Scalar oscillons: from weakly nonlinear models to a generic effective approach

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Based on: D. G. Levkov, VM, Phys. Rev. D 108, 063514 (2023) • arXiv:2306.06171
D. G. Levkov, VM, E. Ya. Nugaev, A. G. Panin, JHEP12(2022)079

Oscillons: introduction

Scalar field theory

$$\partial_t^2 \varphi - \Delta\varphi = -V'(\varphi)$$

Generic lifetimes:

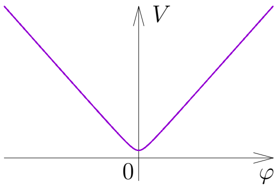
$\gtrsim 10^5$ periods

Example:

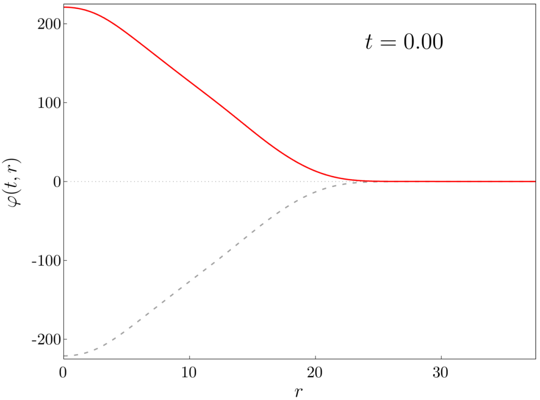
$$V(\varphi) = \sqrt{1+\varphi^2}$$

(axion–monodromy model)

McAllister, Silverstein, Westphal '10



$d = 3$



Oscillons: introduction

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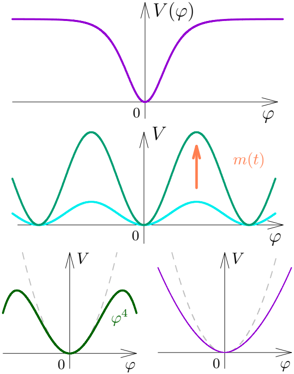
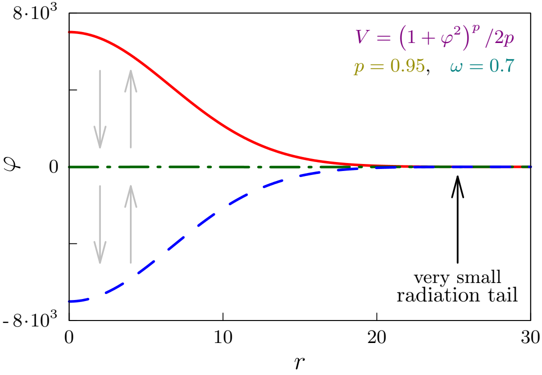
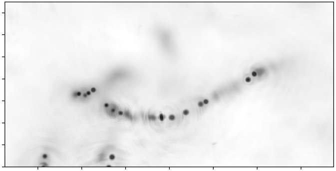
Plethora of theories:





Oscillons in cosmology

- nucleate during generation of axion or ultra-light DM



Kolb, Tkachev '94

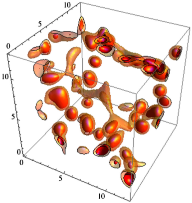
Vaquero, Redondo, Stadler '19

Buschmann, Foster, Safdi '20

- accompany cosmological phase transitions

Dymnikova, Kozel, Khlopov, Rubin '00
Gleiser, Graham, Stamatopoulos '10

- formed by inflaton field during preheating



Amin, Easther, Finkel, Flauger, Herzberg' 12

Hong, Kawasaki, Yamazaki '18

Why are oscillons so long-lived? How to describe them?

Large oscillons, weak nonlinearities

Large oscillons: $R \gg m^{-1}$

- Small repulsion from $\Delta\varphi$

  $\Downarrow$

  Weak attraction expected

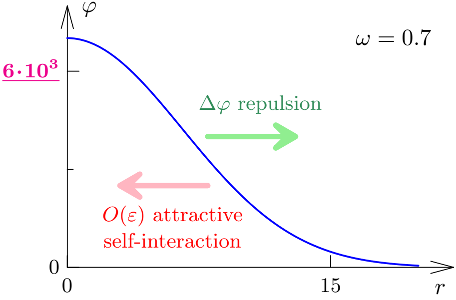
Example: monodromy potentials

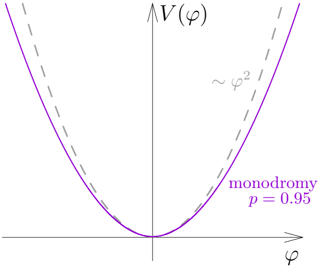
$$V(\varphi) = \frac{1}{2p}\left(1+\varphi^2\right)^p, \qquad p \lesssim 1$$

- Attractive nonlinearity $\quad \varepsilon \equiv 1 - p$
- Large radius: $R^{-2} \sim O(\varepsilon)$.
- Lifetime: up to 10^{14} periods!

  Ollé, Pujolàs, Rompineve '20

- Very strong fields: how to account for small nonlinearities?



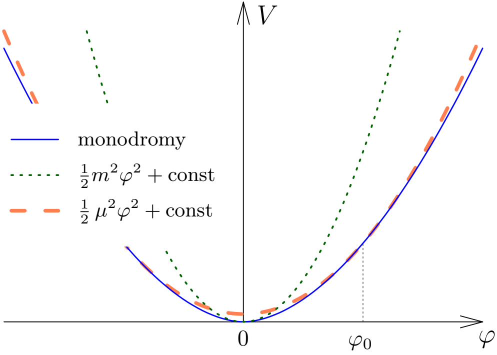


$$\partial_t^2\varphi - \Delta\varphi = -V'(\varphi)$$

- Zero-order approximation: still a **parabola**, but **not** expansion around the vacuum

$$-V'(\varphi) = -\mu^2\varphi - \delta V'(\varphi)$$

$$\delta V \equiv V - \mu^2\varphi^2/2$$



- Wise choice of $\mu \neq m$ to make $\delta V'$ small:

$$\mu^2 = V'(\varphi_0)/\varphi_0$$

for some scale $\varphi_0 \sim \varphi$

- In the end: scale φ_0 — tuned to the oscillon amplitude.

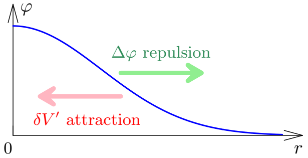
Example: monodromy potential

$$V'(\varphi) = \left(1+\varphi^2\right)^{-\varepsilon}\cdot\varphi = \underbrace{\left(1+\varphi_0^2\right)^{-\varepsilon}}_{\mu^2}\cdot\varphi + \delta V'$$

- Oscillons: $\delta V' \sim \Delta\varphi \sim O(\varepsilon)$

$$\Downarrow$$

- Zero-order approximation:



$$\partial_t^2 \varphi - \cancel{\Delta\varphi} = -\mu^2\varphi - \cancel{\delta V'}$$

linear oscillator →

Action-angle: $\varphi = \sqrt{2I/\mu}\cos\theta$

$$\pi_\varphi \equiv \partial_t \varphi = -\sqrt{2I\mu}\sin\theta$$

Solution: $I(t) = \text{const}, \quad \theta = \mu t.$

- Leading order: restore $\Delta\varphi$ and δV

  $I(t, \boldsymbol{x})$, $\theta(t, \boldsymbol{x})$ now depend on $\boldsymbol{x}$ but **slowly**.

- Classical field action:

$$\mathcal{S} = \int dt\, d^3\boldsymbol{x} \left[\underbrace{\pi_\varphi \partial_t \varphi - \mu^2\varphi^2/2}_{I\partial_t\theta - \mu I} \underbrace{- (\partial_i\varphi)^2/2 - \delta V}_{\text{subleading}} \right]$$

Effective Field Theory (EFT): averaging perturbations

$$\mathcal{S} = \int dt\, d^3\boldsymbol{x}\, \Big[\underbrace{\pi_\varphi \partial_t \varphi - \mu^2\varphi^2/2}_{I\partial_t\theta - \mu I}\underbrace{-(\partial_i\varphi)^2/2 - \delta V}_{subleading}\Big]$$

Averaging over period : $t \longrightarrow \theta$

- $\partial_i I$, $\partial_i \theta$ — slow varying $\Longrightarrow$ moved ***out*** of the averages.

$$(\partial_i\varphi)^2 \longrightarrow \langle(\partial_i\varphi)^2\rangle \overset{t\to\theta}{=} \int_0^{2\pi} \frac{d\theta}{2\pi}(\partial_i\varphi)^2 \approx \frac{(\partial_i I)^2}{4I\mu} + \frac{I}{\mu}(\partial_i\theta)^2 + \cancel{\langle \partial_I\Phi\, \partial_\theta\Phi\rangle \partial_i I \partial_i\theta}$$

$\varphi = \sqrt{2I/\mu}\cos\theta$ symmetry $\theta \to -\theta$

$$\delta V \longrightarrow \langle \delta V\rangle = \int_0^{2\pi} \frac{d\theta}{2\pi}\, \delta V(I,\, \theta)$$

Example: monodromy potential

$$\delta V = \frac{1}{2p}(1+\varphi^2)^p - \frac{\mu^2\varphi^2}{2} \Longrightarrow \langle\delta V\rangle = \frac{1}{2p}\Big(\mathcal{A}_p(\varsigma) - p\mu I\Big)$$

$\varsigma = 2I/\mu$

$\mathcal{A}_p(\varsigma) \equiv \left\langle\left(1+\varsigma\cos^2\theta\right)^p\right\rangle = (1+\varsigma)^{p/2} P_p\left(\frac{1+\varsigma/2}{\sqrt{1+\varsigma}}\right)$

Example: monodromy oscillons

Effective action in the leading order

$$\mathcal{S}_{\text{eff}} = \int dt\, d^3\mathbf{x}\, \left[I\partial_t\theta - \mu I - \frac{(\partial_i I)^2}{8I\mu} - \frac{I(\partial_i\theta)^2}{2\mu} - \frac{\mathcal{A}_p(\varsigma)}{2p} + \frac{\mu I}{2} \right]$$

- Action depends on φ_0 as $O(\varepsilon^2)$
  After second-order corrections — $O(\varepsilon^3)$
- Final step: make "scale" φ_0 and "mass" μ **running**:

$\varphi = \sqrt{2I/\mu}\cos\theta \Longrightarrow \varphi_0^2 = 2I/\mu(\varphi_0^2) \Longrightarrow \mu = \mu(I)$ as planned

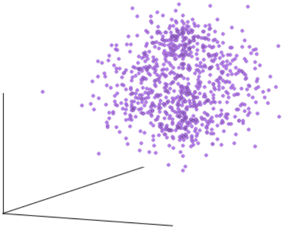
or simply $\varphi_0 = \sqrt{2I}$

- Global symmetry: $\theta \to \theta + \alpha$

$$\Downarrow$$

- Conserved charge: $N = \int d^3\mathbf{x}\, I(t,\mathbf{x})$

$+$ attraction $\Longrightarrow$ solitons!



Oscillons as nontopological solitons

- Stationary ansatz: $I(t, \boldsymbol{x}) = \psi^2(\boldsymbol{x}), \; \theta(t, \boldsymbol{x}) = \omega t$

  or **minimize** energy E at fixed charge N.

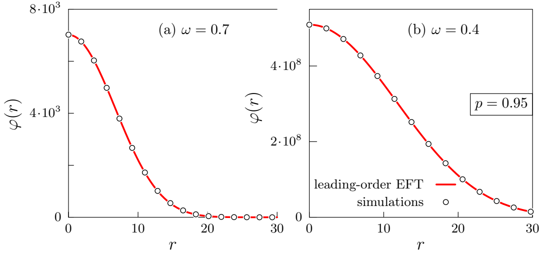
$$\frac{dE}{dN} = \omega$$

Monodromy oscillons profile equation

$$\omega\psi = \mu\psi - \frac{\Delta\psi}{2\mu} + \psi(\partial_i\psi)^2 \frac{\partial_I \mu}{2\mu^2} + \left(\partial_\varsigma \mathcal{A}_p/\mu^2 p - 1/2\right)\left(\mu - \psi^2 \partial_I \mu\right)\psi$$

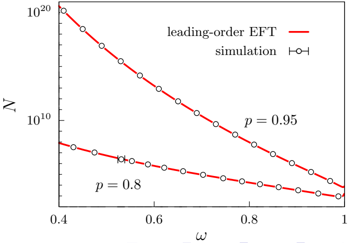
- Field values restored:

$$\varphi(t, \boldsymbol{x}) = \sqrt{2I/\mu(I)}\cos\omega t$$



- Exact adiabatic invariant:

$$N = \int d^3\boldsymbol{x} \int_t^{t+T} \frac{dt}{2\pi} (\partial_t \varphi)^2$$



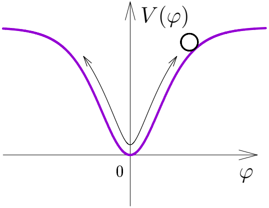
Generalization to arbitrary potentials

- No small nonlinearity, but still **consider large-sized oscillons**

$\Downarrow$

pursue gradient expansion

- Zero order approx.: $\partial_t^2 \varphi - \cancel{\Delta\varphi} = -V'(\varphi) \implies$ Nonlinear oscillator



- Action-angle variables in full nonlinearity

$$\varphi = \Phi(I, \theta), \quad \dot{\varphi} = \Pi(I, \theta)$$

- Hamiltonian: $h = \dot{\varphi}^2/2 + V(\varphi) \equiv h(I)$

- Classical solution: $I = \text{const}, \quad \theta = \Omega t + \text{const}, \quad \boxed{\Omega = \dfrac{\partial h}{\partial I}}$

- Single subleading term in the classical action:

$$\mathcal{S} = \int dt\, d^d\boldsymbol{x} \left(\underbrace{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}_{I\partial_t\theta - h} - \underbrace{\frac{1}{2}(\partial_i\varphi)^2}_{\text{subleading}} \right)$$

- Averaging over period

$$(\partial_i\varphi)^2 \longrightarrow \langle(\partial_i\varphi)^2\rangle = \frac{1}{2\pi}\int_0^{2\pi} (\partial_i\Phi(I,\theta))^2 \, d\theta$$

Generalization to arbitrary potentials

- Slow-varying $\partial_i I$, $\partial_i \theta$ are moved *out* of the average

$$\langle(\partial_i\varphi)^2\rangle \approx \frac{(\partial_i I)^2}{\mu_I(I)} + \frac{(\partial_i\theta)^2}{\mu_\theta(I)} + \langle\partial_I\Phi\,\partial_\theta\Phi\rangle\partial_i I\partial_i\theta$$

$$\mu_I \equiv \left\langle(\partial_I\Phi)^2\right\rangle^{-1}, \quad \mu_\theta \equiv \left\langle(\partial_\theta\Phi)^2\right\rangle^{-1}$$

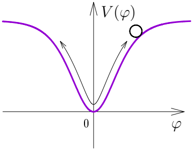
Leading-order effective action for generic potential

$$\mathcal{S}_{\text{eff}} = \int dt\, d^d\boldsymbol{x} \left(I\partial_t\theta - h(I) - \frac{(\partial_i I)^2}{2\mu_I(I)} - \frac{(\partial_i\theta)^2}{2\mu_\theta(I)} \right)$$

- Oscillon profile equation

$$\Omega = \partial h/\partial I$$

$$-\frac{2\psi^2}{\mu_I}\Delta\psi - (\partial_i\psi)^2\frac{d}{d\psi}\left(\psi^2/\mu_I\right) + \Omega\psi = \omega\psi$$



- Existence conditions : *Coleman '85*

$$\Omega(I_0) < m$$

$$h(I_0)/I_0 < m$$

$$I_0 \equiv I(0)$$

Longevity and stability: EFT and beyond

- EFT applicability $\Longleftrightarrow$ oscillon longevity

$$-\frac{2\psi^2}{\mu_I}\Delta\psi - (\partial_i\psi)^2\frac{d}{d\psi}\left(\psi^2/\mu_I\right) + \Omega\psi = \omega\psi$$

$$\Downarrow$$

$$\Omega = \partial h/\partial I$$

$$\omega - \Omega \sim (mR)^{-2} \Longrightarrow \left|\frac{d^2h}{dI^2}\right| = \left|\frac{d\Omega}{dI}\right| \ll \frac{\Omega}{I}$$

potential is close to quadratic!

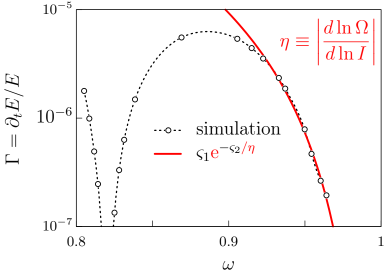
$N = \text{const}$

$\Downarrow$

oscillons — eternal in EFT

$\Downarrow$

evaporation — beyond all orders

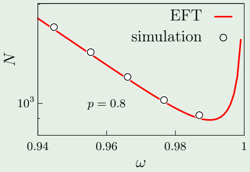


Linear stability

Stable only if

$$dN(\omega)/d\omega < 0$$

Vakhitov-Kolokolov criterion



Higher-order corrections

- Goal: Develop asymptotic expansion in $\varepsilon \sim R^{-2}$:

$$\mathcal{S}_{\text{eff}} = \underbrace{\mathcal{S}^{(1)}_{\text{eff}}}_{\varepsilon^0+\varepsilon^1} + \overbrace{\underbrace{\mathcal{S}^{(2)}_{\text{eff}}}_{\varepsilon^2} + \underbrace{\mathcal{S}^{(3)}_{\text{eff}}}_{\varepsilon^3} + \ldots}^{\text{corrections}}$$

- Field corrections:

$$I = \underbrace{\bar{I}}_{\text{slow}} + \underbrace{\delta I}_{\textit{fast}}, \qquad \theta = \underbrace{\bar{\theta}}_{\text{slow}} + \underbrace{\delta\theta}_{\text{fast}}$$

$$\langle \delta I \rangle = \langle \delta\theta \rangle = 0, \quad \delta I \ll I, \quad \delta\theta \ll \theta$$

- Eqs. for δI, $\delta\theta$:

$$\partial_t \, \delta I = \partial_\theta \varphi (\Delta\varphi - \delta V')$$

$$\partial_t \delta\theta = -\partial_I \varphi (\Delta\varphi - \delta V') + \Big\langle \partial_I \varphi (\Delta\varphi - \delta V') \Big\rangle$$

- Solve order-by-order in δI, $\delta\theta \Longrightarrow$ plug $\delta I(\bar{I}, \bar{\theta})$, $\delta\theta(\bar{I}, \bar{\theta})$ into action + stationary ansatz: $\bar{I} = \psi^2(\boldsymbol{x})$, $\bar{\theta} = \omega t$

Higher-order corrections

Second-order effective action

$$\mathcal{S}_{\rm eff} = \mathcal{S}^{(1)}_{\rm eff} + \mathcal{S}^{(2)}_{\rm eff}$$

$O(\varepsilon^3)$ – sensitive to φ_0

$$\mathcal{S}^{(2)}_{\rm eff} = \int dt\, d^3\boldsymbol{x} \left\{ \frac{1}{2\mu^2}(\Delta\psi + \mu^2\psi)^2 - \mathcal{C}_{1,p}\,(\Delta\psi + \mu^2\psi) + \mathcal{C}_{0,p} \right\}$$

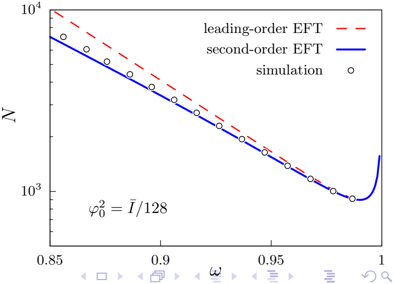
Note. $\sim \varepsilon^2$ contribution, includes 4 spatial derivatives

$\mathcal{C}_{i,p}(\psi^2/\mu)$ — form factors

- $\bar\theta \to \bar\theta + \alpha$ — still a global symmetry
- Test: Detune the "scale" φ_0 to show 2nd order improvement:

$$\cancel{\varphi_0^2 = 2\bar I} \implies \varphi_0^2 = \bar I/128$$

$p = 0.8$

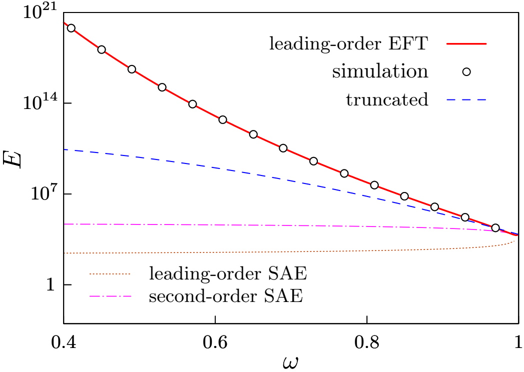


Monodromy: small-amplitude vs. EFT vs. $\varphi^2 \ln \varphi^2$

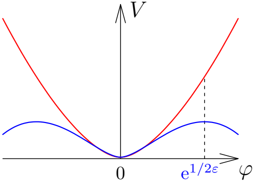
- Small-amplitude expansion: $|\varphi| \ll 1, \quad R \gg m^{-1}$
- Monodromy potential: expansion in ε at $|\varphi| \gg 1$

$$V = \underbrace{\frac{\varphi^2}{2}\left[1 + \varepsilon - \varepsilon \ln \varphi^2\right.}_{\substack{\text{admits} \\ \text{exactly periodic solutions}}} + O(\varphi^{-2}) + O(\varepsilon^2 \ln^2 |\varphi|)\Big] .$$

$d = 3; \quad p = 0.95$

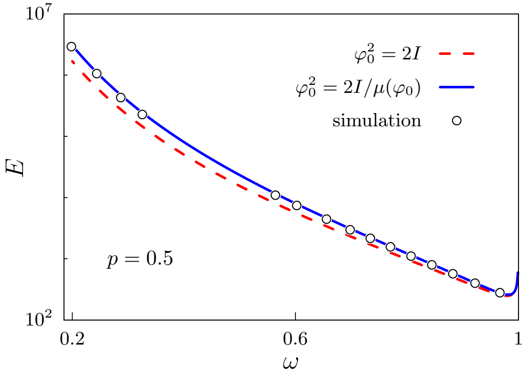


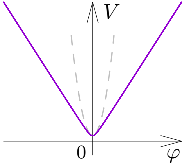
ε-expansion breaks down at $\varepsilon \ln |\varphi| \gtrsim 1$.



Axion-monodromy potential: $V(\varphi) = \sqrt{1+\varphi^2}$

- Significantly nonlinear: $p = 0.5$.
- How does that affect the EFT precisi?





$\delta N/N \lesssim 0.4$

$\Downarrow$

$\delta N/N \lesssim 0.1$

- Proper choice of φ_0 scale cures the method!
- Does not mean the EFT series converge well: $\varepsilon = 0.5$.

EFT.

- Large oscillons — held together by weak nonlinearity
- Parameter of the expansion: $(mR)^{-2} \sim O(\varepsilon)$
- Global $U(1)$ symmetry $\Longrightarrow$ oscillons
- Conditions for existence of long-lived oscillons:

$$V(\varphi) \begin{cases} \text{attractive} \\ \text{nearly quadratic potential} \end{cases}$$

- $\begin{cases} \text{“running mass”} \ \mu \\ \text{expansion in } \Delta\varphi \text{ and } \delta V' \end{cases}$ → great precision!

Perspective.

- Decay of oscillons — nonperturbative in EFT?
- $\begin{cases} \text{large } R \Longleftrightarrow \text{small } p \\ \text{considering slow dynamics} \Longleftrightarrow \text{integrating out high frequencies} \end{cases}$ → EFT applications in QFT ?

THANK YOU FOR YOUR ATTENTION!

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