KK compactifications of Horndeski theories

based on 2405.02281

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Motivation Why compactify Horndeski?

$$R^5 \quad \longrightarrow \quad R^4 \times S^1$$

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no higher derivatives in EOMs	$\longrightarrow$	no higher derivatives in EOMs
* Metric + scalar	$\longrightarrow$	Metric + vector + scalar + scalar

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* Horndeski theory coupled to a U(1) gauge boson and scalar

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Natural way to obtain vector Galileon Lagrangian.

Some uniformity in vector and tensor sectors.

Results

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Vector Galileons

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Non-gauge vector galileons:

Examples: 1307.0077 (G. Tasinato, K. Koyama, and N. Khosravi),.., 1812.11134 (P. Petrov)

Gauge vector galileons:

No-go: 1312.6690 (C. Deffayet, A. E. Gümrükçüoğlu, S. Mukohyama, and Y. Wang)

Latest developments: 2312.14814, 2404.18715 (A. Colléaux, D. Langlois, and K. Noui)

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GW170817: $|\frac{c_T}{c} - 1| < 10^{-15}$ KK compactification: $|\frac{c_T}{c} - 1| \equiv 0$

$$S = \int \mathrm{d}^4 x \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;mn}\pi^{;mn}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{mn}\pi_{;mn} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;mn}\pi^{;mn} + 2\pi_{;mn}\pi^{;ml}\pi_{;l}{}^{n}\right],$$

where π is the Galileon field, $X = g^{mn}\pi_{,m}\pi_{,n}$, $\pi_{,m} = \partial_m\pi$, $\pi_{;mn} = \nabla_n\nabla_m\pi$, $\Box\pi = g^{mn}\nabla_n\nabla_m\pi$, $G_{4X} = \partial G_4/\partial X$.

$$g_{mn} = \begin{pmatrix} g_{\mu\nu} + \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & \phi^2 \end{pmatrix}$$

Horndeski theory → U(1) vector gauge field

$$
\begin{aligned}
S &= \int \mathrm{d}^4x\sqrt{-g}\,(\mathcal{L}_2+\mathcal{L}_3+\mathcal{L}_4+\mathcal{L}_5)\,,\\
\mathcal{L}_2 &= F(\pi,X),\\
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\end{aligned}
$$

$\mathcal{L}_2$, $\mathcal{L}_3$ → no dynamical A_μ

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$\mathcal{L}_4 \rightarrow$ non minimal coupling to A_μ

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$\mathcal{L}_5$, $\mathcal{L}_6$ $\rightarrow$ A_μ is a vector Galileon

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$$\mathcal{S}_T = \int \mathrm{d}\eta\,\mathrm{d}^3x\,a^4\,\left[\frac{1}{2\,a^2}\left(\mathcal{G}_\tau\left(\dot{h}_{ij}\right)^2 - \mathcal{F}_\tau(\partial_k\,h_{ij})^2\right)\right]$$

$$\mathcal{G}_T = 2\left[G_4 - 2XG_{4,X} - X\left(H\dot{\pi}G_{5X} - G_{5\phi}\right)\right]$$

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$$c_T^2 = c_v^2 = 1$$

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$$c_T^2 = c_v^2 \neq 1$$

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allowable Horndeski theory

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 \right),$$

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\mathcal{L}_5 &= G_5(\pi)G^{\mu\nu}\pi_{;\mu\nu}
\end{aligned}
$$

$$\mathcal{S}_{5d} = \int \mathrm{d}^5 x \sqrt{-g_{(5)}} \left(G_4 \, R_{(5)} + G_{4,X} \, \left((\Box \pi)^2 - (\nabla_A \nabla_B \pi)^2 \right) \right)$$

$$\mathcal{S}_{5d} = \int d^5x\,\sqrt{-g_{(5)}}\left(G_4\,R_{(5)} + G_{4,X}\,\left((\Box\pi)^2 - (\nabla_A\nabla_B\pi)^2\right)\right)$$

$$g_{A\,B} = \begin{pmatrix} g_{\mu\,\nu} + \phi^2\,A_\mu\,A_\nu & \phi^2\,A_\mu \\ \phi^2\,A_\nu & \phi^2 \end{pmatrix}$$

$$\mathcal{S}_{5d} = \int d^5x \sqrt{-g_{(5)}} \left(G_4 R_{(5)} + G_{4,X} \left((\Box\pi)^2 - (\nabla_A \nabla_B \pi)^2 \right) \right)$$

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} + \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & \phi^2 \end{pmatrix}$$

$$\mathcal{S}_{4d} = \int d^4x \sqrt{-g}\, \phi\, G_4\, R + \phi\, G_{4,X} \left((\Box\pi)^2 - (\nabla_\mu \nabla_\nu \pi)^2 \right)$$

$$+ G_4 \left(-\frac{1}{4} \phi^3 F^2 - 2\Box\phi \right) + G_{4,X} \left(2 \nabla_\mu \phi \nabla^\mu \pi \, \Box\pi - \frac{1}{2} \phi^3 F_\mu{}^\sigma F_{\nu\sigma} \nabla^\mu \pi \nabla^\nu \pi \right)$$

$$\int \mathrm{d}^5x \, \sqrt{-g_{(5)}} \left(G_5(\pi) \left(R^{AB}_{(5)} - \frac{1}{2} g^{AB}_{(5)} \, R_{(5)} \right) \nabla_A \nabla_B \pi \right)$$

$$\int \mathrm{d}^5x\,\sqrt{-g_{(5)}}\left(G_5(\pi)\,\left(R^{AB}_{(5)}-\frac{1}{2}g^{AB}_{(5)}\,R_{(5)}\right)\nabla_A\nabla_B\pi\right)$$

$$\int \mathrm{d}^4x\,\sqrt{-g}\,\phi\,G_5(\pi)\left(\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}\,R\right)\nabla_\mu\nabla_\nu\pi\right.$$
$$-\frac{1}{2\,\phi}R\,\nabla_\mu\phi\,\nabla^\mu\pi+\frac{1}{\phi}\left(\Box\phi\,\Box\pi-\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\pi\right)$$
$$+\frac{1}{8}\phi\,F^{\mu\nu}\,F^{\sigma\rho}\left(3\,g_{\nu\rho}(-4\,g_{\lambda\mu}\,g_{\beta\sigma}+g_{\lambda\beta}\,g_{\mu\sigma})\,\nabla^\lambda\pi\nabla^\beta\phi\right.$$
$$\left.\left.+\phi\,g_{\sigma\mu}\left(-4\,\nabla_\nu\nabla_\rho\pi+g_{\rho\nu}\Box\pi\right)\right)+\frac{1}{2}\phi^2\,F_{\mu\nu}\,\nabla_\sigma F^{\nu\sigma}\,\nabla^\mu\pi\right)$$

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Thank you for your attention!