Deformed black hole in Sagittarius A

QUARKS
June 10th 2021

Timothy Anson
Laboratoire de Physique des 2 Infinis, Orsay

based on:
2006.06461, TA, E. Babichev, C. Charmousis, M. Hassaine, JHEP01(2021)018
2103.05490, TA, E. Babichev, C. Charmousis, (accepted in PRD)

Introduction

- The Kerr solution describes rotating black holes in general relativity (GR)
- It is interesting to construct deformations of the Kerr spacetime, in order to test general relativity and potentially find signatures of modified theories of gravity
- Usually, *ad hoc* deformations of the Kerr spacetime are constructed **[Psaltis+, 2011; Johannsen, 2013; Papadopoulos+, 2018; ...]**
- Using the disformal map, one can construct deformed versions of the Kerr spacetime which are solutions to higher-order scalar-tensor theories
- We study the post-Newtonian motion of stars around a deformed Kerr black hole and discuss current and future experiments in this context

Table of contents

1. Disforming the Kerr metric

2. Stars orbiting Sgr A*

1. Disforming the Kerr metric

Kerr solution

- Vacuum solution of GR describing a rotating black hole **[Kerr, 1963]**. The metric g verifies $R_{\mu\nu} = 0$.
- In Boyer-Lindquist coordinates, the metric tensor is:

$$\mathrm{d}s_{\mathrm{K}}^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)\mathrm{d}t^2 - \frac{4aMr\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]\mathrm{d}\varphi^2 + \frac{\rho^2}{\Delta}\mathrm{d}r^2 + \rho^2\mathrm{d}\theta^2$$

where M is the mass, a is the angular momentum per unit mass, and

$$\rho^2 = r^2 + a^2\cos^2\theta\ ,$$
$$\Delta = r^2 + a^2 - 2Mr\ .$$

- $R_{\mu\nu\alpha\sigma}R^{\mu\nu\alpha\sigma}$ is singular at $\rho = \sqrt{r^2 + a^2\cos^2\theta} = 0$, so there is a ring singularity at

$$r = 0 \qquad \text{and} \qquad \theta = \frac{\pi}{2}$$

Disforming the Kerr metric

- We start from the Kerr solution g^{K}, and perform the disformal transformation

$$\tilde{g}_{\mu\nu} = g^{\mathrm{K}}_{\mu\nu} - \frac{D}{q^2}\,\partial_\mu\phi\,\partial_\nu\phi\,,$$

$$\phi = q_0\left[t + \int \frac{\sqrt{2Mr(a^2+r^2)}}{\Delta}\mathrm{d}r\right]\,.$$

- When $D=0$, stealth-Kerr solution in degenerate higher-order scalar-tensor theories (DHOST) **[Charmousis+, 2019]**
- The scalar field defines a timelike geodesic direction for the Kerr metric, since we have

$$g^{\mathrm{K}}_{\mu\nu}\partial^\mu\phi\partial^\nu\phi = -q_0^2 \qquad \Rightarrow \qquad \partial^\mu\phi\nabla_\mu\nabla_\nu\phi = 0$$

- Because the DHOST Ia class is stable under the disformal map **[Zumalacárregui+; Bettoni+, 2015; ...]**, we obtain another DHOST solution depending on the constants $\{D, q\}$ **[Achour+, 2020]**

Disformed Kerr metrics

- The line element becomes

$$d\tilde{s}^2 = -\left(1 - \frac{2\tilde{M}r}{\rho^2}\right)dt^2 - 2D\frac{\sqrt{2\tilde{M}r(a^2+r^2)}}{\Delta}dtdr + \frac{\rho^2\Delta - 2\tilde{M}rD(1+D)(a^2+r^2)}{\Delta^2}dr^2$$
$$-\frac{4\sqrt{1+D}\tilde{M}ar\sin^2\theta}{\rho^2}dtd\varphi + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta\right]d\varphi^2 + \rho^2 d\theta^2$$

with $\tilde{M} = M/(1+D)$ and the rescaling $t \to \sqrt{1+D}t$

- The scalar again defines a geodesic direction, since

$$\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -\frac{q_0^2}{1+D}$$

- If $a = 0$, there exists a diffeomorphism $dt \to dT + f(r)dr$ that brings the metric to the form [Babichev+, 2017; Achour+, 2019]

$$d\tilde{s}^2 = -\left(1 - \frac{2\tilde{M}}{r}\right)dT^2 + \left(1 - \frac{2\tilde{M}}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

Regular solutions

- The disformed metric has the following curvature scalars

$$\tilde{R} = -\frac{Da^2 Mr[1 + 3\cos(2\theta)]}{(1+D)\rho^6}, \quad \tilde{R}_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta} = \frac{M^2 Q_2(r,\theta)}{\rho^{12}(r^2+a^2)(1+D)^2},$$

- The solution is not Ricci-flat, but the only singularity is at $\rho = 0$, like Kerr. To verify this, one changes coordinates to

$$t \to v - r - \int \frac{2Mr}{\Delta}\mathrm{d}r\,, \qquad \varphi \to -\Phi - a\int \frac{\mathrm{d}r}{\Delta}$$

- The metric components are regular in these coordinates, and the scalar field reads

$$\phi = q_0\left(v - r + \int \frac{\mathrm{d}r}{1 + \sqrt{\frac{r^2+a^2}{2Mr}}}\right)$$

- The scalar acts as a global time function (stable causality)

- We still have the two commuting Killing vectors ∂_t and ∂_φ associated to axisymmetry
- However, defining $\xi_{(t)} = \tilde{g}_{t\mu}\mathrm{d}x^\mu$ and $\xi_{(\varphi)} = \tilde{g}_{\varphi\mu}\mathrm{d}x^\mu$, we now have

$$\xi_{(t)} \wedge \xi_{(\varphi)} \wedge \mathrm{d}\xi_{(t)} = -D\frac{4a^2\tilde{M}r\sqrt{2\tilde{M}r(a^2+r^2)}\cos\theta\sin^3\theta}{\rho^4}\mathrm{d}t \wedge \mathrm{d}r \wedge \mathrm{d}\theta \wedge \mathrm{d}\varphi$$

- This means we cannot write the metric in a form that is invariant under the reflection $(t, \varphi) \to (-t, -\varphi)$
- It also has an impact on the separability structure of the spacetime, and we no longer have a nontrivial Killing tensor **[Benenti+, 1979,1980]**
- The geodesic equations should be integrated numerically, and the shadows of disformed Kerr spacetimes have been studied **[Long+, 2020]**

Important hypersurfaces in the disformed spacetimes

- Similarly to Kerr, there is a limiting surface for static observers, or ergosphere, defined by $\tilde{g}_{tt} = 0$
- The limiting surface for stationary observers is obtained by solving the equation

$$\tilde{g}_{tt}\tilde{g}_{\varphi\varphi} - \tilde{g}_{t\varphi}^2 = 0$$

- It is a timelike hypersurface, and hence it cannot correspond to an event horizon
- Instead, the horizon surface $r = R(\theta)$ is null and R must verify

$$R'(\theta)^2 + R^2 + a^2 - 2\tilde{M}R + \frac{2\tilde{M}Da^2 R \sin^2\theta}{\rho^2(R,\theta)} = 0$$

- What happens between the stationary limit and horizon surfaces ?

Asymptotically similar to Kerr

- Asymptotically, the Kerr metric with parameters $\{\tilde{M}, \tilde{a}\}$ can be written

$$\mathrm{d}s^2_{\mathrm{Kerr}} = -\left[1 - \frac{2\tilde{M}}{r} + \mathcal{O}\left(\frac{1}{r^3}\right)\right]\mathrm{d}T^2 - \left[\frac{4\tilde{a}\tilde{M}}{r^3} + \mathcal{O}\left(\frac{1}{r^5}\right)\right][x\mathrm{d}y - y\mathrm{d}x]\,\mathrm{d}T$$
$$+ \left[1 + \mathcal{O}\left(\frac{1}{r}\right)\right]\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$

- After a coordinate transformation, one can write the disformal metric as

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}s^2_{\mathrm{Kerr}} + \frac{D}{1+D}\left[\mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}}{r^3}\right)\mathrm{d}T^2 + \mathcal{O}\left(\frac{\tilde{a}^2\tilde{M}^{3/2}}{r^{7/2}}\right)\alpha_i \mathrm{d}T\mathrm{d}x^i + \mathcal{O}\left(\frac{\tilde{a}^2}{r^2}\right)\beta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]$$

with $\alpha_i, \beta_{ij} \sim \mathcal{O}(1)$.

- The physical parameters determined from the asymptotic expansion are

$$\tilde{M} = \frac{M}{1+D}\,, \qquad \tilde{a} = a\sqrt{1+D}$$

Limit $D \to \infty$: noncircular Schwarzschild metric

- In the limit $D \to \infty$, one can obtain a simpler line element. After a coordinate transformation, it reads with $\tilde{\chi} = \tilde{a}/\tilde{M}$

$$
\mathrm{d}\tilde{s}^2_{\mathrm{NCS}} = -\left(1 - \frac{2\tilde{M}}{r}\right)\left(\mathrm{d}T + \frac{2\tilde{\chi}\tilde{M}^2 \sin^2\theta}{r - 2\tilde{M}}\mathrm{d}\varphi\right)^2 + \left(1 - \frac{2\tilde{M}}{r}\right)^{-1}\left(\mathrm{d}r - \sqrt{\frac{2\tilde{M}^3}{r}}\tilde{\chi}\sin^2\theta \mathrm{d}\varphi\right)^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right)
$$

- After the field redefinition $\psi = \sqrt{1+D}\phi/q_0$, the scalar reads

$$
\psi = T + 2\sqrt{2\tilde{M}r} - 4\tanh^{-1}\sqrt{\frac{r}{2\tilde{M}}}\,.
$$

- The metric and scalar can be shown to be solutions to a particular class of DHOST theories

Limit $D \to -1$: quasi-Weyl metric

- In the limit $D \to -1$, we have

$$\mathrm{d}\tilde{s}^2_{\mathrm{QW}} = -\left(1 - \frac{2\tilde{M}r}{\rho^2}\right)\mathrm{d}t^2 + \frac{\rho^2}{r^2+a^2}\mathrm{d}r^2 + 2\sqrt{\frac{2\tilde{M}r}{r^2+a^2}}\mathrm{d}t\mathrm{d}r + \rho^2\mathrm{d}\theta^2 + \left(r^2+a^2\right)\sin^2\theta\mathrm{d}\varphi^2$$

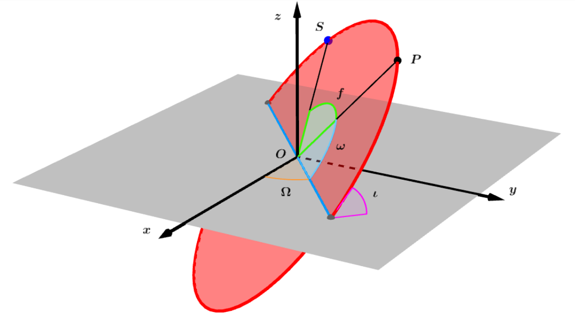
- It is *a priori* a singular limit, since after the redefinition $t \to t/\sqrt{1+D}$ we have

$$\phi = \frac{q_0}{\sqrt{1+D}}\left[t + (1+D)\int \frac{\sqrt{2\tilde{M}r(a^2+r^2)}}{\Delta}\mathrm{d}r\right] .$$

- After the field redefinition $\psi = \sqrt{1+D}\phi/q_0$, the scalar is simply $\psi = t$
- Again, one can isolate the corresponding DHOST theories
- These 2 simpler examples could be useful in understanding the properties of noncircular spacetimes

2. Stars orbiting Sgr A*

Orbit of stars around Sgr A*



- The pericenter precession ($\Delta\omega$) for the star S2 around Sagittarius A* has been measured **[GRAVITY, 2020]**
- In the future, smaller effects ($\Delta\Omega, \Delta\iota$) will be probed (for instance with stars with high eccentricities and shorter orbital periods **[Will, 2007]**)
- This will provide a measurement of J and Q, the spin and quadrupole moment of the black hole, providing a test of the no-hair theorem in GR:

$$Q = -Ma^2$$

Secular variation of orbital elements

- We study the post-Newtonian (PN) motion of stars around a deformed Kerr black hole, using coordinates that are harmonic for the Kerr metric $\Box x^\mu = 0$
- We write the Gauss planetary equations up to 2PN order, and use a two-timescale analysis closely following [Will+, 2016]. A second variable $\Theta = \epsilon u$ is introduced, with $u = \omega + f$. The evolution of orbital parameters X_k can be written

$$\frac{\mathrm{d}X_k}{\mathrm{d}u} = \epsilon Q_k \left(X_l(u), u\right) \ ,$$

- Each parameter is decomposed according to

$$X_k(\Theta, u) = \bar{X}_k(\Theta) + \epsilon Z_k(\bar{X}_l, u) \ , \qquad \bar{X}_k(\Theta) = \langle X_k(\Theta, u) \rangle \ , \qquad \langle Z_k \rangle = 0 \ .$$

- From this we obtain, with $Q_k^0 = Q_k(\bar{X}_l, u)$

$$\frac{\mathrm{d}\bar{X}_k}{\mathrm{d}u} = \epsilon \langle Q_k^0 \rangle + \epsilon^2 \left[\langle Q_{k,l}^0 \int_0^u Q_l^0 \mathrm{d}u' \rangle + \langle Q_{k,l}^0 \rangle \langle u Q_l^0 \rangle - \langle (u + \pi) Q_{k,l}^0 \rangle \langle Q_l^0 \rangle \right] + \mathcal{O}\left(\epsilon^3\right)$$

- For the disformed Kerr metrics in the generic case, we obtain the following expressions at 2PN order, with $\alpha = e\cos\omega$ and $\beta = e\sin\omega$

$$
\begin{aligned}
\frac{\mathrm{d}\bar{p}}{\mathrm{d}u} &= 0\,,\\
\frac{\mathrm{d}\bar{\alpha}}{\mathrm{d}u} &= -\frac{3\tilde{M}\bar{\beta}}{\bar{p}} + 6\tilde{\chi}\bar{\beta}\cos\bar{\iota}\left(\frac{\tilde{M}}{\bar{p}}\right)^{3/2} + \frac{3\tilde{M}^2\bar{\beta}}{4\bar{p}^2}\left(10-\bar{\alpha}^2-\bar{\beta}^2\right) - \frac{3\tilde{M}^2\bar{\beta}\tilde{\chi}^2(5\cos^2\bar{\iota}-1)}{4\bar{p}^2(1+D)}\,,\\
\frac{\mathrm{d}\bar{\beta}}{\mathrm{d}u} &= \frac{3\tilde{M}\bar{\alpha}}{\bar{p}} - 6\tilde{\chi}\bar{\alpha}\cos\bar{\iota}\left(\frac{\tilde{M}}{\bar{p}}\right)^{3/2} - \frac{3\tilde{M}^2\bar{\alpha}}{4\bar{p}^2}\left(10-\bar{\alpha}^2-\bar{\beta}^2\right) + \frac{3\tilde{M}^2\bar{\alpha}\tilde{\chi}^2(5\cos^2\bar{\iota}-1)}{4\bar{p}^2(1+D)}\,,\\
\frac{\mathrm{d}\bar{\iota}}{\mathrm{d}u} &= 0\,,\\
\frac{\mathrm{d}\bar{\Omega}}{\mathrm{d}u} &= 2\tilde{\chi}\left(\frac{\tilde{M}}{\bar{p}}\right)^{3/2} - \frac{3\tilde{M}^2\tilde{\chi}^2\cos\bar{\iota}}{2\bar{p}^2(1+D)}\,.
\end{aligned}
$$

- The dimensionless quadrupole in the disformal case reads

$$q^{(D)} = -\frac{\tilde{\chi}^2}{1+D}$$

Limit $D \to \infty$: noncircular Schwarzschild metric

- In this limit, the quadrupole term disappears and we have

$$\frac{\mathrm{d}\bar{p}}{\mathrm{d}u} = 0 \, ,$$

$$\frac{\mathrm{d}\bar{\alpha}}{\mathrm{d}u} = -\frac{3\tilde{M}\bar{\beta}}{\bar{p}} + 6\tilde{\chi}\bar{\beta}\cos\bar{\iota}\left(\frac{\tilde{M}}{\bar{p}}\right)^{3/2} + \frac{3\tilde{M}^2\bar{\beta}}{4\bar{p}^2}\left(10 - \bar{\alpha}^2 - \bar{\beta}^2\right) + 0 \, ,$$

$$\frac{\mathrm{d}\bar{\beta}}{\mathrm{d}u} = \frac{3\tilde{M}\bar{\alpha}}{\bar{p}} - 6\tilde{\chi}\bar{\alpha}\cos\bar{\iota}\left(\frac{\tilde{M}}{\bar{p}}\right)^{3/2} - \frac{3\tilde{M}^2\bar{\alpha}}{4\bar{p}^2}\left(10 - \bar{\alpha}^2 - \bar{\beta}^2\right) + 0 \, ,$$

$$\frac{\mathrm{d}\bar{\iota}}{\mathrm{d}u} = 0 \, ,$$

$$\frac{\mathrm{d}\bar{\Omega}}{\mathrm{d}u} = 2\tilde{\chi}\left(\frac{\tilde{M}}{\bar{p}}\right)^{3/2} + 0 \, .$$

- We have the Schwarzschild predictions and frame-dragging, but no quadrupole so the no-hair theorem is violated
- Even though the metric is noncircular at this order, it has no influence on the secular variation of parameters

Limit $D \to -1$: quasi-Weyl metric

- In this limit, one cannot use the physical parameter $\tilde{\chi} = \tilde{a}/\tilde{M}$, since there is a divergence when $D \to -1$. Instead, we use the parameter $\chi = a/\tilde{M}$, and in this case

$$
\begin{aligned}
\frac{\mathrm{d}\bar{p}}{\mathrm{d}u} &= 0\,, \\
\frac{\mathrm{d}\bar{\alpha}}{\mathrm{d}u} &= -\frac{3\tilde{M}\bar{\beta}}{\bar{p}} + \frac{3\tilde{M}^2\bar{\beta}}{4\bar{p}^2}\left(10 - \bar{\alpha}^2 - \bar{\beta}^2\right) - \frac{3\tilde{M}^2\bar{\beta}\chi^2}{4\bar{p}^2}\left(5\cos^2\bar{\iota} - 1\right)\,, \\
\frac{\mathrm{d}\bar{\beta}}{\mathrm{d}u} &= \frac{3\tilde{M}\bar{\alpha}}{\bar{p}} - \frac{3\tilde{M}^2\bar{\alpha}}{4\bar{p}^2}\left(10 - \bar{\alpha}^2 - \bar{\beta}^2\right) + \frac{3\tilde{M}^2\bar{\alpha}\chi^2}{4\bar{p}^2}\left(5\cos^2\bar{\iota} - 1\right)\,, \\
\frac{\mathrm{d}\bar{\iota}}{\mathrm{d}u} &= 0\,, \\
\frac{\mathrm{d}\bar{\Omega}}{\mathrm{d}u} &= -\frac{3\tilde{M}^2\chi^2\cos\bar{\iota}}{2\bar{p}^2}\,.
\end{aligned}
$$

- The no-hair theorem is again violated, since in this case there is a free parameter a entering the quadrupole term. In this case $\tilde{\chi} = 0$.

$D+1 \sim \varepsilon$: enhanced Kerr disformation

- We now try to maximize the effects of disformality. From the asymptotic expansion of the disformed metric, this seems to happen when $1+D \ll 1$
- We define $\varepsilon = \tilde{M}/A$, where A is the semimajor axis, and assume that the constant disformal parameter is given by

$$D = -1 + \frac{\tilde{\chi}^2}{\lambda}\varepsilon\,, \qquad \{\lambda, \tilde{\chi}\} \sim \mathcal{O}(1)$$

- We expect 1PN terms to be modified in this case, and indeed we obtain

$$\Delta\bar{\varpi} \equiv \Delta\bar{\omega} + \cos\bar{\iota}\,\Delta\bar{\Omega} = \frac{6\pi\tilde{M}}{\bar{p}}\left[1 + \frac{\lambda}{4(1-\bar{e}^2)}\left(3\cos^2\bar{\iota} - 1\right)\right] + \mathcal{O}\left(\varepsilon^{3/2}\right)$$

Comparison to experiments

- Using the previous expression for the orbit of S2 around Sgr A*, with $\varepsilon_0 = \tilde{M}/A_0$, the GRAVITY constraint implies

$$\left| \frac{\lambda \left(3\cos^2\bar{\iota} - 1\right)}{4(1 - \bar{e}^2)} \right| \lesssim 0.2$$

- If we replace the eccentricity of $S2$ and assume $|3\cos^2\bar{\iota} - 1| \sim 1$, the inequality is saturated for $\lambda_0 \sim 0.2$. To maximize the effects of disformality, we take

$$D_0 = -1 + \frac{\tilde{\chi}^2 \varepsilon_0}{\lambda_0}$$

- For another star with $\varepsilon \neq \varepsilon_0$, we have

$$\Delta\bar{\varpi} = \frac{6\pi\tilde{M}}{\bar{p}} \left[1 + \frac{\varepsilon\lambda_0}{\varepsilon_0} \frac{\left(3\cos^2\bar{\iota} - 1\right)}{4(1 - \bar{e}^2)} \right].$$

- This is valid for $\varepsilon^2 \lesssim 10^{-3} \lesssim \sqrt{\varepsilon}$

Conclusion

- Studying alternatives to the Kerr spacetime allows to test GR
- We have constructed solutions to DHOST theories by performing a disformal transformation of the Kerr spacetime using a geodesic scalar
- While asymptotically very similar to Kerr, the solution presents many interesting properties: noncircularity, horizon not located at constant r and not a Killing horizon, the stationary limit is distinct from the event horizon
- In two particular limits, simpler metrics were obtained, and they could be useful in studying the properties of noncircular spacetimes
- We have calculated the secular variation of orbital parameters for stars around a deformed black hole, and shown that the no-hair theorem of GR is violated in general for these spacetimes
- In the particular limit of $D \sim 1 + \epsilon$, we derived the maximal deformation compatible with current observations, and used it to predict the pericenter precession for other stars, which may be measured in the future

Thank you for your attention.

Regular coordinates

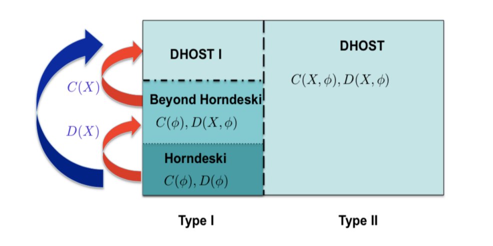
- After the coordinate change

$$t \to v - r - \int \frac{2Mr}{\Delta} \mathrm{d}r \,, \qquad \varphi \to -\Phi - a \int \frac{\mathrm{d}r}{\Delta} \,,$$

the disformed Kerr metric reads

$$\begin{aligned}
\mathrm{d}\tilde{s}^2 = &- \left(1 + D - \frac{2Mr}{\rho^2}\right) \mathrm{d}v^2 + 2 \left(1 + D - \frac{D}{1 + \sqrt{\frac{r^2+a^2}{2Mr}}}\right) \mathrm{d}v\mathrm{d}r \\
&- D \left(1 - \frac{1}{1 + \sqrt{\frac{a^2+r^2}{2Mr}}}\right)^2 \mathrm{d}r^2 \\
&+ \frac{4aMr\sin^2\theta}{\rho^2} \mathrm{d}v\mathrm{d}\Phi + 2a\sin^2\theta \mathrm{d}r\mathrm{d}\Phi + \rho^2\mathrm{d}\theta^2 \\
&+ \frac{\sin^2\theta \left(2a^4\cos^2\theta + 4a^2Mr\sin^2\theta + a^2r^2\left[3 + 2\cos(2\theta)\right] + 2r^4\right)}{2\rho^2} \mathrm{d}\Phi^2 \,.
\end{aligned}$$

Stability of the DHOST class under the disformal map



[Langlois, 2018]

- The Ia subclass can be obtained from Horndeski theories by a disformal transformation of the metric [Ben Achour+; Crisostomi+, 2016; ...]:

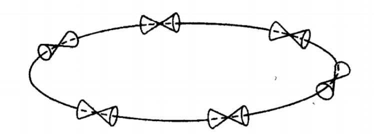
$$\tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X) \partial_\mu \phi \partial_\nu \phi$$

- The theories are different because of the matter coupling:

$$\tilde{S}[\tilde{g}_{\mu\nu}, \phi] + S_{\mathrm{m}}[\tilde{g}_{\mu\nu}, \Psi_{\mathrm{m}}] \xrightarrow{\text{DISFORMAL}} S[g_{\mu\nu}, \phi] + S_{\mathrm{m}}[g_{\mu\nu}, \Psi_{\mathrm{m}}]$$

Stably causal spacetime

- There can exist closed timelike curves in a spacetime, even in GR (Kerr with $a > M$ for example). We want to avoid such pathologies.



[Wald's book]

Theorem: A spacetime $(M_0, g_{\mu\nu})$ is stably causal if and only if there exists a differentiable function f on M_0 such that $\nabla^\mu f$ is a future (past) directed timelike vector field

- We have such a function by construction, the scalar field ϕ itself. It serves as a global time.
- The spacetime is globally causal if the region $r > 0$ is causally disconnected from the region $r < 0$ (where CTCs are present even for Kerr)
- Some of the *ad hoc* deformations of Kerr proposed in the past contain such pathologies [Johannsen, 2013]