Construction of inflationary models with the Gauss–Bonnet term

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based on

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M. Sami, A.V. Toporensky, S.Yu. Vernov,
Phys. Rev. D **102** (2020) 043525 [arXiv:2006.08027]
E.O. Pozdeeva, S.Yu. Vernov, arXiv:2104.04995

Quarks online workshop "Modification of Gravity: Theories and Observations", Moscow, Russia, 11.06.2021

STRING THEORY MOTIVATED GRAVITY

Einstein–Gauss–Bonnet gravity models are motivated by α' corrections in string theories. The most general Lagrangian density at the next to leading order in the parameter α' reads[1]:

$$L_{string} = -\frac{\lambda}{2}\alpha'\xi(\phi)\left[c_1\mathcal{G} + c_2 G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + c_3\Box\phi\phi^{;\mu}\phi_{;\mu} + c_4(\phi^{;\mu}\phi_{;\mu})^2\right],$$

- $\mathcal{G}$ is the Gauss–Bonnet term:

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta},$$

- $G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$ is the Einstein tensor,
- $\alpha' = \lambda_s^2$, where λ_s is the fundamental string length scale;
- c_i are constants (we will consider the case $c_k = 0$, $k = 2, 3, 4$);
- λ is an additional parameter allowing for different species of string theories, $\lambda = -1/4$ for the Bosonic string and $\lambda = -1/8$ for Heterotic string respectively.

[1] D.J. Gross and J.H. Sloan, Nucl. Phys. B **291** (1987) 41;
R.R. Metsaev and A.A. Tseytlin, Nucl. Phys. B **293** (1987) 385.

INFLATIONARY MODELS

The perturbation theory for such types of models has been developed in C. Cartier, J. c. Hwang and E. J. Copeland, *Evolution of cosmological perturbations in nonsingular string cosmologies*, Phys. Rev. D **64** (2001) 103504 [astro-ph/0106197];
J. c. Hwang and H. Noh, *Classical evolution and quantum generation in generalized gravity theories including string corrections and tachyon: Unified analysis*, Phys. Rev. D **71** (2005) 063536 [gr-qc/0412126]

Inflationary models have been proposed:
Z.K. Guo and D.J. Schwarz, Phys. Rev. D **81**, 123520 (2010)
A. De Felice, S. Tsujikawa, J. Elliston, R. Tavakol, JCAP **08** (2011) 021
M. De Laurentis, M. Paolella and S. Capozziello, Phys. Rev. D **91** (2015) 083531,
G. Hikmawan, J. Soda, A. Suroso, and F.P. Zen, Phys. Rev. D **93**, 068301 (2016)
C. van de Bruck and C. Longden, Phys. Rev. D **93** (2016) 063519
S. Koh, B.H. Lee and G. Tumurtushaa, Phys. Rev. D **95** (2017) 123509,
K. Nozari and N. Rashidi, Phys. Rev. D **95** (2017) 123518
S.D. Odintsov and V.K. Oikonomou, Phys. Rev. D **98** (2018) 044039
Z. Yi and Y. Gong, Universe **5** (2019) 200
E.O. Pozdeeva, *Eur. Phys. J.* C **80** (2020) 612

Let us consider the action

$$S = \int d^4x \sqrt{-g}\,[U_0 R + F(C_1 R + C_2 \mathcal{G})]\,, \qquad (1)$$

where F is a double differentiable function, U_0, C_1, and C_2 are constants. A linear function F corresponds to the General Relativity, whereas a nonlinear function F corresponds to the modified gravity.
$F(R)$ and $F(\mathcal{G})$ gravity models are particular cases of this model.
Introducing a field ϕ without kinetic term, action (1) can be rewritten in the following form:

$$S = \int d^4x \sqrt{-g}\,[U_0 R + F'(\phi)(C_1 R + C_2 \mathcal{G} - \phi) + F(\phi)]\,. \qquad (2)$$

Varying action (2) over ϕ, one gets $\phi = C_1 R + C_2 \mathcal{G}$ and the initial $F(R, \mathcal{G})$ model with action (1).

MODELS WITH THE GAUSS–BONNET TERM

$$S=\int d^4x\sqrt{-g}\left(U(\phi)R-\frac{c}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)-\frac{1}{2}\xi(\phi)\mathcal{G}\right). \quad (3)$$

In the spatially flat FLRW universe with the interval

$$ds^2 = -\,dt^2+a^2(t)\left(dx_1^2+dx_2^2+dx_3^2\right),$$

one gets the following equations

$$6H^2U+6HU'\dot\phi=\frac{c}{2}\dot\phi^2+V+12H^3\xi'\dot\phi, \quad (4)$$

$$4\left(U-2H\dot\xi\right)\dot H = -\,c\dot\phi^2-2\ddot U+2H\dot U+4H^2\left(\ddot\xi-H\dot\xi\right), \quad (5)$$

$$c\ddot\phi+3c\,H\dot\phi-6\left(\dot H+2H^2\right)U'+V'+12H^2\xi'\left(\dot H+H^2\right)=0, \quad (6)$$

where $H=\dot a/a$ is the Hubble parameter, primes mean the derivatives with respect to ϕ, dots mean the derivatives with respect to t.

DE SITTER SOLUTIONS

Let us find de Sitter solutions in the model with the Gauss–Bonnet term. We restrict ourselves to de Sitter solutions with a constant ϕ. Substituting $\phi = \phi_{dS}$ and $H = H_{dS}$ into Eqs. (4) and (6), we get:

- The equation for the Hubble parameter at the de Sitter point is the same as in the corresponding model without the Gauss–Bonnet term:

$$H_{dS}^2 = \frac{V_{dS}}{6U_{dS}}. \tag{7}$$

- The value of $\xi'(\phi_{dS})$ is

$$\xi'_{dS} = \frac{3U_{dS}(2U'_{dS}V_{dS} - V'_{dS}U_{dS})}{V_{dS}^2}, \tag{8}$$

  where $A_{dS} \equiv A(\phi_{dS})$ for any function A.

It would be convenient, if all the necessary information on the existence and stability of de Sitter solutions could be obtained from a combination of functions U, V, and ξ dubbed the effective potential V_{eff}.
Stable de Sitter solutions correspond to minima of the effective potential.

THE DYNAMICAL SYSTEM

We cast Eqs. (5) and (6) as the following dynamical system:

$$\begin{aligned}
\dot{\phi} =& \psi, \\
\dot{\psi} =& \frac{1}{2\left(B - 2c\xi' H\psi\right)} \left\{ 2H \left[3B + 2\xi' V' - 6U'^2 - 6cU \right] \psi - 2\frac{V^2}{U} X \right. \\
& \left. + \left[12H^2 \left[\left(2U'' + 3c\right) F' + U'\xi'' \right] - 24\xi'\xi'' H^4 - 3\left(2U'' + c\right) U' \right] \psi^2 \right\}, \\
\dot{H} =& \frac{1}{4\left(B - 2c\xi' H\psi\right)} \left\{ 8c\left(U' - 2\xi' H^2\right) H\psi \right. \\
& \left. - 2\frac{V^2}{U^2}\left(2\xi' H^2 - U'\right) X + \left(4\xi'' H^2 - 2U'' - c\right) c\psi^2 \right\},
\end{aligned} \tag{9}$$

$$B = 3\left(2H^2\xi' - U'\right)^2 + cU, \qquad X = \frac{U^2}{V^2}\left[12H^4\xi' - 12H^2U' + V'\right].$$

We introduce the effective potential $V_{eff}(\phi)$ in the model with the Gauss–Bonnet term, such that

$$V'_{eff}(\phi_{dS}) = X(\phi_{dS}) = 0. \tag{10}$$

Indeed, let

$$V_{eff} = -\frac{U^2}{V} + \frac{1}{3}\xi. \tag{11}$$

we get

$$X(\phi_{dS}) = \frac{1}{3}\xi'_{dS} - 2\frac{U'_{dS}U_{dS}}{V_{dS}} + \frac{V'_{dS}U^2_{dS}}{V^2_{dS}} = V'_{eff}(\phi_{dS}) = 0.$$

De Sitter solutions correspond to extremum points of the effective potential V_{eff}.

E.O. Pozdeeva, M. Sami, A.V. Toporensky, S.Yu. Vernov, *Phys. Rev. D* **100** (2019) 083527 [arXiv:1905.05085]

THE LYAPUNOV STABILITY

To investigate the Lyapunov stability of a de Sitter solution we use the following expansions,

$$H(t) = H_{dS} + \varepsilon H_1(t),\ \phi(t) = \phi_{dS} + \varepsilon\phi_1(t),\ \psi(t) = \varepsilon\psi_1(t),$$

where ε is a small parameter.
The functions $H_1(t)$, $\phi_1(t)$ and $\psi_1(t)$ are not independent. From Eq. (4), we obtain

$$H_1 = \frac{V'_{dS}U_{dS} - U'_{dS}V_{dS}}{2U_{dS}V_{dS}}\left(H_{dS}\phi_1 - \psi_1\right). \tag{12}$$

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From system (9) and Eq. (12), we get:

$$\begin{aligned}\dot{\phi}_1 &= A_{11}\phi_1 + A_{12}\psi_1\,,\\ \dot{\psi}_1 &= A_{21}\phi_1 + A_{22}\psi_1\,,\end{aligned}$$

where

$$A = \left\| \begin{array}{cc} 0, & 1 \\ -\frac{V_{dS}^2 V''_{eff}(\phi_{dS})}{U_{dS}B_{dS}}, & -3H_{dS} \end{array} \right\|$$

Solving the characteristic equation:

$$\det(A - \lambda \cdot I) = \lambda^2 - 3H_{dS}\lambda + \frac{V_{dS}^2 V''_{eff}(\phi_{dS})}{U_{dS}B_{dS}} = 0,$$

we get the following roots:

$$\lambda_\pm = -\frac{3}{2}H_{dS} \pm \sqrt{\frac{9}{4}H_{dS}^2 - \frac{V_{dS}^2}{U_{dS}B_{dS}}V''_{eff}(\phi_{dS})}\,. \qquad (13)$$

A de Sitter solution is stable if real parts of both λ_- and λ_+ are negative.
To get this result, we assume that $H_{dS} > 0$, hence, $\Re e(\lambda_-) < 0$.
In the case of a positive U_{dS}, we see that $B_{dS} > 0$ for $c \geqslant 0$. The condition $\Re e(\lambda_+) < 0$ is equivalent to $V''_{eff}(\phi_{dS}) > 0$.
In the cases $c > 0$ and $c = 0$, a de Sitter solution is stable if $V''_{eff}(\phi_{dS}) > 0$ and unstable if $V''_{eff}(\phi_{dS}) < 0$.
In the case $c < 0$, we see that B_{dS} can be negative. So, in this case de Sitter solution is stable if the $V''_{eff}(\phi_{dS})B_{dS} > 0$.

DIFFERENT FORMS OF THE EFFECTIVE POTENTIAL

We define the effective potential as such a function that its minima correspond to the stable de Sitter solutions and maxima correspond to unstable de Sitter solutions.

The effective potential is not unique:

- We can add a constant to it or multiply it on a positive number.
- If $V_{eff}(\phi) > 0$ for any ϕ, then functions V_{eff}^n and $-1/V_{eff}^n$, where n is a natural number, can be considered as new effective potentials.
- Let $\tilde{U}(\phi) = f(\phi)U(\phi)$ and

$$\tilde{V}(\phi) = \frac{V(\phi)f(\phi)^2 U(\phi)^3}{V^2(\phi) + U(\phi)^3}, \tag{14}$$

then the original and transformed effective potentials are connected as:

$$\tilde{V}_{eff} = \frac{\xi}{3} - \frac{\tilde{U}^2}{\tilde{V}} = V_{eff} + \frac{V}{U}. \tag{15}$$

The structure of de Sitter solutions does not change if $W \equiv \frac{V}{U} = CV_{eff} + W_0$, where $C > -1$.

One should check that $\tilde{V}(\phi_{dS}) > 0$ and $\tilde{U}(\phi_{dS}) > 0$.

SLOW-ROLL APPROXIMATION

We seek inflationary scenarios in the model

$$S = \int d^4x\sqrt{-g}\left(U(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}\xi(\phi)\mathcal{G}\right).$$

In the slow-roll approximation, defined by the following conditions[2]:

$$\dot\phi^2 \ll V, \quad |\ddot\phi| \ll 3H|\dot\phi|, \quad |\ddot U| \ll H|\dot U| \ll H^2 U, \quad 2H|\dot\xi| \ll U, \quad |\ddot\xi| \ll |\dot\xi|H,$$

The evolution equations are:

$$H^2 \simeq \frac{V}{6U}, \tag{16}$$

$$4U\dot H \simeq -\dot\phi^2 - 4H^3\dot\xi + 2H\dot U, \tag{17}$$

$$\dot\phi \simeq -\frac{V' + 12\xi'_{,\phi}H^4 - 12H^2U'_{,\phi}}{3H}. \tag{18}$$

[2] C. van de Bruck and C. Longden, Phys. Rev. D **93** (2016) 063519.

We use the dimensionless parameter $N_e = -\ln(a/a_e)$ as a new measure of time.
The constant a_e is fixed by the condition that the end of inflation happens at $N_e = 0$.
$N_e > 0$ during inflation.
From Eqs. (16)–(18), we get the following leading-order equations:

$$\ln(H)'_{,N} = W_{,\phi} V'_{eff,\phi} \,, \tag{19}$$

$$\phi'_{,N} = 2WV'_{eff,\phi} \,, \tag{20}$$

where $A'_{,N} \equiv \frac{dA}{dN_e}$ for any function A,
$W \equiv V/U$.

SLOW-ROLL PARAMETERS

The slow-roll approximation requires $|\epsilon_i| \ll 1$, $|\delta_i| \ll 1$, and $|\zeta_i| \ll 1$, where the slow-roll parameters are

$$\epsilon_1 = \frac{1}{2}\frac{(H^2)'_{,N}}{H^2} \simeq \frac{1}{2}\frac{W'_{,N}}{W}, \quad \epsilon_{i+1} = -\frac{\epsilon'_{i,N}}{\epsilon_i}, \quad i \geqslant 1, \tag{21}$$

$$\zeta_1 = -\frac{U'_{,N}}{U}, \quad \zeta_{i+1} = -\frac{\zeta'_{i,N}}{\zeta_i}, \quad i \geqslant 1, \tag{22}$$

$$\delta_1 = -\frac{2H^2}{U}\xi'_{,N} \simeq -\frac{V}{3U^2}\xi'_{,N}, \quad \delta_{i+1} = -\frac{\delta'_{i,N}}{\delta_i}, \quad i \geqslant 1. \tag{23}$$

It is easy to get:

$$\epsilon_2 = 2\epsilon_1 - \frac{W''_{,NN}}{W'_{,N}}, \quad \zeta_2 = -\zeta_1 - \frac{U''_{,NN}}{U'_{,N}}, \quad \delta_2 = -2\epsilon_1 - \zeta_1 - \frac{\xi''_{,NN}}{\xi'_{,N}}.$$

INFLATIONARY PARAMETERS

We get the tensor-to-scalar ratio

$$r = 8|2\epsilon_1 + \zeta_1 - \delta_1| = \frac{4}{U}\left(\phi'_{,N}\right)^2 = \frac{8V}{U^2}V'_{eff,N}. \qquad (24)$$

The spectral index of scalar perturbations n_s has the following form:

$$n_s = 1 - 2\epsilon_1 - \zeta_1 + \frac{r'_{,N}}{r} = 1 + \frac{V''_{eff,NN}}{V'_{eff,N}}. \qquad (25)$$

The expression of the amplitude of the scalar perturbations in terms of the effective potential is as follows:

$$A_s = \frac{V}{6\pi^2 U^2 r} = \frac{1}{48\pi^2 V'_{eff,N}}. \qquad (26)$$

E.O. Pozdeeva, M. R. Gangopadhyay, M. Sami, A.V. Toporensky and S.Yu. Vernov, Phys. Rev. D **102** (2020) 043525 [arXiv:2006.08027]
E.O. Pozdeeva, S.Yu. Vernov, arXiv:2104.04995

MODELS WITH $\xi(\phi) = C/V(\phi)$

The choice of the function $\xi = C/V$ is actively studied[3]. In the case of a constant $U = U_0 = M_{Pl}^2/2$,

$$V_{eff} = \frac{C - 3U_0^2}{3V}, \tag{27}$$

and the slow-roll parameters are as follows:

$$\epsilon_1 = \frac{\left(3U_0^2 - C\right) {V'_{,\phi}}^2}{3U_0 V^2}, \quad \epsilon_2 = \frac{4\left(C - 3U_0^2\right)\left(VV''_{,\phi\phi} - {V'_{,\phi}}^2\right)}{3U_0 V^2},$$

$$\delta_1 = \frac{2C}{3U_0^2}\epsilon_1\,, \quad \delta_2 = \epsilon_2\,.$$

So, the inflationary parameters are

$$n_s = 1 + \frac{2\left(3U_0^2 - C\right)\left(2VV''_{,\phi\phi} - 3{V'_{,\phi}}^2\right)}{3U_0 V^2}\,. \tag{28}$$

$$r = \frac{16{V'_{,\phi}}^2\left(3U_0^2 - C\right)^2}{9U_0^3 V^2}. \tag{29}$$

[3]Z.K. Guo and D.J. Schwarz, Phys. Rev. D **81**, 123520 (2010)

In the case of $V = V_0\phi^n$, we obtain, taking into account $\epsilon_1(\phi(0)) = 1$,

$$\phi^2(N_e) = \frac{n(4N_e + n)(3U_0^2 - C)}{3U_0}.$$

$$n_s = 1 - \frac{2(n+2)}{4N_e + n}\,, \qquad r = \frac{16n(3U_0^2 - C)}{3U_0^2(4N_e + n)}\,, \tag{30}$$

Adding of the GB term with $\xi = C/V$ does not change n_s, but changes r.
In the case of $n = 2$, n_s is correct, but $\epsilon_2 = \delta_2 = 2\epsilon_1$. The slow-roll approximation is broken before the end of inflation.
In the case of $n = 4$, the slow-roll approximation is satisfied $\epsilon_2 = \delta_2 = \epsilon_1$, but

$$n_s = 1 - \frac{3}{N_e + 1}. \tag{31}$$

The Planck observation: $n_s = 0.9649 \pm 0.0042$ at 68% CL, implies that $75 < N_e < 96$.
It is the general situation of for GB inflationary models with n_s and r given by (30). E.O. Pozdeeva, *Universe* **7** (2021) 181, arXiv:2105.02772

The $\lambda\phi^4$ potential

Let

$$V = \lambda\phi^4, \qquad \xi = \xi_2\phi^{-2} + \xi_4\phi^{-4} + \xi_6\phi^{-6}, \tag{32}$$

with arbitrary constants λ, ξ_2, ξ_4, and ξ_6.

$$V_{eff} = \frac{\xi_2}{3\phi^2} + \frac{\beta}{3\phi^4} + \frac{\xi_6}{3\phi^6}, \tag{33}$$

where $\beta = \xi_4 - 3U_0^2/\lambda$.
The inflationary parameters are as follows:

$$n_s = 1 + \frac{8\lambda(\xi_2\phi^4 + 6\beta\phi^2 + 15\xi_6)}{3U_0\phi^4}, \qquad r = \frac{64\lambda^2\left(\xi_2\phi^4 + 2\beta\phi^2 + 3\xi_6\right)^2}{9U_0^3\phi^6}. \tag{34}$$

In the generic case, when parameters ξ_2, ξ_6, and β are nonzero, analytical solutions cannot be obtained. We use numeric computations at $\lambda = 0.1$, $\xi_6 = -0.1$, $U_0 = 1/2$, and $\beta = -7.4$.
The choice of β is from the fact to keep $A_s \sim 2.1 \times 10^{-9}$.
The parameter ξ_2 is taken in the range $0 \le \xi_2 \le 0.5$.

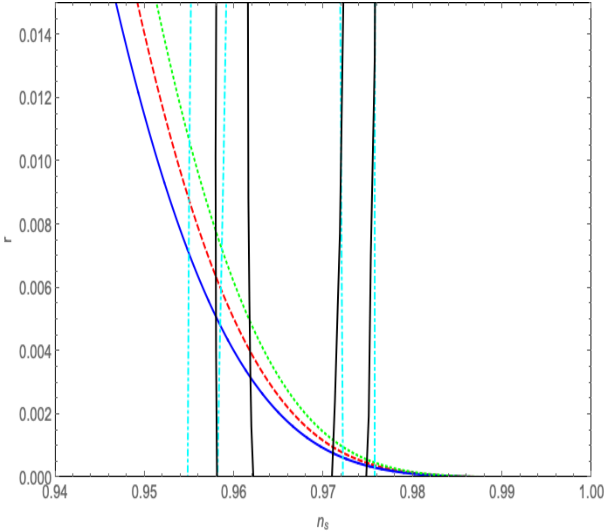

Figure: Blue solid, red dashed and green dot curves correspond to $N_e = 55$, $N_e = 60$, and $N_e = 65$ respectively. The contours correspond to the marginalized joint 68% and 95% CL.

The case of $\beta = 0$

In the case of $\xi_6 \neq 0$ and $\beta = 0$, the inflationary parameters are as follows:

$$n_s = 1 + \frac{8\lambda(\xi_2\phi^4 + 15\xi_6)}{3U_0\phi^4}, \qquad r = \frac{64\lambda^2\left(\xi_2\phi^4 + 3\xi_6\right)^2}{9U_0^3\phi^6}. \tag{35}$$

$$\phi(N_e) = \sqrt[4]{\frac{3\xi_6(3U_0\mathrm{e}^{-16\lambda\xi_2 N_e/(3U_0)} - 8\lambda\xi_2 - 3U_0)}{\xi_2(8\lambda\xi_2 + 3U_0)}} \tag{36}$$

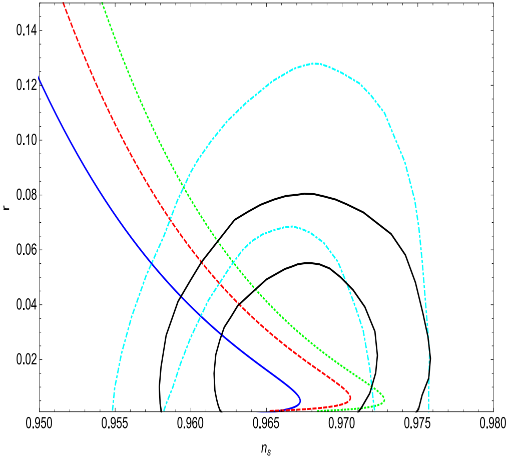

Figure: Parameter space of n_s and r for the model with $V = \lambda\phi^4$ and $\xi = \xi_2\phi^{-2} + \xi_4\phi^{-4} + \xi_6\phi^{-6}$ in the case of $\lambda = 0.1$, $\xi_6 = -0.1$, $U_0 = 1/2$, and $\beta = 0$. Blue solid, red dashed, and green dot curves correspond to $N_e = 55$, $N_e = 60$, and $N_e = 65$ respectively. The contours correspond to the marginalized joint 68% and 95% CL.

INFLATIONARY SCENARIOS WITH THE GIVEN FUNCTIONS $n_s(N_e)$ AND $r(N_e)$

Let the functions $n_s(N_e)$ and $r(N_e)$ coincide in the leading order of $1/N_e$ with parameters of conformal attractor models:

$$n_s = 1 - \frac{2}{N_e + N_0}\,, \tag{37}$$

$$r = \frac{12C_\alpha}{(N_e + N_0)^2}, \tag{38}$$

where $|N_0| \ll 60$ and $C_\alpha > 0$ are constants. The Starobinsky R^2 inflation and the Higgs-driven inflation correspond to $C_\alpha = 1$.

For models without the Gauss–Bonnet term, the case of an arbitrary C_α has been proposed in V. Mukhanov, Eur. Phys. J. C **73** (2013) 2486 and actively used in cosmological attractor approach ($\alpha-$attractor):

R. Kallosh and A. Linde, JCAP **1307** (2013) 002

D. Roest, JCAP **01** (2014) 007

M. Galante, R. Kallosh, A. Linde and D. Roest, Phys. Rev. Lett. **114** (2015) 141302

For models with the GB term, this approach has been proposed in E.O. Pozdeeva, *Eur. Phys. J.* C **80** (2020) 612.

The relation

$$n_s = 1 + \frac{V''_{eff,NN}}{V'_{eff,N}}, \qquad (39)$$

for the given $n_s(N_e)$ is a linear differential equation for $V_{eff}(N_e)$.
Substituting (37) into Eq. (39), we obtain:

$$V'_{eff,N}(N_e) = C_{eff}(N_e + N_0)^{-2} = \frac{C_{eff}}{4}(n_s - 1)^2,$$

$$A_s = \frac{1}{48\pi^2 V'_{eff,N}} = \frac{1}{12\pi^2 C_{eff}(n_s - 1)^2}. \qquad (40)$$

Substituting n_s and r are given by (37) and (38) into

$$n_s = 1 - 2\epsilon_1 - \zeta_1 + \frac{r'_{,N}}{r},$$

we obtain $2\epsilon_1 + \zeta_1 = 0$.
If U is a constant, then the potential V is a constant as well. So, H is a constant and $\epsilon_1 = 0$.
For a nonconstant $U(\phi)$, we obtain that $\zeta_1 = -2\epsilon_1 < -1$ during inflation.

MODEL WITH $U = M_{Pl}^2/2$

An inflationary model constructed in
E.O. Pozdeeva, *Eur. Phys. J.* C **80** (2020) 612 [arXiv:2005.10133]
has the function $U = M_{Pl}^2/2$, the potential

$$\tilde{V} = V_0 \exp\left(-\omega_0 \exp\left(-\sqrt{\frac{2}{3C_\alpha}}\frac{\phi}{M_{Pl}}\right)\right), \qquad (41)$$

and

$$\tilde{\xi} = \xi_0 \exp\left(\omega_0 \exp\left(-\sqrt{\frac{2}{3C_\alpha}}\frac{\phi}{M_{Pl}}\right)\right), \qquad (42)$$

where $V_0 > 0$, $C_\alpha > 0$, ω_0, and ξ_0 are constants.
The effective potential is

$$\tilde{V}_{eff} = \left(\frac{4\xi_0 V_0 - 3M_{Pl}^4}{12V_0}\right)\exp\left(\omega_0 \exp\left(-\sqrt{\frac{2}{3C_\alpha}}\frac{\phi}{M_{Pl}}\right)\right). \qquad (43)$$

Using Eq. (20), we obtain $\phi(N_e)$ in the slow-roll approximation:

$$\phi(N_e) = \frac{\sqrt{6C_\alpha}}{2} M_{Pl} \ln\left(\frac{2\omega_0(3M_{Pl}^4 - 4V_0\xi_0)}{9C_\alpha M_{Pl}^4}(N_e + N_0)\right), \qquad (44)$$

where N_0 is an integration constant.

The conditions that $0 < \epsilon_1 < 1$ during inflation (for $N_e > 0$) and $\epsilon_1 = 1$ at $N_e = 0$ give

$$C_\alpha = \frac{4N_0^2\left(3M_{Pl}^4 - 4V_0\xi_0\right)}{9M_{Pl}^4}\,.$$

The slow-roll parameter

$$\epsilon_2 = \frac{2}{N_e + N_0}, \tag{45}$$

so $\epsilon_2 < 1$ during inflation if $N_0 \geqslant 2$.
If $8\xi_0 V_0 < 3M_{Pl}^4$, then all slow-roll parameters are less than one during inflation.
Inflationary parameters are

$$n_s = 1 - \frac{2}{N_e + N_0} - \frac{2N_0^2}{\left(N_e + N_0\right)^2}, \quad r = \frac{16N_0^2\left(3M_{Pl}^4 - 4V_0\xi_0\right)}{3M_{Pl}^4\left(N_e + N_0\right)^2}\,. \tag{46}$$

The observable values of n_s, obtained by the telescope Planck:

$$n_s = 0.965 \pm 0.04,$$

allows us to restrict values of N_0:

$$2 \leqslant N_0 \leqslant 0.0199 N_e - 0.510 + 0.0102\sqrt{195 N_e^2 - 10000 N_e + 2500}.$$

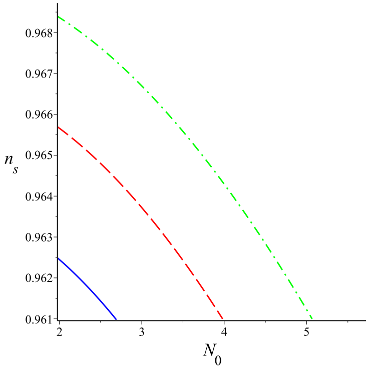

Figure: The inflationary parameter n_s as a function of N_0 for the different numbers of e-foldings during inflation: $N_e = 55$ (blue solid curve), $N_e = 60$ (red dash curve) and $N_e = 65$ (green dash-dot curve).

GENERALIZATION OF INFLATIONARY MODEL

To construct a set of inflationary models with the same functions $n_s(N_e)$ and $A_s(N_e)$ we put the condition that V'_{eff} does not change.
To get the same function $\phi(N_e)$ we add the condition that the function $W = V/U$ does not change.

$$U = \frac{M_{Pl}^2}{2} f(\phi), \quad V = f(\phi)\tilde{V} = V_0 f(\phi) \exp\left(- \omega_0 \exp\left(-\sqrt{\frac{2}{3C_\alpha}} \frac{\phi}{M_{Pl}} \right) \right),$$

$$\xi(\phi) = \left(\xi_0 + \frac{3M_{Pl}^4}{4V_0} (f(\phi) - 1) \right) \exp\left(\omega_0 \exp\left(-\sqrt{\frac{2}{3C_\alpha}} \frac{\phi}{M_{Pl}} \right) \right),$$

where $f(\phi)$ is a double differentiable function.
We do not fix the parameter $r(N_e)$:

$$r(N_e) = \frac{12 C_\alpha}{f \cdot (N_e + N_0)^2}, \qquad (47)$$

hence, the observation data gives restrictions on the function f.
Other restrictions on this function can be obtained from the condition that the slow-roll approximation should be satisfied during inflation.
We the parameters ϵ_i do not depend on f, whereas other slow-roll parameters depend on f.

AN EXPONENTIAL FUNCTION F

Let us consider the case

$$f(\phi) = f_0 \exp\left(\beta\omega_0 \exp\left(-\sqrt{\frac{2}{3C_\alpha}}\frac{\phi}{M_{Pl}}\right)\right),$$

where β is a constant. Using Eq. (44), we get

$$U = \frac{M_{Pl}^2}{2} f_0 \exp\left(\frac{2N_0^2\beta}{N_e + N_0}\right),$$

and

$$r = \frac{16N_0^2\left(3M_{Pl}^4 - 4V_0\xi_0\right)}{3M_{Pl}^4 f_0 (N_e + N_0)^2} \exp\left(-\frac{2N_0^2\beta}{N_e + N_0}\right). \quad (48)$$

$$V = f_0 V_0 \exp\left(\frac{2N_0^2(\beta - 1)}{N_e + N_0}\right),$$

$$\xi = \frac{1}{4V_0}\left(3M_{Pl}^4 f_0 \exp\left(\frac{2\beta N_0^2}{N_e + N_0}\right) - 3M_{Pl}^4 + 4\xi_0 V_0\right) \exp\left(\frac{2N_0^2}{N_e + N_0}\right).$$

To fix f_0 we assume that at the end of inflation $U = M_{Pl}^2/2$, therefore,

$$f_0 = \exp\left(-2N_0\beta\right) \,. \tag{49}$$

Table: Model parameters: β, $J \equiv V_0\xi_0/M_{Pl}^4$, V_0/M_{Pl}^4, ξ_0, and the corresponding values of r.

β	J	V_0/M_{Pl}^4	ξ_0	r
-0.5	0.72	$2.3556 \cdot 10^{-11}$	$3.0565 \cdot 10^{10}$	0.00009614
-0.5	0.5	$1.9630 \cdot 10^{-10}$	$2.5471 \cdot 10^{9}$	0.0008011
-0.3	0.5	$1.9630 \cdot 10^{-10}$	$2.5471 \cdot 10^{9}$	0.001737
-0.1	0.45	$2.3556 \cdot 10^{-10}$	$1.9103 \cdot 10^{9}$	0.004522
-0.1	0.2	$4.31863 \cdot 10^{-10}$	$4.6311 \cdot 10^{8}$	0.00829
0	0.2	$4.3186 \cdot 10^{-10}$	$4.6311 \cdot 10^{8}$	0.0122
0.1	-0.2	$7.4595 \cdot 10^{-10}$	$-2.6812 \cdot 10^{8}$	0.03106
0.1	-0.4	$9.0299 \cdot 10^{-10}$	$-4.4297 \cdot 10^{8}$	0.03760
0.2	-0.4	$9.0299 \cdot 10^{-10}$	$-4.4297 \cdot 10^{8}$	0.0554
0.25	-0.45	$9.4225 \cdot 10^{-10}$	$-4.7758 \cdot 10^{8}$	0.07011

CONCLUSIONS

We analyze the Einstein–Gauss–Bonnet gravity model:

$$S=\int d^4x\sqrt{-g}\left(U(\phi)R-\frac{c}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)-F(\phi)\mathcal{G}\right),$$

• In the case of $U(\phi)>0$, it is possible to introduce the effective potential V_{eff} which can be expressed through the coupling function U, the scalar field potential V and the coupling function with the Gauss–Bonnet term ξ:

$$V_{eff}=\frac{1}{3}\xi-\frac{U^2}{V}.$$

• For $c\geqslant 0$, it is convenient to investigate the structure of fixed points using the effective potential, indeed, the stable de Sitter solutions correspond to minima of the effective potential V_{eff}.
E.O. Pozdeeva, M. Sami, A.V. Toporensky, S.Yu. Vernov, *Phys. Rev. D* **100** (2019) 083527 [arXiv:1905.05085]

• The effective potential V_{eff} can be used to analyze the stability of de Sitter solutions in model with $F(\mathcal{G})$ term ($c=0$).
E.O. Pozdeeva, S.Yu. Vernov, Universe 7 (2021) 149 [arXiv:2104.11111]

CONCLUSIONS

• The effective potential V_{eff} plays an important role in the inflationary scenario construction. The inflationary parameters are

$$n_s = 1 + \frac{V''_{eff,NN}}{V'_{eff,N}}, \qquad A_s = \frac{1}{48\pi^2 V'_{eff,N}}.$$

E.O. Pozdeeva, S.Yu. Vernov, arXiv:2104.04995

• We have generalized the inflationary scenario with a constant $U = M_{Pl}^2/2$ proposed in

E.O. Pozdeeva, *Eur. Phys. J.* C **80** (2020) 612 [arXiv:2005.10133].

• Using the effective potential, we construct sets of inflationary models with nonconstant functions U that is equal to $M_{Pl}^2/2$ at the end of inflation. In distinguish to the cosmological attractor approach, we do not fix $r(N_e)$, but fix $\phi(N_e)$ and $n_s(N_e)$.

• We plan to generalize the Gauss-Bonnet inflationary models with a constant U proposed in

E.O. Pozdeeva, M. R. Gangopadhyay, M. Sami, A.V. Toporensky and S.Yu. Vernov, Phys. Rev. D **102** (2020) 043525 [arXiv:2006.08027]

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Thank for your attention