Massive particle production in the field of massless plane wave

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- **Particle creation by a dynamic classical field. Example: the production of matter particles by the inflaton field ostillation at the end of inflation (reheating stage) [1]**
- **Particle production enhancement effect - parametric resonance**
- **Creation of massive particles by an intense wave of a massless field with resonant amplification [2], small mass m«ω**
- **The purpose of the work is to consider the case of arbitrary masses**

[1] Rubakov V.A. Gorbunov D.S. Introduction to the theory of the early universe. Cosmological perturbations. Inflationary theory. URSS, 2009.

[2] A.Arza. Phys.Rev.D 105 (2022) arXiv:2009.03870

$$\mathcal{H} = g\phi\chi^2$$

Two-scalar model

Equations of motion

$$\Box\,\phi = -g\chi^2,$$

$$(\Box + m_\chi^2)\chi = -2g\phi\chi$$

Monochromatic plane wave

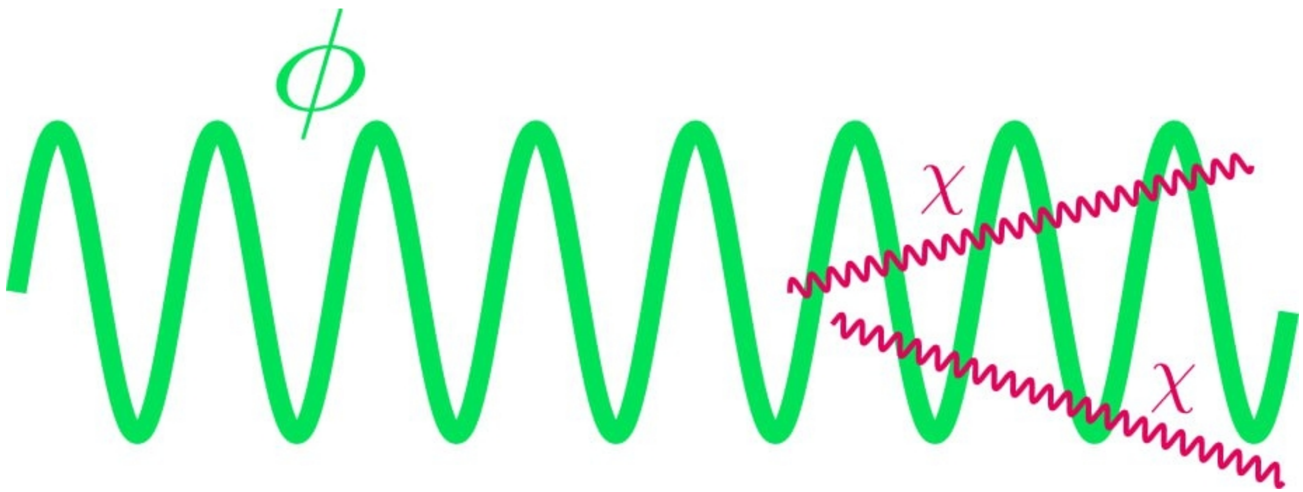
$$\phi(\vec{x},t) = \frac{\sqrt{2\rho_\phi}}{\omega_p}\cos(\vec{p}\cdot\vec{x} - \omega_p t)$$

$$\Omega_{\vec{k}} = \sqrt{k^2 + m_\chi^2} \qquad \omega_{\vec{p}} = \sqrt{p^2 + m_\phi^2}$$

Quantum field

$$\chi = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\Omega_{\vec{k}}}}\left(\chi_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}} + \chi_{\vec{k}}(t)^\dagger e^{-i\vec{k}\cdot\vec{x}}\right)$$

$$\mathcal{H} = g\phi\chi^2$$



Denote

$$A_{\vec{k}} = \chi_{\vec{k}} + \chi^{\dagger}_{-\vec{k}}$$

$$(\partial_t^2 + \Omega_{\vec{k}}^2)A_{\vec{k}} = -\omega_{\vec{p}}^2\alpha\left(\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}}}}A_{\vec{k}-\vec{p}}e^{-i\omega_{\vec{p}}t} + \sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}+\vec{p}}}}A_{\vec{k}+\vec{p}}e^{i\omega_{\vec{p}}t}\right)$$

where $\alpha \equiv \frac{g\sqrt{2\rho_\phi}}{\omega_{\vec{p}}^3}$

Suppose $\chi_{\vec{k}} = a_{\vec{k}}(t)e^{-i\Omega_{\vec{k}}t}$

$$e^{-i\Omega_{\vec{k}}t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}}\dot{a}_{\vec{k}}) + e^{i\Omega_{-\vec{k}}t}(\ddot{a}^{\dagger}_{-\vec{k}} + 2i\Omega_{-\vec{k}}\dot{a}^{\dagger}_{-\vec{k}}) =$$

$$= -\omega_{\vec{p}}^2\alpha\left(\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}+\vec{p}}}}\left(a^{\dagger}_{-\vec{k}-\vec{p}}e^{i(\Omega_{-\vec{k}-\vec{p}}+\omega_{\vec{p}})t} + a_{\vec{k}+\vec{p}}e^{-i(\Omega_{\vec{k}+\vec{p}}-\omega_{\vec{p}})t}\right)+\right.$$

$$\left.+\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}}}}\left(a^{\dagger}_{-\vec{k}+\vec{p}}e^{i(\Omega_{-\vec{k}+\vec{p}}-\omega_{\vec{p}})t} + a_{\vec{k}-\vec{p}}e^{-i(\Omega_{\vec{k}-\vec{p}}+\omega_{\vec{p}})t}\right)\right)$$

The amplitude $a^{\dagger}_{-\vec{k}+\vec{p}}$ - leading, $a^{\dagger}_{-\vec{k}-\vec{p}}$ - subleading

The final equation

- **Approximation (A.Arza, PRD 2022):** $\ddot{a}$ **term neglected** → $m_\chi \ll \omega_p$

$$\dot{a}_{\vec{k}} = -i\sigma_{\vec{k}}\, a^\dagger_{\vec{p}-\vec{k}}\, e^{i\epsilon_{\vec{k}}\, t}$$

$$\sigma_{\vec{k}} = g\sqrt{\frac{\rho_\phi/2}{\omega^2_{\vec{p}}\,\Omega_{\vec{p}-\vec{k}}\,\Omega_{\vec{k}}}}$$

- **Without approximation**

$$e^{-i\Omega_{\vec{k}}t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}}\dot{a}_{\vec{k}}) = \sigma_{p-k}\, a^\dagger_{-\vec{k}+\vec{p}}\, e^{i(\Omega_{-\vec{k}+\vec{p}}-\omega_{\vec{p}})t}$$

$$\sigma_{p-k} = -\omega_p^2\alpha\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}-\vec{p}}}}$$

Solution

- **In the approximation**

$$\epsilon_{p-k} = \Omega_k + \Omega_{p-k} - \omega_p = \epsilon_k$$

$$a_{\vec{k}}(t) = e^{i\epsilon_{\vec{k}}t/2}\left(a_{\vec{k}}(0)\left(\cosh(s^0_{\vec{k}}t) - i\frac{\epsilon_{\vec{k}}}{2s^0_{\vec{k}}}\sinh(s^0_{\vec{k}}t)\right) - i\frac{\sigma_{\vec{p}-\vec{k}}}{2s^0_{\vec{k}}\Omega_{\vec{k}}}a^\dagger_{\vec{p}-\vec{k}}(0)\sinh(s^0_{\vec{k}}t)\right)$$

$$s^0_{\vec{k}} = \frac{1}{2}\sqrt{\frac{\sigma^2_{\vec{p}-\vec{k}}}{\Omega^2_{\vec{k}}} - \epsilon^2_{\vec{k}}}$$

- **Without approximation**

$$a_{\vec{k}}(t) = e^{i\epsilon_{\vec{k}}t/2}\Bigg[a_{\vec{k}}(0)\left(\cosh(s_{\vec{k}}t) - i\frac{\epsilon^2_{\vec{k}}/4 - s^2_{\vec{k}} - \Omega_{\vec{k}}\epsilon_{\vec{k}}}{s_{\vec{k}}(2\Omega_{\vec{k}} - \epsilon_{\vec{k}})}\sinh(s_{\vec{k}}t)\right) - $$
$$- a^\dagger_{\vec{p}-\vec{k}}(0)\cdot i\frac{\sigma_{\vec{p}-\vec{k}}}{s_{\vec{k}}(2\Omega_{\vec{k}} - \epsilon_{\vec{k}})}\sinh(s_{\vec{k}}t)\Bigg],$$

$$s^2_{\vec{k}} = \sqrt{\Omega^2_{\vec{k}}(2\Omega_{\vec{k}} - \epsilon_{\vec{k}})^2 + \sigma^2_{\vec{p}-\vec{k}}} - \Omega_{\vec{k}}(2\Omega_{\vec{k}} - \epsilon_{\vec{k}}) - \frac{\epsilon^2_{\vec{k}}}{4}$$

Production of χ modes in the external φ field

$$s^2 > 0$$

The occupancy number

$$f_{\chi,\vec{k}}(t) = \langle 0 | a^\dagger_{\vec{k}}(t) a_{\vec{k}}(t) | 0 \rangle = \frac{\sigma^2_{\vec{p}-\vec{k}}}{4} \times \begin{cases} \dfrac{\sinh^2(s^0_{\vec{k}} t)}{\left(s^0_{\vec{k}}\right)^2 \Omega^2_{\vec{k}}}, \\ \dfrac{\sinh^2(s_{\vec{k}} t)}{s^2_{\vec{k}}(\Omega_{\vec{k}} - \epsilon_{\vec{k}}/2)^2} \end{cases}$$

The total density

$$n_\chi(t) = \int \frac{d^3 k}{(2\pi)^3} f_{\chi,\vec{k}}(t)$$

Dimensionless parameters $\eta_{\vec{k}} = s_{\vec{k}}/\omega_{\vec{p}}$

$$\vec{k} = \vec{k}_{\parallel} + \vec{k}_{\perp}$$

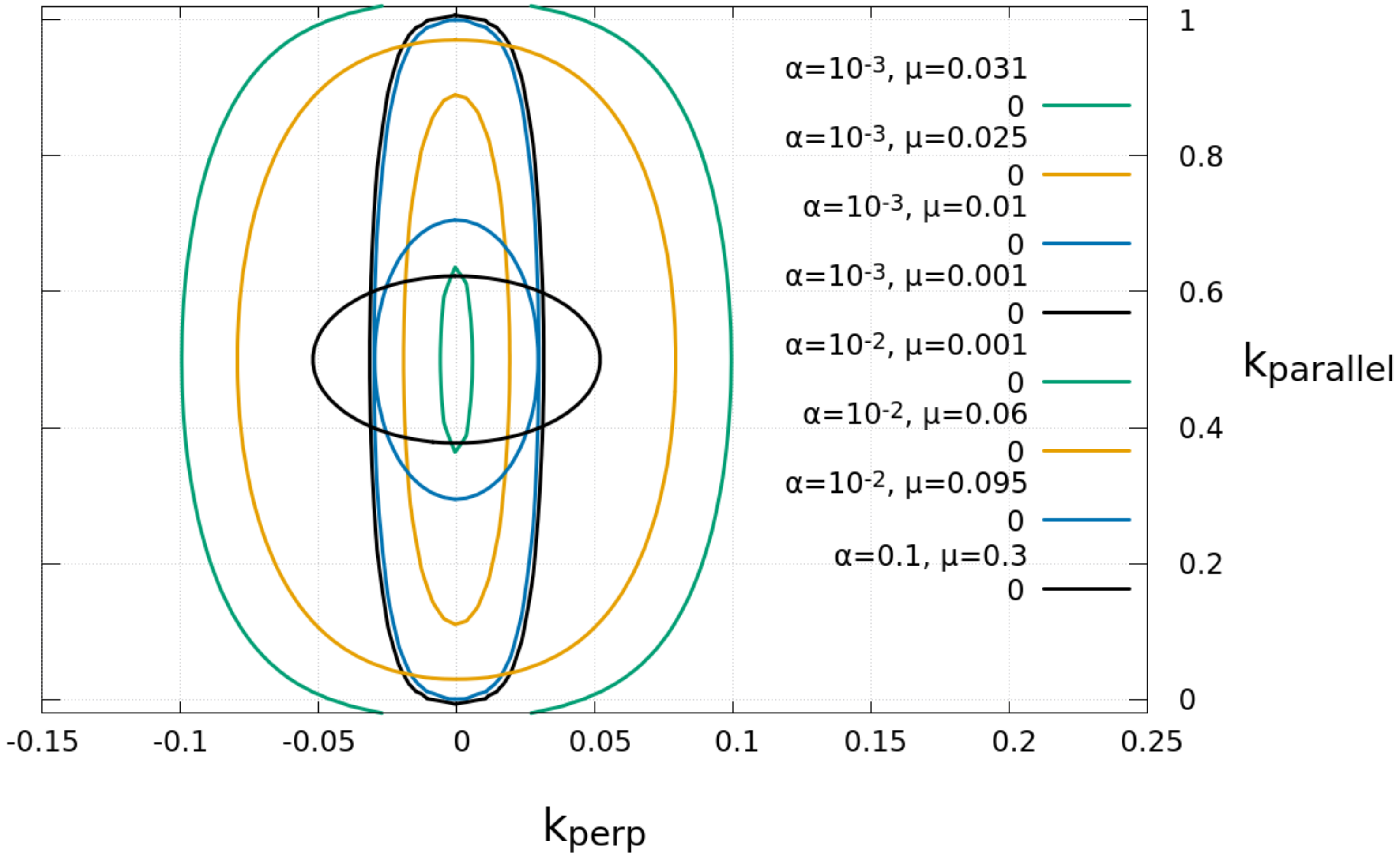
$$\frac{\Omega_{\vec{k}}}{\omega_{\vec{p}}} \equiv \beta_{\vec{\kappa}} = \sqrt{\kappa_{\parallel}^2 + \kappa_{\perp}^2 + \mu^2}$$

$$\frac{\Omega_{\vec{p}-\vec{k}}}{\omega_{\vec{p}}} \equiv \beta_{\vec{v}-\vec{\kappa}} = \sqrt{1 - 2\kappa_{\parallel} + \kappa_{\parallel}^2 + \kappa_{\perp}^2 + \mu^2}$$

$$\eta_{\vec{k}}^2 = \begin{cases} \dfrac{\alpha^2}{4\beta_{\vec{\kappa}}\beta_{\vec{v}-\vec{\kappa}}} - \dfrac{(\beta_{\vec{\kappa}} + \beta_{\vec{v}-\vec{k}} - 1)^2}{4}, & \text{(approximation)} \\ \sqrt{\beta_{\vec{\kappa}}^2 \left(\beta_{\vec{\kappa}} - \beta_{\vec{v}-\vec{k}} + 1\right)^2 + \alpha^2 \beta_{\vec{\kappa}} \beta_{\vec{v}-\vec{\kappa}}^{-1}} - \beta_{\vec{\kappa}} \left(\beta_{\vec{\kappa}} - \beta_{\vec{v}-\vec{k}} + 1\right) - \dfrac{(\beta_{\vec{\kappa}} + \beta_{\vec{v}-\vec{k}} - 1)^2}{4} \end{cases}$$

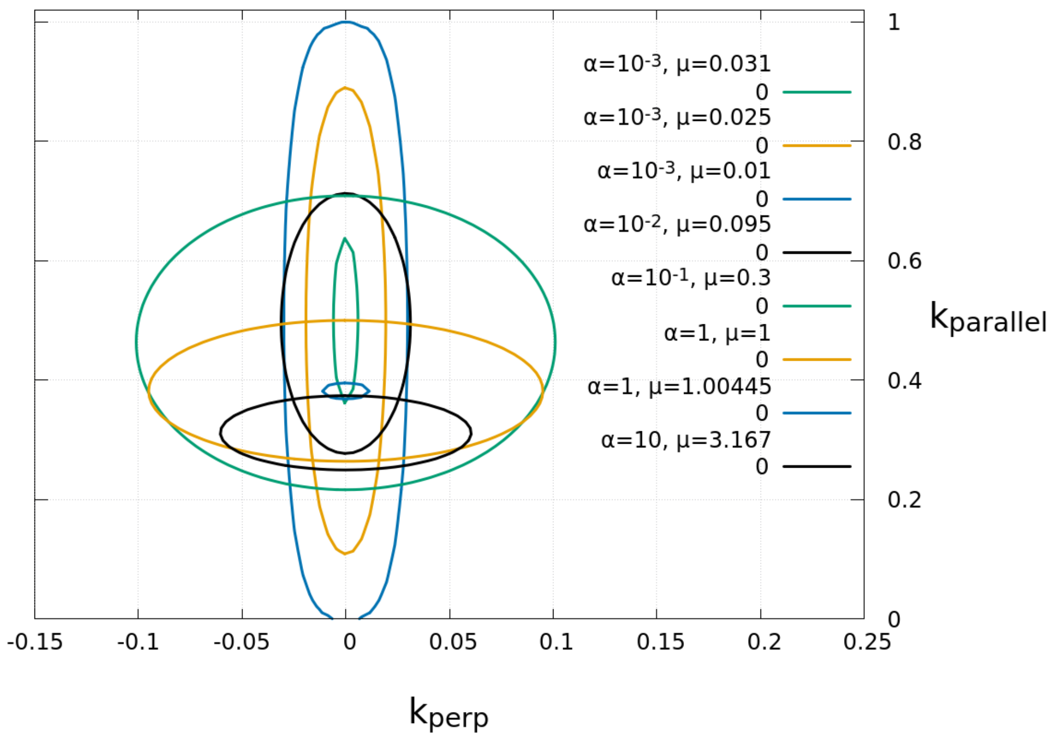
The contours of $\eta_k = 0$ as the function of parallel and perpendicular momenta

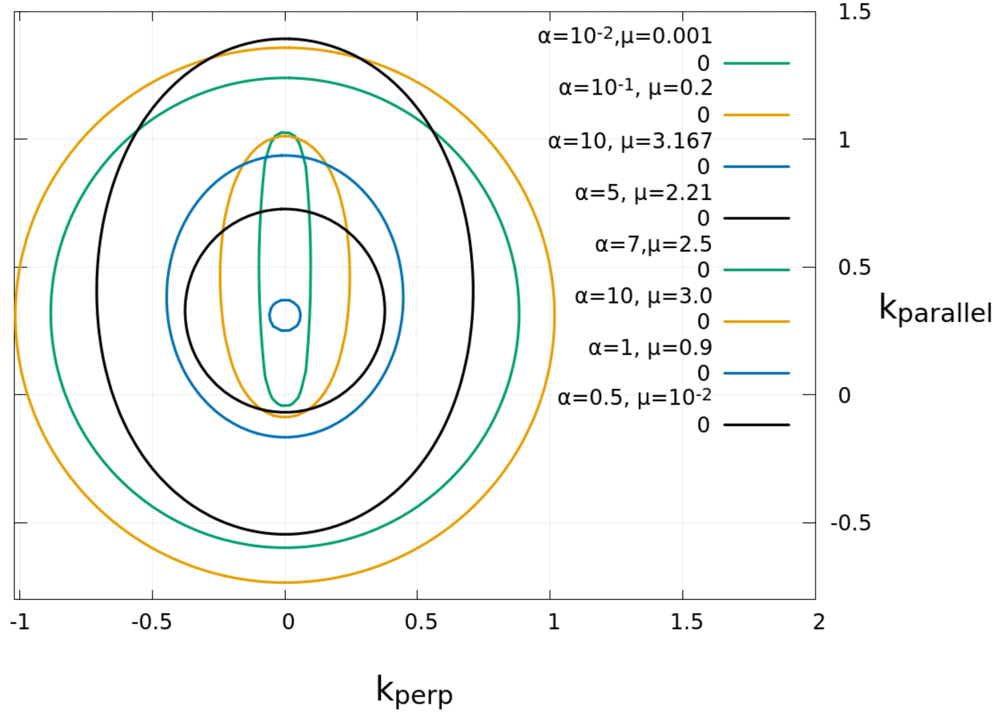
Approximation case



The contours of $\eta_k = 0$ as the function of parallel and perpendicular momenta

Without approximation

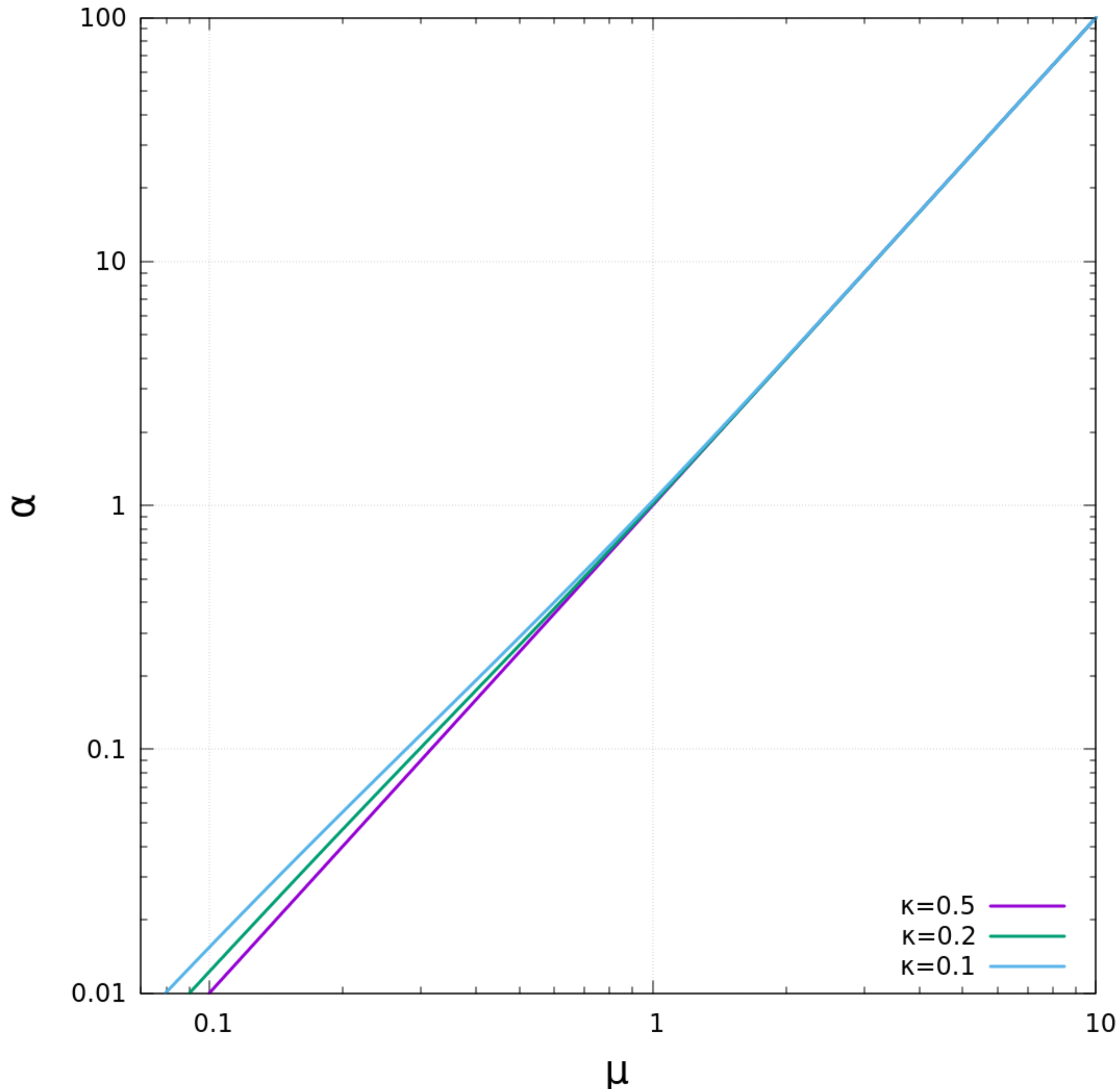




The boundary of stability

$$\eta_{\vec{k}} = 0$$

$$\rho_\phi \geq \frac{m_\chi^4 \omega_p^2}{2g^2}$$



$$\alpha(\mu) = 4\sqrt{\frac{\beta_{v-k}}{\beta_k}(\beta_k + \beta_{v-k} - 1)^2(-3\beta_k + \beta_{v-k} - 1)^2}$$

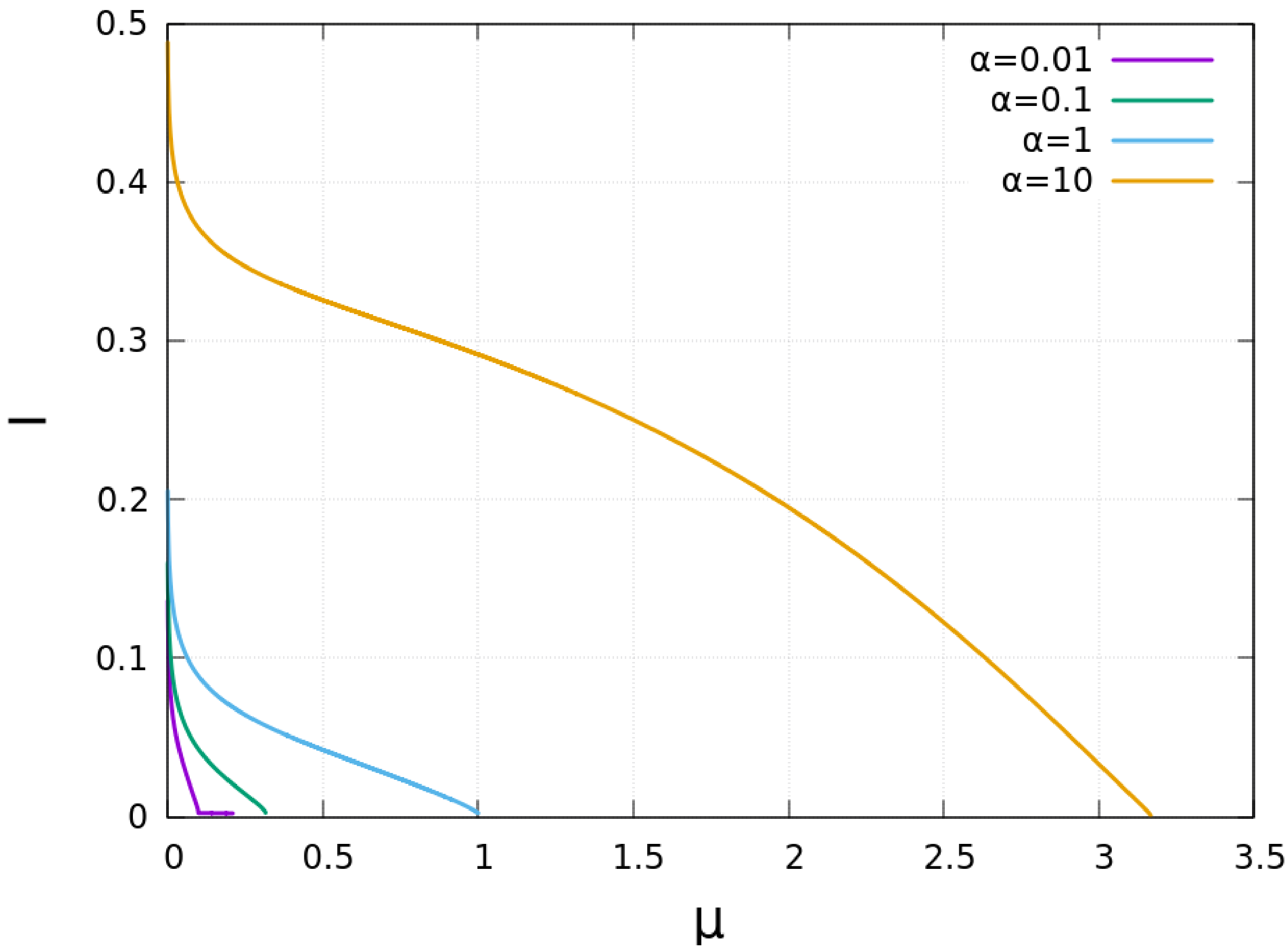
Density of modes

$$n_\chi(t) = \int \frac{d^3k}{(2\pi)^3} f_{\chi,\vec{k}}(t)$$

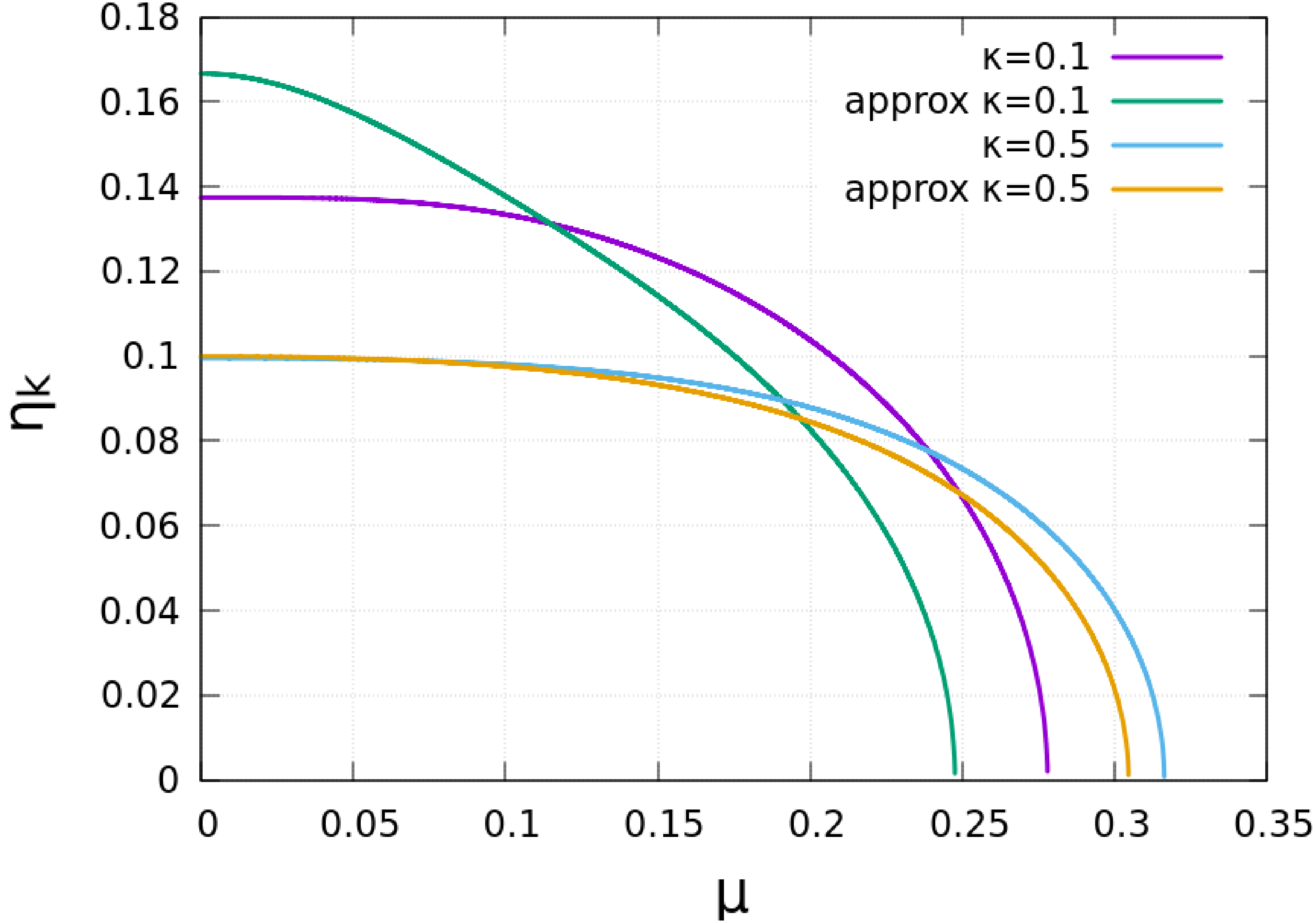
$$n_\chi(t) = \frac{\alpha^2 t^2 \omega_p^5}{(2\pi)^2} \int \kappa_\perp d\kappa_\perp I(\alpha, \mu, \kappa_\perp)$$

$$n_\chi(t) = \alpha^2 t^2 \omega_p^5 \frac{\int d\kappa_\parallel \int \kappa_\perp d\kappa_\perp}{(2\pi)^2} \times \begin{cases} \dfrac{1}{4\beta_{\vec{k}}\beta_{\vec{v}-\vec{k}}}, & \text{(approximation)} \\ \dfrac{\beta_{\vec{k}}}{\beta_{\vec{v}-\vec{k}}(\beta_{\vec{k}} - \beta_{\vec{v}-\vec{k}} + 1)^2}. & \text{(no approx)} \end{cases}$$

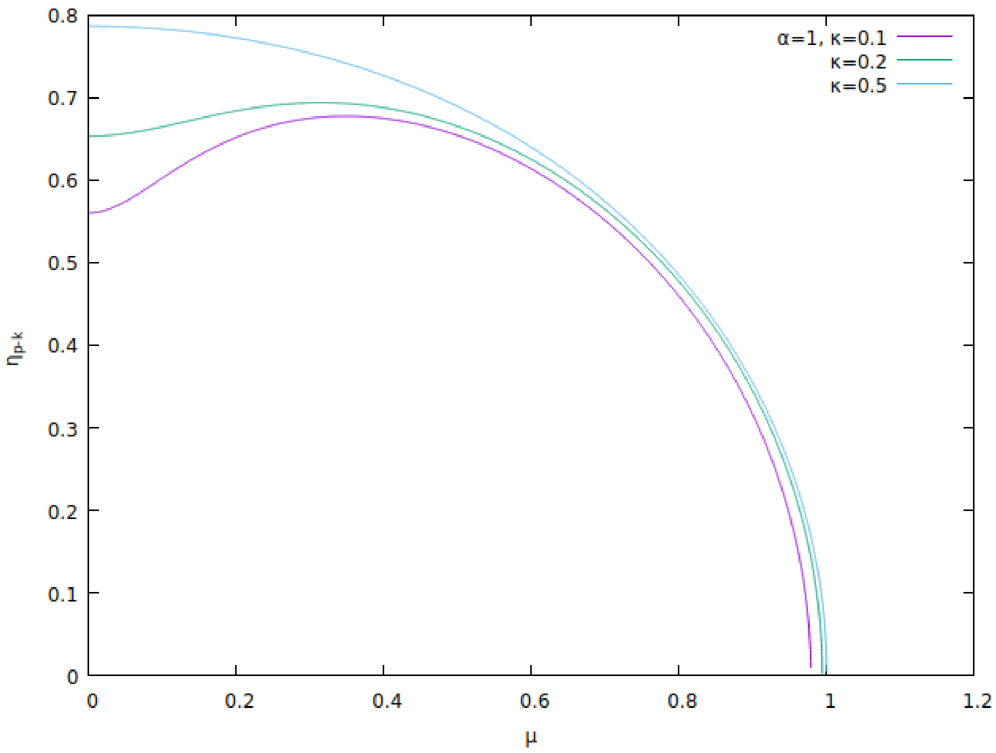
Dependence I(μ) for fixed α

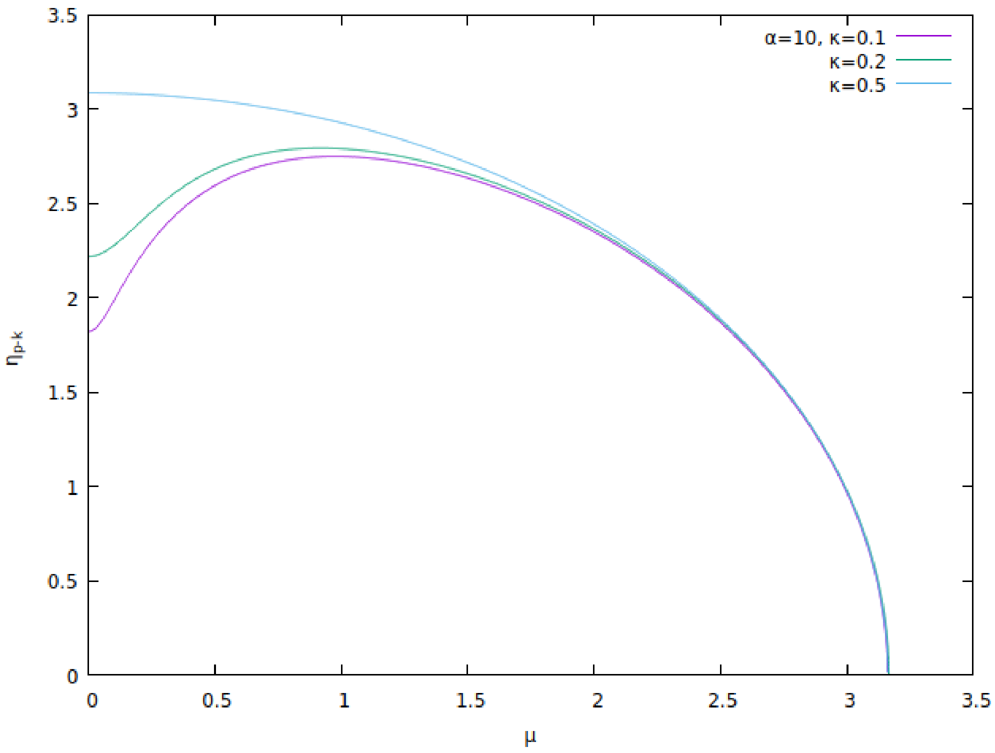


Dependence of η_k on μ at α=0.1 and κ=0.1, 0.5 in the approximation and without it



Dependences of η_k on μ for various α (without approximation)





$\alpha=1$

$\alpha=10$

Solution for a^{+}_{-k}

$$e^{i\Omega_{-\vec{k}}t}(\ddot{a}^{\dagger}_{-\vec{k}} + 2i\Omega_{-\vec{k}}\dot{a}^{\dagger}_{-\vec{k}}) = \sigma_{\vec{k}+\vec{p}}a_{\vec{k}+\vec{p}}e^{-i(\Omega_{\vec{k}+\vec{p}}-\omega_{\vec{p}})t}$$

$$a^{\dagger}_{-\vec{k}} = e^{-i\epsilon_{-k}t/2}\Big[a^{\dagger}_{-\vec{k}}(0)(\cosh(s_{-\vec{k}}t) + i\frac{\frac{-\epsilon^2_{-\vec{k}}}{4} + s^2_{-\vec{k}} + \Omega_{\vec{k}}\epsilon_{-\vec{k}}}{s_{-\vec{k}}(-\epsilon_{-\vec{k}} + 2\Omega_{\vec{k}})}\sinh(s_{-\vec{k}}t))+$$

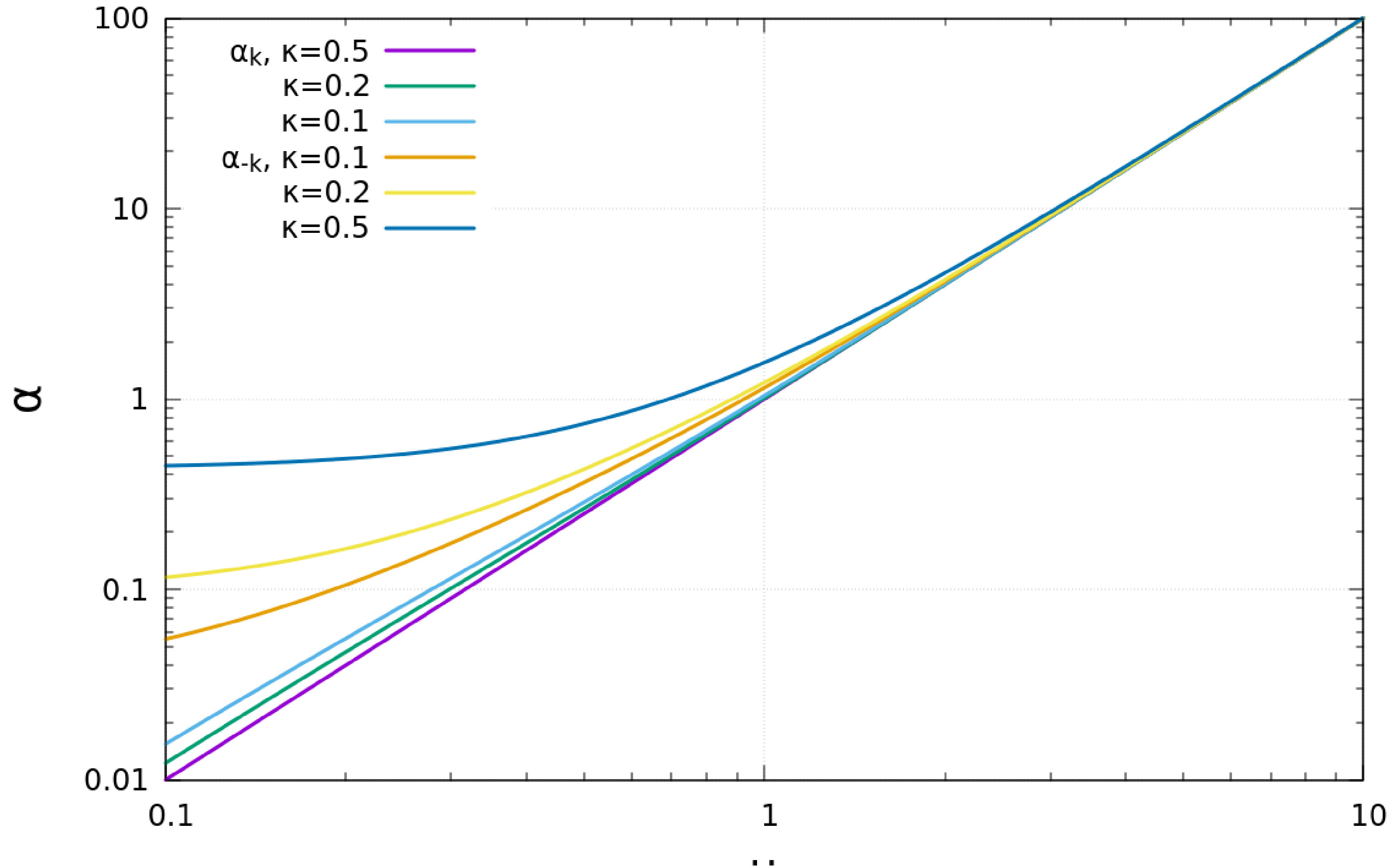
$$+i\frac{-\sigma_{\vec{p}+\vec{k}}}{s_{-\vec{k}}(-\epsilon_{-\vec{k}} + 2\Omega_{\vec{k}})}a_{\vec{p}+\vec{k}}(0)\sinh(s_{-\vec{k}}t)\Big]$$

$$\sigma_{\vec{k}+\vec{p}} = -\omega^2_{\vec{p}}\alpha\sqrt{\frac{\Omega_{\vec{k}}}{\Omega_{\vec{k}+\vec{p}}}}$$

$$\epsilon_{-\vec{k}} = \Omega_{\vec{k}} + \Omega_{\vec{p}+\vec{k}} - \omega_p$$

$$s^2_{-\vec{k}} = -\frac{\epsilon^2_{-\vec{k}}}{4} - 2\Omega^2_{\vec{k}} + \epsilon_{-\vec{k}}\Omega_{\vec{k}} + \sqrt{\Omega^2_{\vec{k}}\epsilon^2_{-\vec{k}} + 4\Omega^4_{\vec{k}} + \sigma^2_{\vec{p}+\vec{k}} - 4\epsilon_{-\vec{k}}\Omega^3_{\vec{k}}}$$

The boundary of stability for a_k and a^+_{-k}



$$\alpha_{-\kappa}(\mu) = \frac{1}{4}\sqrt{\frac{\beta_{\vec{v}+\vec{\kappa}}}{\beta_{\vec{\kappa}}}(\beta_{\vec{\kappa}} + \beta_{\vec{v}+\vec{\kappa}} - 1)^2(-3\beta_{\vec{\kappa}} + \beta_{\vec{v}+\vec{\kappa}} - 1)^2}$$

Standing wave

$$\phi(\xi,t) = 8\frac{\sqrt{2\rho_\phi}}{\omega}\cos(p_x\xi)\cos(p_y\xi)\cos(p_z\xi)\cos(\omega t)$$

The equation $$e^{-i\Omega_{\vec{k}}t}(\ddot{a}_{\vec{k}} - 2i\Omega_{\vec{k}}\dot{a}_{\vec{k}}) = \sigma_{\vec{p}_j-\vec{k}}a^\dagger_{\vec{p}_j-\vec{k}}e^{i(\Omega_{\vec{p}_j-\vec{k}}-\omega)t}$$

$$j = 1,..,4$$

Solution

$$a_{\vec{k}}(t) = \sigma_{\vec{p}_j-\vec{k}}e^{i\epsilon_{\vec{p}_j-\vec{k}}t/2}\Big[a_{\vec{k}}(0)(\cosh(s_{\vec{p}_j-\vec{k}}t) - i\frac{\frac{p_j-\kappa}{4} - s^2_{\vec{p}_j-\vec{k}} - \Omega_{\vec{k}}\epsilon_{\vec{p}_j-\vec{k}}}{s_{\vec{p}_j-\vec{k}}(\epsilon_{\vec{p}_j-\vec{k}} - 2\Omega_{\vec{k}})}\sinh(s_{\vec{p}_j-\vec{k}}t)) -$$

$$-i\frac{\sigma_{\vec{p}_j-\vec{k}}}{s_{\vec{p}_j-\vec{k}}(\epsilon_{\vec{p}_j-\vec{k}} - 2\Omega_{\vec{k}})}a^\dagger_{\vec{p}_j-\vec{k}}(0)\sinh(s_{\vec{p}_j-\vec{k}}t)\Big]$$

$$\vec{\xi} = (\xi_x, \xi_y, \xi_z), \vec{p}_1 = (p_x, p_y, p_z), \vec{p}_2 = (p_x, -p_y, p_z)$$

$$\vec{p}_3 = (-p_x, -p_y, p_z), \vec{p}_4 = (-p_x, -p_y, p_z)$$

Conclusions

- **The decay of φ→χχ occurs at the amplitude of the field φ above the threshold, not only in the case of small masses($m_\chi \ll \omega_p$) - the case of A.Arza, but also at an arbitrary mass**
- **In the case of large masses ($m_\chi \gg \omega_p$), the required threshold amplitude of the field φ is powerfully greater in comparison with the case of small masses**

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Thanks for your attention!