# Electromagnetic hedgehogs

Yulia Galushkina

Institute for Nuclear Research, Moscow, Russia

2024

Based on arXiv 2405.01335 (with E. Nugaev and A. Shkerin)

# Introduction

In our study we considered spherically-symmetric nontopological solitons in (3+1) theory of a complex neutral vector field $V^\mu$ coupled to an electromagnetic field tensor $F^{\mu\nu}$ by a dipole interaction:

$$\mathcal{L}_{int} = -\frac{i\gamma}{2} F^{\mu\nu}(V^*_\mu V_\nu - V^*_\nu V_\mu),$$

where $\gamma$ is dimensionless a coupling constant.

Moving to the non-relativistic limit, we obtain (neglecting zero component of a vector field):

$$i\frac{\gamma}{2}F_{ij}(V^{*i}V^j - V^{*j}V^i) = -i\frac{\gamma}{2M}B^k\epsilon_{ijk}\Psi^{*i}\Psi^j = \frac{\gamma}{2M}\Psi^*(\boldsymbol{B}\cdot\hat{\boldsymbol{S}})\Psi,$$

where $V^i = e^{-iMt}\dfrac{1}{\sqrt{2M}}\Psi^i(p)$ and $\dfrac{\gamma}{2M}\boldsymbol{S}$ is a magnetic moment (Lee, Yang, Phys. Rev. 1962).

# Motivation

- Neutral vector particle with a magnetic moment is a candidate for a feebly interacting DM. The bounds on this type of matter are not as strong as those on charged matter (Barger, Keung, Marfatia, 2010).
- Classical configurations of vector matter with a magnetic moment (Rb atoms) were observed in the experiments with ultracold atoms (Dalfovo, Giorgini, Pitaevskii, Rev. Mod. Phys., 1999).

# Setup

Consider a (3+1)-dimensional theory of the complex massive neutral field $V^\mu$ coupled to the electromagnetic field tensor $F_{\mu\nu}$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}V^*_{\mu\nu}V^{\mu\nu} - \frac{i\gamma}{2}F_{\mu\nu}W^{\mu\nu} - U(V^\nu, V^{*\mu}) \,,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, and $W_{\mu\nu} = V^*_\mu V_\nu - V^*_\nu V_\mu$. Let us choose:

$$U = -M^2 V^*_\mu V^\mu - \frac{\alpha}{2}(V^*_\mu V^\mu)^2 - \frac{\beta}{2}(V^*_\mu V^{*\mu})(V_\nu V^\nu)$$

# Integrating out the electromagnetic field

Lagrangian of the model leads to the equation

$$\partial_\mu F^{\mu\nu} = -i\gamma\partial_\mu W^{\mu\nu},$$

which, in absence of external fields, can be integrated to yield

$$F^{\mu\nu} = -i\gamma W^{\mu\nu}, \quad W_{\mu\nu} = V_\mu^* V_\nu - V_\nu^* V_\mu.$$

We obtain following effective Lagrangian:

$$\mathcal{L} = -\frac{1}{2} V_{\mu\nu}^* V^{\mu\nu} + M^2 V_\mu^* V^\mu + \frac{\tilde{\alpha}}{2}(V_\mu^* V^\mu)^2 + \frac{\tilde{\beta}}{2}(V_\mu^* V^{*\mu})(V_\nu V^\nu)$$

where $\tilde{\alpha} = \alpha - \gamma^2$ and $\tilde{\beta} = \beta + \gamma^2$.

# Dual theory

We can also consider interaction $\frac{i\tilde{\gamma}}{2}\tilde{F}_{\mu\nu}W^{\mu\nu}$, where $\tilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\rho\lambda}F^{\rho\lambda}$ and $\tilde{\gamma}$ is a dimensionless constant. Integrating out the electromagnetic field, we obtain qualitatively the same Lagrangian as in the P-even theory. The effective potential is:

$$\tilde{U} = -M^2 V_\mu^* V^\mu - \frac{\tilde{\alpha}}{2}(V_\mu^* V^\mu)^2 - \frac{\tilde{\beta}}{2}(V_\mu^* V^{*\mu})(V_\nu V^\nu),$$

where $\tilde{\alpha} = \alpha + 4\tilde{\gamma}^2$ and $\tilde{\beta} = \beta - 4\tilde{\gamma}^2$.

# Ansatz

To facilitate the study, we switch to the dimensionless units as follows,

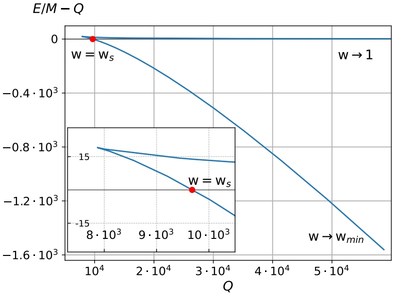
$$V^{\nu} = \frac{M\tilde{V}^{\nu}}{\sqrt{|\tilde{\alpha}|}}\ , \ \ \kappa = \frac{\tilde{\beta}}{|\tilde{\alpha}|}\ , \ \ x_i x_i = \frac{r^2}{M^2}\ , \ \ t = \frac{\tau}{M}\ . \qquad (1)$$

We use the following ansatz (Loginov, PRD, 2015):

$$\tilde{V}_0 = iu(r)\mathrm{e}^{-i\mathrm{w}\tau}\ , \quad \tilde{V}^i = \frac{\tilde{x}^i}{r}v(r)\mathrm{e}^{-i\mathrm{w}\tau}.$$

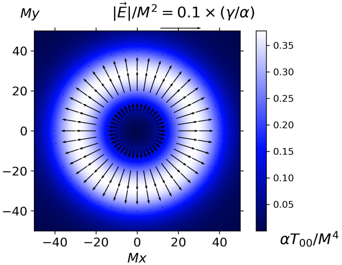
For this ansatz $W^{\mu\nu}$ is nontrivial, so we can localize electromagnetic field.

# Kinematic stability of the solitons




$E/M - Q$ as a function of $Q$ for vector solitons with $\tilde{\alpha} = 1$ and $\kappa = \tilde{\beta}/|\tilde{\alpha}| = -0.55$.

# Electromagnetic hedgehogs




The equatorial cross-section of the electric hedgehog with $\kappa = \beta/\alpha = -0.9$, $\mathrm{w} = 0.998$, and $\gamma \ll \alpha$. The arrows indicate the value $|\vec{E}|/M^2$ and the direction of the electric field. Note that the electromagnetic field decays exponentially (unlike in monopoles).

# Conclusion

- We obtained kinematically stable configuration of localized electromagnetic field in a theory without charges.
- There is a possibility of partial UV-completion of vector model, e. g. with a scalar field (Herdeiro, Radu, dos Santos Costa Filho, 2023).
- Our result can be generalized on the non-abelian theory with an interaction $i(V_\mu^\dagger F_{\mu\nu} V_\nu) + h.c.$
- There is a possibility of Bose-stars with a similar structure. They may demonstrate a wider range of stable configurations.

Thanks for your attention!

This work was supported by grant RSF 22-12-00215

# Appendix 1: Coupling constants

If we consider our vector field as a dark matter candidate, it is natural to set $\alpha, \beta \gg \gamma$. The applicability of semiclassical approach requires $|\alpha|/4\pi \ll 1$, $|\beta|/4\pi \ll 1$.
The strong coupling scale of the theory can be estimated as $\Lambda \sim M\sqrt{4\pi}/\alpha^{1/4}$ (Porrati, Rahman, Nucl. Phys., 2008). On the other hand, the physical field $V^\mu$ scales as $1/\alpha^{1/2}$. Requiring the physical field amplitude stay below the cutoff leads to the bound $\alpha > 1/(4\pi)^2$.

# Appendix 2: Condensate

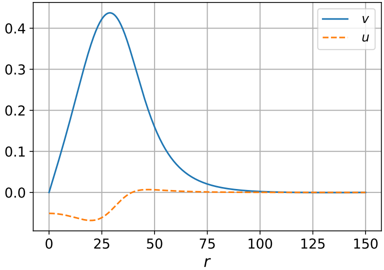
There are two types of relevant condensates. The first one is

$$\tilde{V}_0 = 0, \quad \tilde{V}^1 = \sqrt{\frac{1-\mathrm{w}^2}{\pm 1+\kappa}}\, \mathrm{e}^{-i\mathrm{w}\tau} , \quad \tilde{V}^i = 0 , \quad i = 2,3$$

The second condensate is of the form

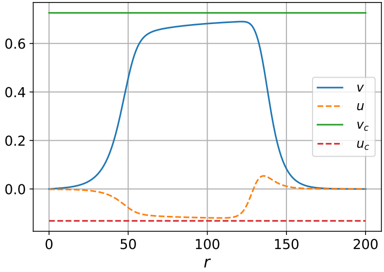
$$\tilde{V}_0 = iu\mathrm{e}^{-i\mathrm{w}\tau} , \quad \tilde{V}^1 = v\mathrm{e}^{-i\mathrm{w}\tau} , \quad \tilde{V}^i = 0 , \quad i = 2,3$$

# Appendix 3: Soliton profiles



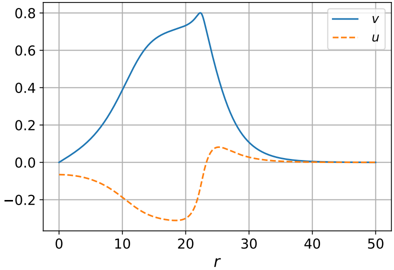

Vector soliton at $\kappa = -0.9$ and $\mathrm{w} = 0.998$. This solution is kinematically stable, $E < MQ$.

# Appendix 3: Soliton profiles




Vector soliton at $\kappa = -0.9$ and $\mathrm{w} = 0.99$. This solution is kinematically stable, $E < MQ$.

# Appendix 3: Soliton profiles




Vector soliton at $\kappa = -0.55$ and $\mathrm{w} = 0.96$. This solution is kinematically stable, $E < MQ$.