Einstein-Cartan Portal to Dark Matter

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References

Mikhail Shaposhnikov, Andrey Shkerin, IT, and Sebastian Zell:

- Einstein-Cartan gravity, matter, and scale-invariant generalization, 2007.16158, JHEP 10 (2020) 177
- Higgs inflation in Einstein-Cartan gravity, 2007.14978, JCAP 02 (2021) 008

Talk by Sebastian Zell [link]

- Einstein-Cartan Portal to Dark Matter, 2008.11686, Phys.Rev.Lett. 126 (2021) 16, 161301

Outline

- Metric and Palatini gravity
- Einstein-Cartan gravity
- Einstein-Cartan portal to dark matter
- Summary

Metric and Palatini gravity

Lowest order action

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{|g|} R$$

Riemann curvature tensor is expressed via connection $\Gamma^{\rho}_{\nu\sigma}$ as

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Metric gravity

- $\Gamma^{\rho}_{\nu\sigma}$ is **symmetric** with respect to lower indices
- $\Gamma^{\rho}_{\nu\sigma}$ is expressed in terms of metric via $g_{\mu\nu\,;\,\alpha} = 0$
- The dynamical variable is $g_{\mu\nu}$, variation with respect to $g_{\mu\nu}$ gives the Einstein equations

Metric and Palatini gravity

Lowest order action

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{|g|} R$$

Riemann curvature tensor is expressed via connection $\Gamma^{\rho}_{\nu\sigma}$ as

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Palatini gravity

- $\Gamma^{\rho}_{\nu\sigma}$ is **symmetric** with respect to lower indices
- The dynamical variables are $g_{\mu\nu}$ and $\Gamma^{\rho}_{\nu\sigma}$,
- Variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$
  variation with respect to $g_{\mu\nu}$ gives the Einstein equations

Metric and Palatini gravity

Lowest order action $$S = \frac{M_P^2}{2}\int d^4x\sqrt{|g|}R$$

Riemann curvature tensor is expressed via connection $\Gamma^{\rho}_{\nu\sigma}$ as

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Metric gravity

$\Gamma^{\rho}_{\nu\sigma}$ is compatible with metric

$$\nabla_\alpha g^{\mu\nu} = 0$$

Palatini gravity

$\Gamma^{\rho}_{\nu\sigma}$ is independent

$$\frac{\delta S}{\delta \Gamma^{\alpha}_{\mu\nu}} \propto \nabla_\alpha g^{\mu\nu}$$

$$\frac{\delta S}{\delta g^{\mu\nu}} \propto R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

without matter Palatini gravity is equivalent to metric gravity

Einstein-Cartan gravity

Einstein-Cartan(-Sciama–Kibble) theory
gauging of the Poincaré group, Utiyama ‘56, Kibble ‘61

Riemann curvature tensor is expressed via connection $\Gamma^{\rho}_{\nu\sigma}$ as

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Symmetry of $\Gamma^{\rho}_{\nu\sigma}$ with respect to lower indices is not assumed.
Torsion tensor: $T^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\nu\sigma} - \Gamma^{\rho}_{\sigma\nu}$

Variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$
Variation with respect to $g_{\mu\nu}$ gives the Einstein equations
On the solution $T^{\rho}_{\nu\sigma} = 0$

Einstein-Cartan pure gravity is equivalent to metric gravity

Einstein-Cartan gravity, Holst term and Nieh-Yan invariant

Symmetry of $\Gamma^{\rho}_{\nu\sigma}$ with respect to lower indices is not assumed.

Torsion tensor: $T^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\nu\sigma} - \Gamma^{\rho}_{\sigma\nu}$

$$S = \boxed{\frac{M_P^2}{2}\int d^4x\sqrt{|g|}R} + \boxed{\frac{M_P^2}{2\bar{\gamma}}\int \mathrm{d}^4x\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}} + \boxed{M^2\int \mathrm{d}^4x\partial_\mu\left(\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)}$$

as in metric gravity — Holst term — Nieh-Yan invariant

$\bar{\gamma}$ — Barbero-Immirzi parameter

Variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$

Variation with respect to $g_{\mu\nu}$ gives the Einstein equations

On the solution $T^{\rho}_{\nu\sigma} = 0$

Einstein-Cartan pure gravity is equivalent to metric gravity

Higgs inflation in EC gravity

Non-minimal coupling to Higgs can be added

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{|g|}R + \frac{M_P^2}{2\bar{\gamma}}\int \mathrm{d}^4x\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + M^2\int \mathrm{d}^4x\partial_\mu\left(\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

M. Långvik, J. Ojanperä, S. Raatikainen, S. Räsänen,
Higgs inflation with the Holst and the Nieh-Yan term, arXiv:2007.12595

M. Shaposhnikov, A. Shkerin, IT, and Sebastian Zell:,
Higgs inflation in Einstein-Cartan gravity, arXiv:2007.14978

Talk by Sebastian Zell [link]

EC gravity action with the Higgs field

Modified kinetic term: essential for inflation and non-perturbative generation of the electroweak scale
[Mikhail Shaposhnikov, Andrey Shkerin, and Sebastian Zell, 2001.09088]
talk by Mikhail Shaposhnikov [link]

$$S_{\text{metric}} = \int d^4x \sqrt{|g|} \left\{ \frac{M_P^2}{2} R - \left[\frac{1}{2\Omega^2} \left(\partial_\mu h \right)^2 + \frac{\lambda}{4} \frac{h^4}{\Omega^4} \right] - \frac{3M_P^2}{4\left(\gamma^2+1\right)} \left(\frac{\partial_\mu \bar{\eta}}{\Omega^2} + \partial_\mu \gamma \right)^2 \right\}$$

$$\gamma = \frac{1}{\bar{\gamma}\Omega^2} \left(1 + \frac{\xi_\gamma h^2}{M_P^2} \right), \quad \bar{\eta} = \frac{\xi_\eta h^2}{M_P^2}$$

Einstein-Cartan portal to dark matter

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{|g|}R + \frac{M_P^2}{2\gamma}\int \mathrm{d}^4x\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + M^2\int \mathrm{d}^4x\partial_\mu\left(\sqrt{|g|}\,\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

+ fermionic part

Fermonic action in Einstein-Cartan gravity

- Appropriate variables:
  - e^a_μ - tetrad field (translations)
  - A^{ab}_μ - spin connection (local Lorentz transformations)
- Fermionic action

$$S = \frac{i}{2}\int \mathrm{d}^4x\sqrt{-g}\left(\bar{\Psi}\gamma^\mu D_\mu\psi - \mathrm{h.c.}\right)$$

$$D_\mu\Psi = \left(\partial_\mu + \frac{1}{8}A_{\mu ab}\left[\gamma^a,\gamma^b\right]\right)\Psi$$

Fermonic action in Einstein-Cartan gravity

Non-vanishing torsion allows introducing new couplings

$$S = \frac{i}{2}\int \mathrm{d}^4x\sqrt{-g}\left(\bar{\Psi}\left(1 - \boxed{i\alpha - i\beta\gamma^5}\right)\gamma^\mu D_\mu\Psi - \mathrm{h.c.}\right)$$

α, β — real parameters

L. Freidel, D. Minic, T. Takeuchi, Quantum gravity, torsion, parity violation and all that, hep-th/0507253.

S. Alexandrov, Immirzi parameter and fermions with non-minimal coupling, 0802.1221.

Fermonic action in Einstein-Cartan gravity

Integrating out torsion one arrives at
new universal four-fermion interaction

$$\mathscr{L}_{4f} = \frac{-3\alpha^2}{16M_P^2} V^\mu V_\mu - \frac{3\alpha\beta}{8M_P^2} V^\mu A_\mu + \frac{3-3\beta^2}{16M_P^2} A^\mu A_\mu$$

This interaction is universal and also affects a new hypothetical singlet particle— DM candidate

$$V^\mu = \boxed{\bar{N}\gamma^\mu N} + \sum_X \bar{X}\gamma^\mu X$$

$$A^\mu = \boxed{\bar{N}\gamma^5\gamma^\mu N} + \sum_X \bar{X}\gamma^5\gamma^\mu X$$

X are SM fermions

DM production via Four-Fermion Interactions

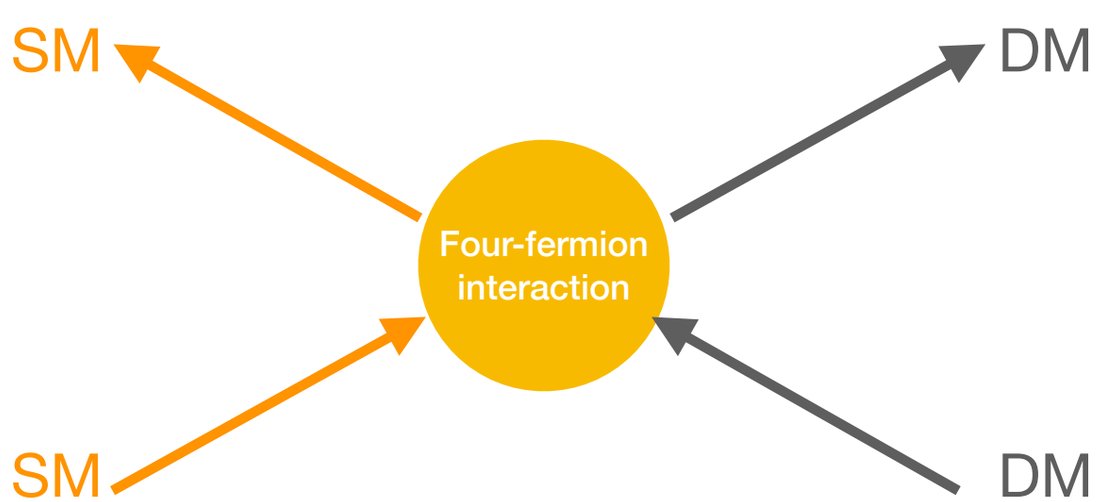
$$\mathscr{L}_{4f} = \frac{-3\alpha^2}{16M_P^2}V^\mu V_\mu - \frac{3\alpha\beta}{8M_P^2}V^\mu A_\mu + \frac{3-3\beta^2}{16M_P^2}A^\mu A_\mu$$

$$V^\mu = \bar{N}\gamma^\mu N + \sum_X \bar{X}\gamma^\mu X$$

$$A^\mu = \bar{N}\gamma^5\gamma^\mu N + \sum_X \bar{X}\gamma^5\gamma^\mu X$$

The four-fermion is universal: affects singlet fermions — DM candidates

Allows for annihilation of the SM particles $\bar{X} + X \to \bar{N} + N$



DM production via Four-Fermion Interactions

$$\mathscr{L}_{4f} = \frac{-3\alpha^2}{16M_P^2} V^\mu V_\mu - \frac{3\alpha\beta}{8M_P^2} V^\mu A_\mu + \frac{3-3\beta^2}{16M_P^2} A^\mu A_\mu$$

Allows for annihilation of the SM particles $\bar{X} + X \to \bar{N} + N$

Kinetic description of N production

$$\left(\frac{\partial}{\partial t} - H q_i \frac{\partial}{\partial q_i}\right) \underbrace{f_N(t, \vec{q})}_{\text{number density}} = R(\vec{q}, T)$$

See, e.g.
Dolgov 2000, hep-ph/0202122

$$R = \frac{1}{2|q|} \sum_X \int \frac{\mathrm{d}^3\vec{p}_1}{(2\pi)^3\, 2E_1} \frac{\mathrm{d}^3\vec{p}_2}{(2\pi)^3\, 2E_2} \frac{\mathrm{d}^3\vec{p}_3}{(2\pi)^3\, 2E_3}$$
$$\times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - q - p_3)\, |\overline{\mathcal{M}}_X|^2\, f_X(p_1) f_{\bar{X}}(p_2)$$

Freeze-in

we assume thermal distributions of the SM particles

DM production via Four-Fermion Interactions

Kinetic description of N production

$$TH\frac{\partial f_N}{\partial T} = -\frac{C_f}{M_{Pl}^4}T^5 r(y), \quad y = E/T, \quad C_f = \text{a combination of } \alpha, \beta$$

$$r(E,T) = \frac{1}{2E}\sum_X \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}$$
$$\times (2\pi)^4\delta^{(4)}\left(p_1+p_2-q-p_3\right)\left(p_1\cdot q\right)\left(p_2\cdot p_3\right)\times f_X\left(p_1\right)f_{\bar{X}}\left(p_2\right)$$

$$r(y) \simeq \frac{1}{24\pi^3}y f_X$$

$$f_N(y) = \frac{C_f T_{\text{prod}}^3 M_0\left(T_{\text{prod}}\right)}{3M_P^4}r(y)$$

DM spectrum carries information about the distribution of the SM fermions at T = T_{prod}

DM production via Four-Fermion Interactions

DM abundance:

$$\frac{\Omega_N}{\Omega_{DM}} \simeq 3.6 \cdot 10^{-2} C_f \left(\frac{M_N}{10\text{keV}}\right)\left(\frac{T_{\text{prod}}}{M_P}\right)^3$$

C_f is different for Majorana and Dirac fermions:

$$C_M = \frac{9}{4}\left\{24\left(1+\alpha^2-\beta^2\right)^2 + 21\left(1-(\alpha+\beta)^2\right)^2\right\},$$

$$C_D = \frac{9}{4}\left\{45\left(1+\alpha^2-\beta^2\right)^2 + 21\left(1-(\alpha+\beta)^2\right)^2 + 24\left(1-(\alpha-\beta)^2\right)^2\right\}.$$

Einstein-Cartan portal to dark matter

- Universal four-fermion interaction of EC gravity can lead to production of singlet fermions
- This mechanism is the most effective at very high temperatures ($\Omega_N \sim T_{\rm prod}^3$)

Einstein-Cartan portal to dark matter and Higgs inflation

$$S = \int d^4x \sqrt{-g} \left\{ -\frac{1}{2}\left(\partial_\mu h\right)^2 - \frac{\lambda}{4}h^4 + \frac{M_P^2}{2}R\left(1 + \frac{\xi h^2}{M_p^2}\right)\right\}$$

- Higgs is the only scalar in the SM
- It can play the role of inflaton
- Original metric Higgs inflation [Bezrukov and Shaposhnikov, 0710.3755]
- Palatini Higgs Inflation [Bauer and Demir 0803.2664]

Advantages of Palatini formulation, see
Bauer and Demir 0803.2664
Shaposhnikov, Shkerin, and Zell, 2002.07105

Einstein-Cartan portal to dark matter

- (Palatini) Higgs inflation: non-minimal coupling ξ

- Almost **instantaneous** preheating in Higgs inflation [DeCross, Kaiser, Prabhu, Prescod-Weinstein, Sfakianakis; Ema, Jinno, Mukaida, Nakayama ; Rubio, Tomberg; Bezrukov, Shepherd]

- We take $T_{prod} = T_{reh}$

$$\frac{\Omega_N}{\Omega_{DM}} \simeq 3.6 \cdot 10^{-2} C_f \left(\frac{M_N}{10\text{keV}}\right)\left(\frac{T_{\text{prod}}}{M_P}\right)^3$$

$$T_{\text{reh}} \simeq \left(\frac{15\lambda}{2\pi^2 g_{\text{eff}}}\right)^{\frac{1}{4}} \frac{M_P}{\sqrt{\xi}}$$

Einstein-Cartan portal to dark matter

- Two "natural" choices of α and β:

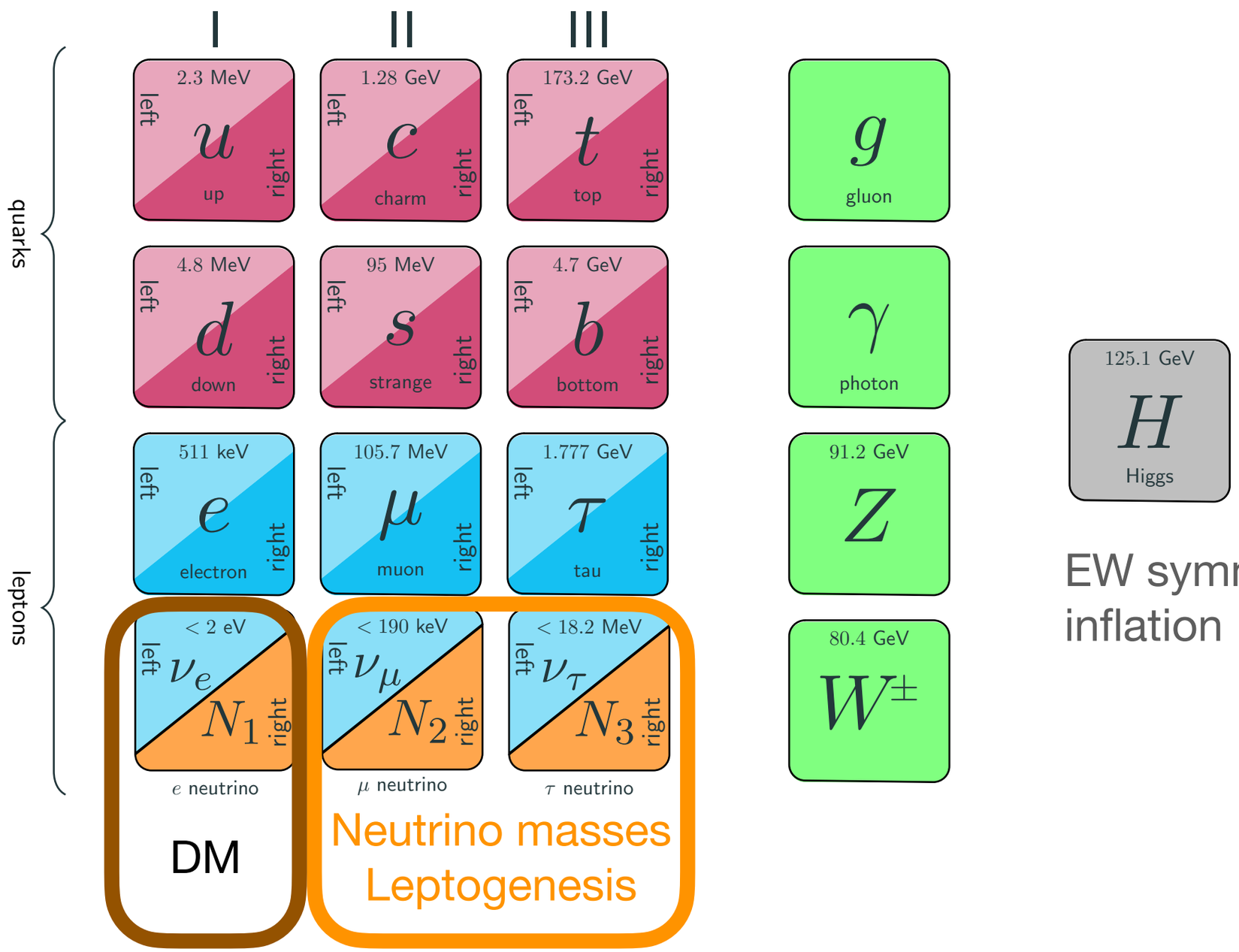
- $\alpha = \beta = 0$
  the correct DM abundance is obtained for $(3-6)\times 10^8$ GeV fermion in Palatini Higgs inflation

- $\alpha \sim \beta \sim \sqrt{\xi}$ (universal UV cutoff $\Lambda \sim M_P/\sqrt{\xi}$)
  the correct DM abundance is obtained for a keV fermion in Palatini Higgs inflation

$$\frac{\Omega_N}{\Omega_{DM}} \simeq 1.4 \frac{\sqrt{\xi}\lambda^{3/4}}{g_{\rm eff}^{3/4}} \frac{(\alpha+\beta)^4}{\xi^2} \left(\frac{M_N}{10\text{keV}}\right) \left(\frac{T_{\rm prod}}{T_{\rm reh}}\right)^3$$

Einstein-Cartan portal to dark matter

and the νMSM

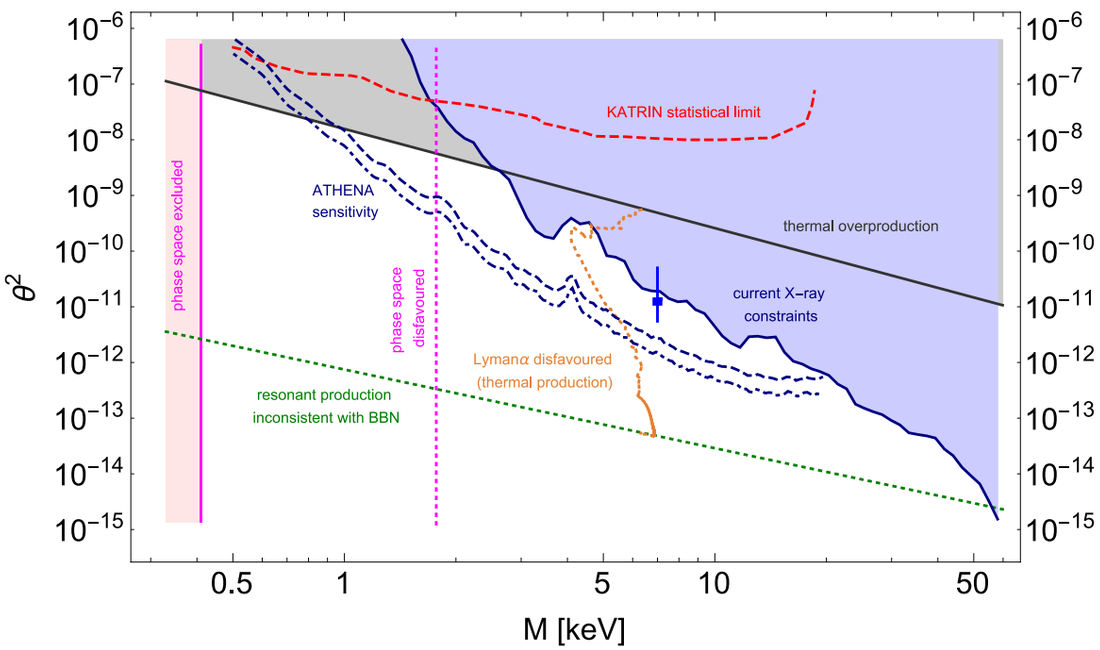


EW symmetry breaking
inflation

Einstein-Cartan portal to dark matter

and the νMSM

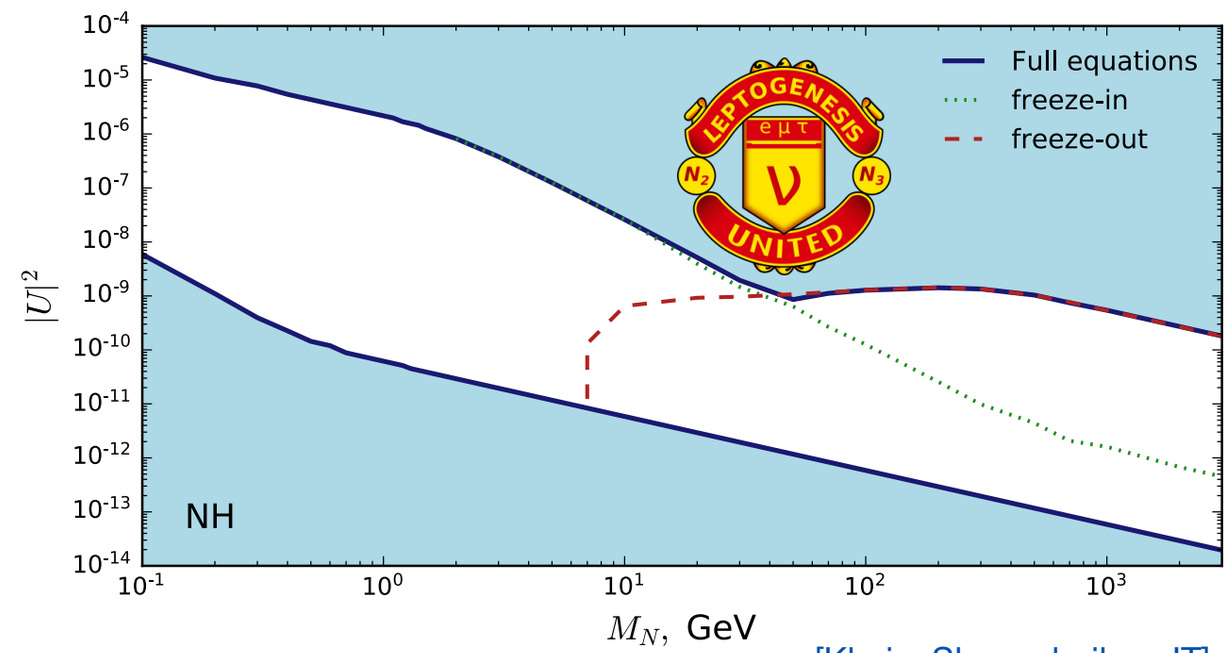
N_1



[from Boyarsky, Drewes, Lasserre, Mertens, Ruchayskiy]

DM can be produced via universal 4-femion interaction
No lower bound on the mixing angle θ^2

$N_{2,3}$



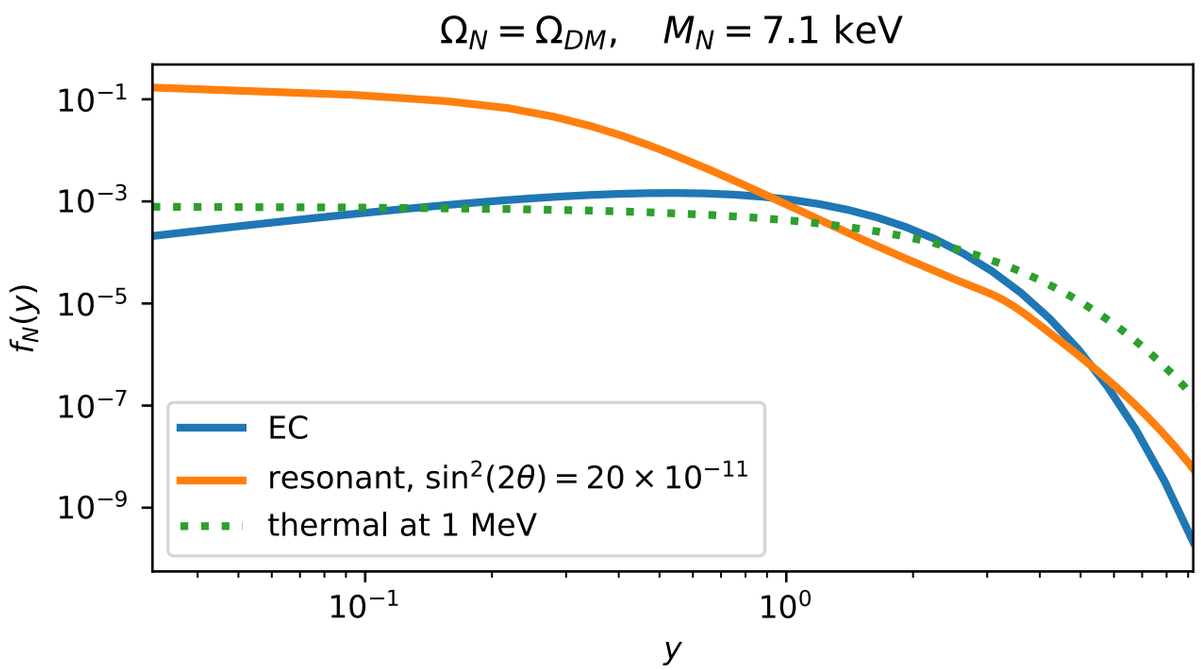
[Klaric, Shaposhnikov, IT]

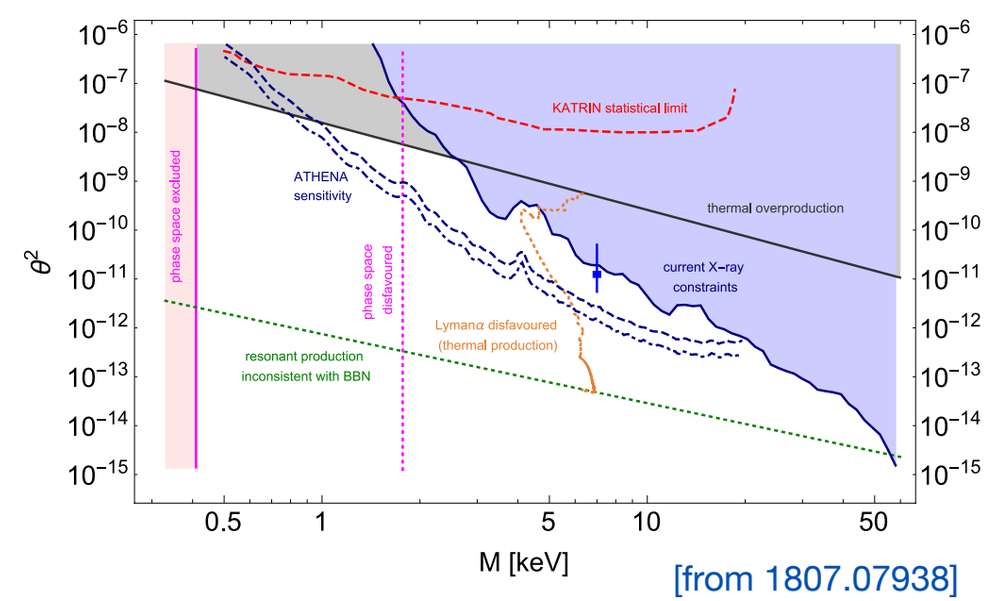
$N_{2,3}$ are also produced,
but $n_{2,3} \simeq 10^{-2} n_{eq} \left(10\text{keV}/M_1\right)$
So leptogenesis is not affected

Einstein-Cartan portal to dark matter

and the νMSM

N_1 – momentum distribution





[from 1807.07938]

Summary

Einstein-Cartan theory

- Pure gravity: equivalent to metric formulation

- **A new universal mechanism for fermion dark matter production**