The HQCD running coupling in strong magnetic field

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Prehistory and motivation

Perturbative methods are not suitable ⇒

Lattice QCD calculations ⇒

Problems in the case of chemical potential ⇒

Holographic duality approach

⇕

Motivated by AdS/CFT duality Maldacena, 1998

Temperature in QCD ⇔ **black hole temperature in (deform.) AdS5**

Thermalization in QCD ⇔ **black hole formation in (deform.) AdS5**

Goal: **describe running coupling constant behavior in a magnetic field**

Holographic model of an anisotropic plasma in a magnetic field at a nonzero chemical potential

$$S = \int \frac{d^5x\sqrt{-g}}{16\pi G_5}\left[R - \frac{f_1(\phi)}{4}F_{(1)}^2 - \frac{f_2(\phi)}{4}F_{(2)}^2 - \frac{f_B(\phi)}{4}F_{(B)}^2 - \frac{\partial_\mu\phi\partial^\mu\phi}{2} - V(\phi)\right]$$

$$A_\mu^{(1)} = A_t(z)\delta_\mu^0, \quad F_{y_1y_2}^{(2)} = q, \quad F_{xy_1}^{(B)} = q_B\,.$$

Ansatz for the metric:

$$ds^2 = \frac{L^2}{z^2}b(z)\left[-g(z)dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}}dy_1^2 + y^{c_Bz^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}}dy_2^2 + \frac{dz^2}{g(z)}\right],$$

$$b(z) = e^{2A(z)}, \quad A(z) = -\,a\ln(bz^2+1)$$

$$a = 4.046, \quad b = 0.01613, \quad c = 0.227, \quad \nu = 1, \quad L = 1$$

Solutions

General form of the boundary conditions:

$$A_t(0) = \mu, \quad A_t(z_h) = 0,$$

$$g(0) = 1, \quad g(z_h) = 0,$$ Aref'eva, Slepov, Rannu, JHEP 2020

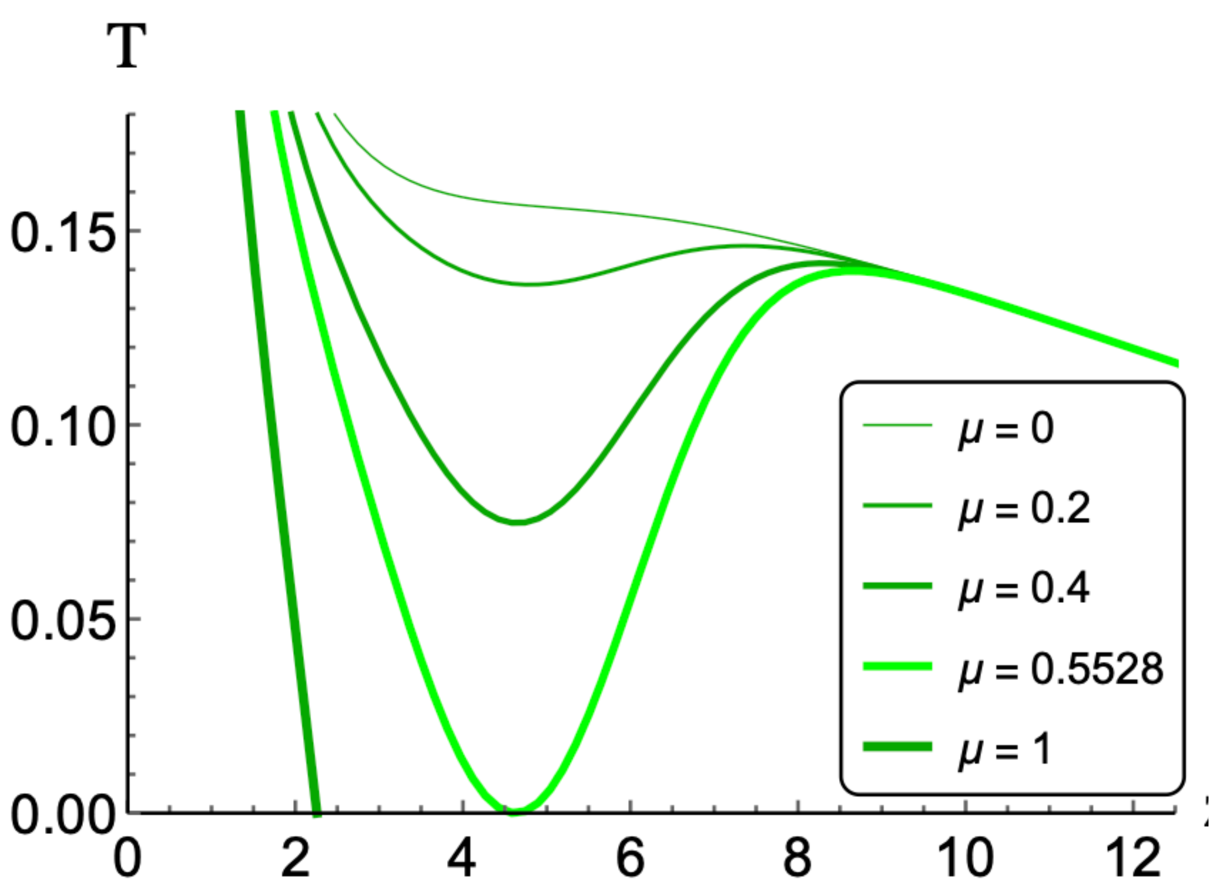
$$\varphi(z)\Big|_{z=z_0} = 0,$$

$$z < z_h, \quad z_0 = 10exp(-z_h/4) + 0.1$$

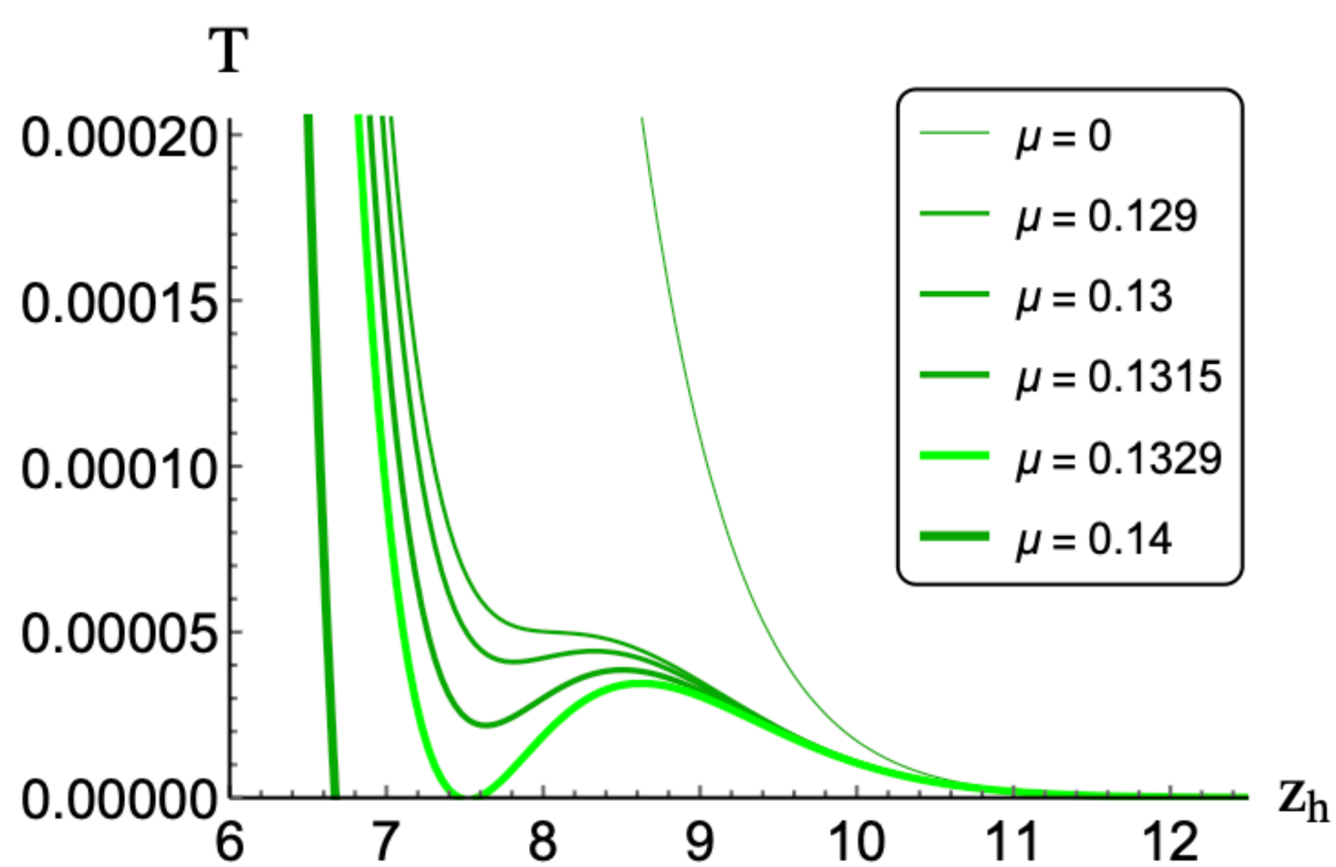
Pavel Slepov's talk

Temperature and Free energy

$$T = \frac{|g'|}{4\pi}\Big|_{z=z_h}, \quad s = \left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}} \frac{e^{c_B z_h^2/2}}{4\left(1+bz_h^2\right)^{3a}}, \quad F = -\int s\,dT = \int_{z_h}^{\infty} s\,T'\,dz_h$$



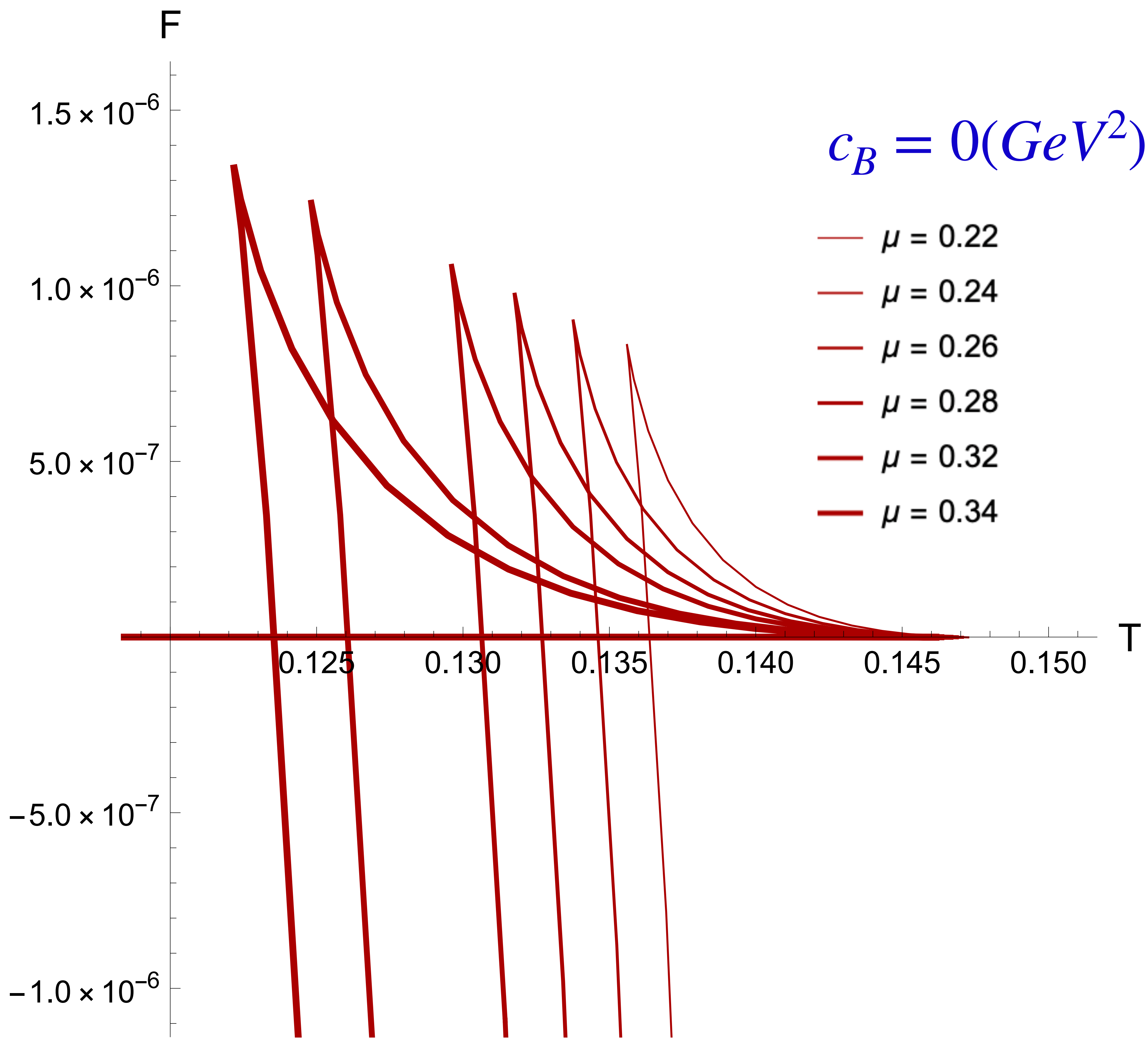
$c_B = -0.001(GeV^2)$



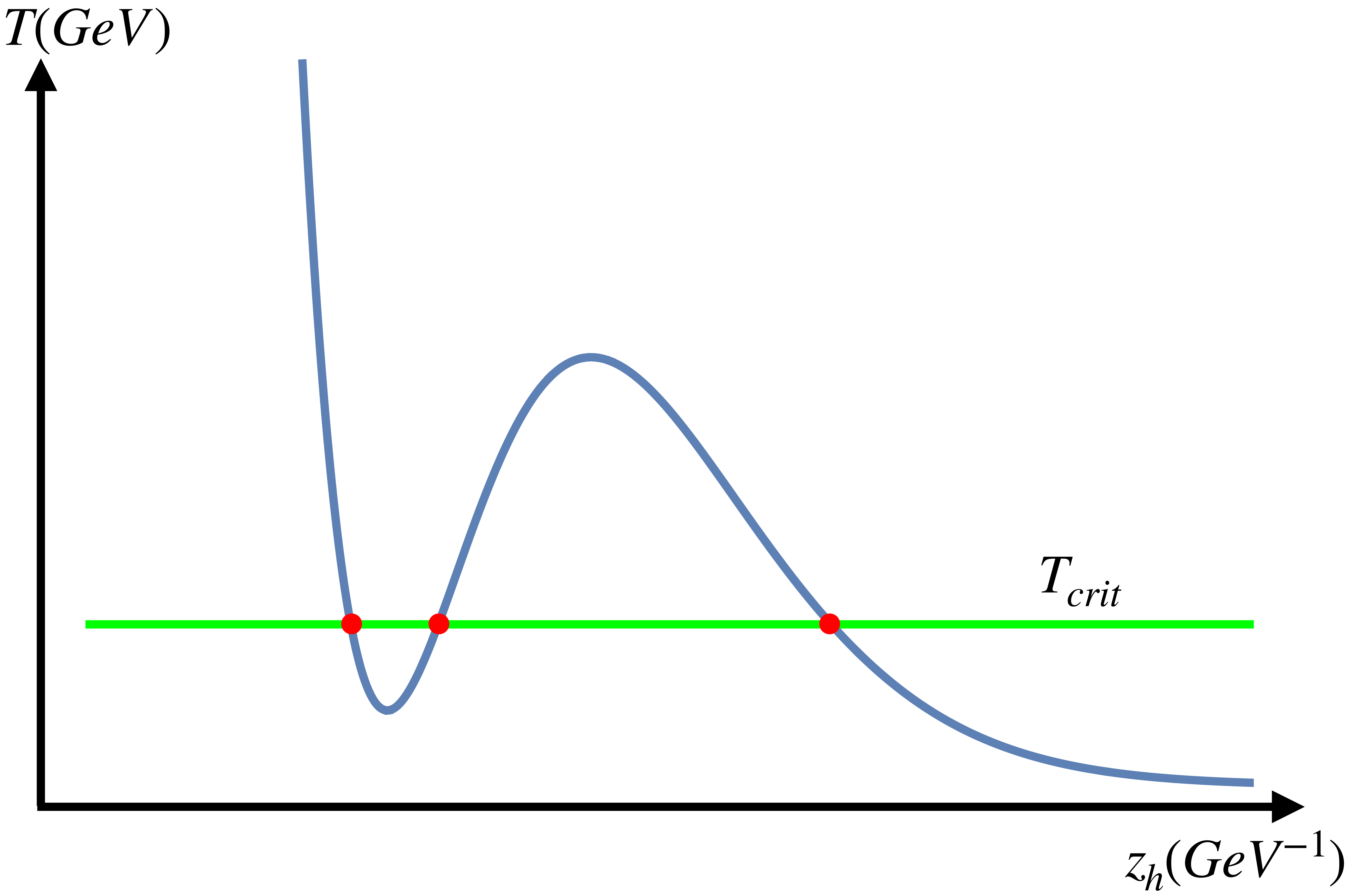
$c_B = -0.1(GeV^2)$

Aref'eva, Ermakov, Rannu, Slepov 2203.12539

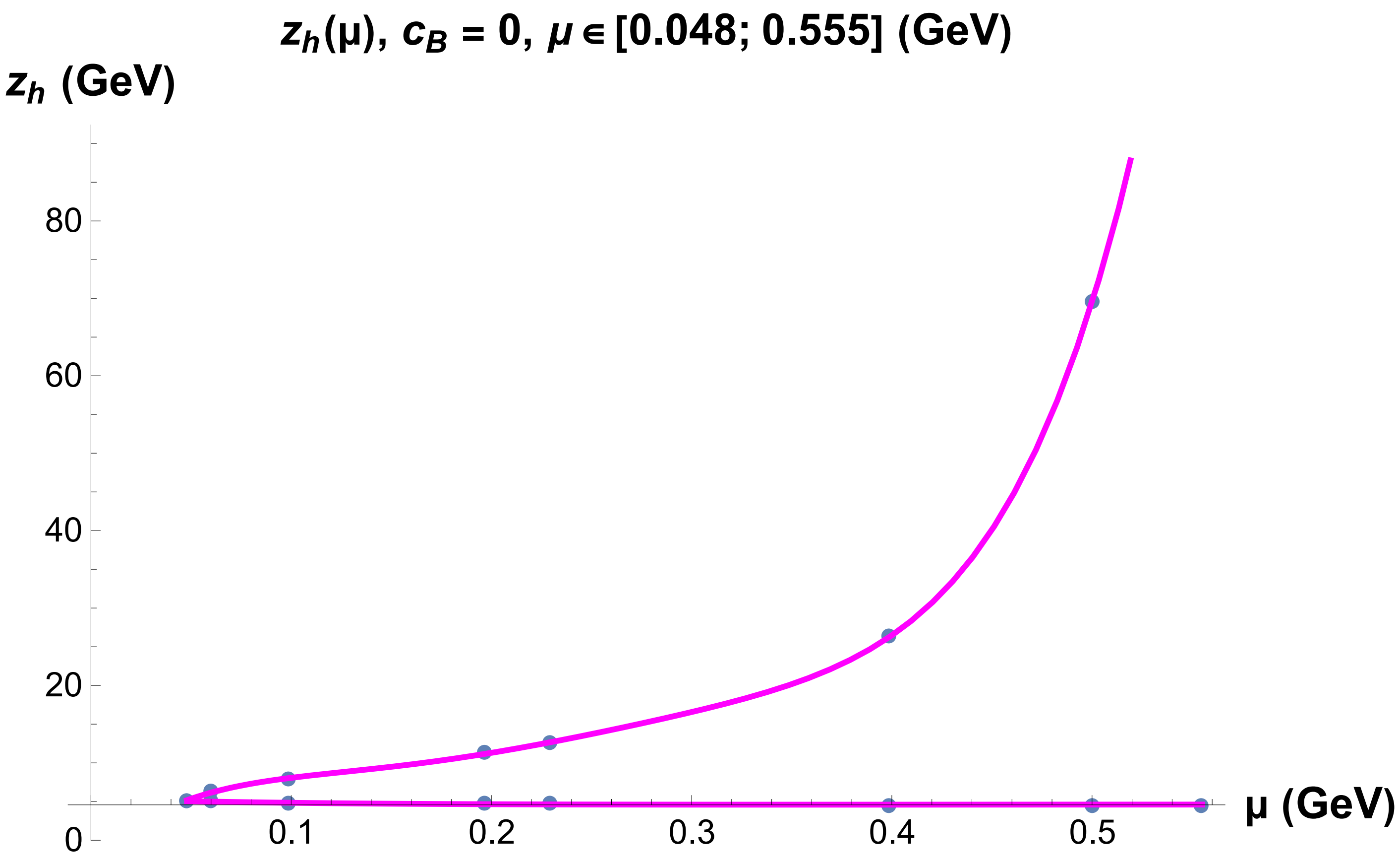
Temperature and Free energy

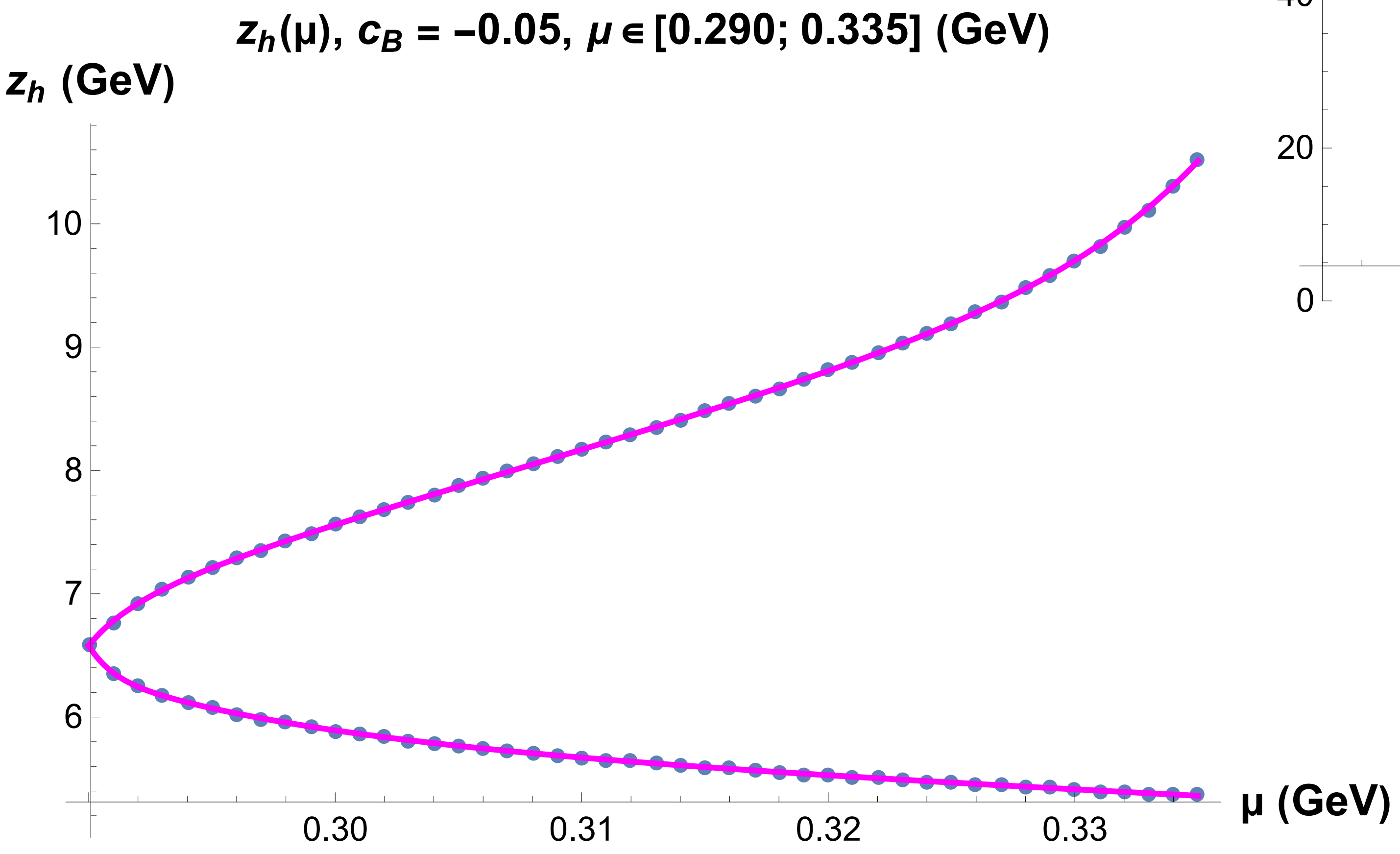


Temperature and Free energy

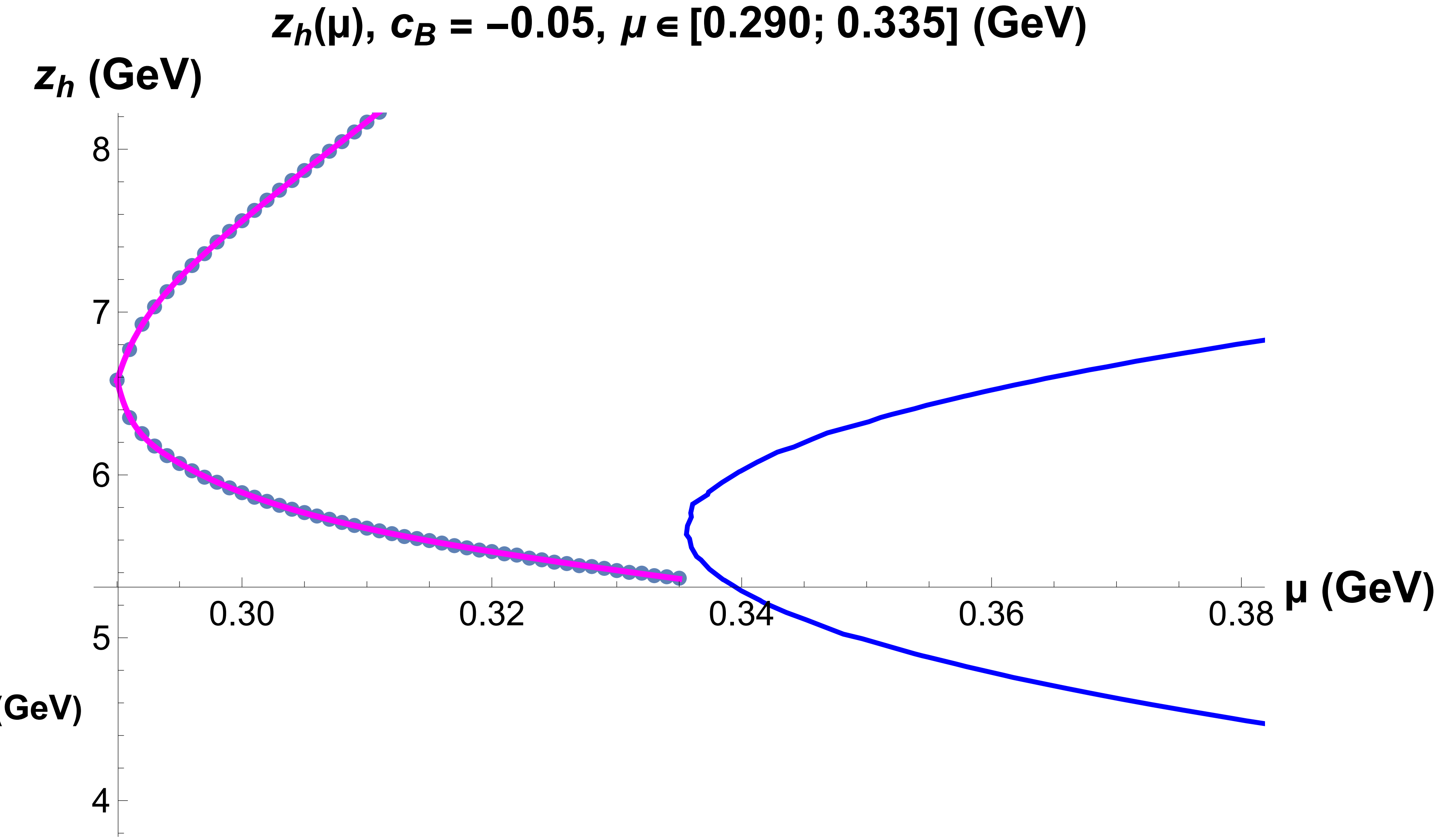


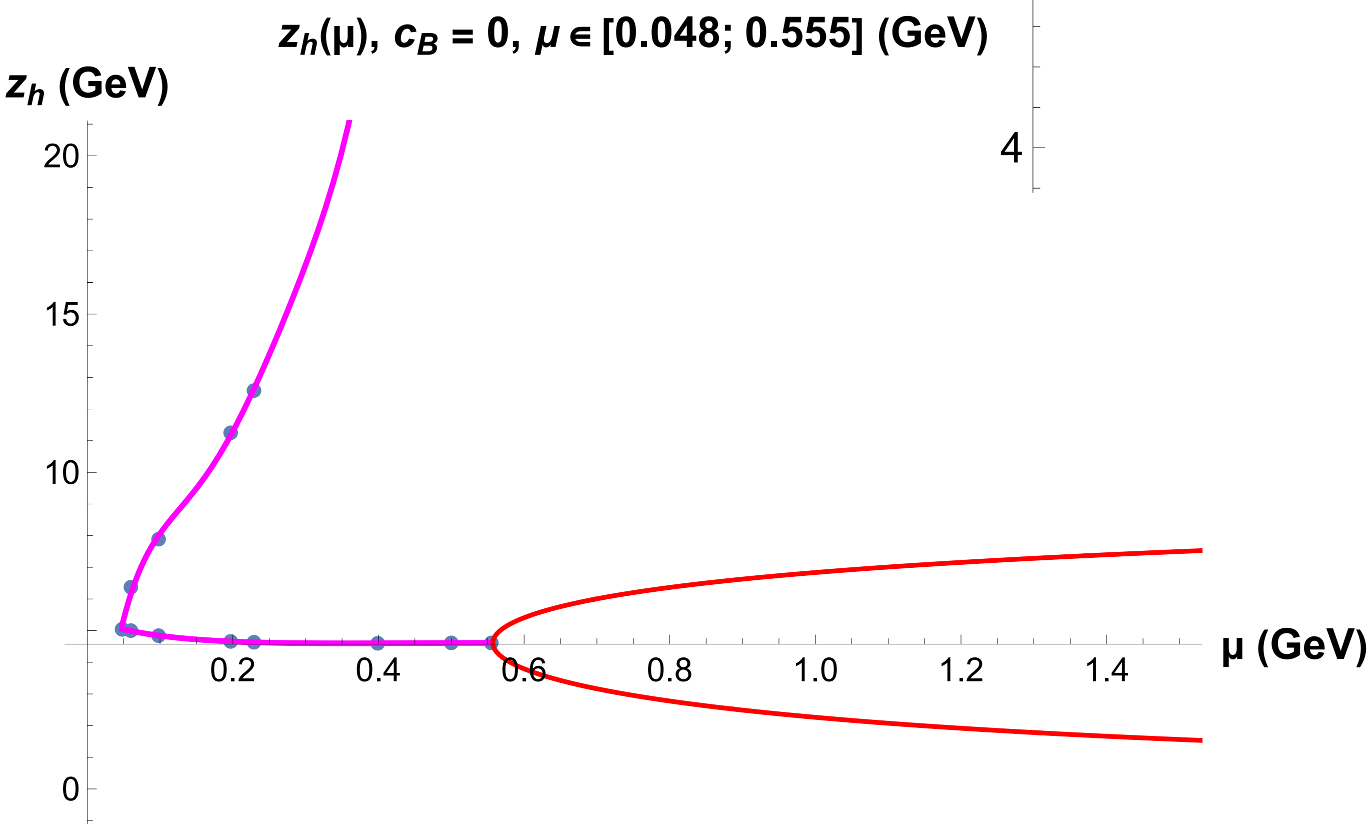
First order Phase transition



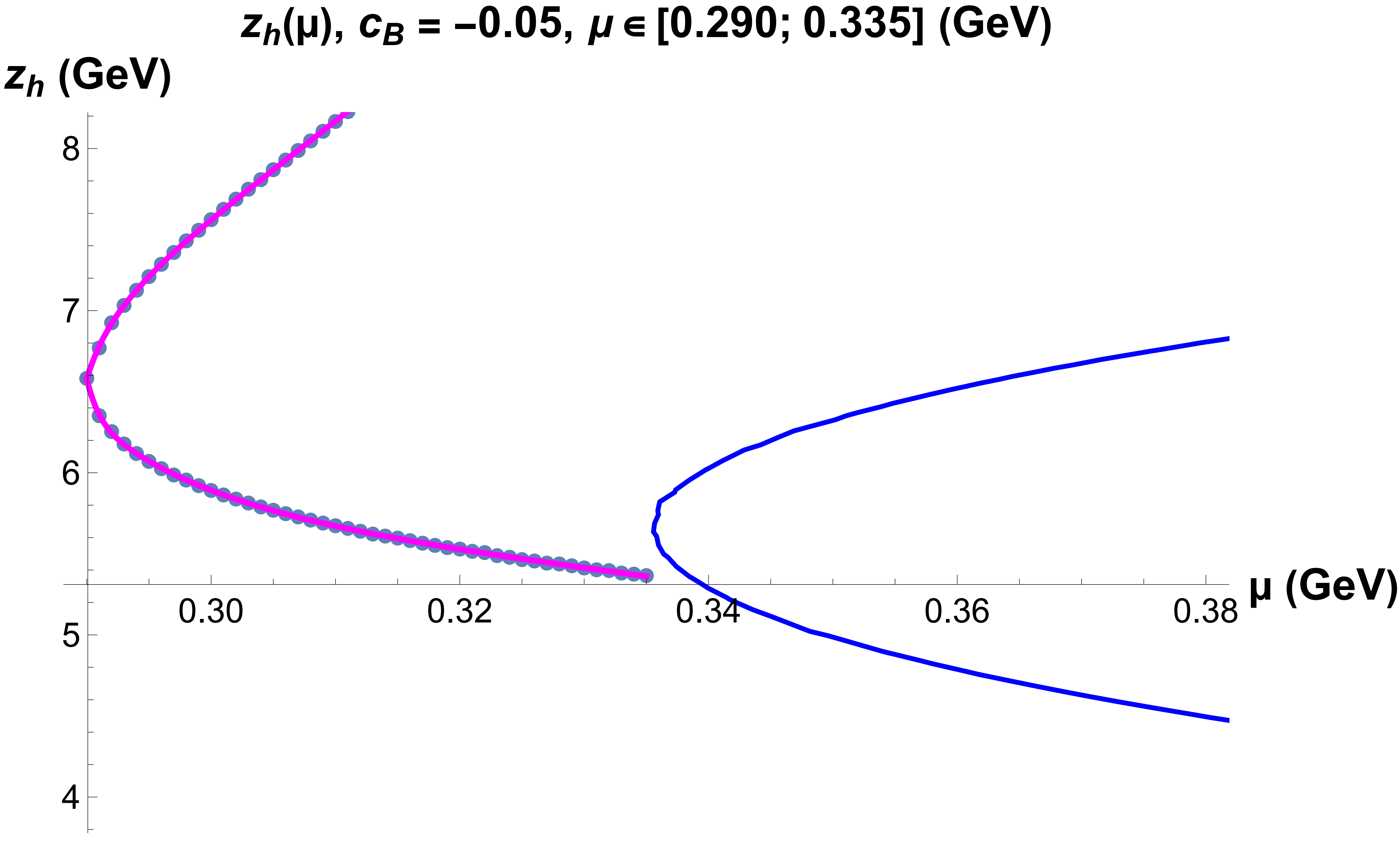


First order Phase transition





First order Phase transition



Holographic RG flows and Beta-Function

$$ds^2 = \frac{L^2}{z^2} b(z) \left[-g(z) dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + y^{c_B z^2} \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)} \right],$$

$$\phi = \phi(z)$$

We have the following holographic dictionary:

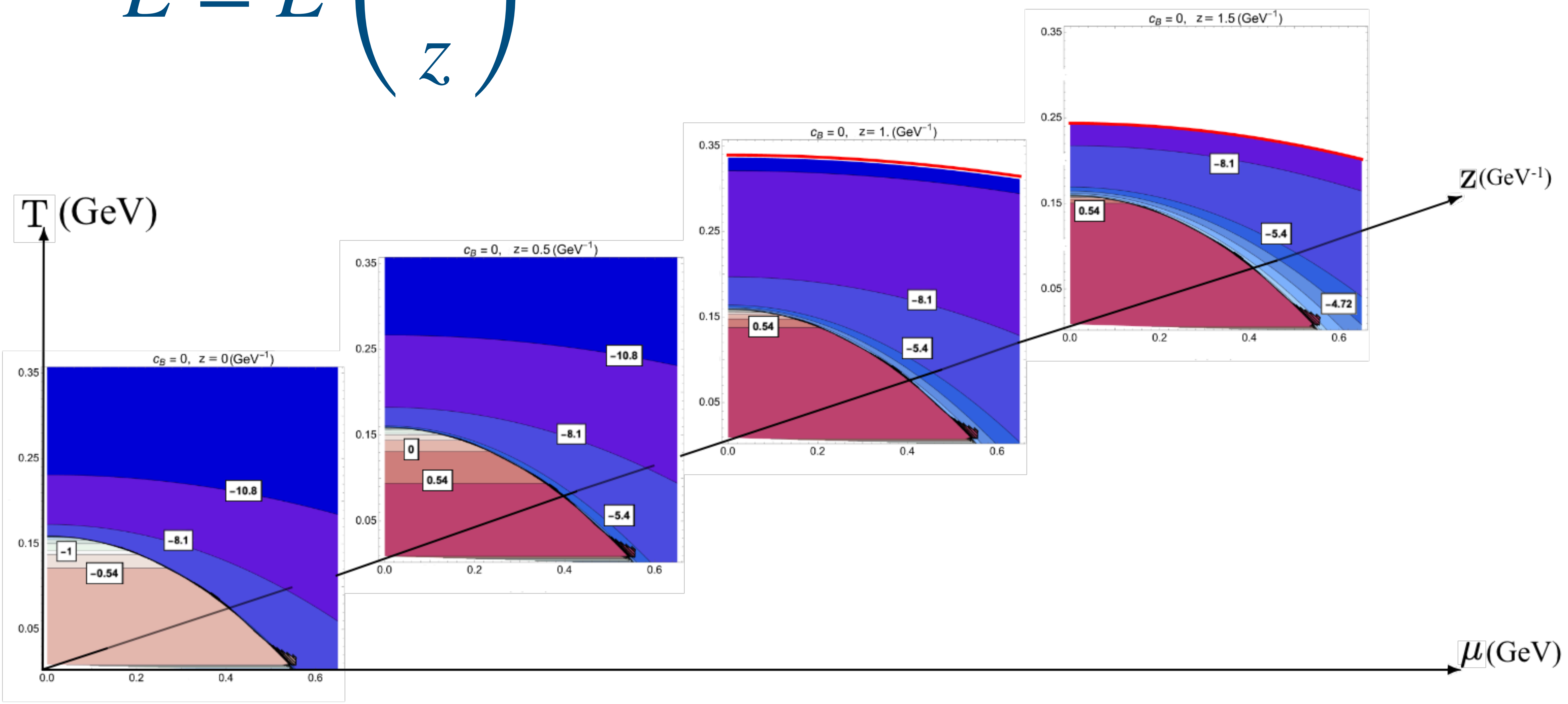
- $\frac{L^2}{z^2} b(z)$ — **corresponds to the energy scale E of the dual field theory**
- $\phi(z) = \log(\alpha)$ — **must be identified as running coupling of the field theory**

- **Connection with β-function in this background (DeWolfe et. al. '14, Kiritsis et.al.'14):**

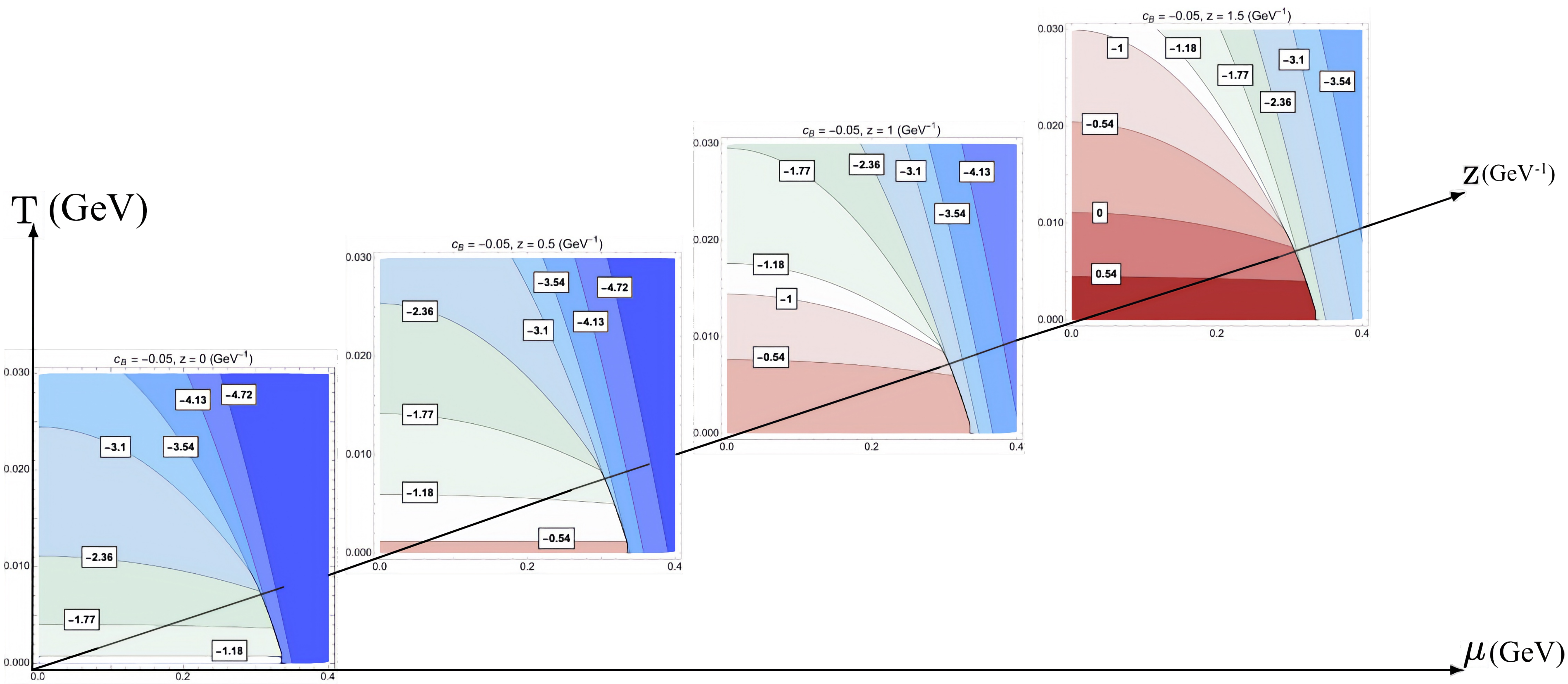
$$\beta = \left. \frac{d\alpha}{d \log E} \right|_{QFT} = \alpha \left. \frac{d\varphi}{d \log B} \right|_{Holo}$$

Running coupling in magnetic field

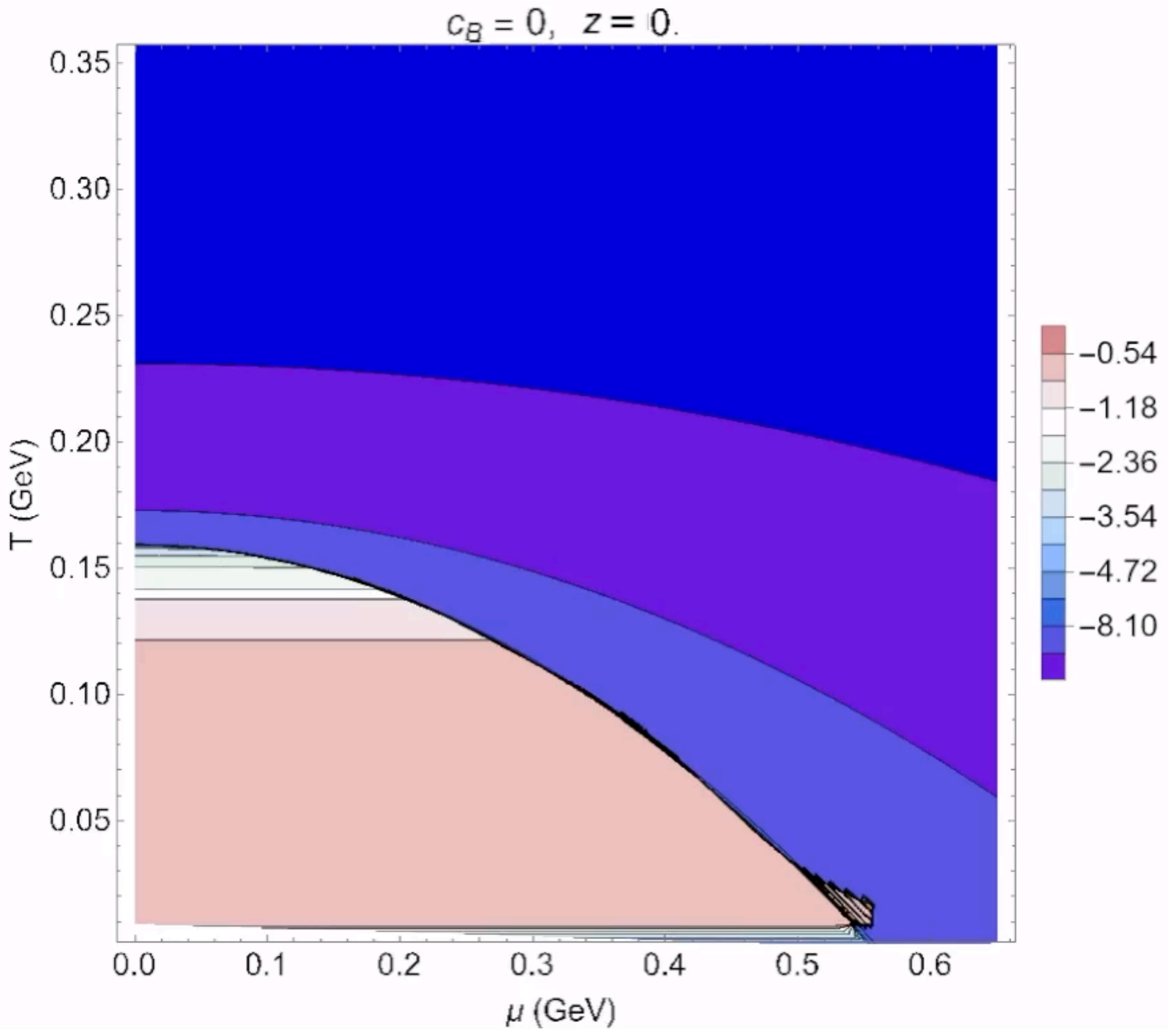
$$E = E\left(\frac{1}{z}\right)$$

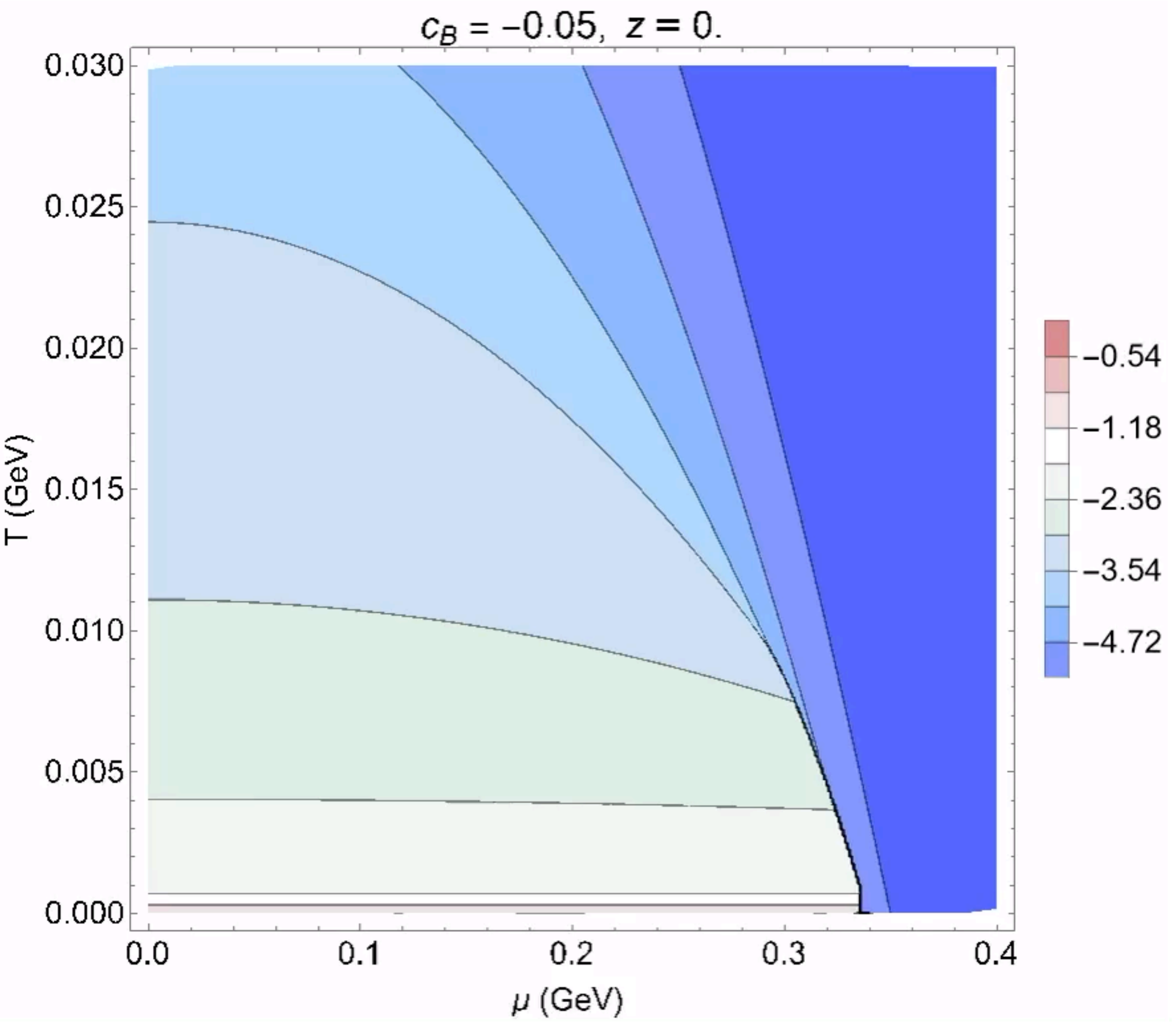


Running coupling in magnetic field



Running coupling in magnetic field





Conclusion

- **Phase transition line was obtained**
- **The phase transition is invariant with respect to changes in the energy scale**
- **Dynamic of the running coupling in magnetic field**
- **Limitations of the method**
- **Coupling constant decreases with increasing magnetic field**

Thank you!