momentum tensor of weylons makes a certain contribution to the change in the metric. Note that turbulent fluctuations of the metric tensor due to the weylon field are significant only in the region of weylon masses close to the Planck mass, $m_W \sim M_{Pl}$. Scholz [ArXiv:1206.1559v6 [gr-qc] 27 Jul 2015] believes that the physical influence of the Weyl vector is determined by its Compton wavelength, limited by the Planck length, that is the direct observation of Weyl curvature is impossible.

Let's use the geodesic equation for a particle with variable mass and momentum:

$$p^\lambda = mcu^\lambda = mc\frac{dx^\lambda}{ds}, \frac{dp^\lambda}{ds} = -\frac{1}{mc}\bar{\Gamma}^\lambda_{\mu\nu}p^\mu p^\nu = -\frac{1}{mc}\Gamma^\lambda_{\mu\nu}p^\mu p^\nu + \frac{2}{mc}p^\lambda\left(w_\nu p^\nu\right) - mcw^\lambda$$

Here, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. We see that the particle is acted upon by a force of geometric origin $f^\lambda = \frac{2}{mc}p^\lambda\left(w_\nu p^\nu\right) - mcw^\lambda$ due to the vector w^λ. Due to the choice of the reference frame, we can neglect the magnitude $\Gamma^\lambda_{\mu\nu}p^\mu p^\nu$ and consider the influence of only the Weyl vector w^λ.

Variability of mass is assumed while maintaining the relations $u_\lambda u^\lambda = 1$ and $p_\lambda p^\lambda = m^2c^2$.

So, the change in mass Δm on a closed loop C is expressed by the formula:

$$\frac{m+\Delta m}{m} = \exp\left(\frac{1}{2}\oint_C w^\nu dx_\nu\right).$$

So, the non-integrability of the Weyl vector can affect ordinary physics in a paradoxical form.

Non-metricity bounds and “second clock” effect see in:

A.D.I. Latorre, G.J. Olmo, M. Ronco, Observable traces of non-metricity: new constraints on metric-affine gravity. Phys. Lett. B 780, 294 (2018). arXiv:1709.04249 [hep-th]

I.P. Lobo, C. Romero, Experimental constraints on the second clock effect. Phys. Lett. B 783, 306 (2018). arXiv:1807.07188 [gr-qc]

What else…

Effective energy-momentum tensor of weylons makes a certain contribution to the change in the metric. Restricting ourselves to the linear approximation, we can obtain a connection between the perturbation of the metric and the energy-momentum tensor

$$g_{\alpha\beta} = g_{0\ \alpha\beta} + h_{\alpha\beta}\ , \quad -\frac{1}{2} g_0^{\alpha\beta} \frac{\partial^2 h_{\mu\nu}}{x^\alpha x^\beta} \equiv \Box_h(x) h_{\mu\nu}(x) = 8\pi \cdot t_{\mu\nu}(x)\ ,$$

where $$t_{\mu\nu}(x) = \left(T_{W\mu\nu}(x) - \frac{1}{2} g_{0\ \mu\nu} \cdot g_0^{\alpha\beta} T_{W\mu\nu}(x) \right).$$

In this regard, problems arise about finding the correlation tensor

$$T_{\alpha\beta\delta\gamma}(x, x') = \left\langle t_{\alpha\beta}(x) \cdot t_{\delta\gamma}(x') \right\rangle$$

and determining the value

$$\left\langle h_{\alpha\beta}(x) h_{\delta\gamma}(x') \right\rangle = (16\pi)^2\, \Box_h^{-1}(x)\, \Box_h^{-1}(x') T_{\alpha\beta\delta\gamma}(x, x').$$

Note that turbulent fluctuations of the metric tensor due to the weylon field are significant only in the region of weylon masses close to the Planck mass, $m_W \sim M_{PL}$. If the mass of the weylon is $m_W \ll M_{PL}$, then the influence of the weylon energy-momentum tensor on the metric can be neglected: $\left\langle h_{\alpha\beta}(x) h_{\delta\gamma}(x') \right\rangle \approx 0$.

D. M. Ghilencea. ***Standard Model in Weyl conformal geometry.***
Eur. Phys. J. C (2022) 82:23

$$\mathcal{L}_0 = \sqrt{g}\left[\frac{1}{4!}\frac{1}{\xi^2}\tilde{R}^2 - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{\eta^2}\tilde{C}_{\mu\nu\rho\sigma}^2\right]$$

In $\mathcal{L}_0$ we replace $\tilde{R}^2 \rightarrow -2\phi_0^2\tilde{R} - \phi_0^4$ with ϕ_0 a scalar

Ghilencea wrote:

The Einstein–Proca action in (8) is a broken phase of $\mathcal{L}_0$ of (7). After ω_μ decouples from (8), below m_ω the Einstein-Hilbert action is obtained as a 'low-energy' effective theory of Weyl gravity [28,29]. Hence, Einstein gravity appears to be the "Einstein gauge"-fixed version of the Weyl action. However, *the breaking is more profound and is not the result of a mere 'gauge choice':* it is accompanied by a Stueckelberg mechanism and by a transition from Weyl to Riemannian geometry: indeed, when massive ω_μ decouples then $\tilde{\Gamma}$ of (2) is replaced by Γ.

In the other case, when ω_μ is very light ($\alpha \ll 1$), it does not decouple in the flat space-time limit and may exist at low energies provided that non-metricity bounds (usually TeV-like [38–40]) do not forbid this. It could also act as a dark matter candidate, see e.g. [41].

9. Conclusion

The main purpose of this report, in addition to the overview, is to voice the idea that the role of the dark photon in the expansion of the SM can in some sense be played by the Weyl meson, whose origin is rooted in the Weyl-Dirac theory of gravity, which is a simple generalization of Einstein's general theory of relativity. At the same time, the Weyl meson must be distinguished from the Weyl vector in order to avoid the need to explain the absence of the "second clock" effect. Let us note the following.

1) The phenomenological model of the connection between gravity and the Standard Model can be based on the integrable Weyl geometry, while retaining the concept of a vector meson.

2) The Weyl vector meson B^{α} (aka weylon) does not coincide with the Weyl vector w^{α}; moreover, there can be several types of Weyl mesons with different masses.

3) The interaction of the Weyl meson B^{α} with other particles can be regulated (turned off, replacing it with the Weyl vector); at the same time, Weyl mesons do not directly interact with fermions.

4) Weyl mesons can interact with scalars; the only example of such a scalar in the Standard Model is the Higgs boson;

5) Weyl meson can kinetically mix with Z-boson, without contradicting the fact that the photon is massless.

6) Questions about the mass of the Weyl meson and its lifetime, as well as experimental and astrophysical constraints, can be studied in a similar way as it is done for the dark photon.

Thank you for your attention!