Neutrino Spin Effects in Gravitaional Scattering

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and

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References

- M. Dvornikov, Neutrino spin oscillations in a magnetized Polish doughnut, JCAP 09 (2023) 039, arxiv:2307.10126.
- M. Deka and M. Dvornikov, Spin oscillations in neutrino gravitational scattering, to be published in Phys.Atom.Nucl. (2024), arxiv:2311.14475.

Outline

- Motivation
- Formulation in brief
- Some key details
- Results
- Conclusion
- Future plans

Motivation

- Neutrinos carry nonzero magnetic moment, μ. Experimental upper bound $\sim 10^{-11} - 10^{-12}\mu_{\mathrm{B}}$.
- Nonzero magnetic moment of neutrinos leads to interaction with the electromagnetic fields.
- Dirac neutrinos are left handed, i.e. their spins are opposite to their momenta.
- If a neutrino spin precesses in an external field, i.e. its spin direction changes with respect to its momentum, it becomes right-handed.

$\Rightarrow$ Neutrino Spin Oscillations. Voloshin et al., 1986.

Motivation

- Accretion disks in SMBH in some galaxies can be sources of both photons and high energy neutrinos. Berezinsky & Ginzburg, 1981

- Before arriving at the observer, these neutrinos move in strong gravitational field near BH, and their spins precess in the presence of external fields.

- Right-handed neutrinos are considered to be sterile in the standard model.

- We shall observe an effective reduction of the initial neutrino flux.

Formulation in brief

- We consider a uniform flux of left-polarized neutrinos propagating in parallel to the equatorial plane of a spinning black hole, $(r, \theta, \phi)_s = (\infty, \pi/2, 0)$.
- Their motion in the presence of a gravitational field of a rotating BH can be described exactly. Gralla et al., 2018.
- We consider a thick realistic accretion disk, a Polish doughnut, surrounding the BH. Abramowicz et al., 1978.
- We consider both toroidal and poloidal magnetic fields inside the accretion disk.
- At the end, we look at the probability distribtuons of neutrinos at the observer position on the $(\theta, \phi)_{\rm obs}$ plane.

Kerr Metric

- We describe the spacetime of a spinning black hole in Kerr metric.
- Boyer-Lindquist coordinates, $x = (t, r, \theta, \phi)$:

$$
\begin{aligned}
ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = \left(1 - \frac{rrg_g}{\Sigma}\right) dt^2 + 2\frac{rr_g a \sin^2\theta}{\Sigma} dt d\phi - \frac{\Sigma}{\Delta} dr^2 \\
&- \Sigma d\theta^2 - \frac{\Xi}{\Sigma} \sin^2\theta d\phi^2
\end{aligned}
$$

$$
\Delta = r^2 - rr_g + a^2, \;\; \Sigma = r^2 + a^2\cos^2\theta, \;\; \Xi = (r^2 + a^2)\Sigma + rr_g a^2 \sin^2\theta
$$

- BH mass: $M = r_g/2$.
- BH spin: $J = Ma(0 < a < M)$.

Particle Trajectory in Kerr Spacetime

- The radial and polar potentials are given by

$$R = \left[(r^2 + a^2)E - aL\right]^2 - \Delta\left[Q + (L - aE)^2\right] \tag{1}$$

$$\Theta = Q + \cos^2\theta\left(a^2E^2 - \frac{L^2}{\sin^2\theta}\right) \tag{2}$$

Q is Carter constant.

- Integral equations along the particle trajectories,

$$\int \frac{dr}{\pm\sqrt{R}} = \int \frac{d\theta}{\pm\sqrt{\Theta}} \tag{3}$$

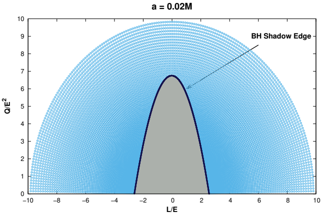
$$\phi = a\int \frac{dr}{\Delta\sqrt{R}}\left[(r^2 + a^2)E - aL\right] + \int \frac{d\theta}{\sqrt{\Theta}}\left[\frac{L}{\sin^2\theta} - aE\right] \tag{4}$$

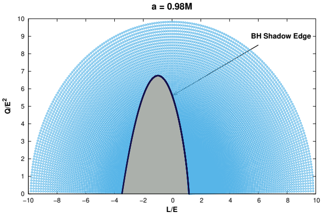
Black Hole Shadow Curve

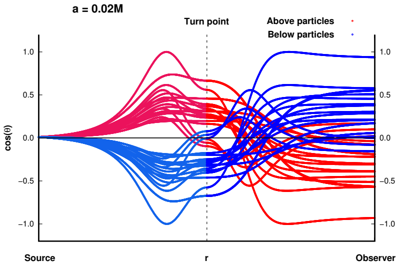
The edge of BH shadow between the captured and escaping neutrinos satisfy: $R(\tilde{r}) = R'(\tilde{r}) = 0$.

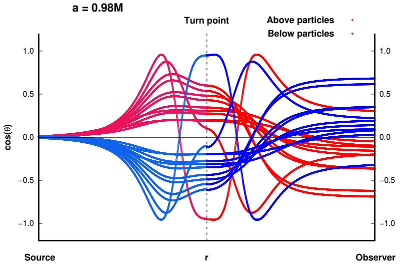
$$\frac{L}{E} = -\frac{\tilde{r}^2(\tilde{r}-3M) + a^2(\tilde{r}+M)}{a(\tilde{r}-M)}, \quad \frac{\sqrt{Q}}{E} = \frac{\tilde{r}^{3/2}}{a(\tilde{r}-M)}\sqrt{4a^2M - \tilde{r}(\tilde{r}-3M)^2} \tag{5}$$

$$r_{\pm} = 2M\left[1 + \cos\left(\frac{2}{3}\arccos(\pm\frac{a}{M})\right)\right], \quad r_- < \tilde{r} < r_+, \tag{6}$$






a = 0.02M
Turn point
Above particles
Below particles
1.0
0.5
0.0
−0.5
−1.0
cos(θ)
Source
r
Observer


a = 0.98M
Turn point
Above particles
Below particles
1.0
0.5
0.0
−0.5
−1.0
cos(θ)
Source
r
Observer

Neutrino spin evolution

- The covariant equation for the neutrino spin four-vector (Dvornikov, 2013; Pomeransky and Khriplovich, 1998),

$$\frac{DS^\mu}{D\tau} = 2\,\mu\left(F^{\mu\nu}S_\nu - U^\mu U_\nu F^{\nu\lambda}S_\lambda\right) + \sqrt{2}G_F\frac{\epsilon^{\mu\nu\lambda\rho}}{\sqrt{-g}}G_\nu U_\lambda S_\rho,\ \frac{DU^\mu}{D\tau} = 0. \quad (7)$$

- Make a boost to a local Minkowskian rest frame so that the Vierbein vectors, e_a^μ, diagonalize the metric:

$$x_a = e_a^\mu x_\mu,\quad \eta_{ab} = e_a^\mu e_b^\nu g_{\mu\nu},\quad \eta_{ab} = (1,-1,-1,-1) \quad (8)$$

- The neutrino polarization three-vector can then be defined

$$\frac{d\boldsymbol{\zeta}}{dt} = 2(\boldsymbol{\zeta}\times\boldsymbol{\Omega}),\quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_{\rm g} + \boldsymbol{\Omega}_{\rm em} + \boldsymbol{\Omega}_{\rm matter}. \quad (9)$$

- $\boldsymbol{\Omega}$ incorporates the neutrino interactions with external fields including gravity, and can be explicitly calculated in a given metric.

Effective Schrödinger Equation

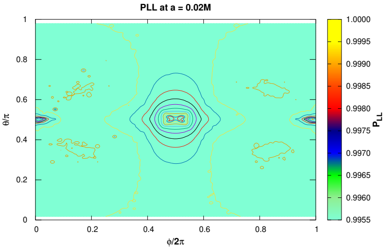
- Instead, we solve the effective Schrödinger equation for the neutrino polarization,

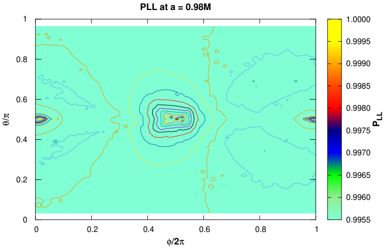
$$i\,\frac{d\psi}{dr} = H_r\,\psi \tag{10}$$

$$H_r = -\mathcal{U}_2(\boldsymbol{\sigma}.\boldsymbol{\Omega}_r)\mathcal{U}_2^{\dagger}, \qquad \mathcal{U}_2 = \exp(i\pi\sigma_2/4)$$

- We use two-step Adams-Bashforth method to solve for ψ.
- For an incoming left polarized neutrino, $\psi^T_{-\infty} = (1, 0)$.
- For an outgoing neutrino, it becomes, $\psi^T_{+\infty} = (\psi^{(R)}_{+\infty}, \psi^{(L)}_{+\infty})$.
- The probability of a neutrino being left polarized: $P_{\mathrm{LL}} = |\psi^{(L)}_{+\infty}|^2$.

$$\Omega = \Omega_g \sout{+ \Omega_{em} + \Omega_{matter}}$$



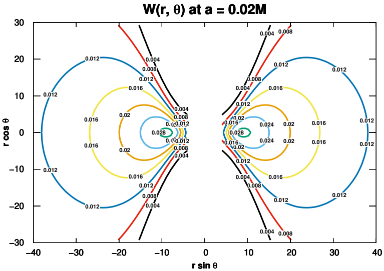


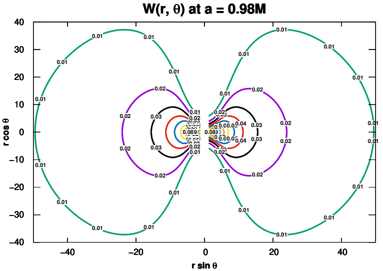
Magnetic fields in the Accretion Disk

- We consider a thick accretion disk surrounding the BH (Polish doughnut).
- All disk parameters depend on r and θ.
- We assume that the specific angular momentum of a neutrino $l_0 = L/E =$ const in the disk.
- The form of the disk depends on the potential,

$$W(r,\theta) = \frac{1}{2}\ln\left|\frac{g_{tt}g_{\phi\phi} - g_{t\phi}^2}{g_{\phi\phi} + 2l_0 g_{t\phi} + l_0^2 g_{tt}}\right| \tag{11}$$

- Toroidal magnetic field is inherent in Polish doughnut. Komissarov, 2006


W(r, θ) at a = 0.02M
r cos θ
r sin θ
0.004
0.008
0.012
0.016
0.02
0.024
0.028


W(r, θ) at a = 0.98M
r cos θ
r sin θ
0.01
0.02
0.03
0.04
0.089

Poloidal Fields

- For stability, both toroidal and poloidal fields need to be included. Tayler, 1973.
- In this work, we use two different models of poloidal fields inside the accretion disk.

Model 1 (Wald, 1974)

$$A_t = Ba\left[1 - \frac{rr_g}{2\Sigma}(1+\cos^2\theta)\right]$$
$$A_\phi = -\frac{B}{2}\sin^2\theta\left[r^2 + a^2 - \frac{a^2 rr_g}{\Sigma}(1+\cos^2\theta)\right] \tag{12}$$

Model 2 (Fragile & Meier, 2009)

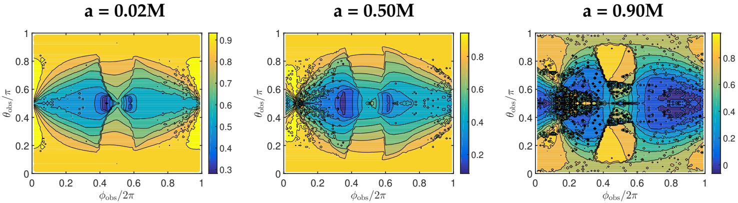
$$A_\phi = b\rho \tag{13}$$

Numerical Parameters

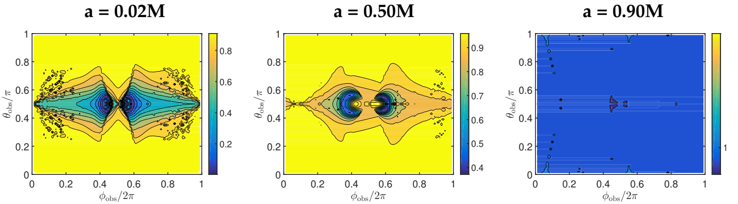
- The mass of SMBH is $10^8 M_{\odot}$. The BH spin is $0 < a < 0.9M$.
- The maximal strength of both poloidal and toroidal fields is 320 G. It is 1% of the Eddington limit for this BH mass. Beskin, 2010.
- The maximal matter density of hydrogen plasma is 10^{18} cm^{-3}. Such density can be found in some AGN. Jiang *et al.*, 2019.
- We consider Neutrino magnetic moment, $\mu = 10^{-13}\mu_{\mathrm{B}}$. It is below the best astrophysical constraint. Viaux *et al.*, 2013.

Results

Toroidal + Model 1 Poloidal field



Toroidal + Model 2 Poloidal field



Conclusion

- We find that only gravitational interaction does not cause the spin-flip of the ultrarelativistic neutrinos in their gravitational scattering.

- The source of neutrino spin oscillations is the neutrino interactions with the moderately strong extrenal magnetic field of the accretion disk.

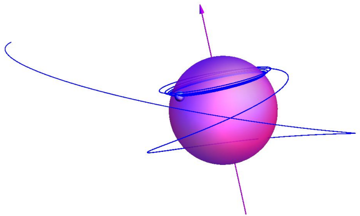
Future Plans

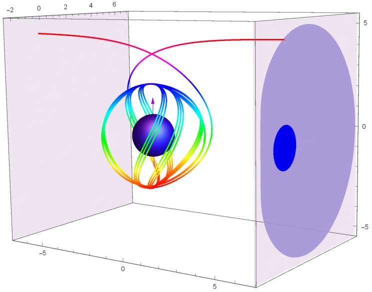
- Extend oue studies to neutrino beams which are not parallel to the equatorial plane.
- Also study counter-rotating accretion disks.
- More accurate solution methods to Schrödinger Equation, e.g. instead of order 2, we shall implement order 3 or 4, etc.
- To make the code more efficient and optimized so that the statistics can be increased multi-fold within a reasonable computational time.

Thank you!

Extras

Dokuchaev and Nazarova, 2020





Particle Trajectory in Kerr Spacetime

- We use the Hamilton-Jacobi approach to describe the geodesic of a particle of mass, m. Later we take $m \to 0$.
- The solution of Hamilton-Jacobi equation leads to,

$$S = -\frac{1}{2}m^2\lambda - Et + L\phi + \int dr\frac{\sqrt{R}}{\Delta} + \int d\theta\sqrt{\Theta} \tag{14}$$

where,

$$\begin{aligned}
&\int \frac{dr}{\pm\sqrt{R}} = \int \frac{d\theta}{\pm\sqrt{\Theta}} \\
&R = \left[(r^2+a^2)E - aL\right]^2 - \Delta\left[Q + (L-aE)^2\right] \\
&\Theta = Q + \cos^2\theta\left(a^2E^2 - \frac{L^2}{\sin^2\theta}\right) \\
&\phi = a\int \frac{dr}{\Delta\sqrt{R}}\left[(r^2+a^2)E - aL\right] + \int \frac{d\theta}{\sqrt{\Theta}}\left[\frac{L}{\sin^2\theta} - aE\right]
\end{aligned} \tag{15}$$

Neutrino spin evolution in curved spacetime

- We consider neutrino as a Dirac particle with nonzero magnetic moment, μ.
- Weakly interacts with the background matter.
- Four velocity of a neutrino is parallel transported along geodesics.
- The covariant equation for the neutrino spin four vector in curved spacetime (Pomeransky and Khriplovich, 1998; Dvornikov, 2013; Dvornikov, 2023),

$$\frac{DS^\mu}{D\tau} = 2\,\mu\left(F^{\mu\nu}S_\nu - U^\mu U_\nu F^{\nu\lambda}S_\lambda\right) + \sqrt{2}G_F\frac{\epsilon^{\mu\nu\lambda\rho}}{\sqrt{-g}}G_\nu U_\lambda S_\rho, \quad \frac{DU^\mu}{D\tau} = 0.$$

$$\begin{aligned} DS^\mu &= dS^\mu + \Gamma^\mu_{\alpha\beta}S^\alpha dx^\beta \\ G_F &= 1.17\times 10^{-5}\text{GeV}^{-2} : \text{Fermi constant} \\ G_\mu &: \text{covariant effective potential.} \end{aligned}$$

We introduce a locally Minkowskian coodinates,

$$x_a = e_a^{\mu} x_{\mu}, \tag{16}$$

where $e_a^{\mu}(a = 0 \cdots 3)$ are the vierbein vectors satisfying the relations

$$e_a^{\mu} e_b^{\nu} g_{\mu\nu} = \eta_{ab}, \quad e_{\mu}^{a} e_{\nu}^{b} \eta_{ab} = g_{\mu\nu} \tag{17}$$

Here $e_{\mu}^{a} e_{\mu}^{a}$ are the inverse vierbein vectors ($e_a^{\mu} e_{\nu}^{a} = \delta_{\nu}^{\mu}$ and $e_a^{\mu} e_{\mu}^{b} = \delta_a^b$) and $\eta_{ab} = \text{diag}(1, -1, -1, -1)$.

$$\begin{aligned} e_0^{\mu} &= \left(\sqrt{\frac{\Xi}{\Sigma\Delta}}, 0, 0, \frac{arr_g}{\sqrt{\Sigma\Delta\Xi}}\right), \quad e_1^{\mu} = \left(0, \sqrt{\frac{\Delta}{\Sigma}}, 0, 0\right), \\ e_2^{\mu} &= \left(0, 0, \frac{1}{\sqrt{\Sigma}}, 0\right), \quad e_3^{\mu} = \left(0, 0, 0, \frac{1}{\sin\theta}\sqrt{\frac{\Sigma}{\Xi}}\right) \end{aligned} \tag{18}$$

$$\frac{d\boldsymbol{\zeta}}{dt} = 2(\boldsymbol{\zeta} \times \boldsymbol{\Omega}), \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{g}} + \boldsymbol{\Omega}_{\mathrm{em}} + \boldsymbol{\Omega}_{\mathrm{matt}} \tag{19}$$

$$\begin{aligned}
\boldsymbol{\Omega}_{\mathrm{g}} &= \frac{1}{2U^t}\left[\boldsymbol{b}_g + \frac{1}{1+u^0}(\boldsymbol{e}_g \times \boldsymbol{u})\right] \\
\boldsymbol{\Omega}_{\mathrm{em}} &= \frac{\mu}{U^t}\left[u^0\boldsymbol{b} - \frac{\boldsymbol{u}(\boldsymbol{u}\boldsymbol{b})}{1+u^0} + (\boldsymbol{e} \times \boldsymbol{u})\right] \\
\boldsymbol{\Omega}_{\mathrm{matt}} &= \frac{G_{\mathrm{F}}}{\sqrt{2}U^t}\left[\boldsymbol{u}\left(g^0 - \frac{(\boldsymbol{g}\boldsymbol{u})}{1+u^0}\right) - \boldsymbol{g}\right]
\end{aligned} \tag{20}$$

Here $u^a = (u^0, \boldsymbol{u}) = e^a_\mu U^\mu$, $U^\mu = \frac{dx^\mu}{d\tau}$ is the four velocity in the world co-ordinates and τ is the proper time. $G_{ab} = (\boldsymbol{e}_g, \boldsymbol{b}_g) = \gamma_{abc}u^c$, $\gamma_{abc} = \eta_{ad}e^d_{\mu;\nu}e^\mu_b e^\nu_c$ are the Ricci rotation coefficients, the semicolon stays for the covariant derivative, and $f_{ab} = e^\mu_a e^\nu_b F_{\mu\nu} = (\boldsymbol{e}, \boldsymbol{b})$ is the electromagnetic field tensor in the locally Minkowskian frame, and $F_{\mu\nu}$ is an external electromagnetic field tensor. μ is the neutrino magnetic moment, and $G_{\mathrm{F}} = 1.17 \times 10^{-5}$ Gev^{-2} is the Fermi constant. $g^a = (g^0, \boldsymbol{g}) = e^a_\mu G^\mu$, G^μ is the contravariant effective potential of the neutrino electroweak interaction with a background matter.

Toroidal Fields

- The electromagnetic field tensor

$$F_{\mu\nu} = E_{\mu\nu\alpha\beta} U_f^\alpha B^\beta, \quad E^{\mu\nu\alpha\beta} = \frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}} \tag{21}$$

- The four vector fluid velocity in the disk and toroidal magnetic field are

$$U_f^\mu = (U_f^t, 0, 0, U_f^\phi), \qquad U_f^t = \sqrt{\left|\frac{\mathcal{A}}{\mathcal{L}}\right| \frac{1}{1 - l_0\Omega}}, \quad U_f^\phi = \Omega U_f^t \tag{22}$$

$$B^\mu = (B^t, 0, 0, B^\phi), \qquad B^\phi = \sqrt{\frac{2p^{(\mathrm{tor})_m}}{|\mathcal{A}|}}, \quad B^t = l_0 B^\phi \tag{23}$$

- The angular velocity in the disk

$$\Omega = -\frac{g_{t\phi} + l_0 g_{tt}}{g_{\phi\phi} + l_0 g_{t\phi}} \tag{24}$$

and

$$\mathcal{L} = g_{tt} g_{\phi\phi} - g_{t\phi}^2, \quad \mathcal{A} = g_{\phi\phi} + 2 l_0 g_{t\phi} + l_0^2 g_{tt} \tag{25}$$

Toroidal Fields

- The disk density ρ and the magnetic pressure $p_m^{(\mathrm{tor})}$ have the form,

$$\rho = \left[\frac{\kappa-1}{\kappa}\frac{W_{\mathrm{in}}-W}{K+K_m\mathcal{L}^{\kappa-1}}\right]^{\frac{1}{\kappa-1}}, \quad p_m^{(\mathrm{tor})} = K_m\mathcal{L}^{\kappa-1}\left[\frac{\kappa-1}{\kappa}\frac{W_{\mathrm{in}}-W}{K+K_m\mathcal{L}^{\kappa-1}}\right]^{\frac{\kappa}{\kappa-1}} \tag{26}$$

Poloidal Fields

- For stability, both toroidal and poloidal fields need to be included. Tayler, 1973.
- In this work, we use two different models of poloidal fields inside the accretion disk.

Model 1 (Wald, 1974)

$$A_t = Ba\left[1 - \frac{rr_g}{2\Sigma}(1+\cos^2\theta)\right]$$

$$A_\phi = -\frac{B}{2}\sin^2\theta\left[r^2 + a^2 - \frac{a^2 r r_g}{\Sigma}(1+\cos^2\theta)\right]$$

$B \propto r^{-5/4}$ (Blandford & Payne, 1982)

Model 2 (Fragile & Meier, 2009)

$$A_\phi = b\rho$$