Probing ALPs, dark photons and nonlinear electrodynamics with radiofrequency cavities

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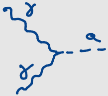
Photophilic Feebly Interacted Particles

Axion-Like-Particles (ALPs)

$$\mathcal{L}_{ALP} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 - \frac{1}{4}g_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$



- Pseudoscalars. QCD or motivated by string theory
- May compose (a part of) Dark Matter

Dark Photons (DPs)

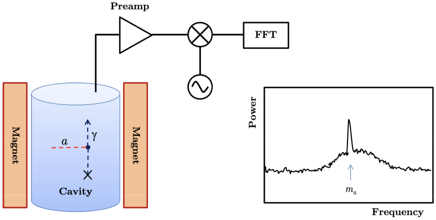
$$\mathcal{L}_{DP} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}\left(F'_{\mu\nu}\right)^2 + \frac{1}{2}m_{A'}^2\left(A'_\mu\right)^2 + \frac{\varepsilon}{2}F'_{\mu\nu}F^{\mu\nu}$$

- May be a renormalizable mediator to the Dark Sector

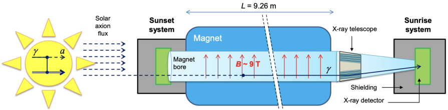
- scalar ALP (dilaton)
- millicharged scalars/fermions

ALPs and DP probes

- Laboratory detection
  - Detection of dark matter ALPs - Haloscopes



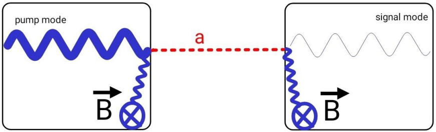
  - Detection of ALPs from Sun - Helioscopes



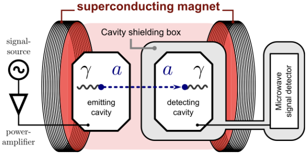
- Both production and detection in laboratory - Light-Shining-through-Wall (LSW)
  Lasers (ALPS-I,ALPS-II,OSCAR) or resonant RF cavities

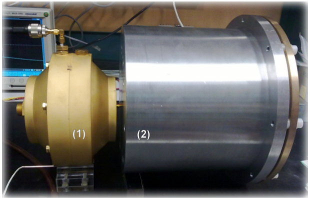
LSW - RF cavities

ALP

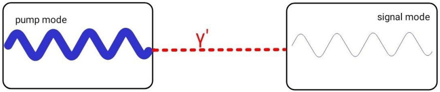


CROWS, Betz et al 2013, arXiv:1310.8098





Dark Photons



DarkSRF, Romanenko et al 2023, arXiv:2301.11512

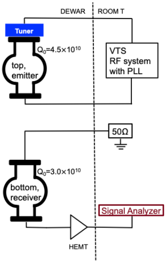
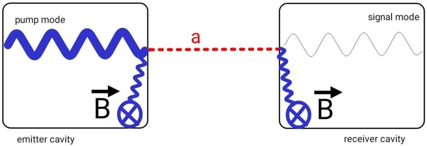


FIG. 1. Left: The experimental setup for the Dark SRF experiment consisting of two 1.3 GHz cavities. Right: A sketch of the Dark SRF electronic system.

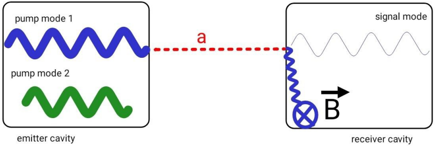
4 types of ALP LSW setups

- 2 components of EM field – magnetic field or an extra EM mode
- Superconducting cavity (pure Nb) — allows higher mode amplitude up to $B_a \sim 0.1$ T and $Q \sim 10^{12}$ at $T \sim 1$ K   Romanenko et al 2020
  No magnetic field in SC walls - extra mode if superconductor

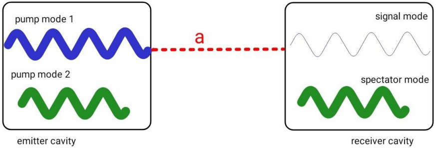
standard scheme (Hogeeven 1992)



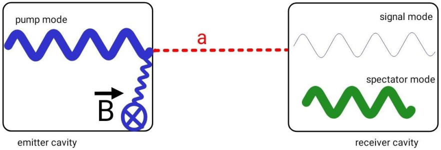
Salnikov, PS, Kirpichnikov Fitkevich 2021



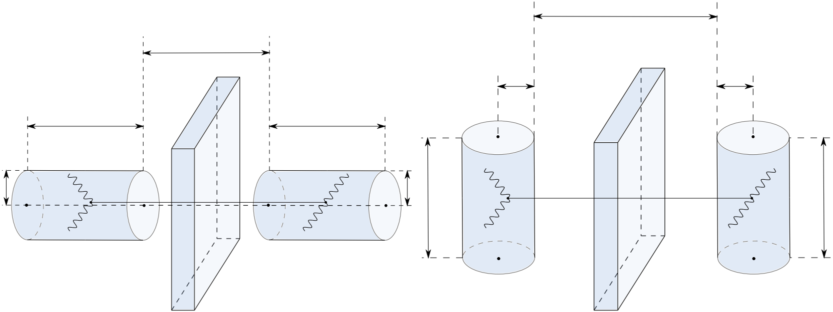
Gao, Harnik 2021



in Salnikov, PS, Kirpichnikov Fitkevich 2023



LSW geometry



Theory

Equations of motion: $$(\partial_\mu \partial^\mu + m_a^2)\, a = g_{a\gamma\gamma}(\vec{E} \cdot \vec{B})$$

$$(\vec{\nabla} \cdot \vec{E}) = \rho_a \; , \qquad [\vec{\nabla} \times \vec{B}] = \dot{\vec{E}} + \vec{j}_a \; ,$$

$$\rho_a = -g_{a\gamma\gamma}(\vec{\nabla} a \cdot \vec{B}) \; , \quad \vec{j}_a = g_{a\gamma\gamma}([\vec{\nabla} a \times \vec{E}] + \dot{a}\vec{B}) \; .$$

ALP production:

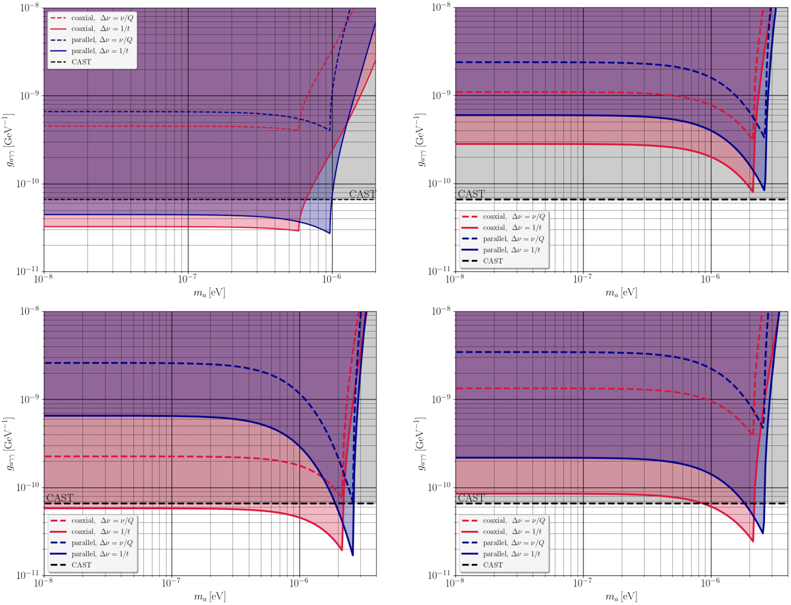
$$a(t, \vec{x}) = g_{a\gamma\gamma}\, E_0^{\rm em} B_{\rm ext} \int\limits_{V_{em}} d^3\vec{x}' \mathcal{E}_z^{em}(\vec{x}') \frac{e^{ik_a|\vec{x}-\vec{x}'|-i\omega t}}{4\pi|\vec{x}-\vec{x}'|}, \qquad k_a = \sqrt{\omega^2 - m_a^2}$$

ALP detection: $$G = -\frac{Q_{\rm rec}}{\omega_s} \cdot \frac{1}{V_{rec}} \int\limits_{V_{rec}} d^3x\, \mathcal{E}_{rec}^{z*}(\vec{x}) j_a^z(\vec{x}),$$ G - signal mode ampl

$$\mathcal{G} = \int\limits_{V_{\rm rec}} \frac{d^3x}{V_{\rm rec}} \int\limits_{V_{\rm em}} \frac{d^3x'}{V_{\rm em}}\, \mathcal{E}_{rec}^{z*}(\vec{x}) \mathcal{E}_{em}^{z}(\vec{x}') \frac{e^{ik_a|\vec{x}-\vec{x}'-\vec{L}|}}{4\pi\omega|\vec{x}-\vec{x}'-\vec{L}|}.$$

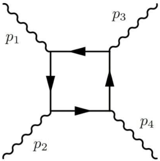
$$g_{a\gamma\gamma} = \left[\frac{2T\,{\rm SNR}}{\omega^3 Q_{\rm rec} E_{0,{\rm em}}^2 B_{\rm ext}^4 V_{\rm em}^2\, V_{\rm rec}|\mathcal{G}|^2}\right]^{\frac{1}{4}} \left(\frac{\Delta\nu}{t}\right)^{\frac{1}{8}},$$

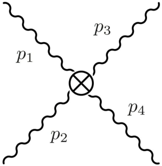
Sensitivity of 4 setups



Nonlinear electrodynamics: Euler-Heisenberg Lagrangian

Interactions with virtual electrons are integrated out (the limit $p \ll m_e$) $\rightarrow$ 4-photon effective interaction





Effective Euler-Heisenberg Lagrangian

$$\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{90m_e^4}\left((F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2\right) + O(\alpha^4).$$

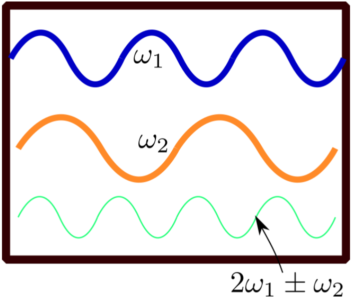
H.Euler and B. Kockel (1935), W.Heisenberg and H.Euler (1936)
No experimental proof in 2024!

- The contribution of the same type to $\mathcal{L}_{eff}$ from ALPs, dilatons and millicharges
- can be tested in a single RF cavity

Single RF cavity

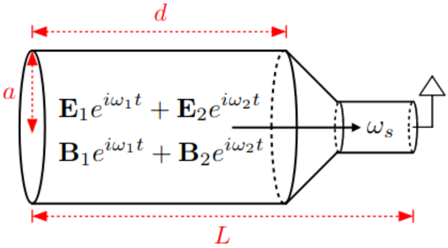
Nonlinear theory with 4-photon interaction
EM modes in radio-frequency cavities: possibility for generation of a mode of combined frequency G. Brodin, M. Marklund, L. Stenflo. PRL (2001)

- pump modes: ω_1, ω_2
  $E_p = E_1\, e^{i\omega_1 t} + E_2\, e^{i\omega_2 t}$
- signal modes:
  $2\omega_{1(2)} \pm \omega_{2(1)}$, $3\omega_{1(2)}$.
- Resonant amplification of signal modes
- Q up to $10^{10} - 10^{12}$ in superconducting cavities



Partial solutions for $\omega_s = 2\omega_1 - \omega_2$
D. Eriksson, G. Brodin, M. Marklund, L. Stenflo. PRA (2004)
hard to make an experiment at the technology level of 2004

Z. Bogorad, A. Hook, Y. Kahn, Y. Soreq. PRL (2020)



(focused on axion searches)

$\omega_1 = TE_{011}$, $\omega_2 = TM_{010}$
$\omega_s = 2\omega_1 - \omega_2 = TM_{020}$,
if $d = 3.112a$

(SQMS, Fermilab) B. Giaccone et al. arXiv:2207.11346 (2022)

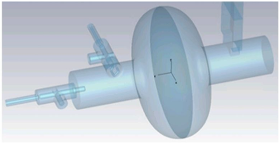
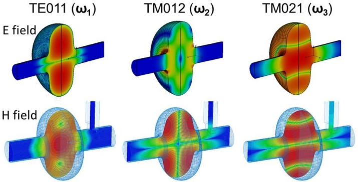


FIG. 9. RF geometry for the three-mode axion search. The design of this cavity is still under study.



Theory

Modified wave equations

$$\Box E = \frac{\partial}{\partial t}\,\mathrm{rot} M + \nabla\,\mathrm{div} P - \frac{\partial^2 P}{\partial t^2}$$
$$\Box H = \frac{\partial}{\partial t}\,\mathrm{rot} P - \nabla\,\mathrm{div} M + \Delta M$$

Vacuum polarization and magnetisation vectors

$$P = 16\varepsilon\left[\left(E^2 - H^2\right)E + 7/2(E\cdot H)H\right]$$
$$M = 16\varepsilon\left[\left(E^2 - H^2\right)H - 7/2(E\cdot H)E\right], \varepsilon = \frac{\alpha^2}{90 m_e^4}.$$

EM field: *pump* mode (initially given) + *signal* mode (we are looking for),
$E = E^p + E^{sig}$, $H = H^p + H^{sig}$
Hierarchy: $E^{sig} \sim \varepsilon\left(E^p\right)^3 \ll E^p$.

Calculation of the signal mode resonant generation

(A) Solve the classical EoMs obtained from EFT Lagrangian (EFT = nonlinear ED). Perturbative limit in solving differential equations $\to$ numerical solution for given sets of modes
Resonance: a signal mode grows with time. Stops at $t \sim Q \cdot T$.

D. Eriksson, G. Brodin, M. Marklund, L. Stenflo. PRA (2004)
Z. Bogorad, A. Hook, Y. Kahn, Y. Soreq. PRL (2020)

Open points in works on (A):

- Only $2\omega_1 - \omega_2$ signal mode considered previously, what happened with $3\omega_1$ or $2\omega_1 + \omega_2$?
- K. Shibata. EPJ D (2020) No 3rd harmonics ($3\omega_1$) generation in 1D cavity
- I. Kopchinskii, P.S. PRA (2022) Analytical solutions: ($3\omega_1$) and ($2\omega_1 + \omega_2$) are not generated in 1D and 3D rectangular cavities for any sets of modes (combinatoric proof)

Open questions in “classical” approach (A)

Open questions:

- What is the reason for the absence of the resonance for $(3\omega_1)$ and $(2\omega_1 + \omega_2)$? The 3rd harmonics is resonantly amplified in $\lambda\varphi^4$ theory!
- Res. generation of $2\omega_1 - \omega_2$: what is the quantum description? Naively, final state of $3 \to 1$ particle process should have energy $2\omega_1 + \omega_2$.
- Is it OK to use “classical” effective theory in case of $N \sim 1$ signal quanta in the final state?

Hope that the quantum amplitude calculation will shed a light on these points

Signal mode generation as QFT perturbative process

- Signal mode generation in nonlinear electrodynamics = nonlinear interaction between quanta of pump modes. Elementary process includes 4 quanta.
- $P \propto |\langle f|U(t_f - t_i)|i\rangle|^2$
- $|i\rangle$ and $|f\rangle$ are not plane waves but cavity eigenmodes (=linear combinations of plane waves, $|i\rangle = \Sigma_n c_n^i |k_n\rangle$). S-matrix formalism is still applicable since the theory is linear. 2 options:
    - $P \propto \Sigma_{nm} (c_m^f)^* c_n^i \langle k_m|\mathrm{S}|k_n\rangle$ – OK if small number of terms in a sum
    - $P \propto |\langle f|S|i\rangle|^2$ in general case
- $T_{fi} = 2\pi\delta\left(\Sigma_i \omega_i - \omega_f\right) M_{fi} = 2\pi\delta(0) M_{fi}$ for the resonance condition
    - No dissipation: $2\pi\delta(0) \to T_{int}$ (see any QFT textbook)
    - Dissipation: $2\pi\delta(0) \to T_{1/2diss} = \frac{Q}{\omega_f}$
- Mean number of signal photons in steady regime $N_s = \frac{Q^2}{\omega_f^2}|M_{fi}|^2$.

$3 \to 1$ merging process in 1D

2 quanta of cavity mode $\omega_n + 1$ quantum ω_p, arbitrary polarizations

$$|i\rangle = \left|1_n^i\right\rangle \otimes \left|1_n^j\right\rangle \otimes \left|1_p^l\right\rangle = a_{i,n}^+ a_{j,n}^+ a_{l,p}^+ |0\rangle \,, \qquad |f\rangle = \left|1_{2n+p}^s\right\rangle = a_{s,2n+p}^+ |0\rangle$$

$$T_{fi} = \langle f|S|i\rangle = i\frac{\alpha^2}{90m_e^4} \int\limits_{-\infty}^{+\infty} \mathrm{d}t \iint \mathrm{d}S \int\limits_0^{L_x} \mathrm{d}x \, \langle f|(\mathbf{EE})^2 - 2\mathbf{B}^2\mathbf{E}^2 + (\mathbf{BB})^2 + 7(\mathbf{BE})^2|i\rangle \,,$$

$$\langle(\mathbf{EE})^2\rangle = \langle(\mathbf{BB})^2\rangle = -\frac{1}{2}\langle\mathbf{B}^2\mathbf{E}^2\rangle = $$
$$= 2\pi\delta(0)\frac{\sqrt{(2n+p)n^2p}\,\pi^2}{L_x^3}\left[\delta_{ij}\delta_{ls}(1+2\delta_{is}) + (1-\delta_{ls})(1-\delta_{ij})\right],$$

and $\langle(\mathbf{EB})^2\rangle = 0$.
Finally, vanishes. What is the reason?

$3 \to 1$ merging. Plane wave decomposition approach

The 4-point amplitude $\langle f|\mathrm{S}|i\rangle = \left\langle 1^s_{2n+p}\middle|\mathrm{S}\middle|1^i_n, 1^j_n, 1^l_p\right\rangle$ decomposes to 16 plane wave amplitudes

$$\frac{(-1)^{\pm}}{(2i)^4}\left\langle \pm k^s_{2n+p}\middle|\mathrm{S}\middle|\pm k^i_n, \pm k^j_n, \pm k^l_p\right\rangle,$$

where $(-1)^{\pm} = 1$ in case of even number of sign $+$ in the amplitude and -1 otherwise. The energy-momentum conservation shows that 14 amplitudes are zero, the remained ones are

$$\frac{1}{(2i)^4}\left\langle k^s_{2n+p}\middle|\mathrm{S}\middle|k^i_n, k^j_n, k^l_p\right\rangle = \frac{1}{(2i)^4}\left\langle -k^s_{2n+p}\middle|\mathrm{S}\middle|-k^i_n, -k^j_n, -k^l_p\right\rangle$$

- $\left\langle k^s_{2n+p}\middle|\mathrm{S}\middle|k^i_n, k^j_n, k^l_p\right\rangle$ — amplitude for $3 \to 1$ parallel plane wave merging. Lorentz scalar, depend only on scalar products $(k^j_\mu k^l_\mu)$, vanishes for parallel momenta
- The same idea for $(3\omega_1)$ and $(2\omega_1 + \omega_2)$ in 3D

$2 \to 2$ scattering

- $2\omega_1 - \omega_2$ cannot be final state of $3 \to 1$ merging due to the energy conservation.
- Classical pump modes are not states with fixed number of particles but the coherent states $\left|\xi^{\lambda}_{npq}\right\rangle = e^{-\frac{|\xi|^2}{2}} \sum\limits_{i=0}^{\infty} \frac{\xi^i}{i!} \left(a^{+\lambda}_{npq}\right)^i |0\rangle$
- Idea: Elementary process – $2 \to 2$ scattering, sum into coherent states
- 2TE011 $\to$ TM110+TM130 for concreteness

Initial and final states for $2 \to 2$,

$$|i\rangle = \left|2^{\mathrm{TE}}_{011}\right\rangle = \frac{1}{\sqrt{2}} \left(a^{\mathrm{TE}+}_{011}\right)^2 |0\rangle\,, \qquad |f\rangle = \left|1^{\mathrm{TM}}_{110}\right\rangle \otimes \left|1^{\mathrm{TM}}_{130}\right\rangle = a^{\mathrm{TM}+}_{110} a^{\mathrm{TM}+}_{130} |0\rangle\,.$$

The matrix element for $2 \to 2$,

$$\mathrm{M}_{2\to 2} = i\frac{2\alpha^2}{45 m_e^4} \int\limits_V \mathrm{d}^3x \, \langle f| \, \mathbf{E}^4 - 2\mathbf{B}^2\mathbf{E}^2 + \mathbf{B}^4 + 7(\mathbf{BE})^2 \, |i\rangle =$$

$$= \left\langle \mathbf{E}^4 \right\rangle - 2\left\langle \mathbf{B}^2\mathbf{E}^2 \right\rangle + \left\langle \mathbf{B}^4 \right\rangle + 7\left\langle (\mathbf{BE})^2 \right\rangle,$$

$2 \to 2$ scattering, 2TE011 $\to$ TM110+TM130

Cavity dimensions $L_x : L_y : L_z = 1 : 1 : r$
Energy conservation for $2 \to 2$: $2\omega_{011} = \omega_{110} + \omega_{130}$
Nonzero result for $r = \sqrt{\sqrt{5} - 2}$

$$\mathrm{M}_{2\to 2} = \left\langle \mathbf{E}^4 \right\rangle - 2\left\langle \mathbf{B}^2\mathbf{E}^2 \right\rangle + \left\langle \mathbf{B}^4 \right\rangle + 7\left\langle (\mathbf{BE})^2 \right\rangle,$$

$$\left\langle \mathbf{E}^4 \right\rangle = -\frac{1}{8V}\sqrt{\omega_{011}^2\omega_{110}\omega_{130}}, \qquad \left\langle \mathbf{B}^4 \right\rangle = \frac{1}{4V}\frac{\pi^4}{L_z^4}r^2\frac{2r^2-3}{\sqrt{\omega_{011}^2\omega_{110}\omega_{130}}},$$

$$\left\langle \mathbf{B}^2\mathbf{E}^2 \right\rangle = \frac{1}{8V}\frac{\pi^2}{L_z^2}\left((r^2-1)\sqrt{\frac{\omega_{110}\omega_{130}}{\omega_{011}^2}} - 4r^2\sqrt{\frac{\omega_{011}^2}{\omega_{110}\omega_{130}}}\right),$$

$$\left\langle (\mathbf{BE})^2 \right\rangle = \frac{1}{8V}\frac{\pi^2}{L_z^2}r^2\left(\sqrt{\frac{\omega_{110}\omega_{130}}{\omega_{011}^2}} - 3\sqrt{\frac{\omega_{011}^2}{\omega_{110}\omega_{130}}}\right).$$

Even single pump mode (TE011) produce two signal modes (TM110 and TM130) by a nonlinear interaction
Bose enhancement if TM110 already excited

$2\omega_1 - \omega_2$ generation by coherent states

the initial and final states read,

$$|i\rangle = \left|\xi_{011}^{\text{TE}}\right\rangle \otimes \left|\eta_{110}^{\text{TM}}\right\rangle = e^{-\frac{|\xi|^2+|\eta|^2}{2}} \sum_{i,j=0}^{\infty} \frac{\xi^i \eta^j}{i!j!} \left(a_{011}^{\mathrm{TE}+}\right)^i \left(a_{110}^{\mathrm{TM}+}\right)^j |0\rangle \, ,$$

$$|f\rangle = \left|\xi_{011}^{\text{TE}}\right\rangle \otimes \left|\eta_{110}^{\text{TM}}\right\rangle \otimes \left|1_{130}^{\text{TM}}\right\rangle = |i\rangle \otimes \left|1_{130}^{\text{TM}}\right\rangle = a_{130}^{\text{TM}+} |i\rangle \, .$$

The parameters ξ, η are associated with the mean number of quanta in the pump modes,

$$\langle N_{\text{TE}_{011}} \rangle = |\xi|^2, \qquad \langle N_{\text{TM}_{110}} \rangle = |\eta|^2.$$

$$\text{M}_{\text{coh}} = \xi^2 \eta^* \times \sqrt{2}\, \text{M}_{2\to 2}.$$

$$P_{\text{coh}} = 2 \left\langle N_{\text{TE}_{011}} \right\rangle^2 \cdot \left\langle N_{\text{TM}_{110}} \right\rangle \cdot P_{2\to 2}.$$

$$\langle N_s \rangle = P_{\text{coh}} = G_1^2 \times \left(\frac{\alpha^2}{90 m_e^2}\right)^2 Q^2 F_0^6 L_z^4,$$

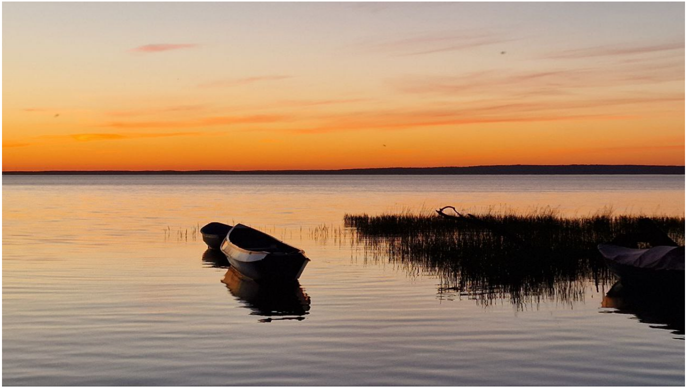
$$G_1^2 = \frac{4}{(10)^{3/2} \pi r^3 (1+r^2)^2} \left[5 + 2\sqrt{5} - \frac{7}{4}\left(\sqrt{1+r^2} + \sqrt{2} r\right)^2\right]^2.$$

The same result as in "classical" approach   I.Kopchinskii, P.S. PRA (2022)

Conclusions

- We developed technique for perturbative calculations in cavities in nonlinear electrodynamics
- $3\omega_1$ and $2\omega_1 + \omega_2$ do not resonate due to plane wave decomposition, LI and photon zero mass
- $2 \to 2$ is a crucial elementary process for $2\omega_1 - \omega_2$ resonant generation, number of quanta does not conserve
- Straightforward generalization to: other geometries, other initial states (squeezed etc), mean values of other operators etc..

Thank you for your attention![1]



[1]The talk is supported by RSF grant 21-72-1015