Primordial Black Holes from an early Matter Phase

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Quarks2021-Dark matter, 23 June 2021

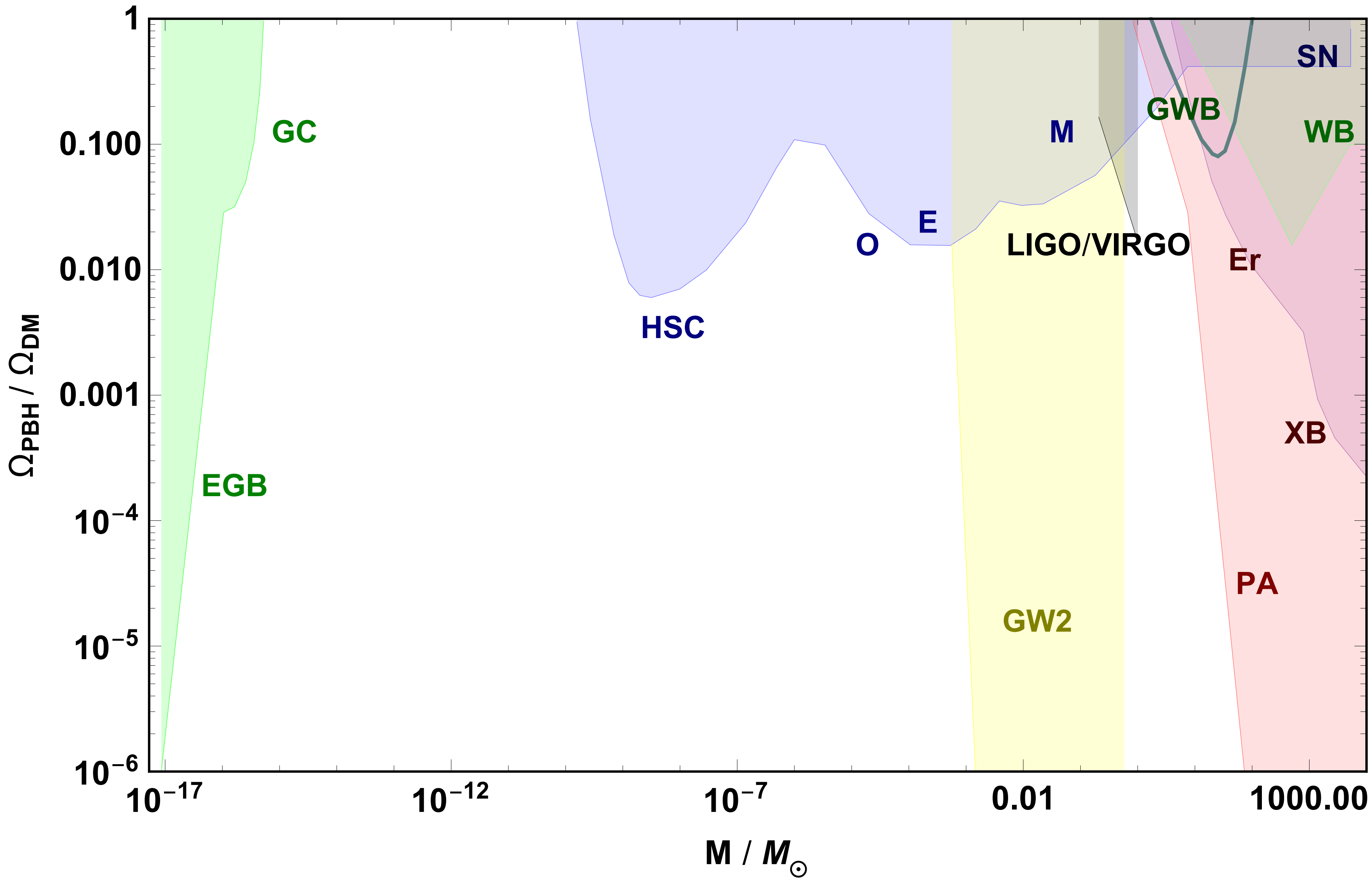
Dark Matter Candidates

- Axions & ALPs
- Sterile Neutrinos
- WIMPs
- Dark Atoms
- Mirror Dark Matter
- WIMPzillas
- MACHOs
- Primordial Black Holes
- ???

Primordial Black Holes

- PBH=BH formed before recombination (z>>1100)
- Formation can take place in radiation domination or an early matter domination (Khlopov Polnarev '80)
- BH formation within a Hubble region $\delta\rho/\rho \sim 1$ (Carr 1975)
- Large perturbations could be provided in specific inflation models (Carr, Lidsey 1991)
- Cold Dark Matter candidate
- Potential solution to the supermassive BH problem (Bean,Magueijo 2002)

PBH Constraints



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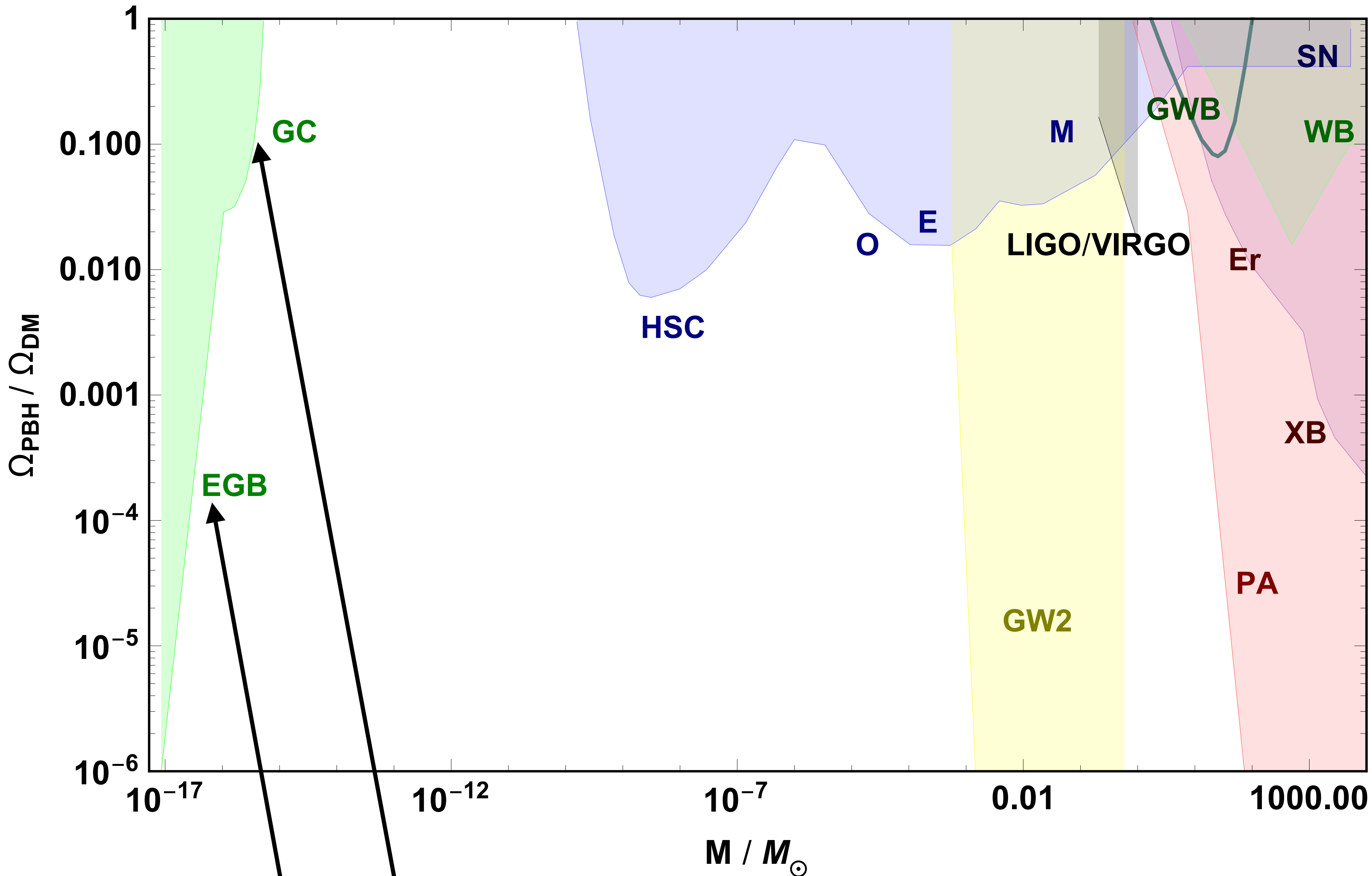
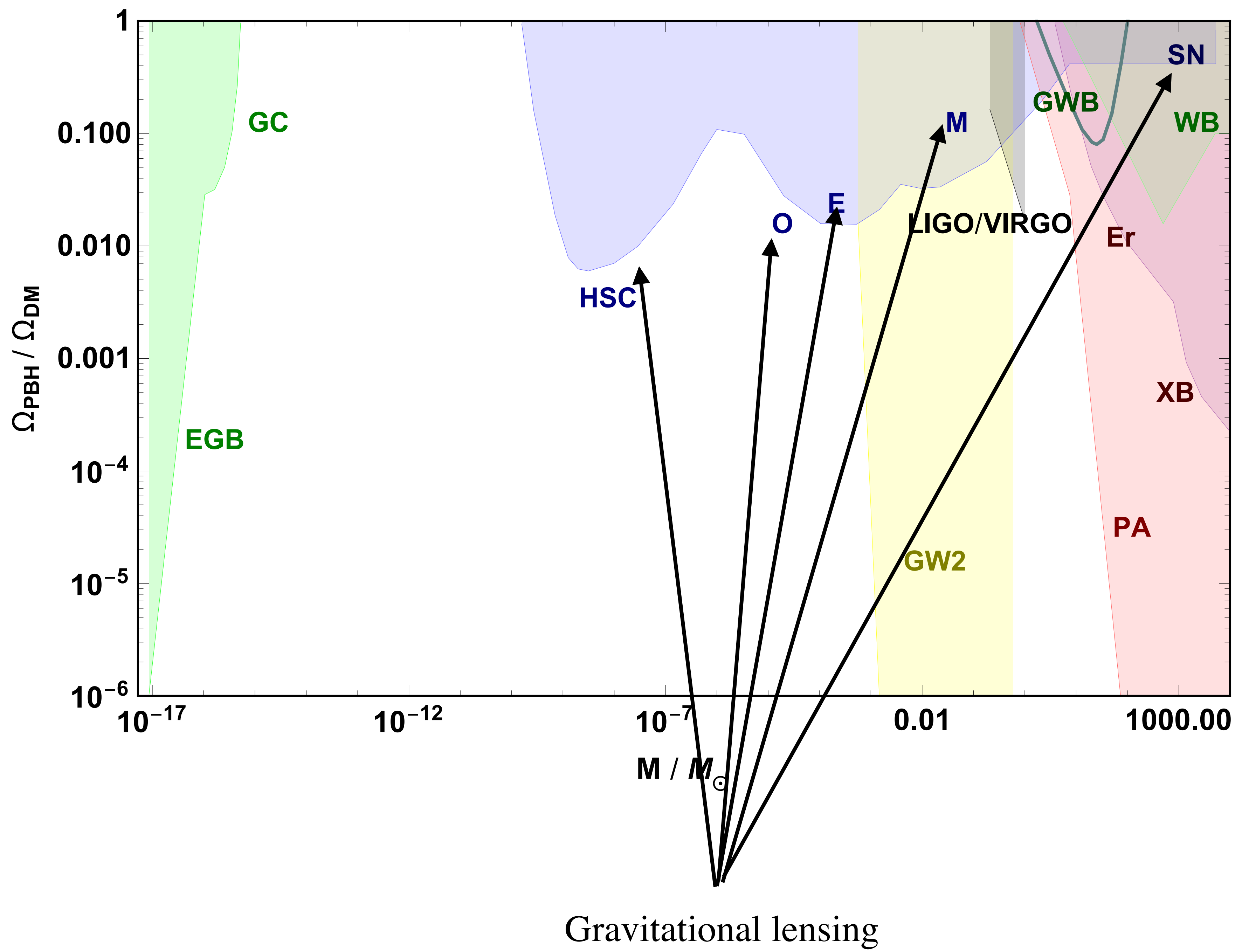


e± from galactic center

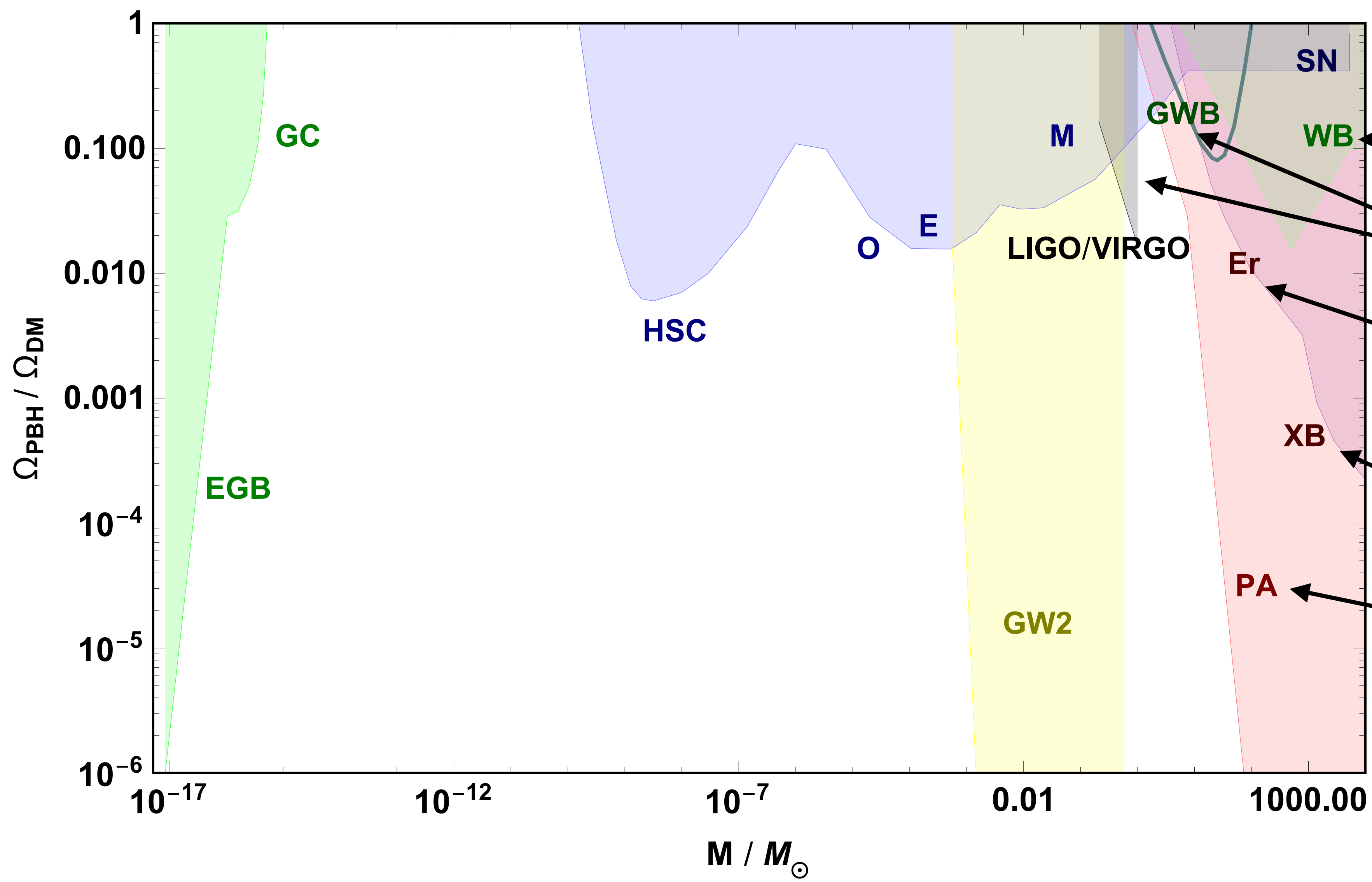
Extra galactic γ-rays

For M<10^13 grams distortions in CMB and BBN constraints

PBH Constraints



PBH Constraints



Wide binary disruption

GW

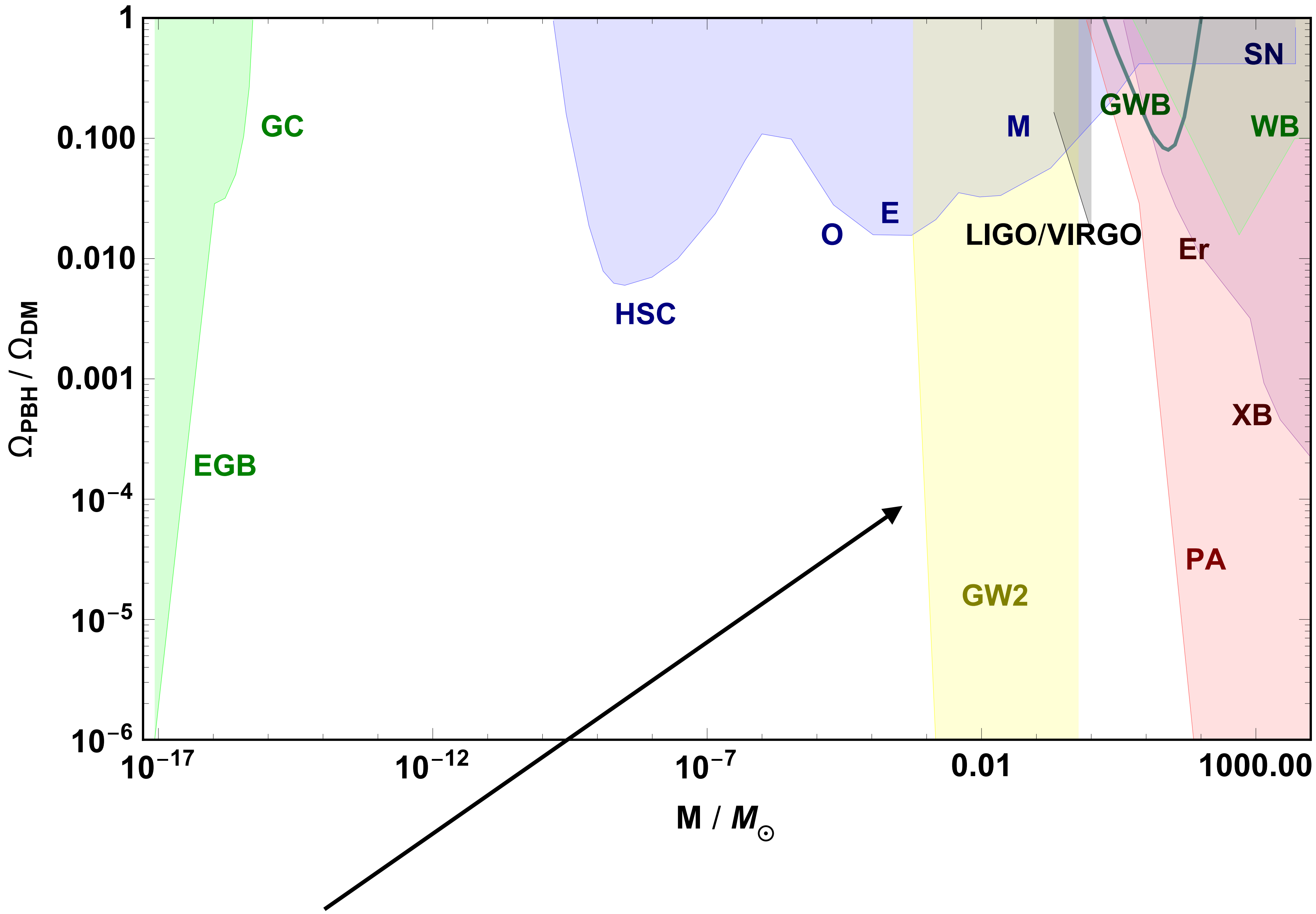
Eridanus II stellar cluster disruption

X-ray binary

CMB Planck constraints for M>10^33 grams

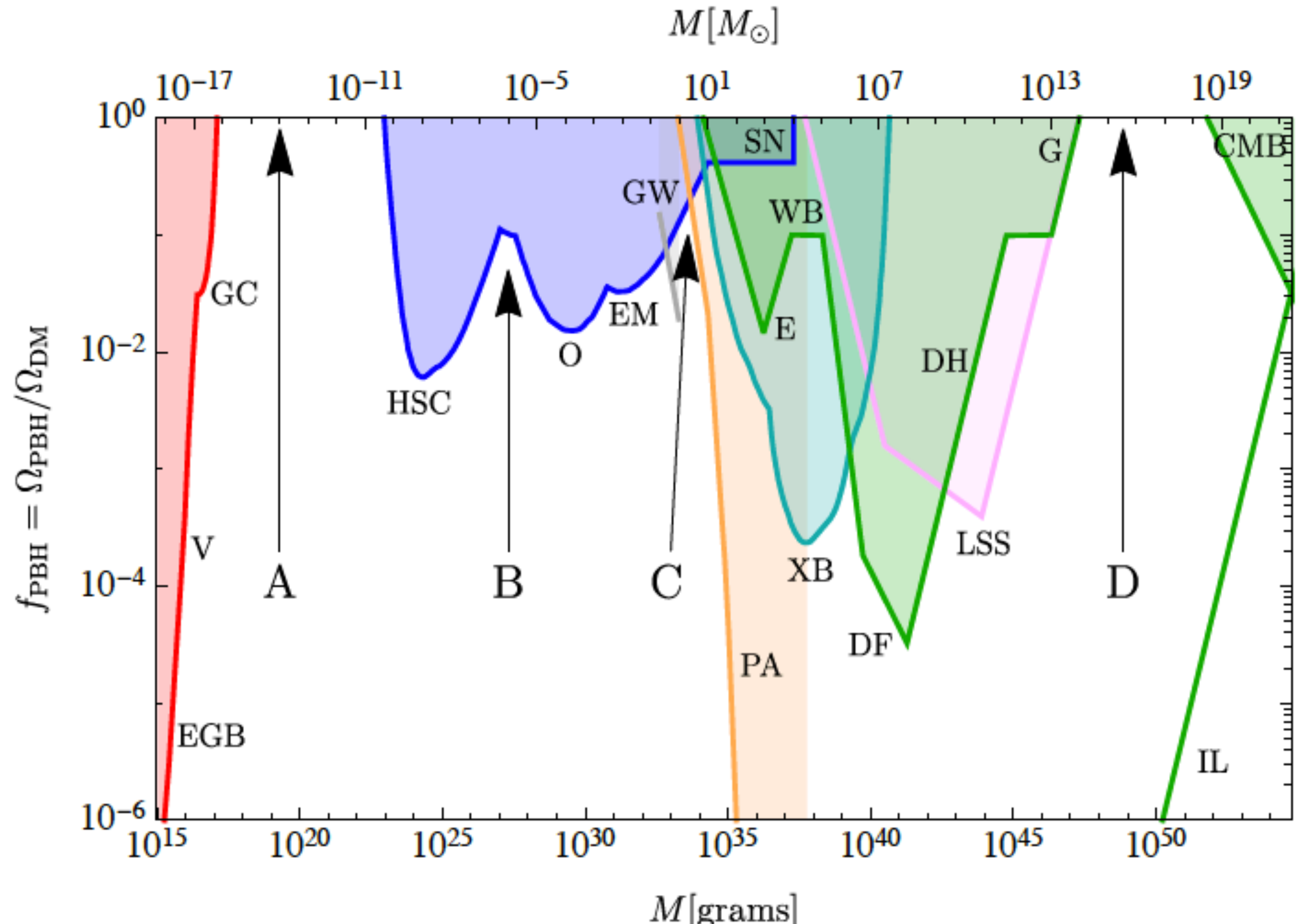
Accretion produces radiation that ionizes atoms thus changing the opacity

PBH Constraints



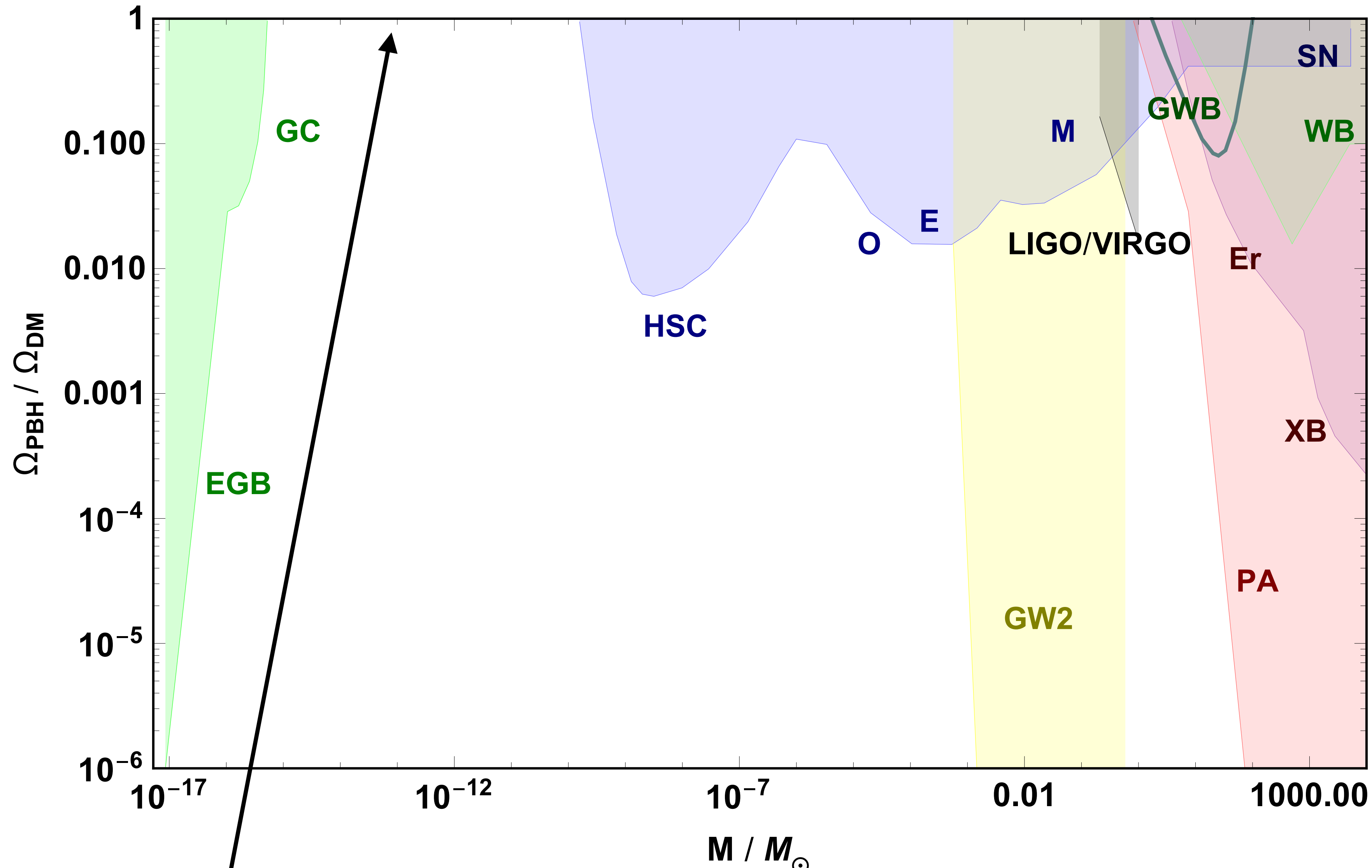
Pulsar Timing Array

PBH Constraints



Carr Kuhnel 2020

PBH Constraints



What about this region?

- Femtolensing constraints M<10^-13 Msun are disputed because the Gamma Ray Bursts sources are not point-like.

- There have been constraints from PBH destroying neutron stars. PBH can hit a NS and get trapped by losing energy via dynamical friction.

Destroying Neutron Stars

As it crosses the neutron star, the PBH should lose enough energy to become gravitationally bound to the star

Two sources of energy loss:

- Matter Accretion
- Dynamical Friction

$$E_{\rm loss} = \frac{4G^2 m_{\rm BH}^2 M}{R^2} \left\langle \frac{\ln \Lambda}{v^2} \right\rangle$$

$$t_{\rm loss} \simeq 4.1 \times 10^4 {\rm yr} \left(\frac{m_{\rm BH}}{10^{22}{\rm g}} \right)^{-3/2}$$

Capela Pshirkov Tinyakov '13,
Loeb Pani '14

The effect of Rotation I

The accretion is never perfectly spherical because the neutron star rotates usually with high frequencies.

The conditions for Bondi accretion are valid as long as the angular momentum of an infalling piece of matter is much smaller than the keplerian one in the innermost last stable orbit

The mass of the black hole must be larger than

$$M_{\rm crit} = \frac{1}{12^{3/2}} \left(\frac{3}{4\pi \rho_c} \right)^2 \left(\frac{\omega_0}{G} \right)^3 \frac{1}{\psi^3} \qquad M_{\rm crit} = 2.2 \times 10^{46} P_1^{-3} \ {\rm GeV}$$

CK, Tinyakov '13

viscosity of nuclear matter saves Bondi

$$\frac{\partial}{\partial t} l - \frac{C_0 M^2}{4\pi \rho r^2} \frac{\partial}{\partial r} l = \frac{1}{\rho r^2} \frac{\partial}{\partial r} \left[\rho \nu r^4 \frac{\partial}{\partial r} \left(\frac{1}{r^2} l \right) \right].$$

It subtracts angular momentum at the initial stage where the black hole is still small

in the final stages Bondi accretion is not valid but the star is seconds away from destruction!

The effect of Rotation II

A maximally spinning black hole will stop the accretion

$$a = J/GM^2$$

$$\frac{1}{a}\frac{da}{dt} = \frac{1}{J}\omega_0 r_s^2 \frac{dM}{dt} - \frac{g(a)}{G^2M^3} - \frac{2}{M}\frac{dM}{dt}$$

$$a_{\rm max} = 2 \times 10^{-23} T_5^4 / P_1^{10}$$

The PBH spins up and at the last stages it spins down

Temperature Considerations

Radiation from in falling matter can in principle impede further accretion in two ways:

Reduce viscosity

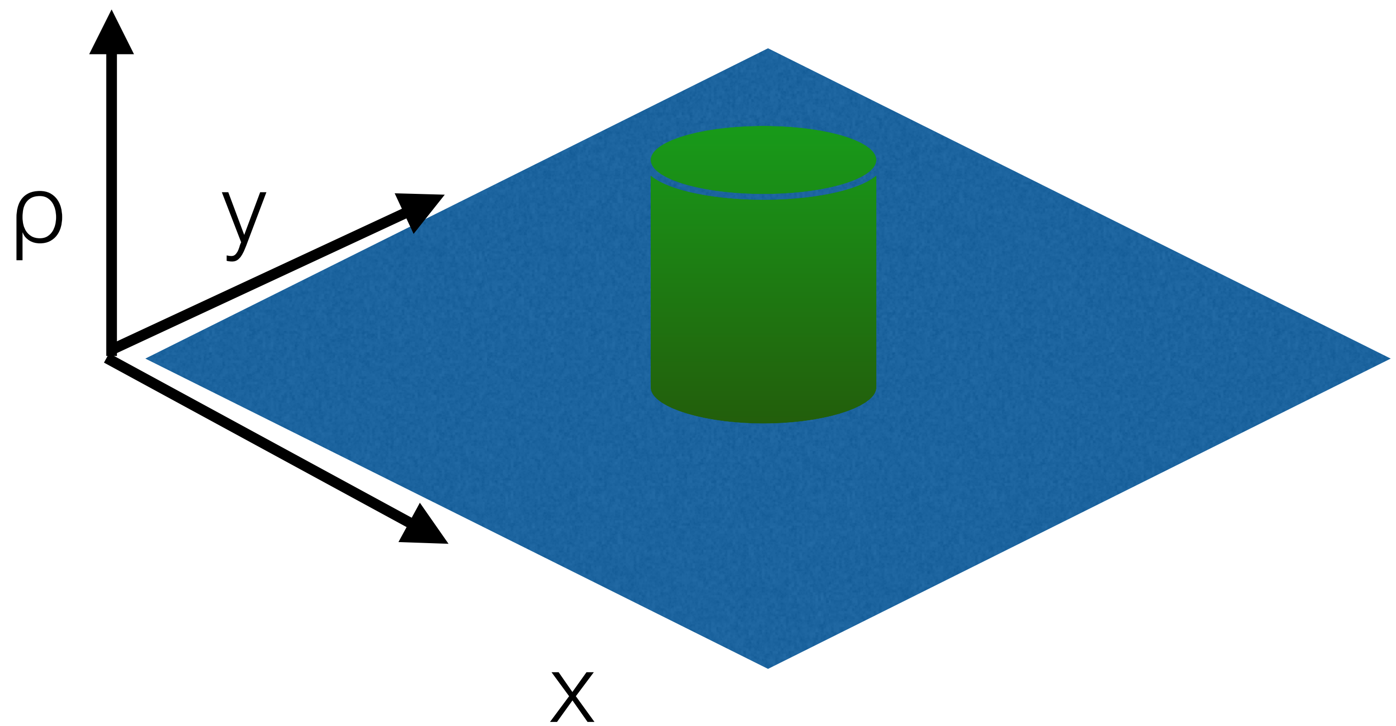
Increase radiation pressure

e-e Bremsstrahlung close to the horizon is the dominant radiation mechanism

$$\epsilon = \frac{L_{ee}}{dM/dt} \simeq 5 \times 10^{-12} T_5 \left(\frac{M}{M_0}\right) \qquad \delta T = \frac{L_{ee}}{4\pi k r} \simeq 458 \left(\frac{M}{M_0}\right)^2 \left(\frac{r_B}{r}\right) \mathrm{K}$$

Punchline: the region around 10^-12 solar mass PBH is probably open

Top Hat Spherical Collapse



- The Friedmann Equation gives

$$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = \frac{2GM_i}{r} - \mathcal{K} \qquad \mathcal{K} = 8\pi G\rho_{\mathrm{H}}(t_i)r_i^2\,\delta\,/3$$

$$r(\theta) = \frac{GM_i}{\mathcal{K}}(1-\cos\theta)$$

$$t(\theta) = \frac{GM_i}{\mathcal{K}^{3/2}}(\theta-\sin\theta)$$

- $\theta=\pi$ maximum expansion
- $\theta=2\pi$ collapse

$$1+\delta_{\mathrm{max}} = 5.55 \qquad \delta_{\mathrm{lin}} = 1.062$$

Formation of PBH

- Actual perturbations are not completely spherical

- To form a PBH, the hoop conjecture should be satisfied

$$\mathcal{C} \lesssim 2\pi r_s$$

- In a RD Universe, the pressure plays a double role:
1. It makes the collapsing perturbation more spherical, so it easier the satisfy the hoop conjecture
2. The pressure impedes the collapse, so large perturbations are needed in order for M to be larger than the Jeans mass.

- In a eMD Universe, there is no pressure. The lack of pressure from one hand facilitates the collapse but the same time small deviations from sphericity can grow larger thus making harder to satisfy the hoop conjecture.

GW Production

- To first order in perturbation theory scalar, vector and tensor perturbations are decoupled. This means that no GW can be produced from scalar perturbations in that order. One needs to go to 2nd order

$$\mathrm{d}s^2 = a^2(\eta)\left[-\left(1+2\Phi^{(1)}+2\Phi^{(2)}\right)\mathrm{d}\eta^2 + 2V_i^{(2)}\mathrm{d}\eta\mathrm{d}x^i + \left\{\left(1-2\Psi^{(1)}-2\Psi^{(2)}\right)\delta_{ij}+\frac{1}{2}h_{ij}\right\}\mathrm{d}x^i\mathrm{d}x^j\right]$$

$$\hat{\mathcal{T}}_{ij}{}^{lm}G^{(2)}_{lm} = \kappa^2\hat{\mathcal{T}}_{ij}{}^{lm}T^{(2)}_{lm}$$

Acquaviva Bartolo Matarrese Riotto '02
Baumann Steinhardt Takahashi Ichiki '07

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

$$S_{\mathbf{k}} = \int\frac{\mathrm{d}^3q}{(2\pi)^{3/2}}e_{ij}(\mathbf{k})q_iq_j\left(2\Phi_{\mathbf{q}}\Phi_{\mathbf{k}-\mathbf{q}} + \frac{4}{3(1+w)}\left(\mathcal{H}^{-1}\Phi'_{\mathbf{q}}+\Phi_{\mathbf{q}}\right)\left(\mathcal{H}^{-1}\Phi'_{\mathbf{k}-\mathbf{q}}+\Phi_{\mathbf{k}-\mathbf{q}}\right)\right)$$

Kohri Terada '18

$$\Phi''_{\mathbf{k}}(\eta) + \frac{6(1+w)}{(1+3w)\eta}\Phi'_{\mathbf{k}}(\eta) + wk^2\Phi_{\mathbf{k}}(\eta) = 0 \qquad \Phi_{\mathbf{k}} = \Phi(k\eta)\phi_{\mathbf{k}}$$

$$\langle\phi_{\mathbf{k}}\phi_{\mathbf{k}'}\rangle = \delta(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\left(\frac{3+3w}{5+3w}\right)^2\mathcal{P}_\zeta(k)$$

GW Production

- Perturbation theory is not valid through the whole collapse process.

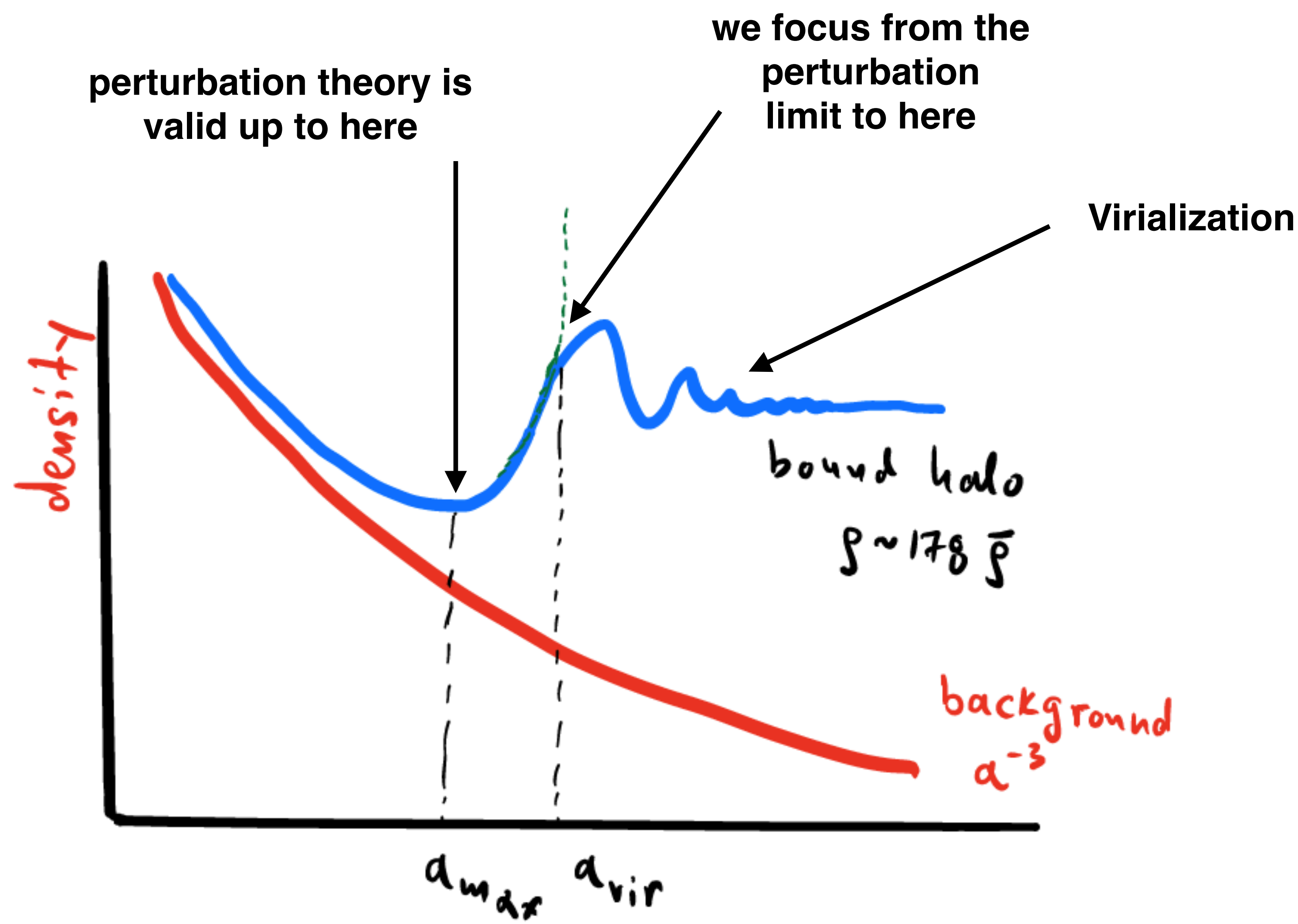
- The turnaround in linear theory happens at δ_{lin}=1.062 and perturbations theory is not trustworthy soon after the turnaround since $\delta >> 1$

- The part of the collapse from maximum expansion to black hole or halo formation is not covered by perturbation theory. This part could potentially give a strong signal especially in a virialization process where shell crossing and oscillations can induce large quadrupole moments

- We implement a nonlinear approach, hopefully capturing more accurately the form of the produced GW.

Collapse in early Matter Domination

- The absence of pressure magnifies deviations from sphericity leading to the formation of the so called Zel'dovich pancakes. Different shells start oscillating and crossing each other, forming eventually a virialized bound halo.



- If the collapsing pancake satisfies at some point the hoop conjecture, a PBH forms and there is no further virialization stage.

Zel'dovich Pancakes

$$r_i = a(t) q_i + b(t) p_i(q_i)$$

scaling factor from Hubble

growing gravitational Instability

initial deviation

$$D_{ik} = \frac{\partial r_i}{\partial q_k} = a(t)\delta_{ik} + b(t)\frac{\partial p_i}{\partial q_k} = \mathrm{diag}(a - \alpha b, a - \beta b, a - \gamma b)$$

perturbation entering the horizon $a(t_q) q = H^{-1}(t_q)$

$$M = \int \rho a^3 d^3 r = \bar{\rho} a^3 \int d^3 q$$

mass conservation $\rho(a - \alpha b)(a - \beta b)(a - \gamma b) = \bar{\rho} a^3$

$$\delta_L \equiv \left(\frac{\rho - \bar{\rho}}{\bar{\rho}}\right)_L = (\alpha + \beta + \gamma)\frac{b}{a}$$

δ ~ a in MD $b \propto a^2$

at turnaround $\dot{r}_1(t_{\max}) = 0$ $r_1(t_{\max}) = a(t_{\max}) q_1 - \alpha b(t_{\max}) q_1 = \frac{1}{2} a(t_{\max}) q_1$ $\frac{b(t_{\max})}{a(t_{\max})} = \frac{1}{2\alpha}$

Pancake evolution

$$r_1(t) = \frac{3}{2} t_q^{1/3} t^{2/3} \left(1 - \frac{1}{2}\left(\frac{t}{t_{\max}}\right)^{2/3}\right) \qquad r_2(t) = \frac{3}{2} t_q^{1/3} t^{2/3} \left(1 - \frac{\beta}{2\alpha}\left(\frac{t}{t_{\max}}\right)^{2/3}\right)$$

$$r_3(t) = \frac{3}{2} t_q^{1/3} t^{2/3} \left(1 - \frac{\gamma}{2\alpha}\left(\frac{t}{t_{\max}}\right)^{2/3}\right) \qquad t_{\max} = \left(\frac{\alpha+\beta+\gamma}{2\alpha\delta_L(t_q)}\right)^{3/2} t_q$$

Doroshkevich probability density for deviations from sphericity '70

$$\mathcal{F}_{\mathrm{D}}(\alpha,\beta,\gamma)d\alpha d\beta d\gamma = -\frac{27}{8\sqrt{5}\pi\sigma_3^6} \times \exp\left[-\frac{3}{5\sigma_3^2}\left((\alpha^2+\beta^2+\gamma^2) - \frac{1}{2}(\alpha\beta+\beta\gamma+\gamma\alpha)\right)\right]$$
$$\times (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)d\alpha d\beta d\gamma,$$

GW Production

Quadrupole

$$Q_{ij} = -I_{ij}(t) + \frac{1}{3}\delta_{ij}\mathrm{Tr}I(t)$$

Moment of Inertia

$$I_{ij} = \frac{1}{5}M\begin{pmatrix} r_2^2 + r_3^2 & 0 & 0 \\ 0 & r_1^2 + r_3^2 & 0 \\ 0 & 0 & r_1^2 + r_2^2 \end{pmatrix}$$

GW quadrupole radiation

$$\frac{dE_e}{dt} = \frac{G}{5c^5}\sum_{ij}\dddot{Q}_{ij}(t)\dddot{Q}_{ji}(t)$$

GW Production

We focus on the time interval between t_{max} and t_{col} or t_{BH} whichever happens first

We break the interval in N subintervals $[t_i, t_i + \delta t]$

$$dE_{\rm GW}(\alpha,\beta,\gamma) = \sum_N \frac{1}{1+z_N}\frac{4\pi G}{5c^5}\omega^7 \sum_{ij}|\tilde{Q}^N_{ij}(\omega)|^2 \frac{V_{\rm com}(t_0)}{\frac{4\pi}{3}q^3}\mathcal{F}_{\rm D}(\alpha,\beta,\gamma)\,d\alpha\, d\beta\, d\gamma\, d\ln\omega$$

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redshift due to expansion

$$\tilde{Q}_{ij}^N(\omega) = \frac{1}{2\pi}\int_{t_N}^{t_N+\delta t} Q_{ij}(t)e^{-i\omega t}dt$$

number of sources

probability

GW Production

We focus on the time interval between t_{max} and t_{col} or t_{BH} whichever happens first

We break the interval in N subintervals $[t_i, t_i + \delta t]$

$$dE_{\rm GW}(\alpha,\beta,\gamma) = \sum_N \frac{1}{1+z_N}\frac{4\pi G}{5c^5}\omega^7 \sum_{ij} |\tilde{Q}^N_{ij}(\omega)|^2 \frac{V_{\rm com}(t_0)}{\frac{4\pi}{3}q^3}\mathcal{F}_{\rm D}(\alpha,\beta,\gamma)d\alpha d\beta d\gamma\, d\ln\omega$$

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$$\tilde{Q}^N_{ij}(\omega) = \frac{1}{2\pi}\int_{t_N}^{t_N+\delta t} Q_{ij}(t)e^{-i\omega t}dt$$

number of sources

probability

To save computational time we take N=1. This introduces a horizontal error ~2

We have to integrate over α, β, γ

We need to insert a step function so the reheating takes place after the collapse

If we want to form PBH, the hoop conjecture should me satisfied

GW Production

$$\Omega_{\rm GW}(t_0, f_0) = \frac{1}{\rho_{\rm crit}(t_0)} \int_0^\infty \int_{-\infty}^{\alpha} \int_{-\infty}^{\beta} d\alpha d\beta d\gamma \sum_N \frac{1}{1+z_N} \frac{4\pi G}{5c^5} \sum_{ij} |\tilde{Q}_{ij}^N (2\pi f_0(1+z_N))|^2$$
$$\times (2\pi f_0(1+z_N))^7 \Theta\left(t_{\rm rh} - t_{\rm col}(\alpha,\beta,\gamma)\right) \left(\frac{4\pi}{3} q^3\right)^{-1} \mathcal{F}_{\rm D}(\alpha,\beta,\gamma).$$

PBH production

$$\beta_0 = \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma\, \Theta(1 - h(\alpha,\beta,\gamma)) \mathcal{F}_{\rm D}(\alpha,\beta,\gamma)$$

$$\beta_0(\sigma) \;=\; 0.056\, \sigma^5$$

GW Production

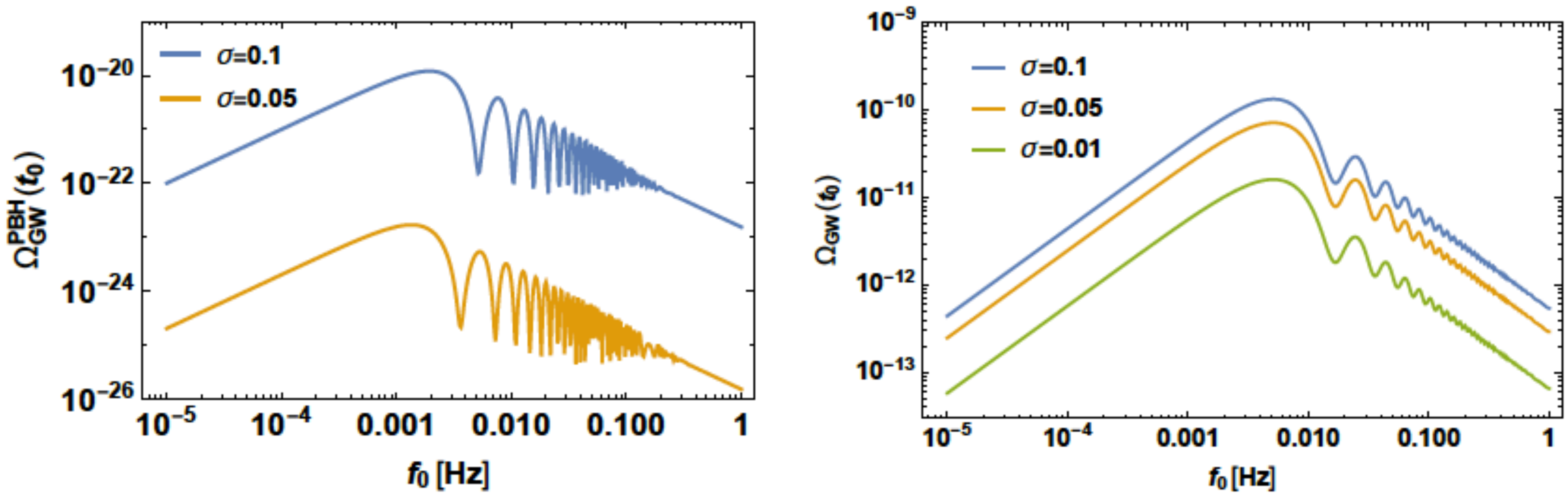
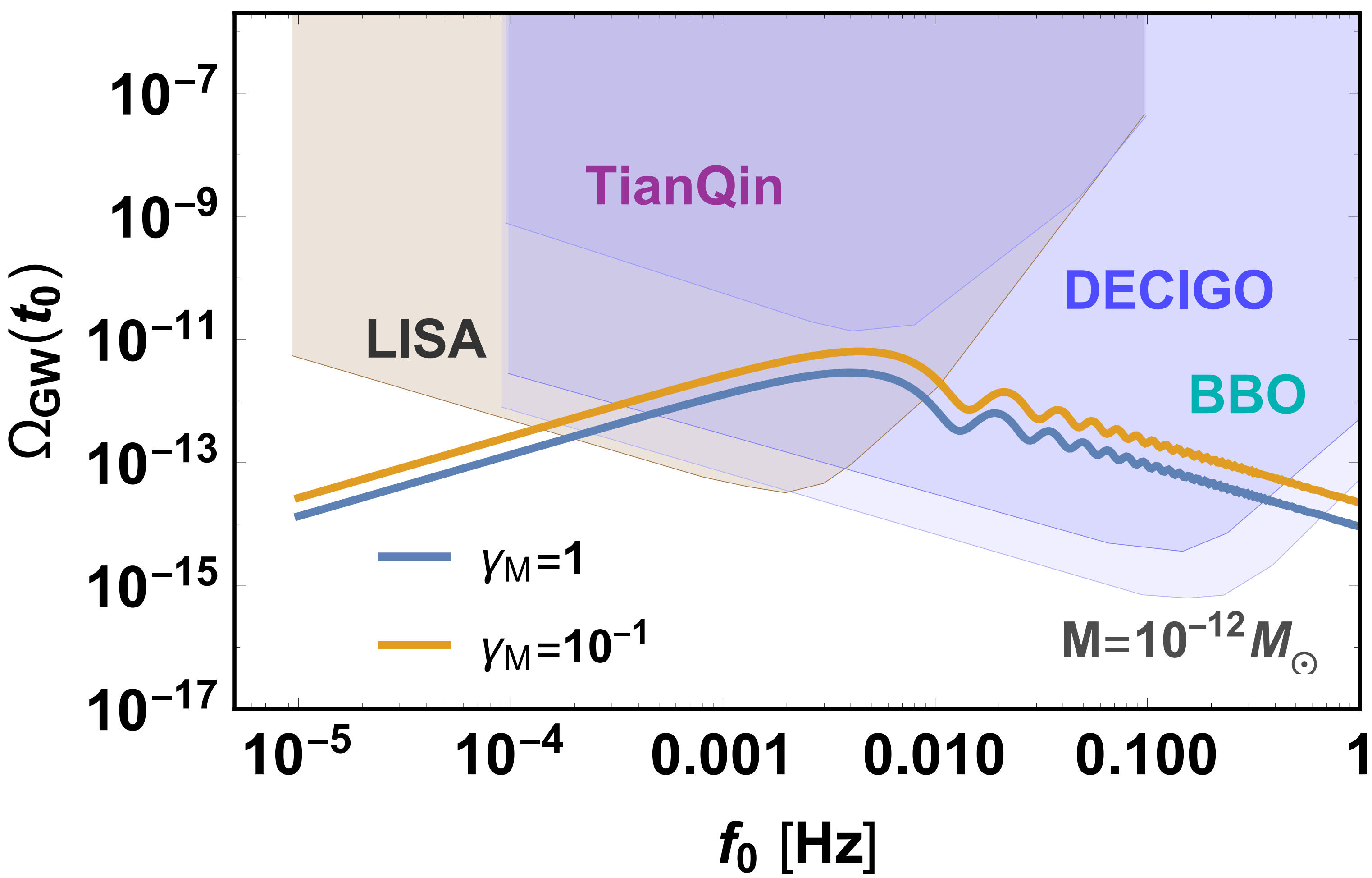


Figure 3: *Left panel.* The GW signal produced only by the regions that collapse into PBH for two σ values, 0.1 and 0.05, and reheating temperature $T_{\rm rh} = 10^4$ GeV. The horizon mass is $M = 10^{-12} M_\odot$. *Right panel.* The entire GW signal for the same horizon mass and temperature.

Dalianis CK '20

Configurations that do not lead to PBH formation create a much stronger background signal.
The PBH formation satisfies the hoop conjecture and it does not allow large deviations from sphericity. It is the large deviations that create the larger quadrupole and thus GW signal.

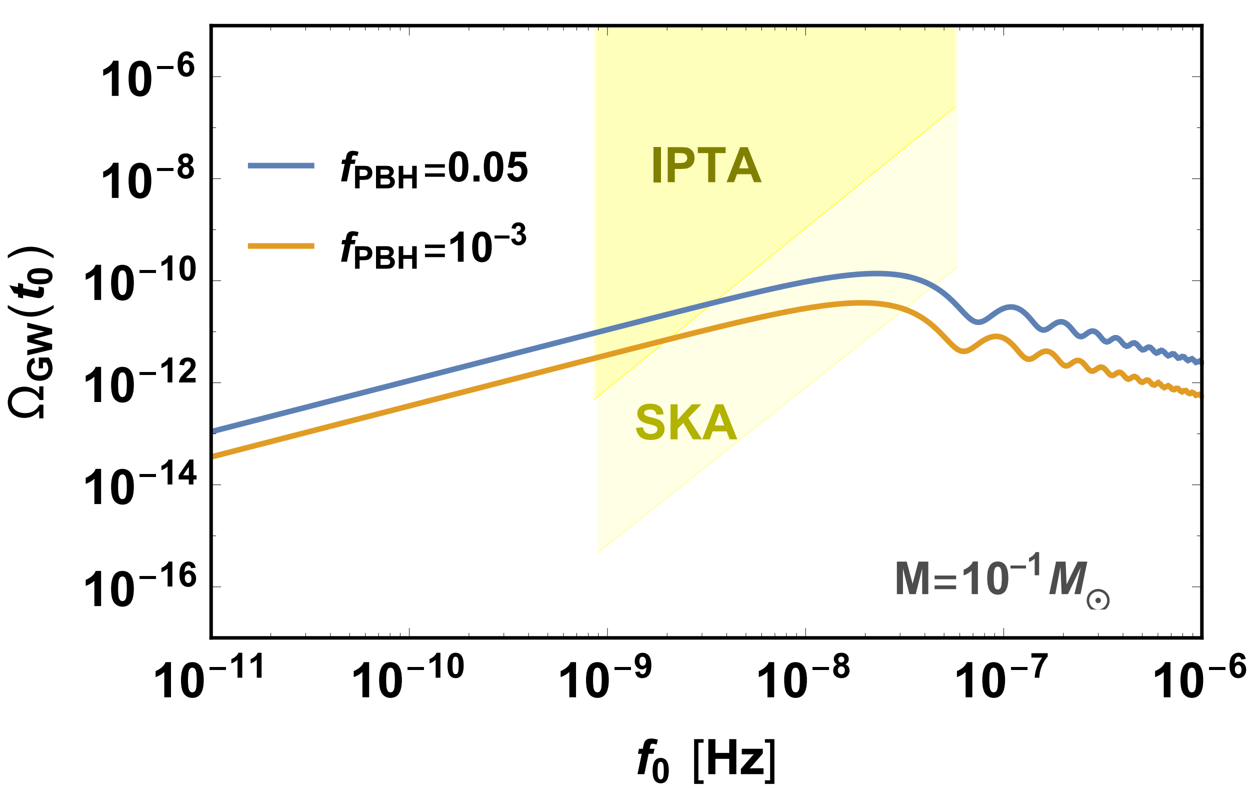
GW Signal for 100% DM contribution

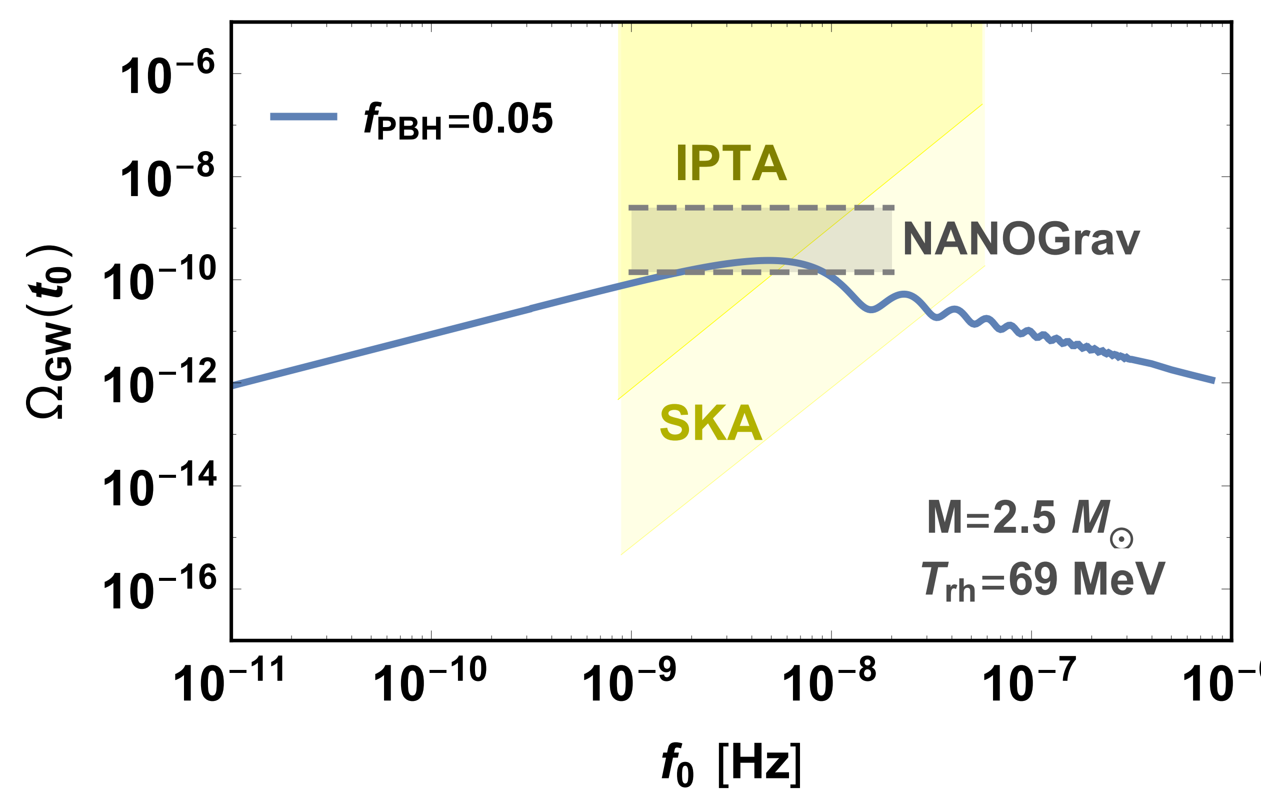


This is the region where PBH could consist 100% of the dark matter abundance.

Smaller γ_M create larger signal

PTA Detection





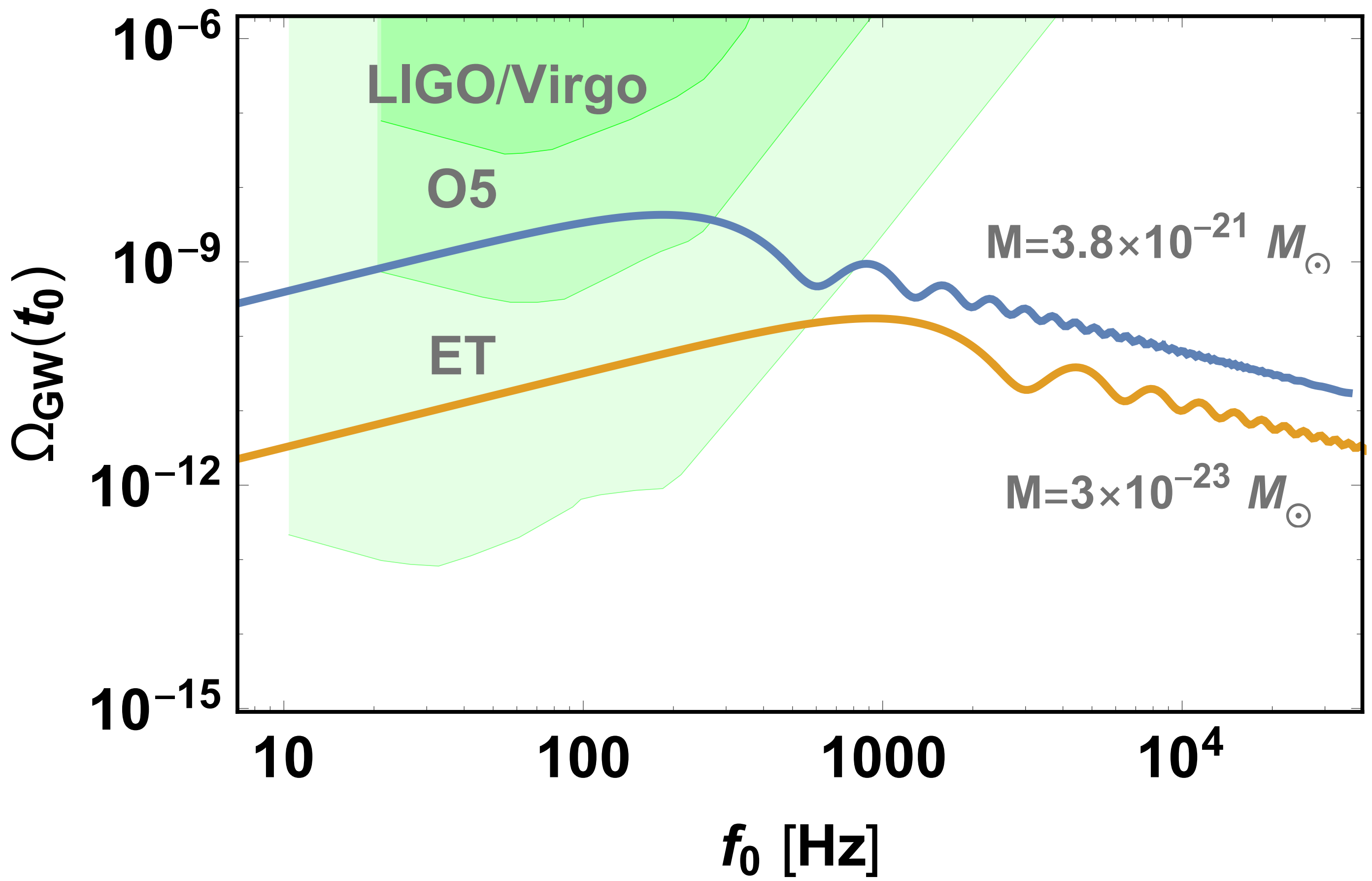
PBH formed in eMD avoid exclusion from PTA unlike the corresponding scenarios in RD.

in RD $\beta \approx \mathrm{Erfc}\left[\frac{\delta_c}{\sqrt{2}\,\sigma}\right]$ there is a threshold in σ to produce sufficient number of PBH

in eMD $\beta_0(\sigma) = 0.056\,\sigma^5$ We can build easier a PBH population. Therefore for the same number of PBH, RD produces stronger GW signal than eMD.

Harada Yoo Kohri Nakao Jhingan '16

Evaporating PBH at LIGO and ET



- Evaporating PBH can be probed in LIGO/Virgo & ET

- The peak can be at a different place compared to PBH formed in RD because γ_M can be much smaller

- If the evaporation leaves a Planck remnant, these PBH could explain 100% of DM relic abundance

Conclusions

Primordial Black Holes
- could account (fully or partially) for the DM relic abundance
- provide the seeds for the supermassive black holes observed in the Universe

Gravitational Wave Production in a eMD era
- Perturbation theory fails after maximum expansion
- Zel'dovich method is valid until violent relaxation
- eMD formed PBH can avoid PTA constraints because it is easier to make BH
- Can be distinguished from RD formation
- Can be tested in current and future interferometers

Future Directions
- Study the full formation period i.e. add:

1. Spectrum of expanding perturbation up to maximum expansion and comparison with perturbation theory
2. Virilization process (we have started performing numerical simulations)

- The total signal is crucial in order to recognise the scenario and distinguish it from other RD formation mechanisms.