QUARKS-2020 ONLINE WORKSHOPS - 2021

Modified gravity in nonsingular cosmology

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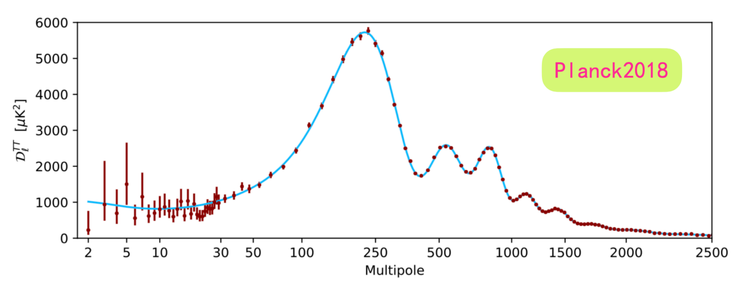
Outline

A motivation for studying nonsingular cosmology

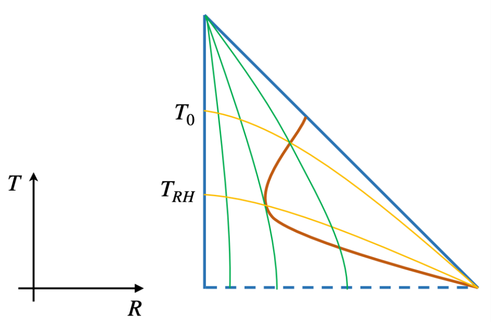
Inflation is so successful

- Inflation is a possible solution to the horizon, flatness, entropy, homogeneity, isotropy and primordial monopole problems.
- Inflation predicts nearly scale-invariant scalar perturbation, which is consistent with the CMB observations.
- Inflation sets the initial conditions of hot big bang expansion (RD, MD).
- ...



Inflation is so successful, but...

- Inflation may not be the only solution to the horizon, flatness, ..., problems. (ekpyrosis, genesis, ...)
- Inflation is geodesically incomplete in the past.
- The prediction of scale-invariant power spectrum of scalar perturbation depends on the initial states of perturbation modes. (“trans-Planckian” problem)



Trans-Planckian problem

- In unitary gauge, the quadratic action of scalar perturbation ζ is

$$S_\zeta^{(2)} = \int d^3x dt\, a^3 Q_s \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial_i \zeta)^2}{a^2} \right] .$$

$$u_k'' + \left(c_s^2 k^2 - \frac{z''}{z} \right) u_k = 0 \,,$$

$$u_k = z\zeta_k, \quad z = a\sqrt{2Q_s}$$

- In the slow-roll inflation, $z''/z \sim a''/a \simeq (2 + \mathcal{O}(\epsilon))/\tau^2$, $c_s = 1$. Thus the solution of Eq. (1) is

$$u_k(\tau) = \frac{\sqrt{-\pi\tau}}{2} \left[\alpha(k) H_\nu^{(1)}(-k\tau) + \beta(k) H_\nu^{(2)}(-k\tau) \right] ,$$

Wronskian condition: $|\alpha|^2 - |\beta|^2 = 1$.

Trans-Planckian problem

- Requiring the perturbation modes coincide with the Minkowski solution ($u_k \simeq \frac{1}{\sqrt{2k}} e^{-ik\tau}$) in the infinite past ($-k\tau \gg 1$) gives

$$|\alpha| = 1\,, \qquad |\beta| = 0\,,$$

$$\Rightarrow u_k = \frac{1}{\sqrt{2k}} e^{-ik\tau} \left(1 - \frac{i}{k\tau}\right),$$

  which corresponds to the Bunch-Davis (BD) vacuum.

- Setting the initial state of perturbation modes as the BD state is essential for inflation to predict the scale-invariant primordial perturbations. ($P_\zeta = \frac{k^3}{2\pi^2}|\zeta_k|^2$)
- It seems natural, since

$$u_k'' + \left(k^2 - \cancel{\frac{z''}{z}}\right) u_k = 0\,.$$

Trans-Planckian problem

- However, the wavelength of perturbation modes $\lambda \sim a$, we will inevitably have $\lambda \lesssim 1/\Lambda$ before some $\tau = \tau_\Lambda$, where $\Lambda \lesssim M_{\mathrm{P}}$ is the cutoff scale below which the higher-order derivative operators are negligible.
- When $\lambda \simeq \mathcal{O}(1/\Lambda)$, the neglected higher-order derivative operators will be no longer negligible.

$$L/M_{\mathrm{P}}^2 \sim \frac{\lambda_4}{\Lambda^2}(R^{(3)})^2 + \frac{\lambda_6}{\Lambda^4}(\nabla_i R^{(3)})^2 + \cdots ,$$

$$(R^{(3)})^2 \sim k^4 \zeta_k^2, \quad (\nabla_i R^{(3)})^2 \sim k^6 \zeta_k^2.$$

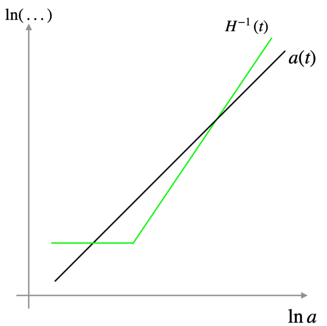
$$u_k'' + \left(\sim \sum_p \frac{k^{2p}}{\Lambda^{2p}}\right) k^2 u_k = 0, \quad \lambda \lesssim 1/\Lambda, \quad p \geqslant 0$$

- Requiring the perturbation modes to coincide with the Minkowski solution is not sufficiently reasonable.

Trans-Planckian Censorship Conjecture (TCC)

- "*a field theory consistent with a quantum theory of gravity does not lead to a cosmological expansion where any perturbation with length scale greater than the Hubble radius trace back to trans-Planckian scales at an earlier time*" (A. Bedroya and C. Vafa, JHEP 09 (2020) 123)
  The TCC suggests that inflation can only last for a limited e-folding number

$$N = \int_{t_i}^{t_f} H dt < \ln \frac{M_{\rm P}}{H_f} \, .$$



Trans-Planckian Censorship Conjecture (TCC)

- Neglecting the higher-order operators, we have $|u_k| = 1/\sqrt{2k}$ for the sub-horizon perturbation modes (it is also approximately valid for $\lambda \lesssim 1/H$). On sub-horizon scale

$$|\zeta_k| \sim |u_k|/a \sim 1/a\,.$$

At the *beginning* of inflation, the spectrum of the sub-horizon perturbation ($\lambda \ll 1/H$) is

$$P_\zeta^{1/2}(t_i, k) = P_\zeta^{1/2}(t_f, k)\frac{a_f}{a_i} \sim \frac{H_f}{\sqrt{\epsilon_f}M_{\mathrm{P}}}\frac{a_f}{a_i},$$

provided this perturbation mode crossed the horizon ($k \simeq a_f H_f$) at t_f.

- The validity of perturbation theory requires that $P_\zeta^{1/2}(t_i, k) < 1$, otherwise the spacetime will be distorted by lots of black holes on the sub-Hubble scale.

$$P_\zeta^{1/2}(t_i, k) < 1 \Rightarrow \boxed{\int_{t_i}^{t_f} H dt < \ln\frac{\sqrt{\epsilon_f}M_{\mathrm{P}}}{H_f}}\,.$$

Trans-Planckian Censorship Conjecture (TCC)

- A similar analysis for the tensor perturbation gives exactly the same constraint as that of the TCC.
- The TCC might suggest the existence of a pre-inflationary or alternative evolution.

Minkowski in past-complete pre-inflation

The **Raychaudhuri equation** for the null geodesics

$$\frac{d\theta}{d\eta} \leqslant -\frac{\theta^2}{2} - R_{\mu\nu} k^\mu k^\nu \,.$$

The **null convergence condition** (NCC)

$$R_{\mu\nu} k^\mu k^\nu \geq 0 \quad \Rightarrow \quad d\theta/d\eta + \theta^2/2 < 0 \,.$$

Back along η the null rays will converge (past-incompletion of inflation), which indicates that the wavelengths of perturbation modes will be blueshifted, hence $\lambda \lesssim 1/\Lambda$ is inevitable.

Thus, only if the pre-inflationary era is past-complete,[1] it is possible to set the initial state of perturbations in the infinite past as the Minkowski vacuum.

[1] Necessary condition but not sufficient.

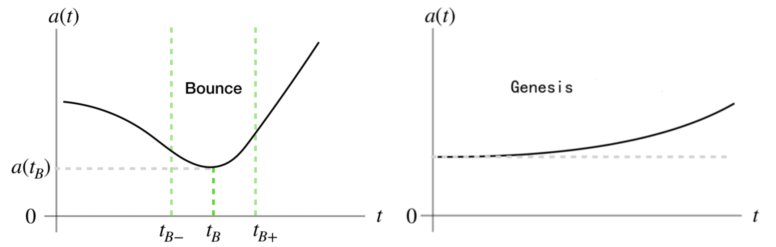
Minkowski in past-complete pre-inflation

The past-completion requires that the converging null rays must be reversed (i.e., $d\theta/d\eta > 0$) at least for some period. Hence, the NCC (or NEC) must be violated, i.e.,

$$R_{\mu\nu}k^{\mu}k^{\nu} < 0,$$

or equivalently, $\dot{H} > 0$, since $R_{\mu\nu}k^{\mu}k^{\nu} = -2\dot{H}(k^0)^2$ for the spatially flat FLRW metric.

Two scenarios

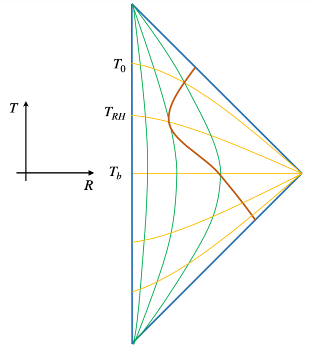


In the infinite past, we have an asymptotic Minkowski spacetime.

Minkowski in past-complete pre-inflation

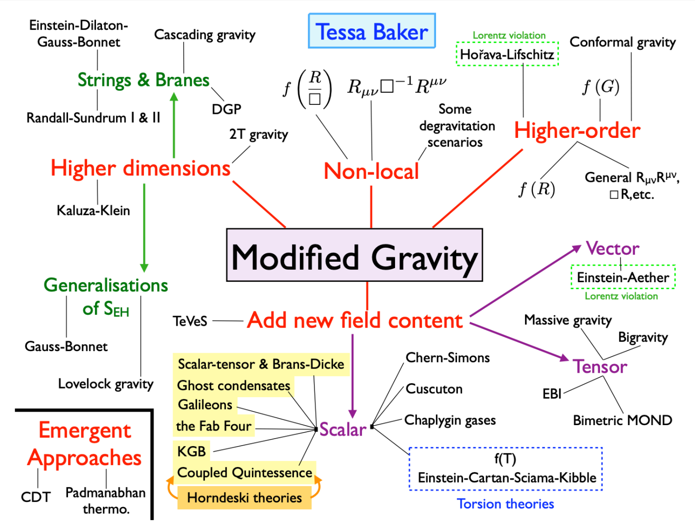
- The k/Λ-suppressed higher-order derivative operators are negligible.
- Such scenarios are consistent with the TCC, since the sub-Planckian perturbation modes may never have the opportunity to cross their horizon.
- All perturbation modes are in a flat Minkowski space in the infinite past. A past-complete pre-inflationary evolution may automatically prepare the initial state required for the inflationary perturbations at the CMB window.

(Y. Cai, Y.-S. Piao, Sci. China Phys. Mech. Astron. 63 no. 11, (2020) 110411, arXiv:1909.12719.)



Modified gravity in nonsingular cosmology

Modified gravity



Credit: arXiv: 1512.05356

Modified gravity in nonsingular cosmology

- The study of inflationary cosmology suggests that we should pay attention not only to the evolution of background but also to the evolution of perturbations in the study of nonsingular cosmology.
- We will pay special attention to scalar-tensor theories.
- In unitary gauge, the quadratic action of scalar perturbation ζ is

$$S_\zeta^{(2)} = \int d^4x a^3 Q_s \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2}\right] ,$$

$$Q_s > 0, \qquad c_s^2 > 0 .$$

$$\begin{aligned} Q_s < 0 \quad &\rightarrow \quad \text{ghost instability,} \\ c_s^2 < 0 \quad &\rightarrow \quad \text{gradient instability.} \end{aligned}$$

Modified gravity in nonsingular cosmology

$$S=\int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R+P(\phi,X)\right]\,,\quad X=\nabla_\mu\phi\nabla^\mu\phi=-\dot{\phi}^2$$

$$S_\zeta^{(2)}=\int d^4xa^3Q_s\left[\dot{\zeta}^2-c_s^2\frac{(\partial\zeta)^2}{a^2}\right]\,,$$

$$Q_s=c_1=\epsilon M_p^2+2\frac{\dot{\phi}^4}{H^2}P_{XX}\,,\quad c_2=M_p^2\,,\quad c_3=\frac{a}{H}M_p^2\,,\quad \gamma=H\,.$$

$$c_s^2=\frac{2\dot{H}M_p^2}{2\dot{H}M_p^2-4\dot{\phi}^4P_{XX}}=1+\frac{2\dot{\phi}^4P_{XX}}{\dot{H}M_p^2-2\dot{\phi}^4P_{XX}}=\frac{\epsilon}{Q_s}M_p^2\,.$$

Thus we may have $Q_s>0$, but we will have $c_s^2<0$ during the period of NEC violation ($\epsilon<0$).

Modified gravity in nonsingular cosmology

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + P(\phi, X) + G(\phi, X)\Box\phi\right], \quad X = \nabla_\mu\phi\nabla^\mu\phi$$

We have

$$Q_s = c_1 = \frac{1}{4\gamma^2 M_p^2}\Big[12H\dot{\phi}^3 G_X M_p^2 - 24H\dot{\phi}^5 G_{XX}M_p^2 - 4\dot{\phi}^4 M_p^2\left(G_{\phi X} - 2P_{XX}\right)$$
$$+4\dot{\phi}^2 G_X M_p^2\ddot{\phi} + 12\dot{\phi}^6 G_X^2 - 4\dot{H}M_p^4\Big]$$

$$c_2 = M_p^2, \quad c_3 = \frac{a}{\gamma}M_p^2, \quad \gamma = H - \frac{\dot{\phi}^3 G_X}{M_p^2}.$$

$$c_s^2 = \frac{H\gamma - \gamma^2 - \dot{\gamma}}{\gamma^2 Q_s}M_p^2$$

$\boxed{\cancel{\text{NEC}} \neq \text{instabilities}}$

D. A. Easson, I. Sawicki and A. Vikman, JCAP 1111, 021 (2011); A. Ijjas and P. J. Steinhardt, Phys. Rev. Lett. 117, no. 12, 121304 (2016).

Modified gravity in nonsingular cosmology

"no-go" theorem

- "no-go" theorem for cubic Galileon: healthy nonsingular cosmological models based on the cubic Galileon does not exist.
  M. Libanov, S. Mironov and V. Rubakov, JCAP 1608, 08, 037 (2016)
- "no-go" theorem for full Horndeski theory.
  T. Kobayashi, Phys. Rev. D 94, no. 4, 043511 (2016)
- loophole: strong gravity in the past Yulia Ageeva's talk
- Horndeski $\Rightarrow$ beyond Horndeski$\Rightarrow$ Degenerate Higher-Order Scalar-Tensor (DHOST) theories

The EFT method

The EFT method

The EFT of cosmological perturbation (inflation, dark energy, ...) in unitary gauge

$$
\begin{aligned}
S \quad = \quad & \int d^4x\sqrt{-g}\Big[\frac{M_p^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} \\
& + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K \delta g^{00} - m_4^2(t)\left(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}\right) + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00} \\
& - \bar{m}_4^2(t)\delta K^2 + \frac{\bar{m}_5(t)}{2}R^{(3)}\delta K + \frac{\bar{\lambda}(t)}{2}(R^{(3)})^2 + \ldots \\
& - \frac{\tilde{\lambda}(t)}{M_p^2}\nabla_i R^{(3)}\nabla^i R^{(3)} + \ldots\Big] ,
\end{aligned}
$$

C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan and L. Senatore, JHEP 0803, 014 (2008);

G. Gubitosi, F. Piazza and F. Vernizzi, JCAP 1302, 032 (2013);

J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, JCAP 1308, 025 (2013)

The EFT method

$$
\begin{aligned}
S \quad = \quad & \int d^4x\sqrt{-g}\Big[\frac{M_p^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} \\
& + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K \delta g^{00} - m_4^2(t)\left(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}\right) + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00} \\
& - \bar{m}_4^2(t)\delta K^2 + \frac{\bar{m}_5(t)}{2}R^{(3)}\delta K + \frac{\bar{\lambda}(t)}{2}(R^{(3)})^2 + \ldots \\
& - \frac{\tilde{\lambda}(t)}{M_p^2}\nabla_i R^{(3)}\nabla^i R^{(3)} + \ldots\Big],
\end{aligned}
$$

$$
\begin{aligned}
& ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\ , \\
& R = R^{(3)} - K^2 + K_{\mu\nu}K^{\mu\nu} + 2\nabla_\mu\left(Kn^\mu - n^\nu\nabla_\nu n^\mu\right) \\
& \delta g^{00} = g^{00} + 1\,, \quad \delta K_{\mu\nu} = K_{\mu\nu} - h_{\mu\nu}H \\
& \delta K^{\mu\nu} = K^{\mu\nu} - h^{\mu\nu}H, \quad \delta K = \delta K^\mu_\mu = K^\mu_\mu - 3H
\end{aligned}
$$

Background equation:

$$
\begin{aligned}
3M_p^2\left[f(t)H^2 + \dot{f}(t)H\right] \quad &= \quad c(t) + \Lambda(t)\,, \\
-M_p^2\left[2f(t)\dot{H} + 3f(t)H^2 + 2\dot{f}(t)H + \ddot{f}(t)\right] \quad &= \quad c(t) - \Lambda(t)
\end{aligned}
$$

The EFT method

$$
\begin{aligned}
S \quad = \quad & \int d^4x\sqrt{-g}\Big[\frac{M_p^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} \\
& + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K \delta g^{00} - m_4^2(t)\left(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}\right) + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00} \\
& - \bar{m}_4^2(t)\delta K^2 + \frac{\bar{m}_5(t)}{2}R^{(3)}\delta K + \frac{\bar{\lambda}(t)}{2}(R^{(3)})^2 + \ldots \\
& - \frac{\tilde{\lambda}(t)}{M_p^2}\nabla_i R^{(3)}\nabla^i R^{(3)} + \ldots\Big],
\end{aligned}
$$

Cubic Galieon: $f = 1\,, \quad m_4^2 = \tilde{m}_4^2 = \bar{m}_4^2 = \bar{m}_5 = \bar{\lambda} = \tilde{\lambda} = 0$

Horndeski: $m_4^2 = \tilde{m}_4^2\,, \quad \bar{m}_4^2 = \bar{m}_5 = \bar{\lambda} = \tilde{\lambda} = 0$

beyond Horndeski: $m_4^2(t) \neq \tilde{m}_4^2(t)\,, \quad \bar{m}_4^2 = \bar{m}_5 = \bar{\lambda} = \tilde{\lambda} = 0$

The EFT method

The quadratic action of tensor perturbation is

$$S_{\gamma}^{(2)} = \frac{M_p^2}{8}\int d^4x a^3 Q_T \left[\dot{\gamma}_{ij}^2 - c_T^2 \frac{(\partial_k \gamma_{ij})^2}{a^2}\right] ,$$

where

$$Q_T = f + 2\left(\frac{m_4}{M_p}\right)^2 > 0\,, \qquad c_T^2 = \frac{f}{Q_T} > 0\,.$$

The quadratic action of scalar perturbation ζ is

$$S_{\zeta}^{(2)} = \int d^4x a^3 \left[c_1 \dot{\zeta}^2 - \left(\frac{\dot{c}_3}{a} - c_2\right)\frac{(\partial\zeta)^2}{a^2} + \frac{c_4}{a^4}(\partial^2\zeta)^2 - \frac{16\tilde{\lambda}(t)}{M_p^2 a^6}\left(\partial^3\zeta\right)^2\right] ,$$

$c_{1,3,4}$ are complicated, while $c_2 = M_p^2 f(t)$.

$$Q_s = c_1, \quad c_s^2 = \left(\frac{\dot{c}_3}{a} - c_2\right)/c_1$$

Stability of the scalar perturbation requires:

$$\boxed{c_1 > 0, \quad \dot{c}_3 - a c_2 > 0\,.}$$

"No-go"

- $c_1 > 0, \quad \dot{c}_3 - ac_2 > 0$

$$c_3\Big|_{t_f} - c_3\Big|_{t_i} > \int_{t_i}^{t_f} ac_2 dt = M_p^2 \int_{t_i}^{t_f} af(t)dt \, .$$

  The RHS corresponds to the affine parameter of the graviton geodesics, which should be divergent. Thus c_3 must cross 0 at sometime t with $t_i < t < t_f$.

- For the Horndeski theory $(m_4^2(t) = \tilde{m}_4^2(t))$,

$$c_3 = \frac{aM_p^2}{\gamma} Q_T^2 \, ,$$

$$\gamma = H\left(1 + \frac{2m_4^2}{M_p^2}\right) - (1/2)m_3^3/M_p^2 \, , \quad Q_T = f + 2\left(\frac{m_4}{M_p}\right)^2$$

  There is "no-go" or strong gravity.

M. Libanov, S. Mironov and V. Rubakov, JCAP 1608, 08, 037 (2016)

T. Kobayashi, Phys. Rev. D 94, no. 4, 043511 (2016)

Avoiding "No-go" in beyond Horndeski theory

- In beyond Horndeski theory, $m_4^2(t) \neq \tilde{m}_4^2(t)$. Considering the EFT operator $\frac{\tilde{m}_4^2(t)}{2} R^{(3)} \delta g^{00}$, e.g.,

$$\begin{aligned} S_{eff} &= \int d^4x \sqrt{-g} \Big[\frac{M_p^2}{2} R - \Lambda(t) - c(t) g^{00} \\ &\quad + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} + \frac{\tilde{m}_4^2(t)}{2} R^{(3)} \delta g^{00} \Big] \,, \end{aligned}$$

where we have set $f(t) = 1$.

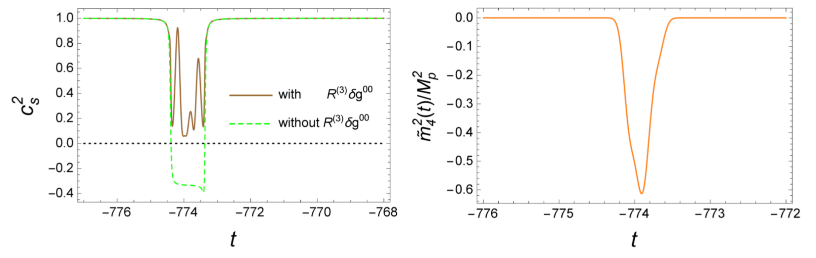
$$c_3 = \frac{a M_p^2}{\gamma} \left(1 + \frac{2\tilde{m}_4^2}{M_p^2} \right) , \quad \gamma = H - (1/2) m_3^3 / M_p^2$$

$$m_4^2(t) = 0, \qquad Q_T = 1 \,.$$

If at that time we have $2\tilde{m}_4^2 / M_p^2$ cross -1, the c_3 will cross 0 naturally without the divergence of γ. Hence, the "no-go" can be avoided.

Y. Cai, Y. Wan, H. G. Li, T. Qiu and Y. S. Piao, JHEP 1701, 090 (2017), arXiv:1610.03400; P. Creminelli, D. Pirtskhalava, L. Santoni and E. Trincherini, JCAP 1611, 11, 047 (2016), arXiv:1610.04207.

Avoiding "No-go" in beyond Horndeski theory



(a) c_s^2 (b) $\tilde{m}_4^2/M_p^2$

Y. Cai, Y. Wan, H. G. Li, T. Qiu and Y. S. Piao, JHEP 1701, 090 (2017), arXiv:1610.03400

Avoiding "No-go" in beyond Horndeski theory

An example of the EFT of nonsingular cosmology can be given as

$$S=\int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R+P(\phi,X)\right)+S_{\delta g^{00}R^{(3)}}\,,$$

where $S_{\delta g^{00}R^{(3)}}=\int d^4x\sqrt{-g}L_{\delta g^{00}R^{(3)}}$,

$$\begin{aligned}L_{\delta g^{00}R^{(3)}} &= \frac{f_1(\phi)}{2}\delta g^{00}R^{(3)}\\ &= \frac{f}{2}R-\frac{X}{2}\int f_{\phi\phi}d\ln X-\left(f_\phi+\int\frac{f_\phi}{2}d\ln X\right)\Box\phi\\ &+ \frac{f}{2X}\left[\phi_{\mu\nu}\phi^{\mu\nu}-(\Box\phi)^2\right]-\frac{f-2Xf_X}{X^2}\left[\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu-(\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu\right]\end{aligned}$$

where $f(\phi,X)=f_1\left(1+X/f_2\right)$.

The covariant lagrangian belongs to the beyond Horndeski theory.

Y. Cai and Y. S. Piao, JHEP 1709, 027 (2017), arXiv:1705.03401; R. Kolevatov, S. Mironov, N. Sukhov and V. Volkova, JCAP 1708, no. 08, 038 (2017), arXiv:1705.06626.

Summary

Summary

- Due to the past-incompleteness of inflation, physics beyond inflation should be considered to set the initial states for the inflationary perturbations.
- In nonsingular cosmology, the spacetime may be Minkowskian in the infinite past. Hence, it is natural to pick out the Minkowski vacuum state as the initial state of perturbations.
- The EFT method is powerful for studying instability problems of perturbations in nonsingular cosmology.
- These instability problems of perturbations in nonsingular cosmology can be avoided in beyond Horndeski theory or more general theories (e.g., DHOST).

Thanks!