Structure Formation with nonrelativistic scalar fields

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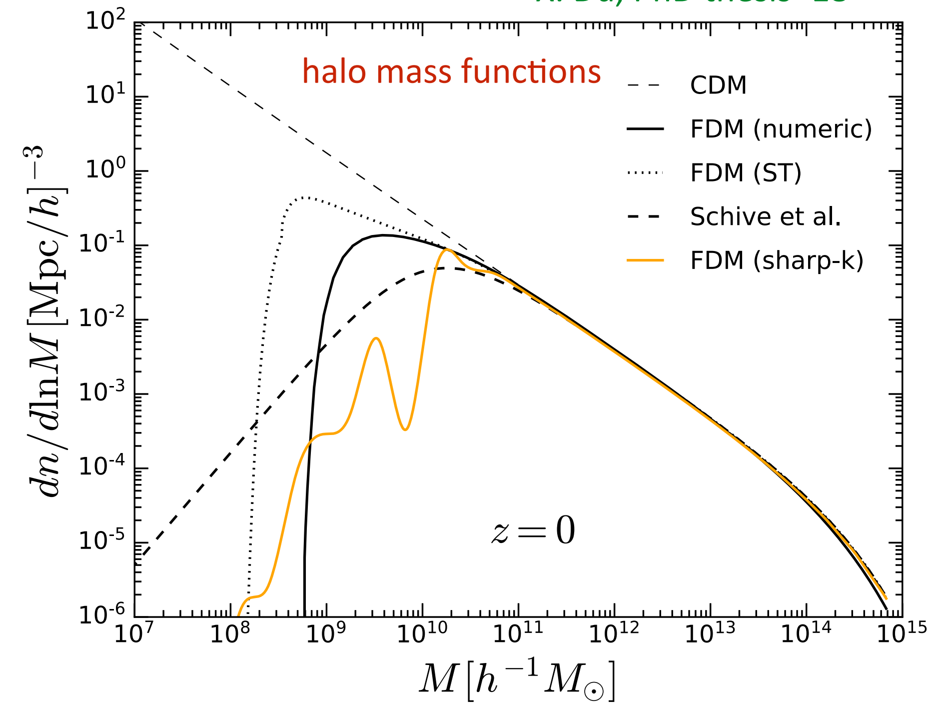
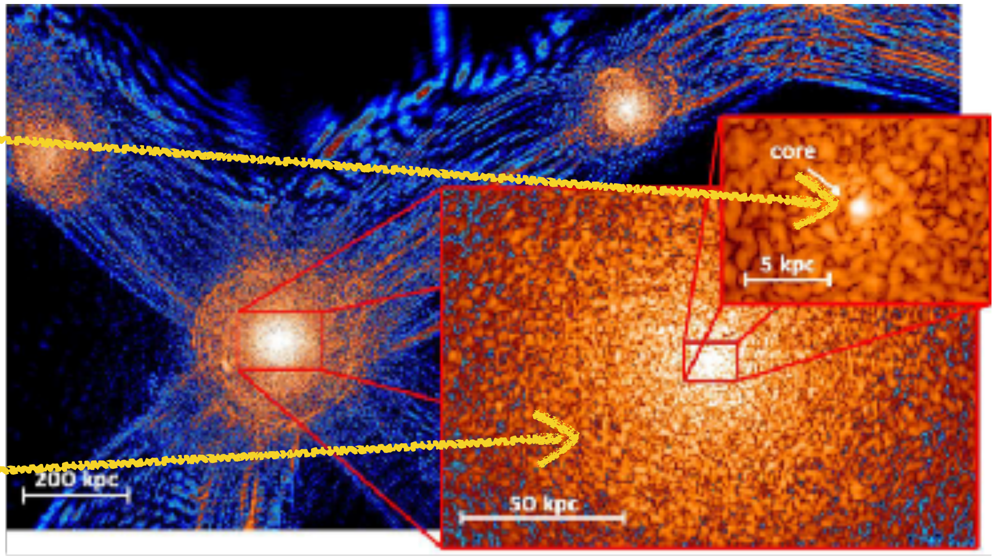
Axion DM phenomenology

X. Du, PhD thesis ´18



Ultralight axions („fuzzy dark matter“, FDM Hu+ ‘00)

- Suppression of small-scale perturbations *(„WDM-like“)*
  - high-z luminosity functions (Bozek+ ’15, Schive+ ’16, Corasaniti+ ’17, Menci+ ’17)
  - Lyman-α forest (Iršič+ ’17, Armengaud+ ’17, Rogers+ ‘20) → $m \gtrsim 10^{-20}$ eV
  - reionization (Bozek+ ’15; Schneider ’18; Lidz, Hui ’18)



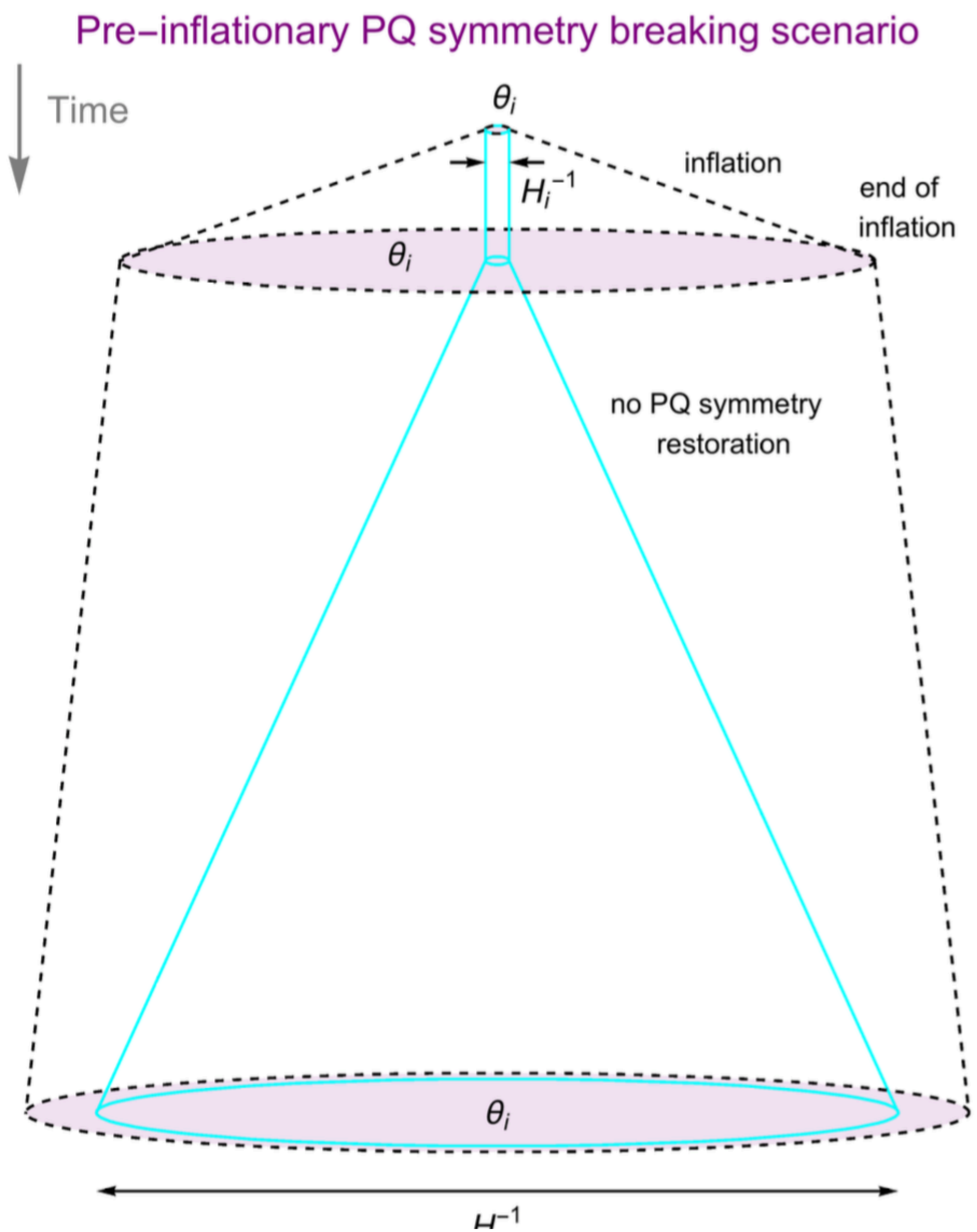
Schive+ ’14

- Formation of coherent solitonic halo cores *(„SIDM-like“)*
  - cusp-core etc., halo substructure (Marsh,Silk ’13, Schive+ ’14, Marsh,Pop ’15, Calabrese,Spergel ’16, Du+ ’16)
- Incoherent interference patterns and granularity on scales of λ_{dB} ~ 1 … 100 kpc
  - „quasi-particle relaxation“ → dynamical friction / heating / diffusion (Hui+ ’17, Bar-Or ´18, Marsh & JN ´18) *(„PBH-like“)*

QCD axions

- Formation of axion miniclusters (Hogan,Rees ’88; Kolb,Tkachev ’93/94; Zurek+ ’07)
  - relevant for direct detection experiments
  - potentially observable in fast radio bursts, tidal streams, microlensing (Tkachev ’15, Tinyakov+ ’16, Fairbairn+ ’17)
- Formation of axion stars (Tkachev ’86; Levkov+ ’18, Eggemeier & JN ‘19)

I. Ultralight axions

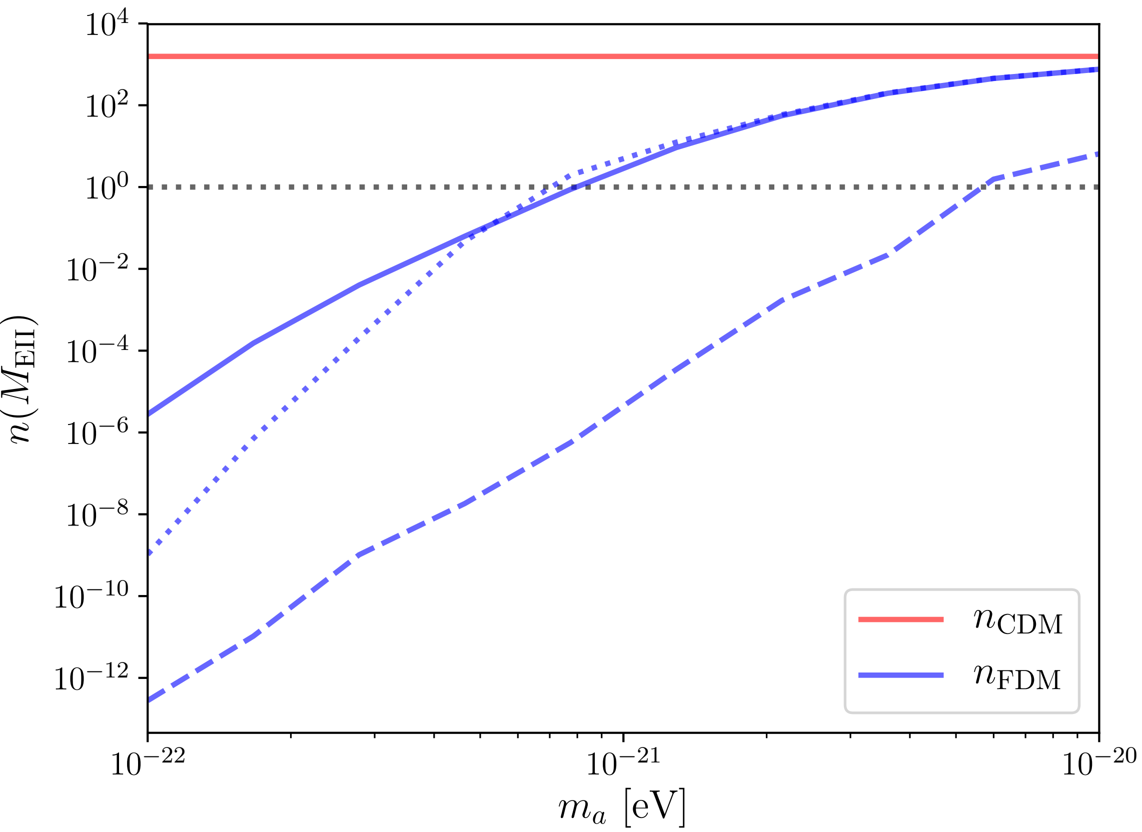


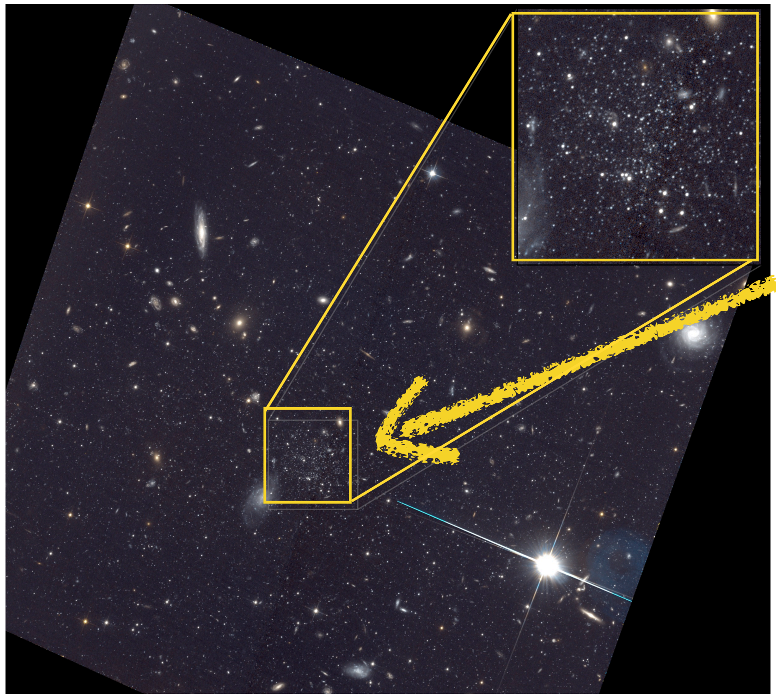
K. Saikawa

Gravitational heating constraints: Star cluster in UFD Eridanus II

(Marsh & JN ´19, PRL 123, 051103)

expected number of subhalos:



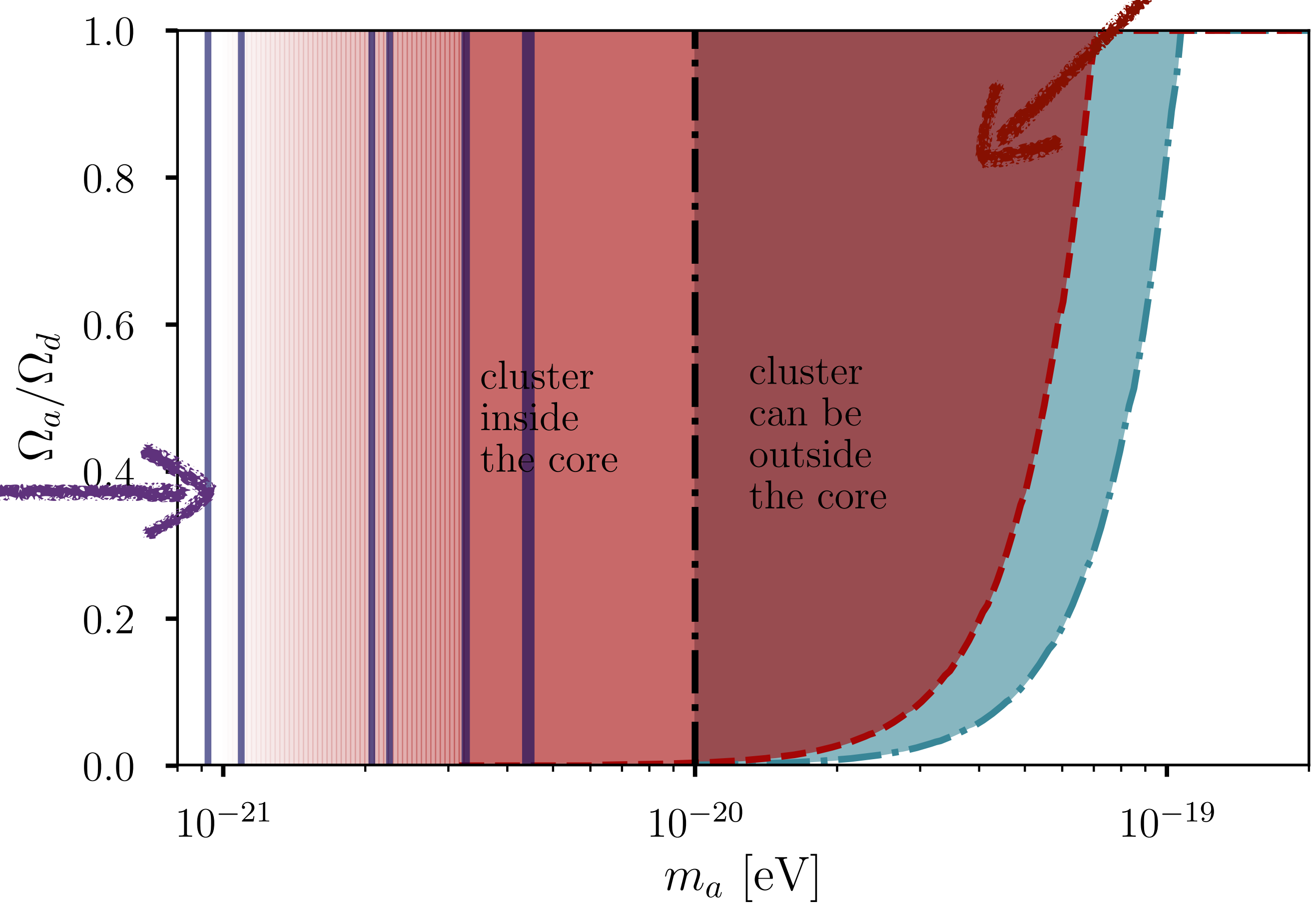


Simon+ ´20

central star cluster

diffusion / heating

resonance



many uncertainties, need simulations!

In the Newtonian limit, ULAs obey the Schrödinger-Poisson (SP) equations:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2a^2 m} \nabla^2 \psi + mV\psi$$

$$\nabla^2 V = 4\pi G a^2 \delta\rho = \frac{4\pi G}{a} \rho_0 (|\psi|^2 - 1)$$

Dynamics of gravitationally interacting random waves equivalent to collisionless matter on large length scales / short time scales.

Madelung / fluid formulation:

$$\dot{\rho} + \nabla(\rho \mathbf{v}) = 0 \qquad \dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla(Q + V)$$

$$\mathbf{v} = m^{-1} \nabla S \qquad Q = -\frac{\hbar^2}{2m^2} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}$$

„quantum pressure“

„Quantum Reynolds number“ (compare advection and quantum pressure terms):

$$\frac{Q/R}{v^2/R} \simeq \frac{\hbar^2}{m^2 R^3} \frac{R}{v^2} = \left(\frac{\lambda_{\rm dB}}{R}\right)^2$$

Gravitational relaxation time (e.g. Levkov+ '18):

$$\tau \sim 10^{-2} \times \left(\frac{\lambda_{\rm dB}}{R}\right)^{-3} t_{\rm cr} \;,\; t_{\rm cr} = R/v$$

Simulations with bosonic dark matter

Different scales / physics require different numerical methods.

1. **N-body with modified initial conditions**:
CDM-like dynamics, linear / weakly nonlinear scales → useful for large-scale structure constraints on FDM (Ly alpha forest, reionization, high-z luminosity functions etc.) or QCD axion miniclusters

2. **Madelung (fluid) formulation** (SPH, PM, or finite volume):
same as above, includes „quantum pressure“ effects, resolution requirements and validity unclear

4. **Schrödinger-Poisson formulation** (finite difference or pseudo-spectral):
full wave-like dynamics, requires phase resolution, can only handle relatively small boxes, nonlinear scales → useful for isolated halos or small cosmological boxes

5. **Hybrid method** (N-body on coarse grids, Schrödinger-Poisson on finest grid):
dynamics CDM-like on large scales, wave-like on small (nonlinear) scales → useful for zoom-in simulations in cosmological boxes

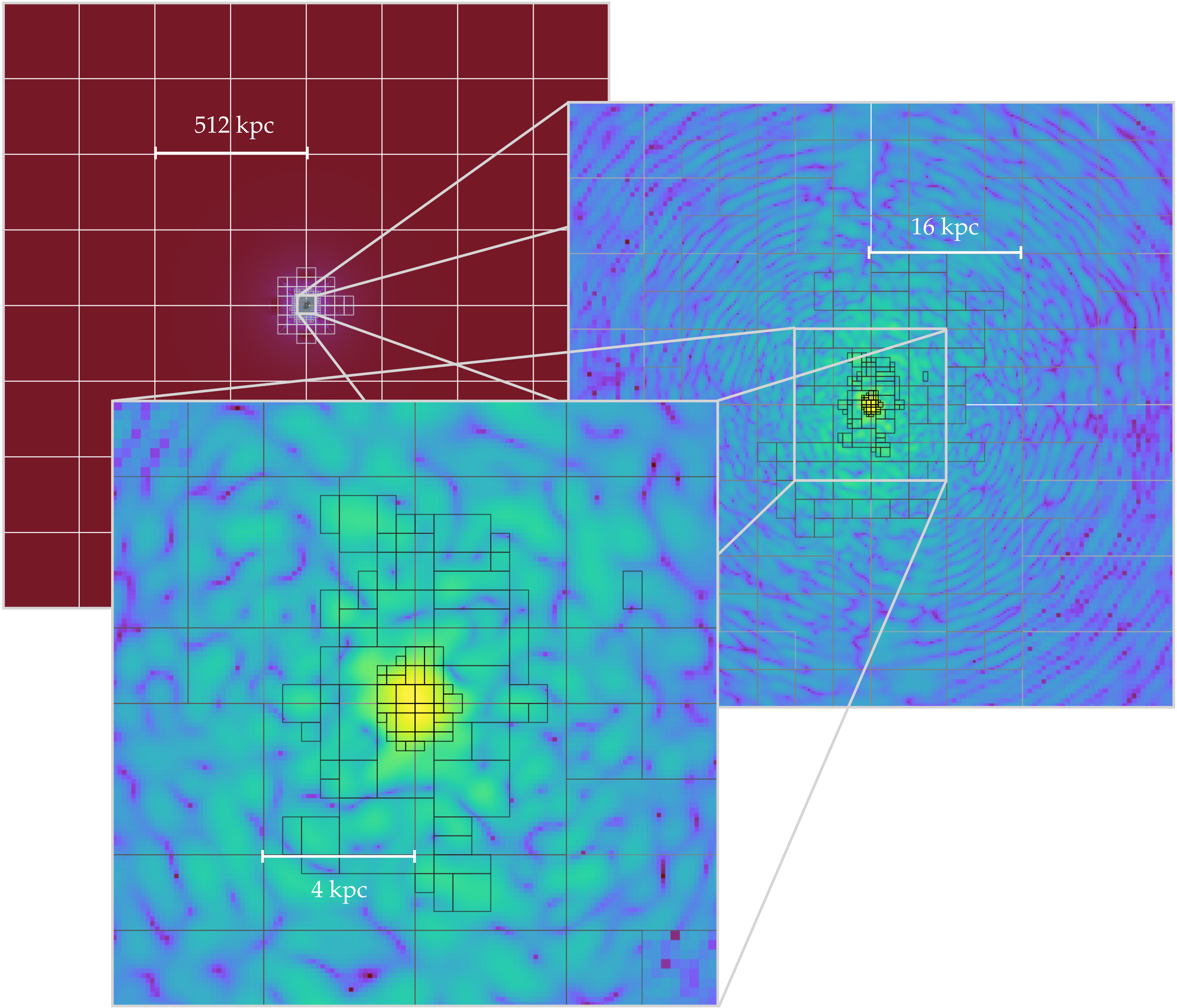
AxioNyx

(Schwabe, Gosenca, Behrens, JN, Easther ’20, PRD 102, 083518)

Public code for mixed FDM+CDM+hydro simulations

based on AMReX

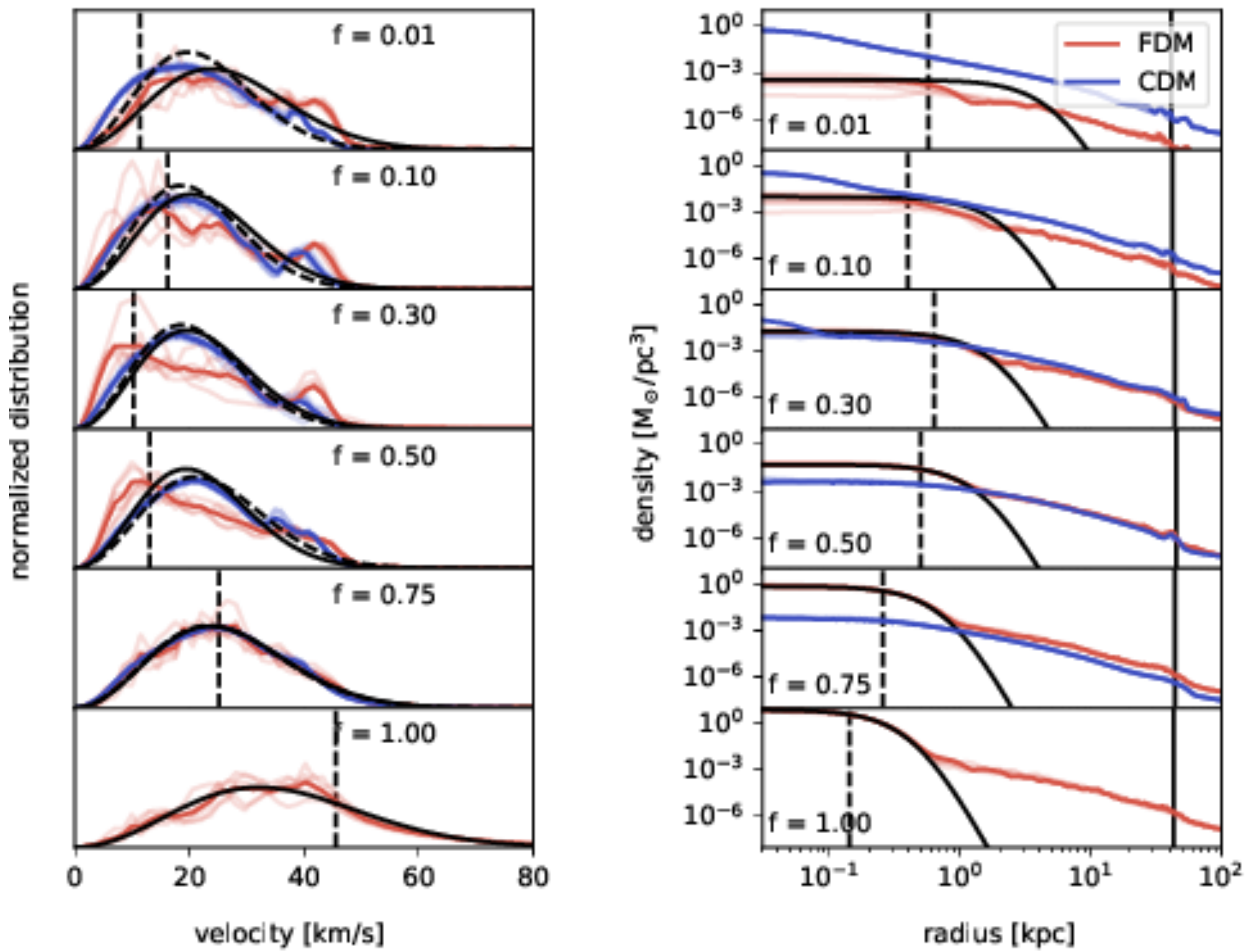
2nd and 6th order pseudo-spectral Schrödinger solvers



Spherical Collapse

Schroedinger-Vlasov correspondence:

- Maxwellian FDM Powerspectrum of central region coincides well with particle velocity dispersion.
- Outer radial density profiles have constant FDM/CDM density ratio



For f>0.1 a soliton forms with velocity $v_c = \frac{2\pi}{7.5}\frac{\hbar}{mr_c}$ close to maximum in spectrum indicated by dashed line

New Hybrid Method

Goal:

- AMR simulation
- Particle method on low resolution levels
- Finite-difference method on finest level
- Important: Boundary conditions between methods

Madelung transformation: $\Psi = A \exp[-iSm/\hbar]$

Initial phase: $\nabla \cdot v_0 = a^{-1} \nabla^2 S_0$

Phase evolution: $\frac{\mathrm{d}S_i}{\mathrm{d}t} = \frac{1}{2} {v_i}^2 - V(x_i)$

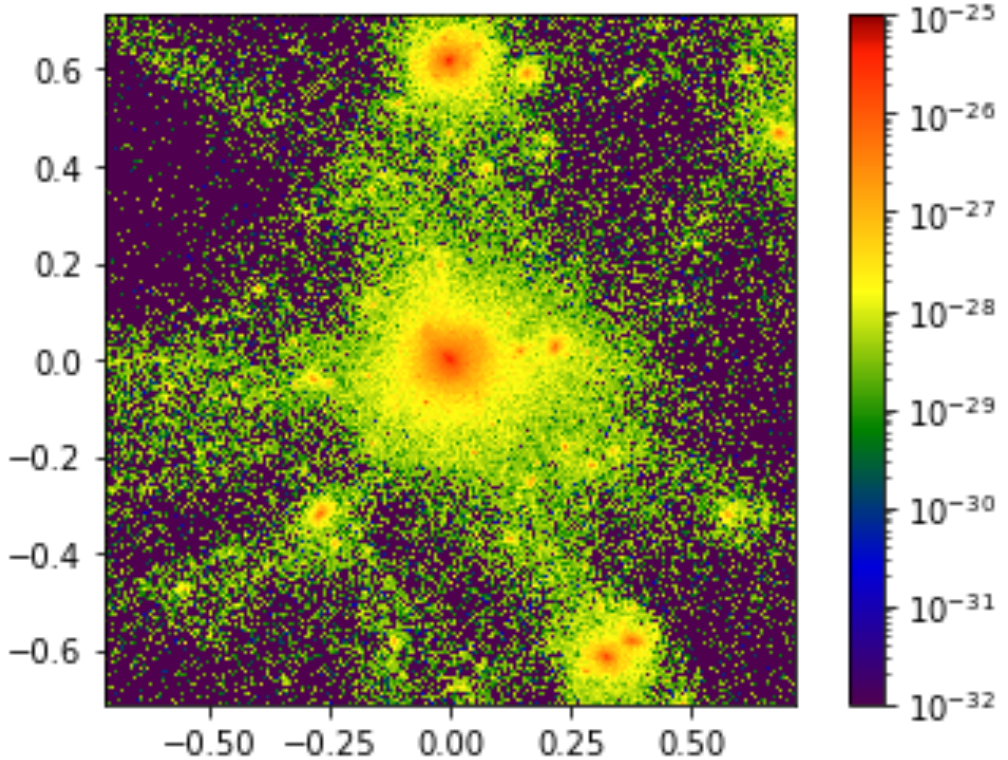
Construction of wavefunction: $\Psi(x) = \sum_i W(x - x_i) A_i e^{i(S_i + v_i \cdot a(x - x_i))m/\hbar}$

Gauss kernel: $W(x - x_i) = \frac{\gamma^{3/2} \Delta^3 x}{\pi^{3/2}} \exp\left[-\gamma(x - x_i)^2\right] \theta(x - x_i)$

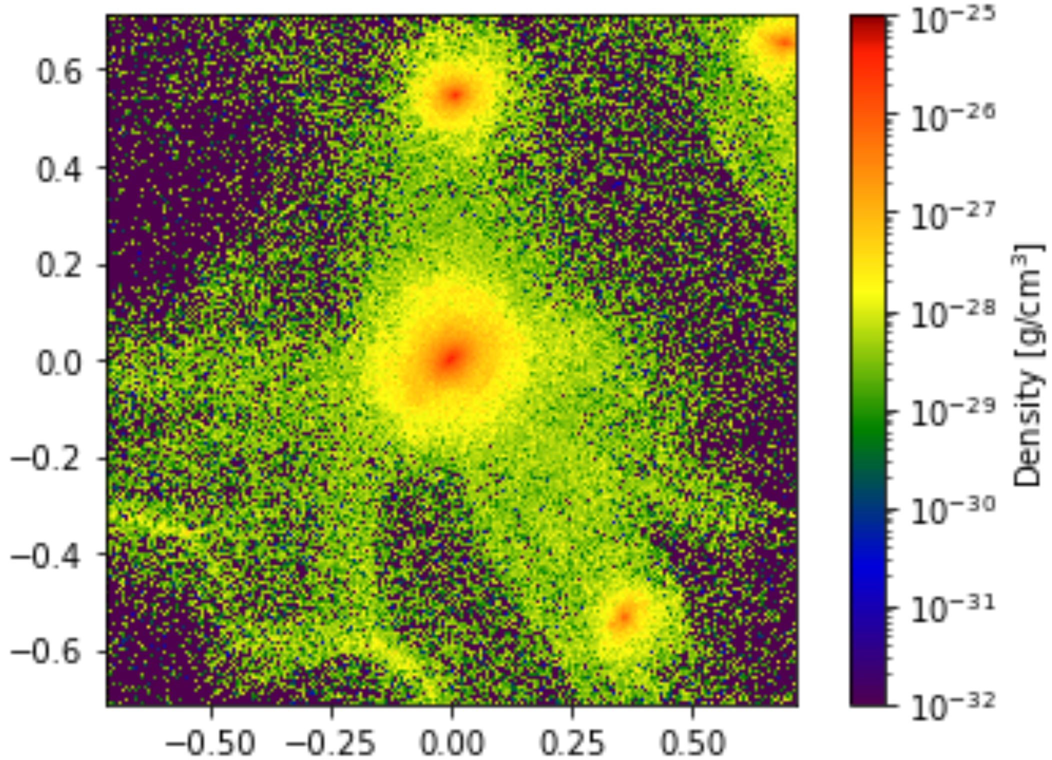
The AGORA halo in FDM (B. Schwabe)

- AGORA High-resolution Galaxy Simulations Comparison Project (Kim + ’14, ApJS 210, 14)
- „proof-of-concept“ halo: M = 1.7 x 10^{11} M_{sol} , quiescent merger history
- re-run with FDM initial conditions and hybrid N-body / SP method (Schwabe, JN ’21, in prep.)

Axionyx N-body run with CDM initial conditions

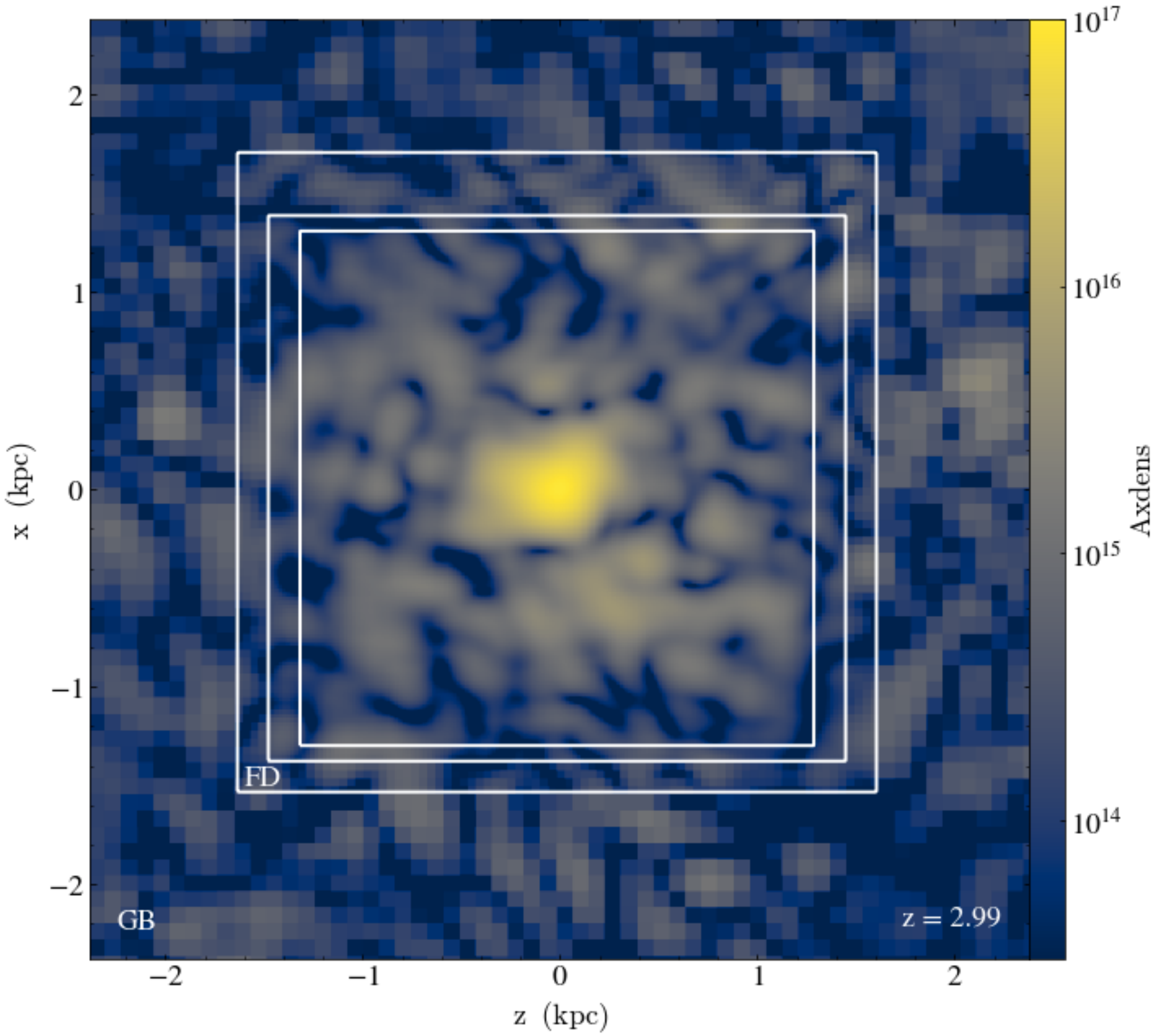


Axionyx N-body run with FDM initial conditions



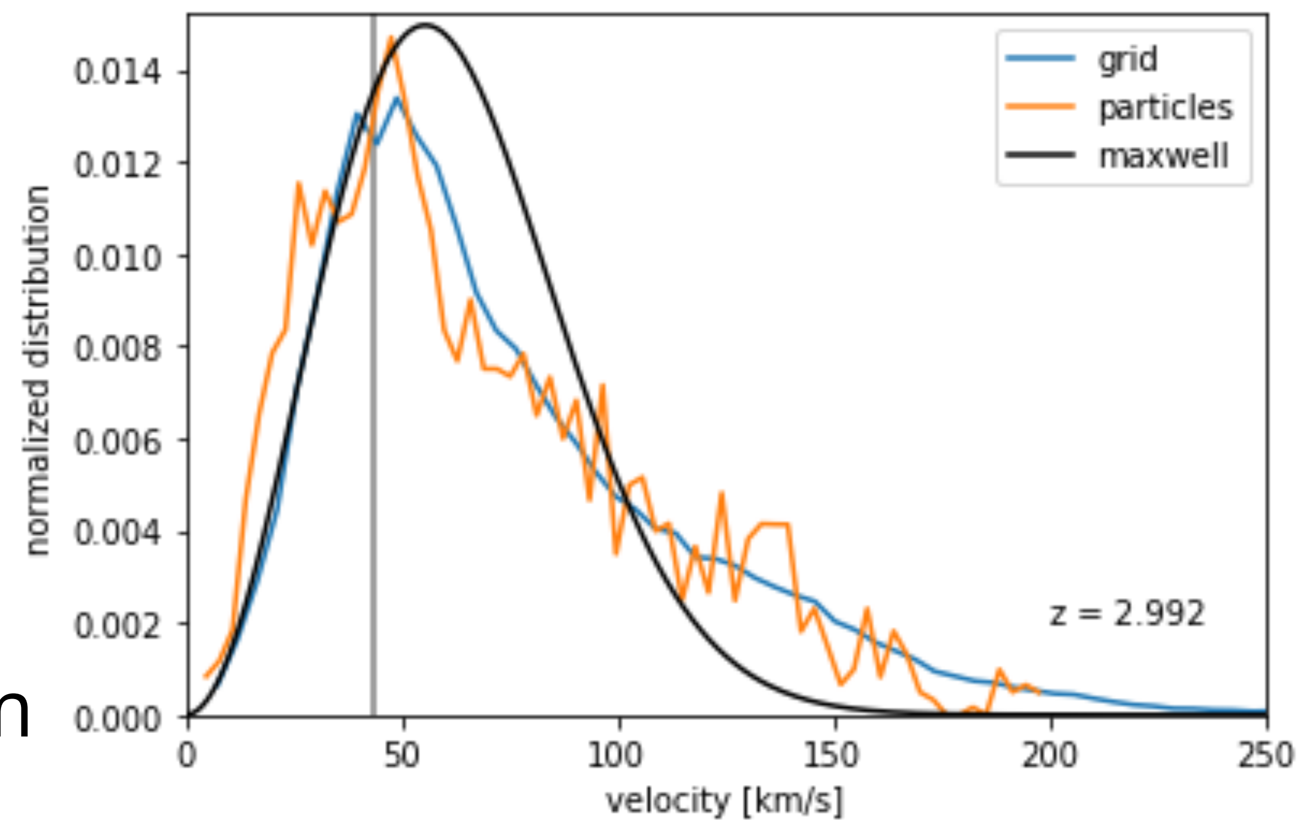
The AGORA halo in FDM (B. Schwabe)

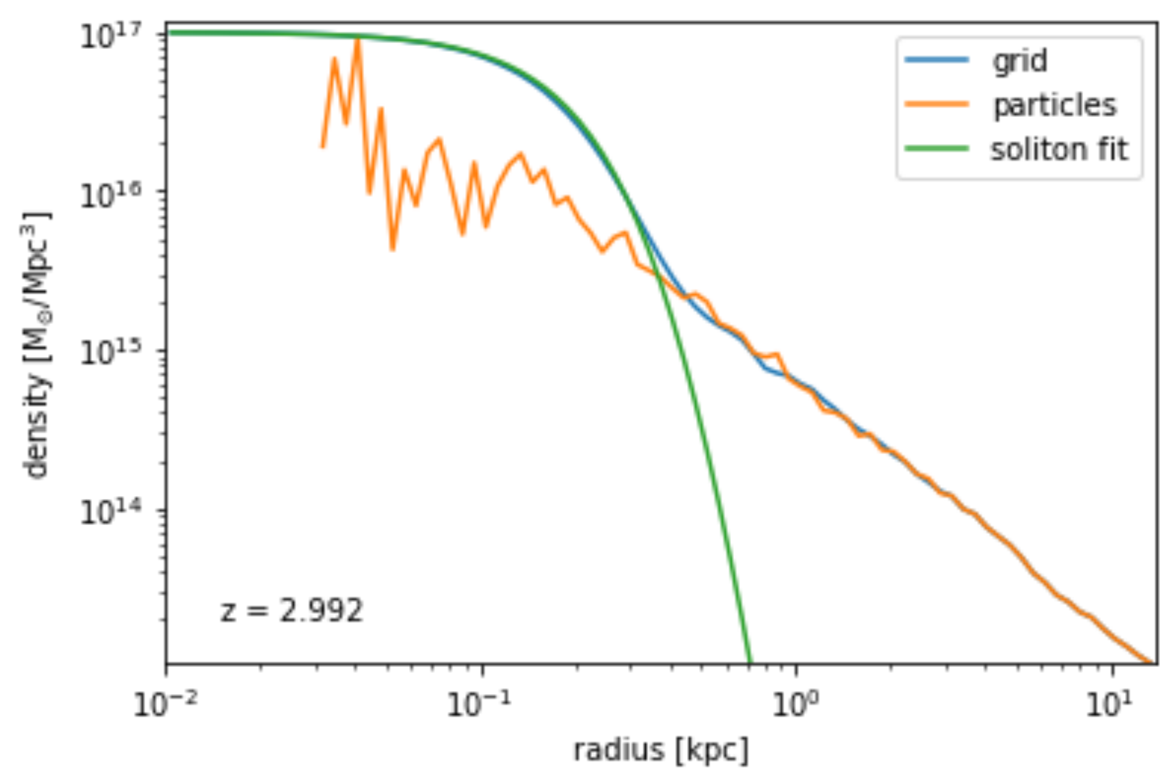
- Restart FDM N-body simulation at z=3:
- Reconstruct wavefunction at amr level 11 in the inner most halo region (virial radius at 50kpc)
- Add 3 finite difference levels



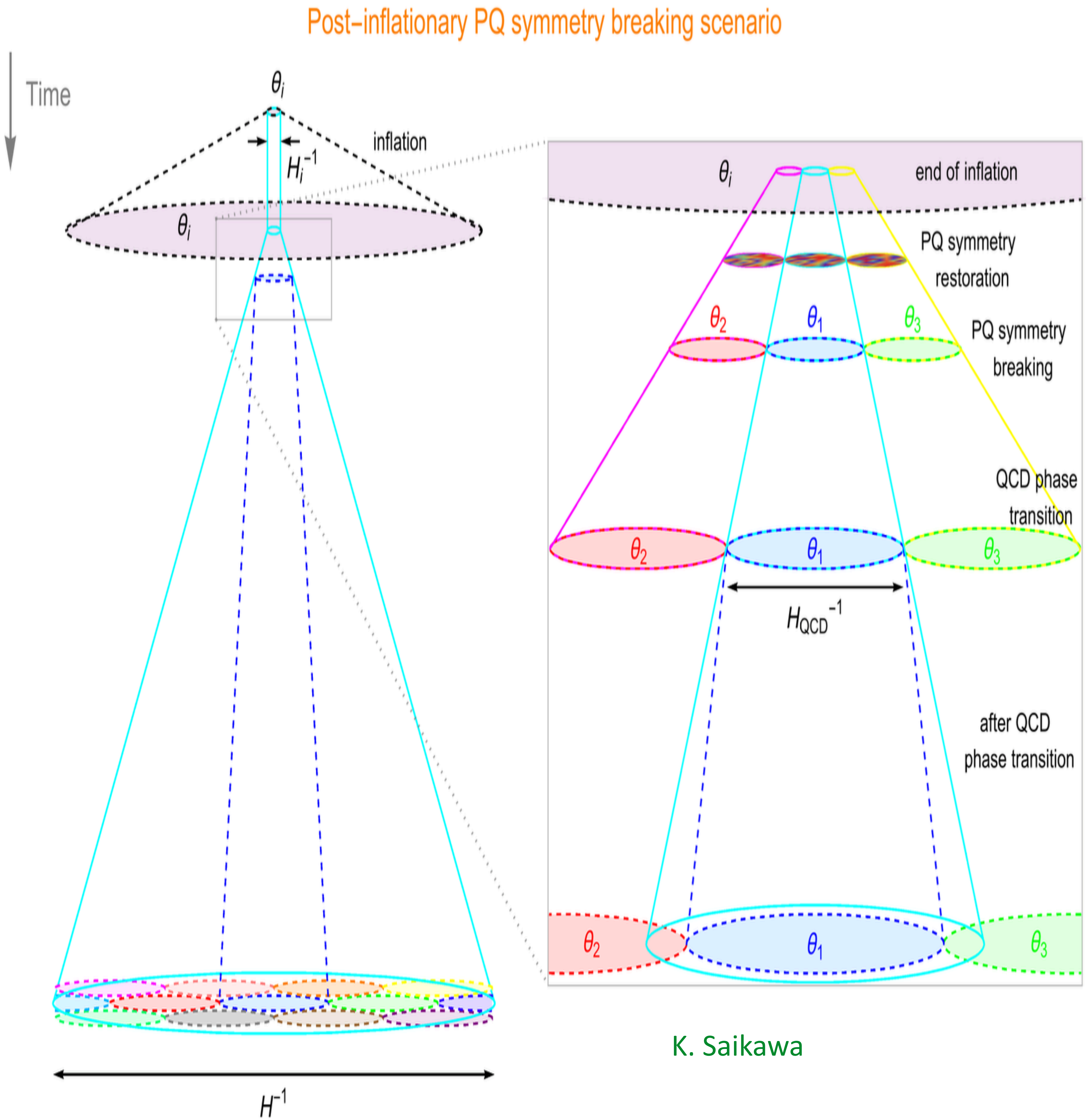
The AGORA halo in FDM (B. Schwabe)

- FDM Powerspectrum and underlying particle velocity dispersion correspond well to each other.
- Halo velocities have not yet relaxed into Maxwell spectrum (but do so later on)
- Soliton velocity (grey line) at peak in spectrum
- FDM radial density profile with soliton core and NFW outer tail





II. QCD axions



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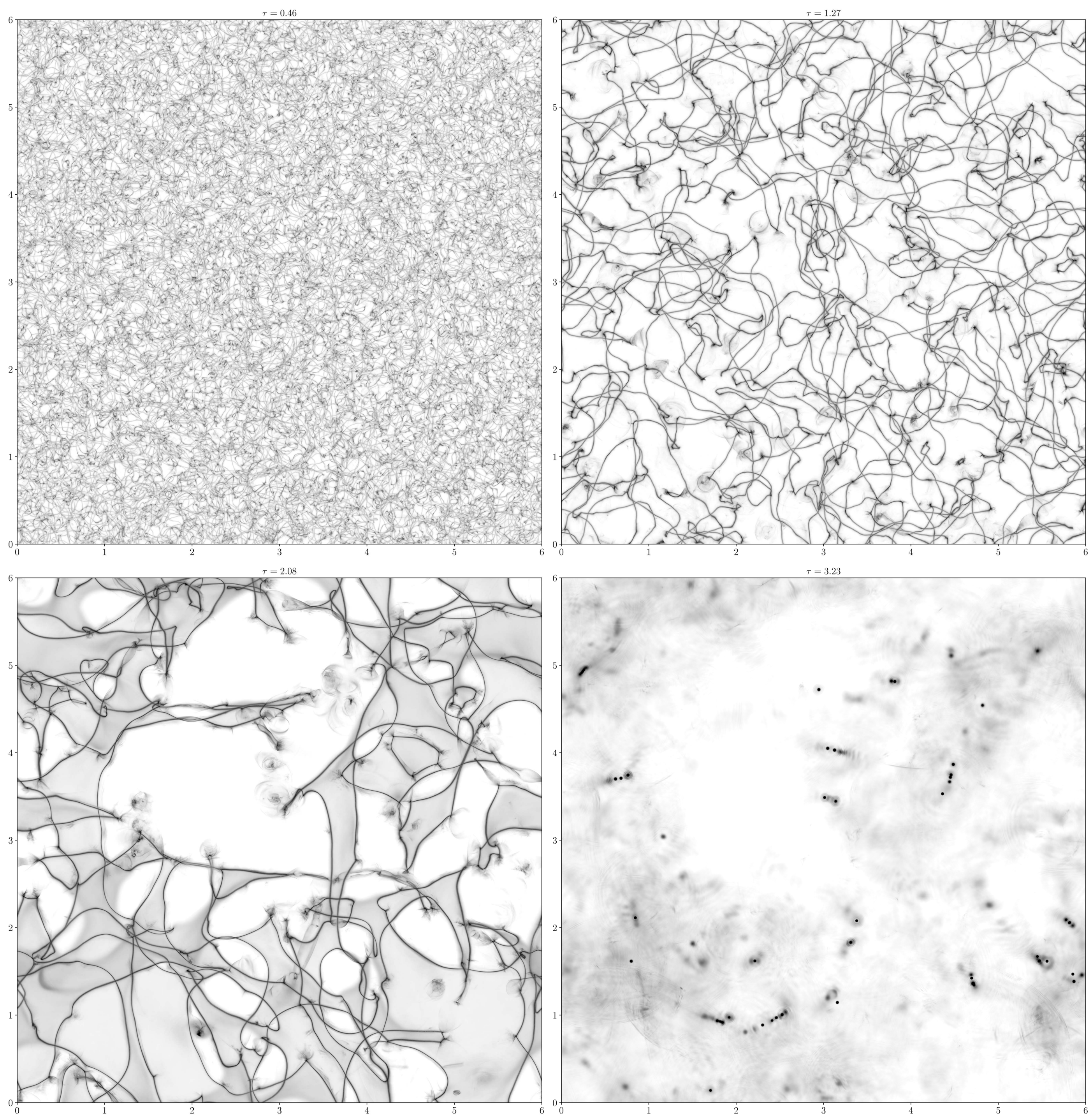
Formation of axion miniclusters

(see also talks by Javier Redondo and David Ellis)

N-body simulations of nonlinear density perturbations after QCD phase transition

1. Initial conditions from simulations of the complex axion field (Vaquero, Redondo, Stadler ´18):



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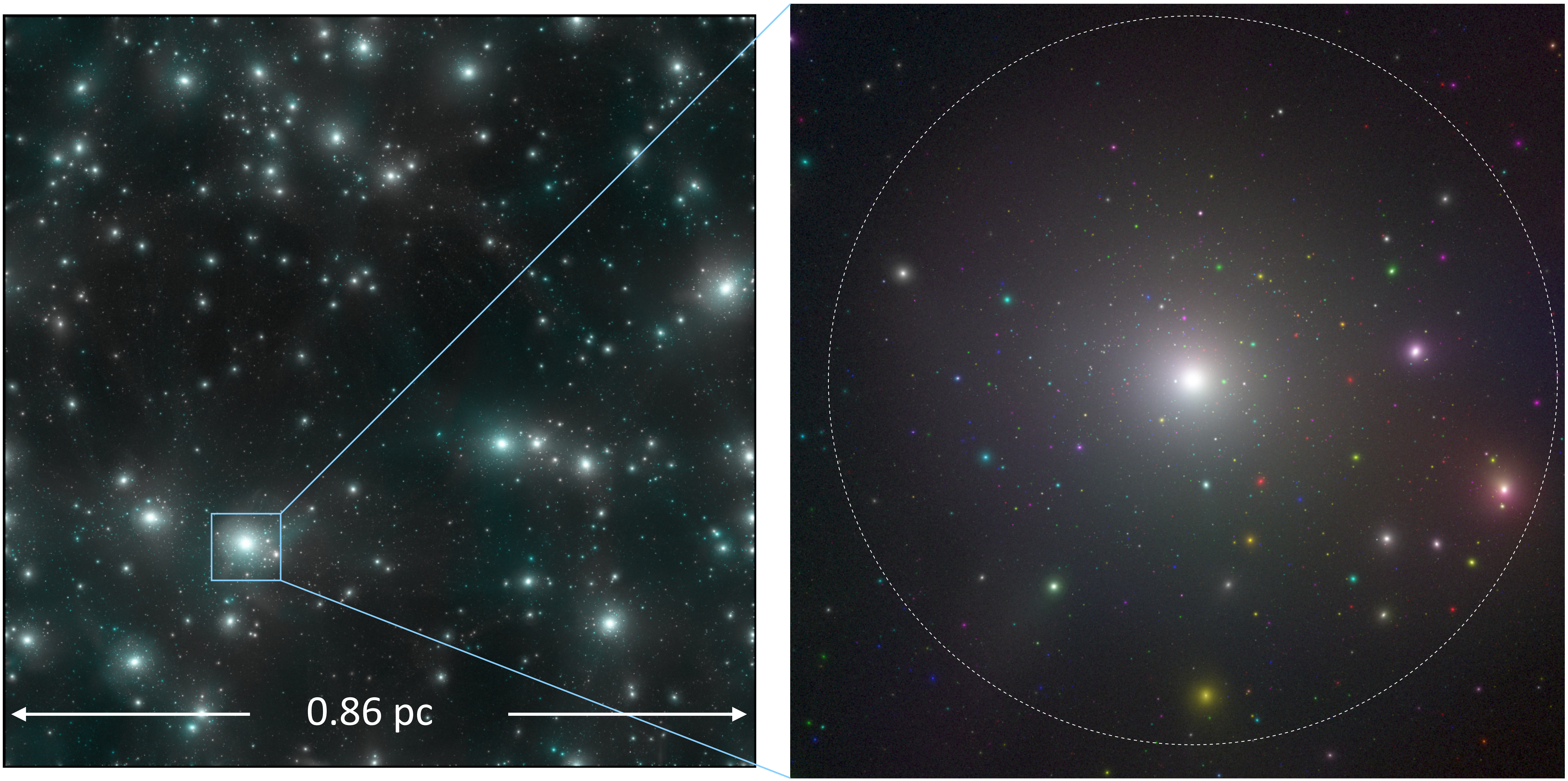
N-body simulations of nonlinear density perturbations after QCD phase transition

2. 1024^3 particle simulation of gravitational evolution (Eggemeier+ '20, PRL 125, 041301):



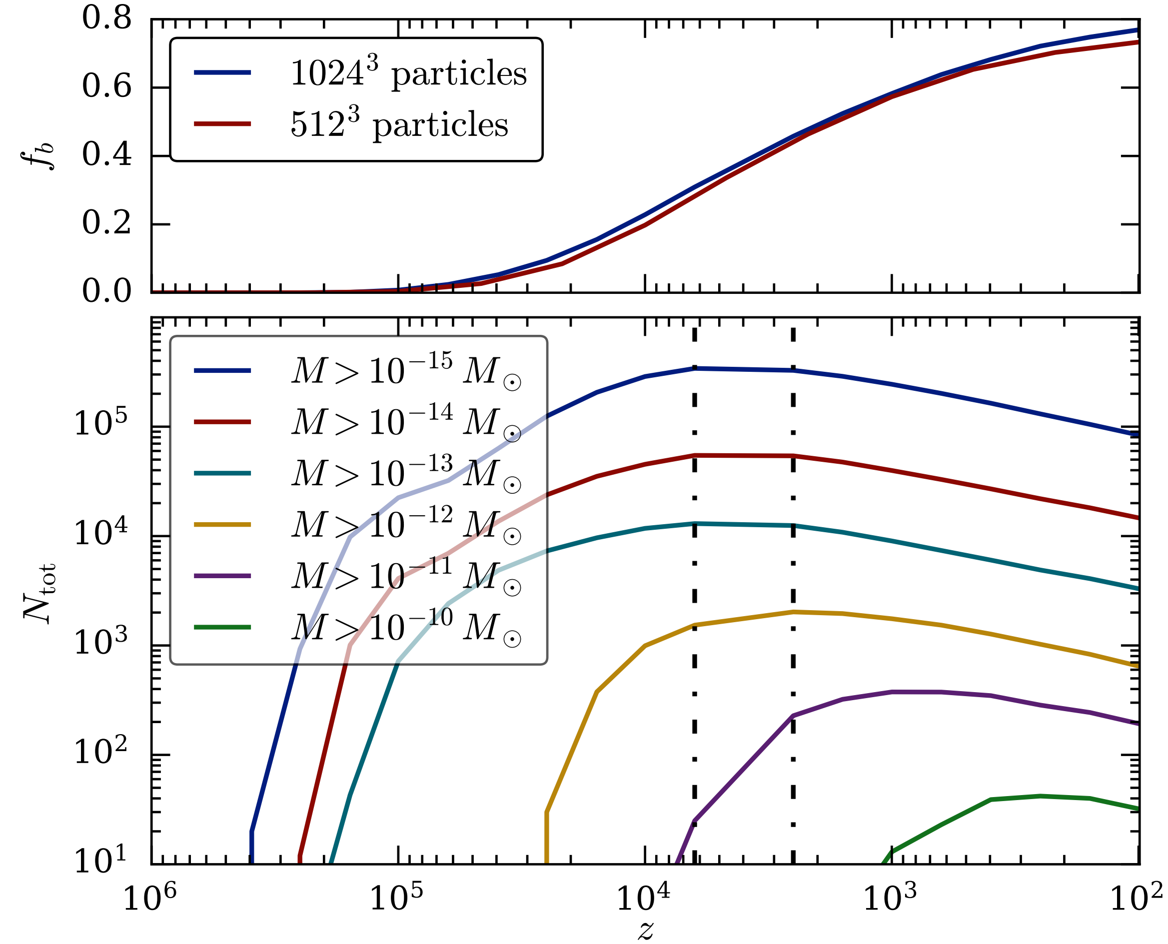
Small-box simulation challenge, part I

Formation of axion miniclusters

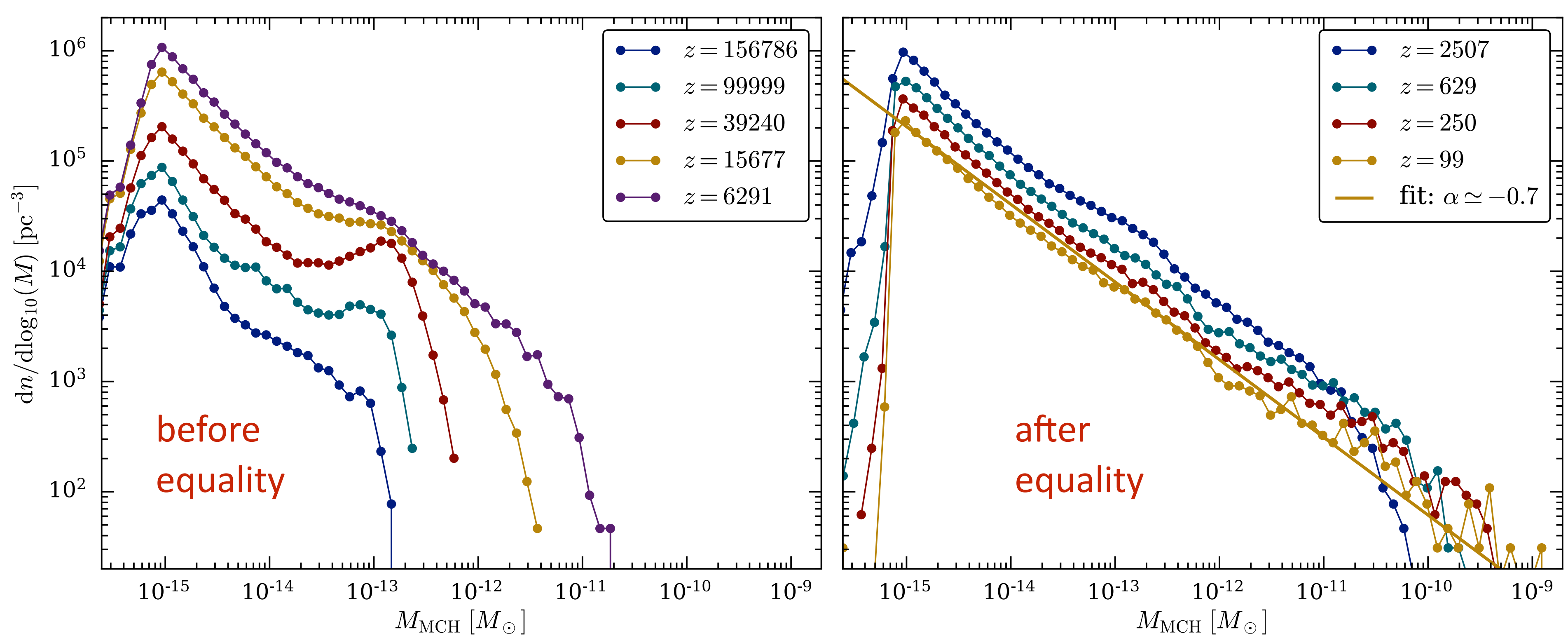
(Eggemeier+, ’20, PRL 125, 041301)

bound fraction:

number of miniclusters:

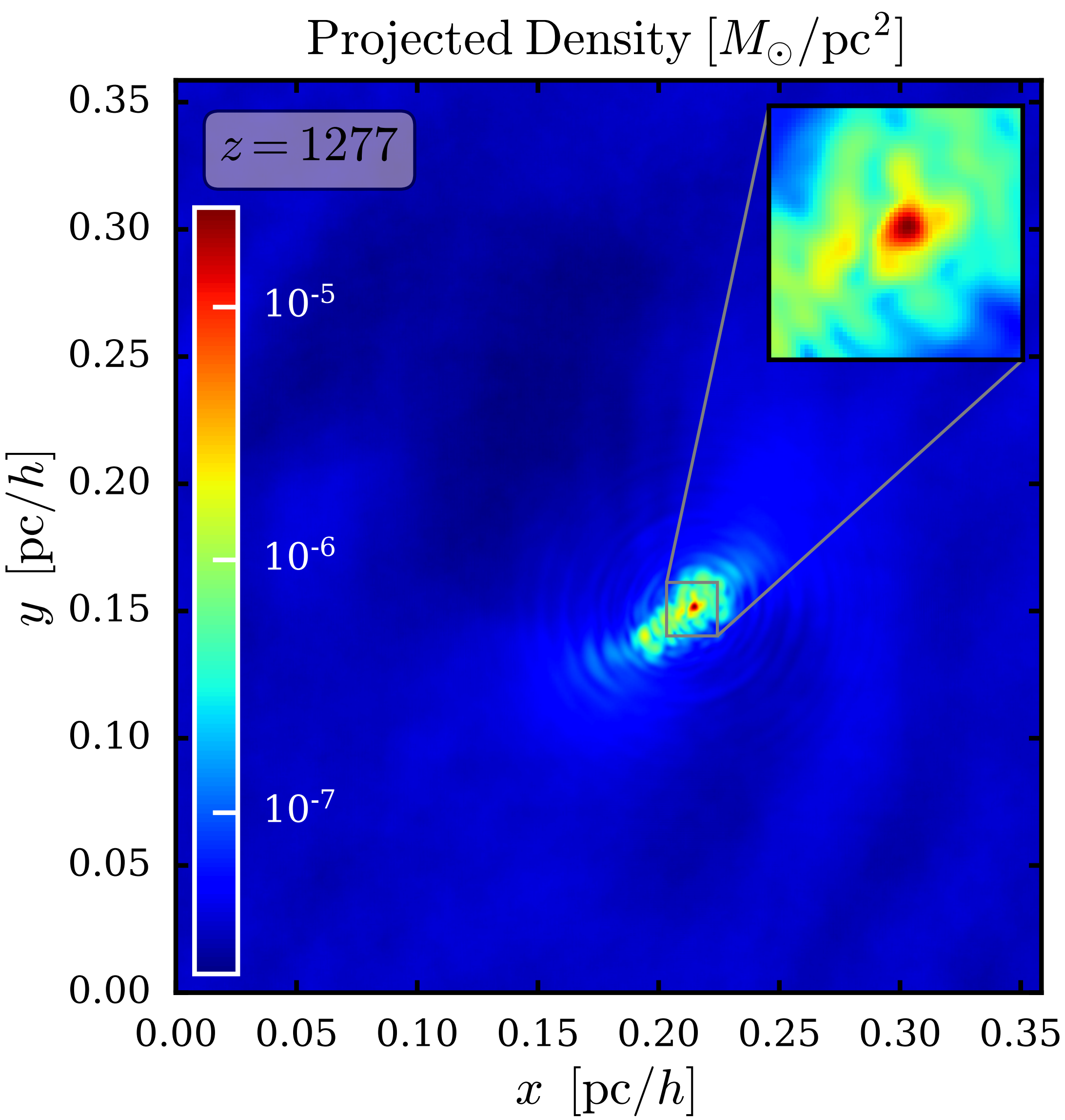


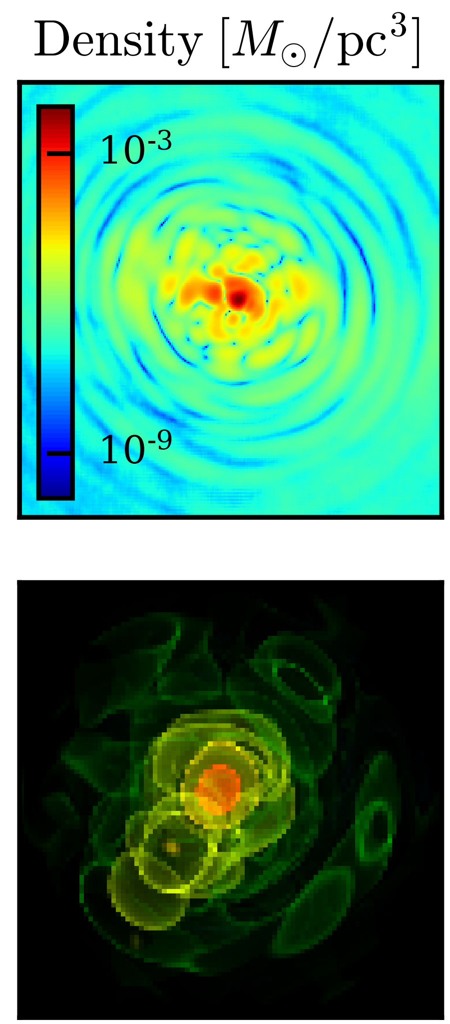
halo mass function:

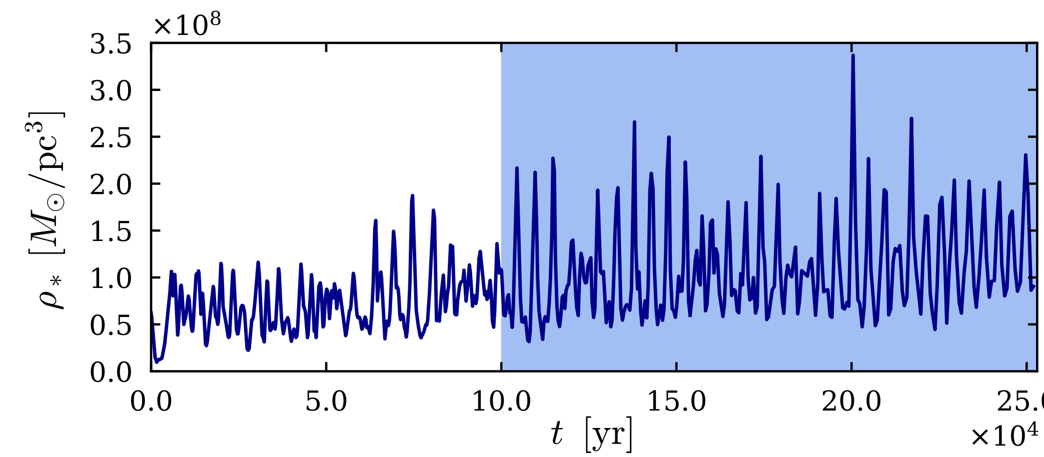


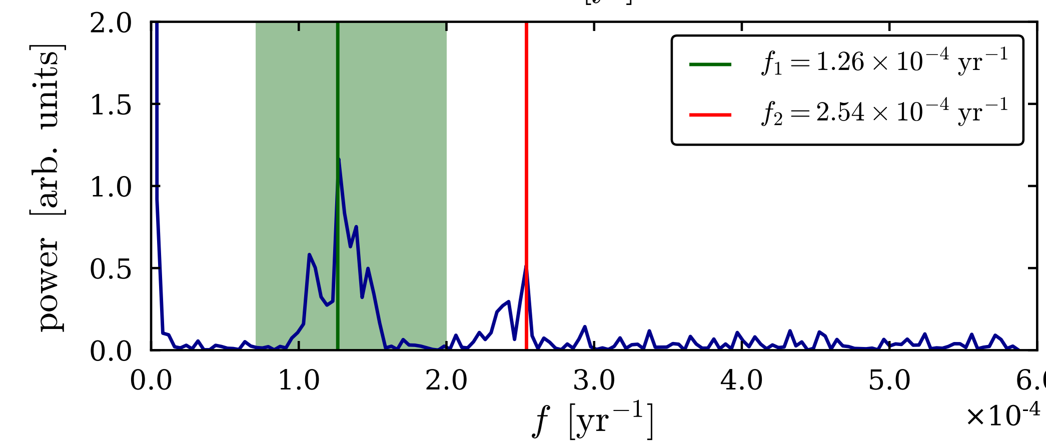
Axion star formation in miniclusters

(Eggemeier, JN ’19, PRD 100, 063528)



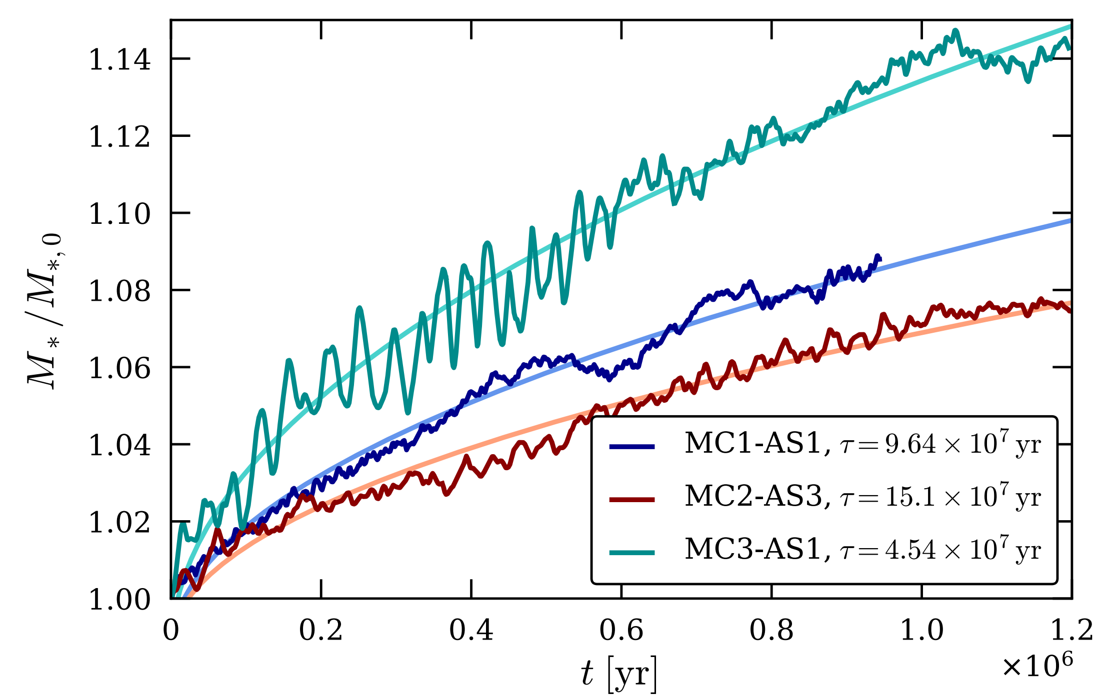






Mass grows as $t^{1/2}$ in agreement with wave condensation (Levkov et al. 2018)

prediction: growth ~ $t^{1/8}$ after reaching virial temperature of minicluster



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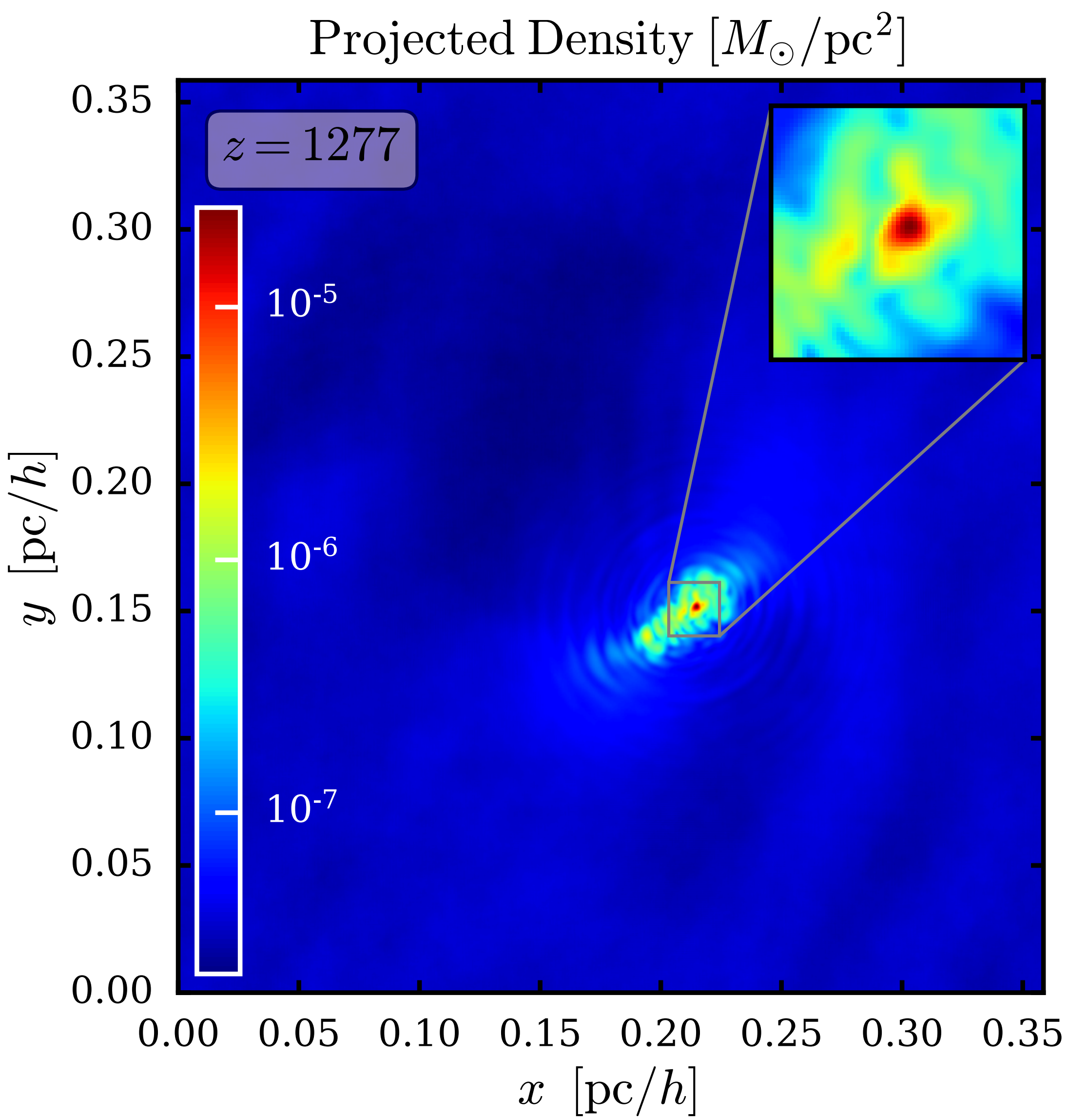
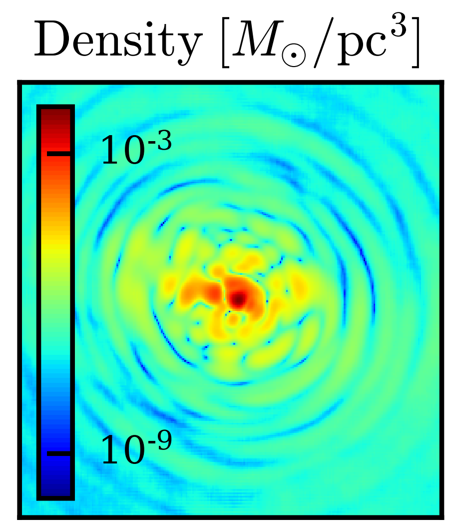
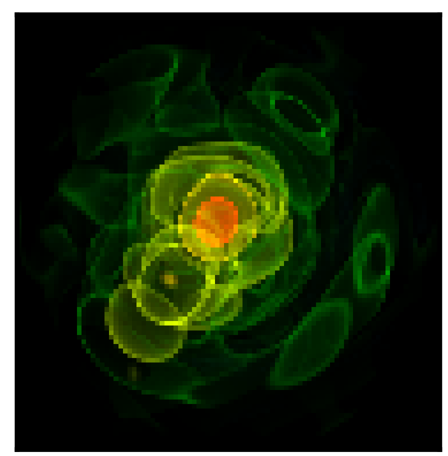
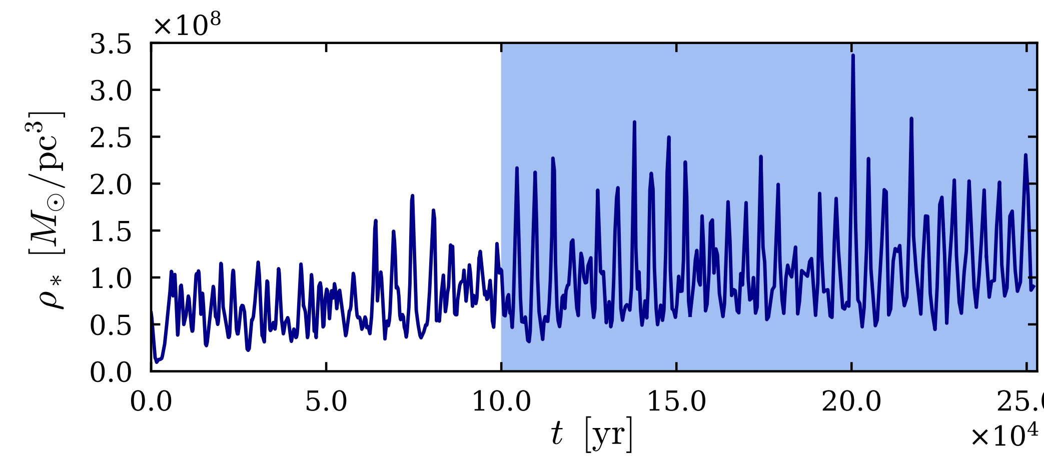
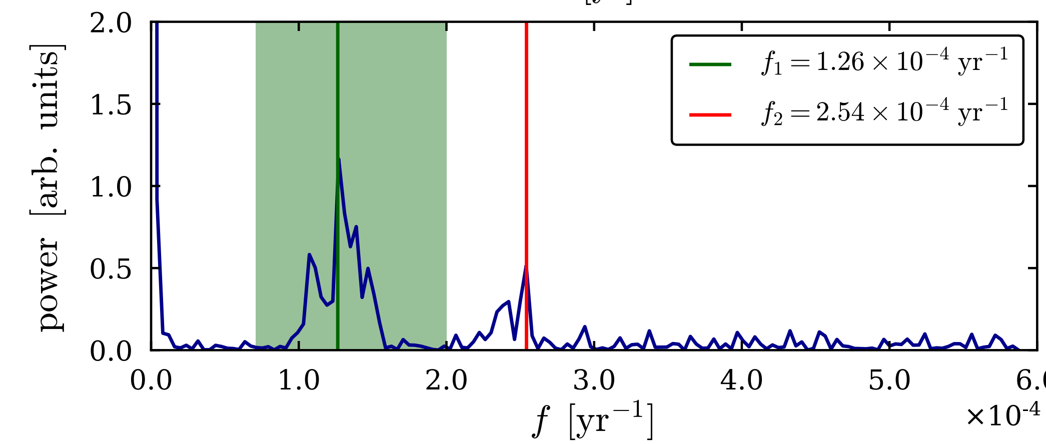
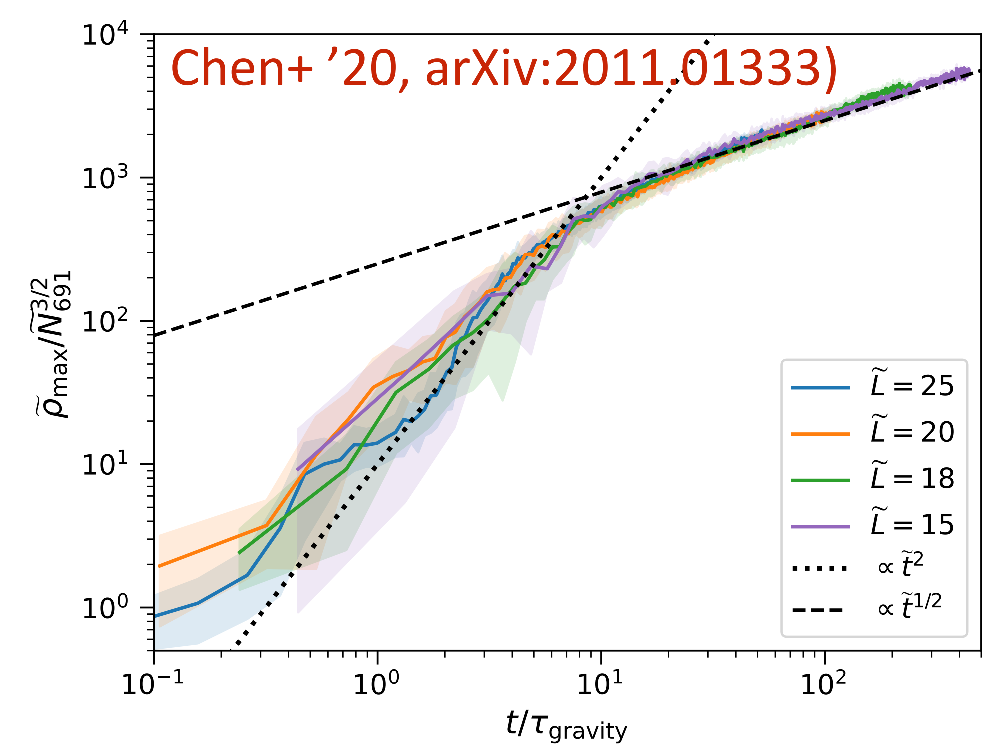










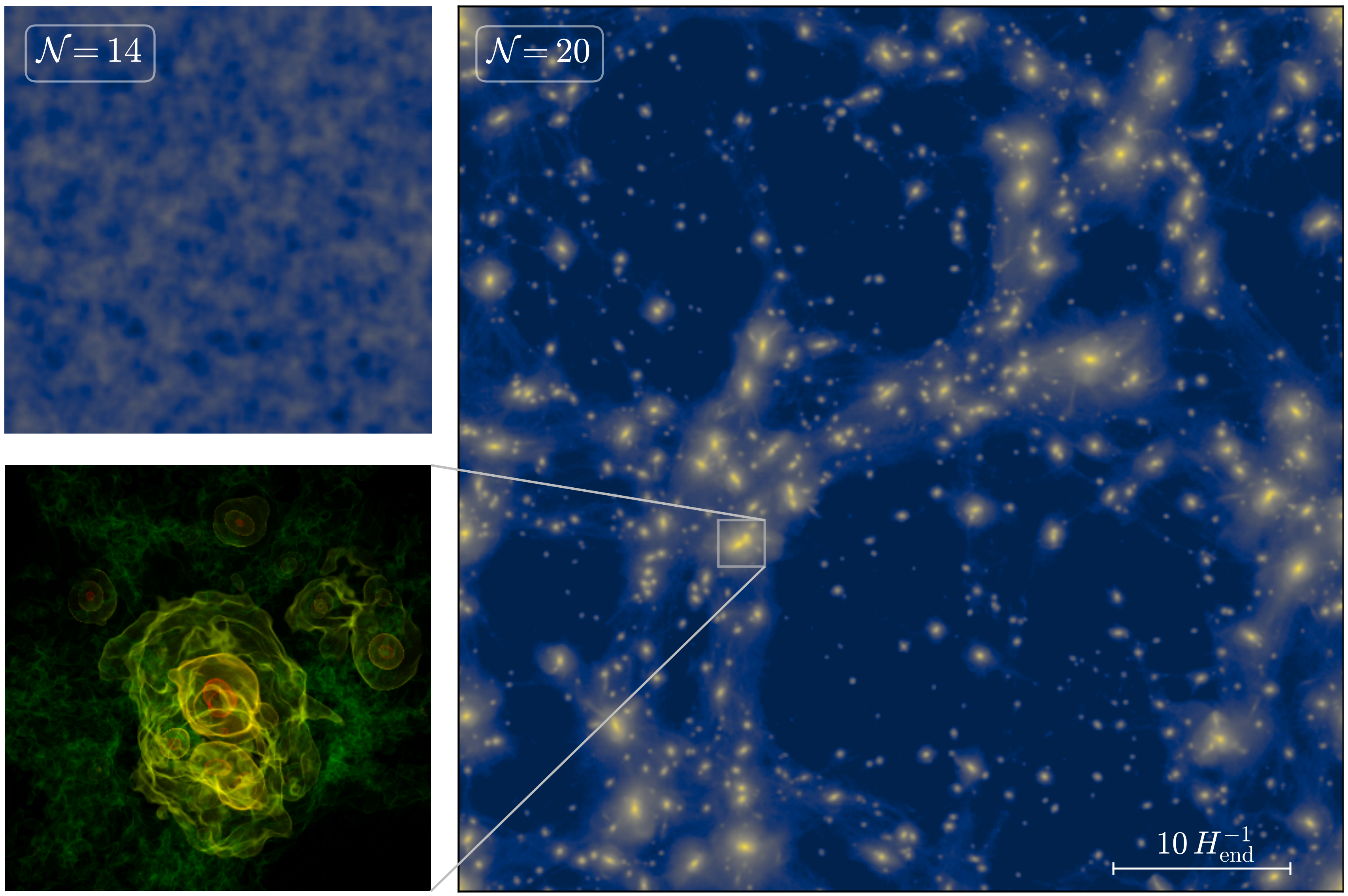


Mass grows as $t^{1/2}$ in agreement with wave condensation (Levkov et al. 2018)

confirmed: growth ~ $t^{1/8}$ after reaching virial temperature of minicluster

Gravitational structure formation during Early Matter Domination (B. Eggemeier)

Small-box simulation challenge, part II



- Early matter dominated phase after the end of inflation: inflaton condensate is cold, nonrelativistic, gravitationally unstable → SP equations work (Musoke+ ’19)

- Formation of inflaton clusters and inflaton stars (JN, Easther ’20; Eggemeier+ ’21)

N-body simulation with physical box size ~10^{-20} m

Gravitational structure formation during Early Matter Domination (B. Eggemeier)

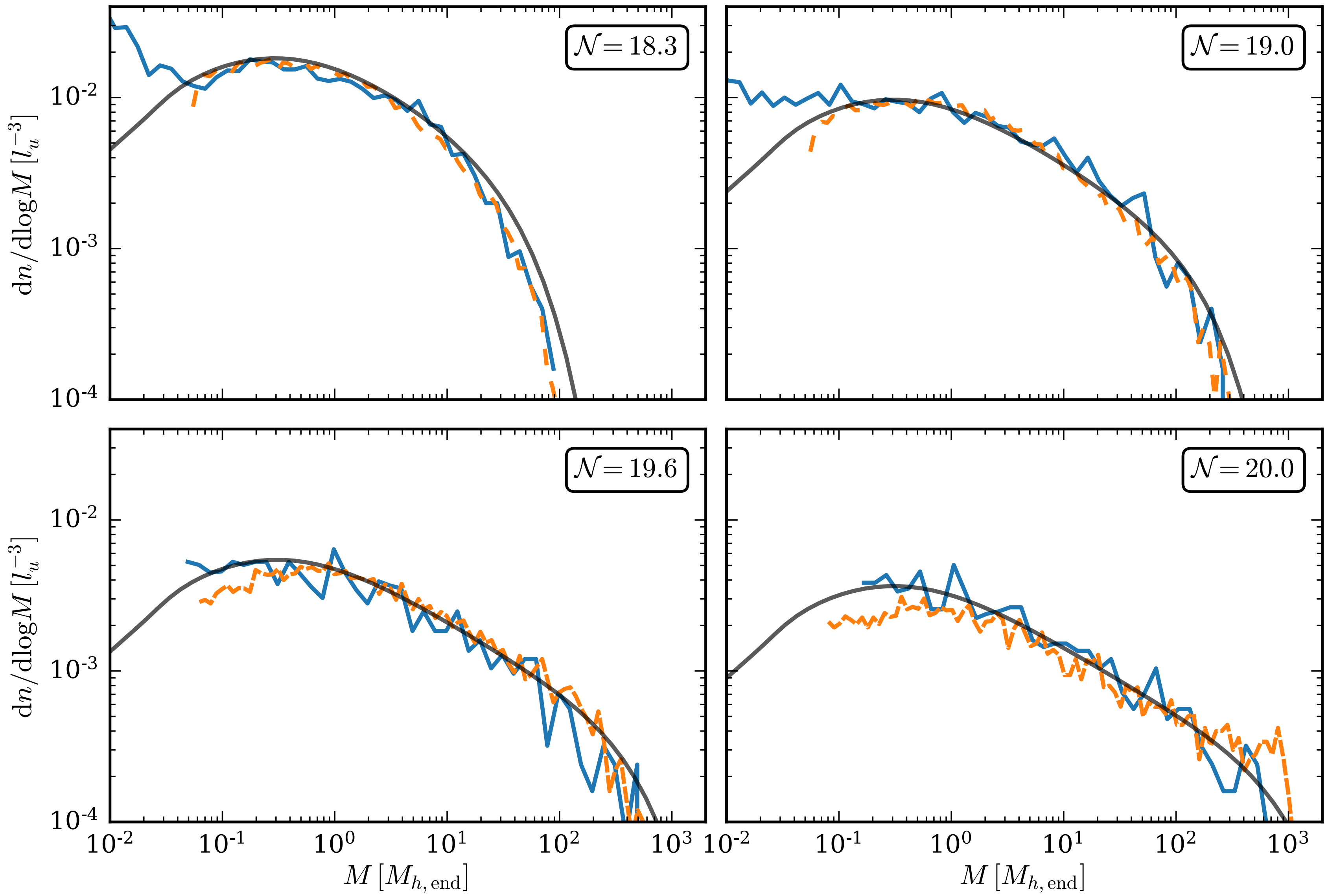
Inflaton Halo Mass Function

N-body:
spurious halos below $\sim 0.1\, M_{h,\mathrm{end}}$

Peak-Patch:
less low-mass and more high-mass halos than expected at $\mathcal{N} = 20$

Press-Schechter:

$$\frac{\mathrm{d}n}{\mathrm{d}\ln M} = \frac{1}{6}\frac{\bar{\rho}}{M}\nu f(\nu)\frac{\Delta^2(1/R)}{\delta_c^2}$$



Mass distributions in general agreement

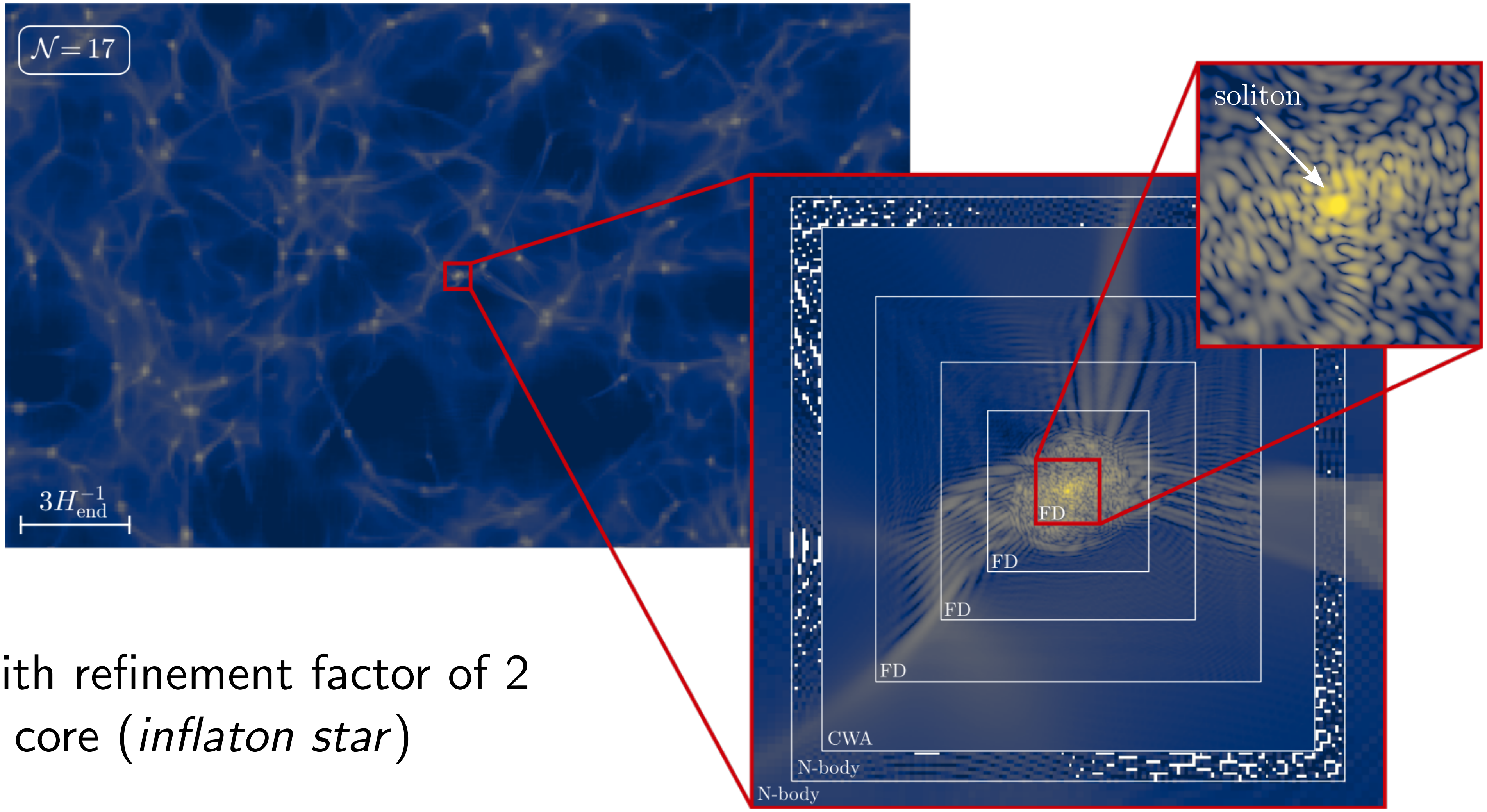
Gravitational structure formation during Early Matter Domination (B. Eggemeier)

Zooming into isolated halos

Hybrid simulations performed with **AxioNyx**: N-body on large scales, SP with FD on small scales
At the boundary: **Classical Wave Approximation** (CWA)

$L = 50\, l_u$, 512^3 root grid

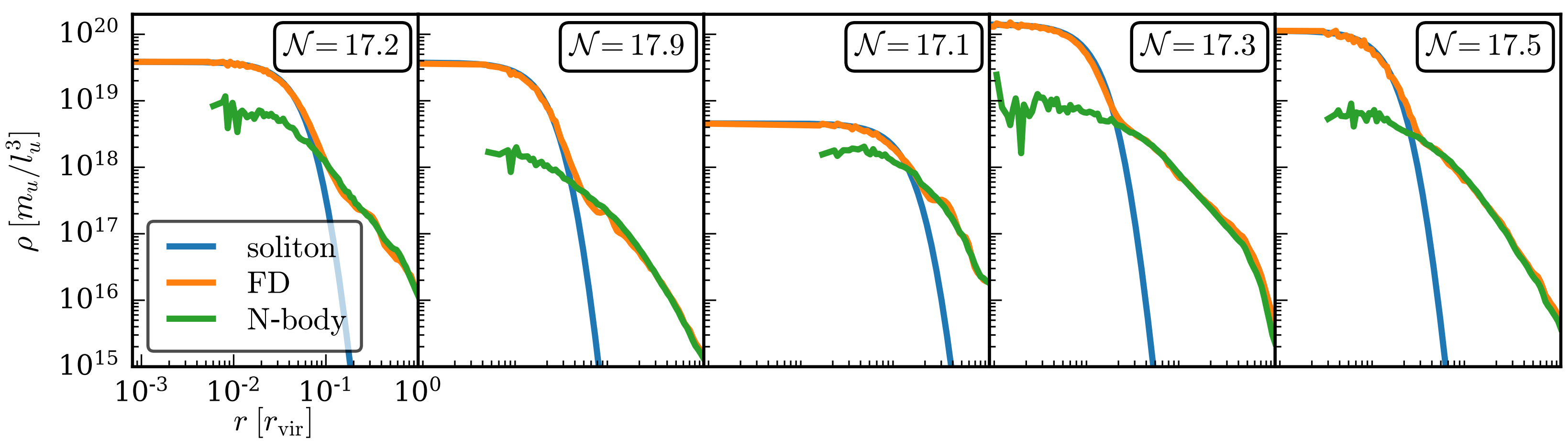
Same ICs as before but 2 additional static refinement levels centered on Lagrangian patch of selected halo



In total, up to 8 levels with refinement factor of 2
$\rightarrow$ formation of solitonic core (*inflaton star*)

Gravitational structure formation during Early Matter Domination (B. Eggemeier)

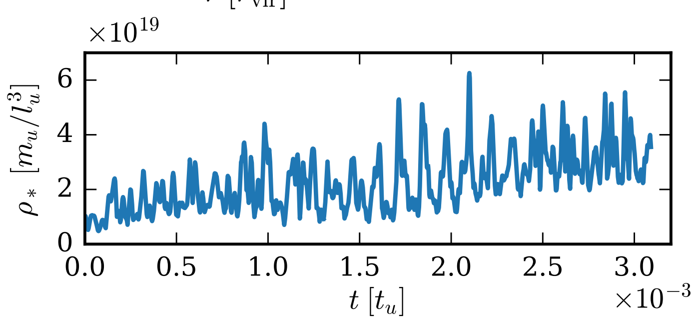
Inflaton Stars: profiles, oscillation, and mass growth

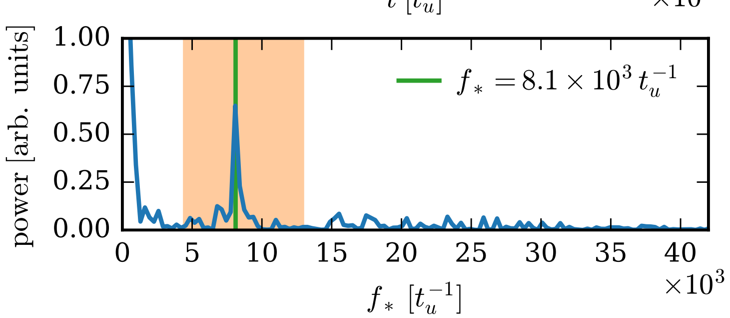


soliton profile:

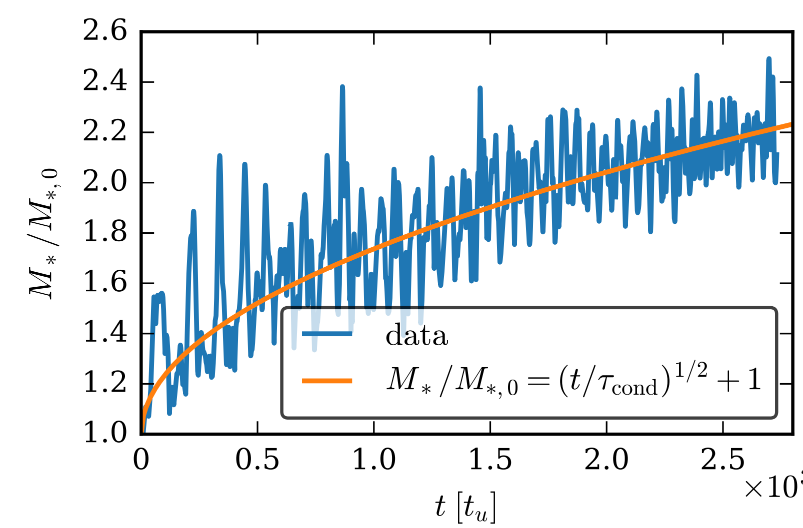
$$\rho_*(r) \simeq \rho_0 \left(1 + 0.091 \left(\frac{r}{r_*}\right)^2\right)^{-8}$$

core-halo relation: $M_* \sim M_h^{1/3}$





Results are consistent with previous FDM/ALP simulations



Quasinormal soliton mode:

$$f_* = 5.2 \times 10^4 \left(\frac{\rho_*}{10^{21}\, m_u/l_u^3}\right)^{1/2} t_u^{-1}$$

Mass growth determined by

$$\tau_{\rm cond} \simeq \frac{\sqrt{2}b}{12\pi^3} \left(\frac{m}{\hbar}\right)^3 \frac{v_{\rm vir}^6}{G^2 \rho^2 \log \Lambda}, \quad \Lambda \sim \frac{m r_{\rm vir}}{\hbar v_{\rm vir}}$$

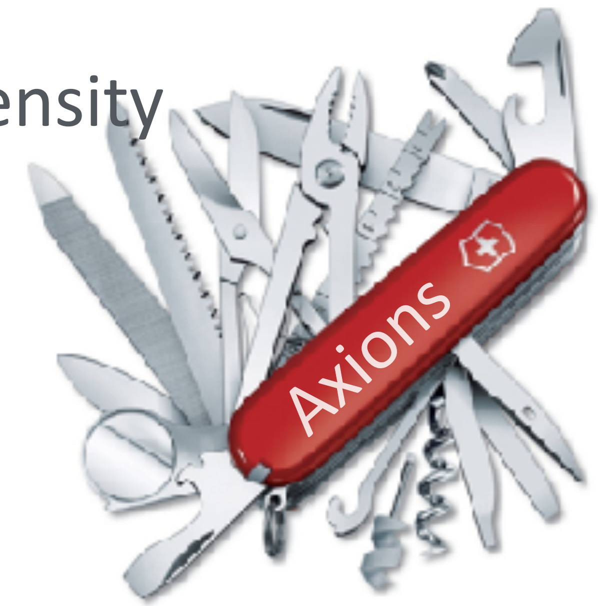
random final thoughts

- Physics of soliton (boson star) formation by classical wave condensation, mass saturation, core-halo relation etc. broadly understood

  important details still missing, e.g. is there always one soliton per halo? do they condense or remain as residuals of initial coherence? importance of oscillations?...

- You come for the solitons but stay for the granules

  lots of unexplored territory for relaxation effects by large density fluctuations, similarities to PBH constraints



- FDM is already an endangered DM species

  mass window closing, with Ly-alpha forest and gravitational heating on one side and BH superradiance on the other

  ...but you can always retrain and work on QCD axion stars or inflaton stars!