Asymptotically flat hairy black holes in massive bigravity

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History of the problem

- Ghost-free bigravity /Hassan, Rosen 2011/
- Hairy black holes in bigravity, not asymptotically flat /M.S.V. 2012/
- Stability analysis of the embedded Schwarzschild /Brito, Cardoso, Pani, 2013/, /Babichev, Fabbri, 2013/
- Asymptotically flat hairy black holes /Brito, Cardoso, Pani, 2013/
- No asymptotically flat hairy black holes *(general analysis of theory structure)* /Torsello, Kocic, Mortsell, 2017/
- Asymptotically flat hairy black holes – detailed analysis. *(many new results)* /R. Gervalle, M.S.V., Phys.Rev. D102 (2020) 124040/

Ghost-free bigravity

The ghost-free bigravity /Hassan and Rosen 2011/

$$S[\mathrm{g},\mathrm{f}] = \frac{1}{2\kappa_1}\int R(\mathrm{g})\sqrt{-\mathrm{g}}\,d^4\mathrm{x} + \frac{1}{2\kappa_2}\int R(\mathrm{f})\sqrt{-\mathrm{f}}\,d^4\mathrm{x} - \frac{\mathrm{m}^2}{\kappa_1+\kappa_2}\int \mathcal{U}\sqrt{-\mathrm{g}}\,d^4\mathrm{x} + S_{\mathrm{mat}}[g,\text{matter}]$$

where $\mathcal{U} = b_0 + \sum_{n=1}^{4} b_k\,\mathcal{U}_k$ is constructed from $\hat{\gamma} = \sqrt{\hat{\mathrm{g}}^{-1}\hat{\mathrm{f}}}$ as

$$\mathcal{U}_1 = \sum_a \lambda_a = [\gamma],\quad \mathcal{U}_2 = \sum_{a<b}\lambda_a\lambda_b = \frac{1}{2!}([\gamma]^2 - [\gamma^2]),$$

$$\mathcal{U}_3 = \sum_{a<b<c}\lambda_c\lambda_b\lambda_c = \frac{1}{3!}([\gamma]^3 - 3[\gamma][\gamma^2] + 2[\gamma^3]),$$

$$\mathcal{U}_4 = \lambda_1\lambda_2\lambda_3\lambda_4 = \det(\hat{\gamma})\,.$$

m is the FP mass and flat space is a solution if

$$b_0 = 4c_3 + c_4 - 6,\quad b_1 = 3 - 3c_3 - c_4,$$

$$b_2 = 2c_3 + c_4 - 1,\quad b_3 = -(c_3 + c_4),\quad b_4 = c_4.$$

Equations

$$\mathrm{g}_{\mu\nu} = \frac{1}{\mathrm{m}^2}\, g_{\mu\nu}, \quad \mathrm{f}_{\mu\nu} = \frac{1}{\mathrm{m}^2}\, f_{\mu\nu},$$

the lengthscale is the Compton wavelength $1/m$ and the field equations become dimensionless

$$G_{\mu\nu}(g) = \kappa_1\, T_{\mu\nu}(g,f) + \kappa_1\, T^{\mathrm{mat}}_{\mu\nu}, \qquad G_{\mu\nu}(f) = \kappa_2\, \mathcal{T}_{\mu\nu}(g,f),$$

with

$$\kappa_1 = \frac{\boldsymbol{\kappa}_1}{\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2} \equiv \cos^2\eta, \qquad \kappa_2 = \frac{\boldsymbol{\kappa}_2}{\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2} \equiv \sin^2\eta.$$

A physical matter does not see $f_{\mu\nu}$. We shall set $T^{\mathrm{mat}}_{\mu\nu} = 0$.

Flat space $g_{\mu\nu} = f_{\mu\nu} = \eta_{\mu\nu}$ is a solution. Small fluctuations $\delta g_{\mu\nu}$ and $\delta f_{\mu\nu}$ around flat space describe

$$\begin{aligned} h^{(0)}_{\mu\nu} &= \kappa_1 \delta\mathrm{f}_{\mu\nu} + \kappa_2 \delta\mathrm{g}_{\mu\nu} && \text{massless graviton} \\ h_{\mu\nu} &= \delta\mathrm{f}_{\mu\nu} - \delta\mathrm{g}_{\mu\nu} && \text{massive graviton} \end{aligned}$$

Vacuum solutions

If $g_{\mu\nu} = f_{\mu\nu}$ then

$$G^{\mu}_{\nu}(g) = 0, \qquad G^{\mu}_{\nu}(f) = 0,$$

$\Rightarrow$ any vacuum metric is a solution. For example, the "bald Schwarzschild"

$$\begin{aligned} g^{\rm S}_{\mu\nu} dx^{\mu} dx^{\nu} &= f^{\rm S}_{\mu\nu} dx^{\mu} dx^{\nu} \\ &= -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2 d\Omega^2 \end{aligned}$$

with r in units of $1/m$ and M in units of $(\mathbf{M}_{\rm Pl}/m)\,\mathbf{M}_{\rm Pl}$. Its perturbations grow in time (instability) if $M < 0.43$

$$g_{\mu\nu} = g^{\rm S}_{\mu\nu} + \delta g_{\mu\nu}(t, x^k), \qquad f_{\mu\nu} = g^{\rm S}_{\mu\nu} + \delta f_{\mu\nu}(t, x^k)$$

/Brito, Cardoso, Pani, 2013/, /Babichev, Fabbri, 2014/.
The graviton mass m should not be too small.

Self acceleration

Setting $f_{\mu\nu} = C^2 g_{\mu\nu}$ with a constant C the bigravity equations reduce to

$$G_{\mu\nu} + \boldsymbol{\Lambda}\, g_{\mu\nu} = 0,$$

whose solution is de Sitter (AdS is also possible) $\Rightarrow$ self acceleration driven by the effective cosmological constant

$$\boldsymbol{\Lambda} \sim \mathrm{m}^2 \kappa_1 \sim 1/R_{\mathrm{Hub}}^2$$

motivation for massive gravitons. To agree with the observations, either the graviton mass has to be very small (small m is better than just small $\boldsymbol{\Lambda}$),

$$\mathrm{m} \sim 1/R_{\mathrm{Hub}}$$

or κ_1 be very small, which is more interesting for us.

Hierarchy between the two gravitational couplings

Assume that κ_1 is very small $\Rightarrow$ hierarchy

$$\kappa_1 \ll \kappa_2 = 1 - \kappa_1 \sim 1$$

Hierarchy is needed to remove the instability in the scalar sector by assuming /Akrami, Hassan ... 2015/

$$\frac{\kappa_1}{\kappa_2} \approx \kappa_1 \leq \left(\frac{\mathrm{M_{ew}}}{\mathrm{M_{Pl}}}\right)^2 \sim 10^{-34} \ll 1,$$

with $\mathrm{M_{ew}} \sim 100$ GeV. This is an upper bound for κ_1, hence

$$\kappa_1 = \gamma^2 \times 10^{-34} \qquad \text{with} \qquad \gamma \in [0,1] \qquad \text{and}$$

$$\frac{1}{\mathrm{m}} \sim \sqrt{\kappa_1}\, R_{\mathrm{Hub}} \sim \gamma \times 10^{-17}\, R_{\mathrm{Hub}} \sim \gamma \times 10^{6}\ \text{km} \sim \underline{\gamma \times \text{Solar size}}$$

$$\Rightarrow mc^2 \approx \frac{1}{\gamma}\, 10^{-16}\mathrm{eV} > 10^{-23}\ \mathrm{eV}\ (\textit{to be commented on later})$$

Constructing hairy black holes

The ansatz and equations

$$
\begin{aligned}
ds_g^2 &= -Q(r)^2 dt^2 + \frac{dr^2}{N(r)^2} + r^2 d\Omega^2 \,, \\
ds_f^2 &= -q(r)^2 dt^2 + \frac{dU(r)^2}{Y(r)^2} + U(r)^2 d\Omega^2 .
\end{aligned}
$$

Q^2, N^2, q^2, Y^2 must all show a simple zero at some $r = r_H > 0$.

- The horizon is common for both metrics. The surface gravity and the Hawking temperature are also the same.
- Horizon radius r_H measured by the g-metric can differ from the radius $U(r_H) \equiv U_H$ measured by the f-metric.

Independent equations:

$$
\begin{aligned}
N' &= \mathcal{D}_N(r, U, N, Y), \\
Y' &= \mathcal{D}_Y(r, U, N, Y), \\
U' &= \mathcal{D}_U(r, U, N, Y).
\end{aligned}
$$

For a given r_H the parameter $U_H = U(r_H)$ completely characterizes the boundary conditions at the horizon $r = r_H$.

Integration – hairy black holes /M.S.V. 2012/

Choosing a value of U_H and integrating the equations starting from $r = r_H$ towards large values of r, one finds that

- Either solutions approach for $r \to \infty$ the proportional AdS, which is an asymptotic attractor: all perturbations decay for $r \to \infty$
- Or solutions become singular at a finite $r_{\rm sing} > r_H$.

Trying randomly many different r_H, U_H does not give anything else. Expanding around flat space at infinity yields

$$\frac{A}{r} + Be^{-mr} + \boxed{Ce^{+mr}}$$

$\Rightarrow$ flat space is not attractor. To suppress the growing mode one should set $C = 0$ and integrate from both sides (horizon and infinity) using $\boxed{U_H, A, B}$ as input parameters. This requires a fine tuning for $r_H, U_H \Rightarrow$ additional information is needed.

Change of stability of Schwarzschild – bifurcation

$$g^{\rm S}_{\mu\nu}dx^\mu dx^\nu = f^{\rm S}_{\mu\nu}dx^\mu dx^\nu \;=\; -\left(1-\frac{r_H}{r}\right)\,dt^2 + \frac{dr^2}{1-r_H/r} + r^2 d\Omega^2.$$

Perturbations around this "bald Schwarzschild"

$$g_{\mu\nu} = g^{\rm S}_{\mu\nu} + e^{i\omega t}\delta g_{\mu\nu}(r,\vartheta,\varphi), \qquad f_{\mu\nu} = g^{\rm S}_{\mu\nu} + e^{i\omega t}\delta f_{\mu\nu}(r,\vartheta,\varphi)$$

admits a negative mode solution with $\omega^2 < 0$ if $r_H < 0.86$.
/Brito, Cardoso, Pani, 2013/, /Babichev, Fabbri, 2013/
This negative mode becomes a static zero mode for $r_H = 0.86$, providing a perturbative approximation of a new static solution which bifurcates with the bald Schwarzschild.
This yields yields the input values for the parameters

$$r_H = U_H = 0.86, \qquad A = -\frac{r_H}{2}, \qquad B = 0.$$

We shall call this the GL (Gregory-Laflamme) point. Changing r_H iteratively yields "fully fledged" hairy solutions.

Numerical procedure

- Desingularization of the equations at the horizon to be able to start exactly at $r = r_H$.
- Using converging integral equation for $r > r_{\max} \gg r_H$.
- Integration from two sides in the region $[r_H, r_{\max}]$: from $r = r_H$ toward large r and from $r = r_{\max}$ toward small r via the multiple shooting untill the two solutions match.

Asymptotically flat Hairy Black Holes

Solutions

- Are labeled by values of r_H and depend on the theory parameters c_3, c_4 and $\kappa_1 \cos^2\eta$, $\kappa_2 = \sin^2\eta$.

- Close to the GL point ($r_H = 86$) solutions are very close to Schwarzschild. If r_H deviates from 0.86, solutions deviate from Schwarzschild in the near horizon region where the massive hair is located.

- Solutions cease to existe beyond a minimal r_H^{min} or maximal r_H^{max} values of the event horizon size. These values depend on c_3, c_4, η.

Solution profiles

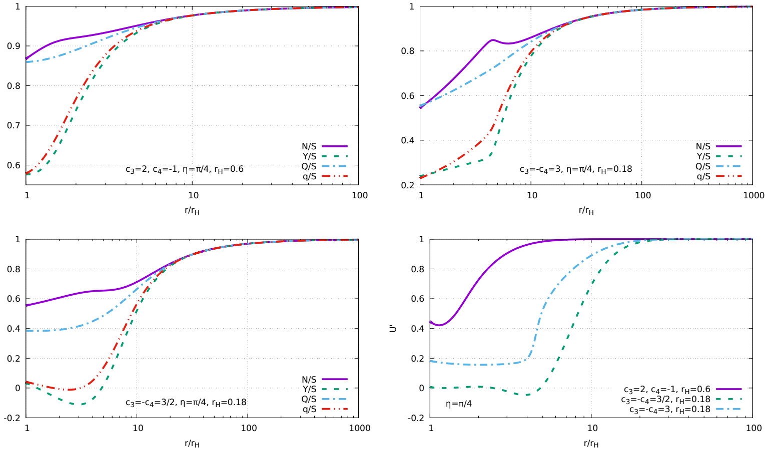


Figure: Profiles of N/S, Y/S, Q/S, q/S with $S = \sqrt{1 - r_H/r}$ and that of U' for various values of r_H, c_3, c_4.

Duality

The bigravity is invariant under

$$g_{\mu\nu} \leftrightarrow f_{\mu\nu}, \qquad \kappa_1 \leftrightarrow \kappa_2, \qquad b_k \leftrightarrow b_{4-k}$$

which translates to

$$\kappa_1 \leftrightarrow \kappa_2,\ \ Q \leftrightarrow q,\ \ N \leftrightarrow Y,\ \ U \leftrightarrow r,\ \ c_3 \to 3 - c_3,\ \ c_4 \to 4c_3 + c_4 - 6$$

If for some η, c_3, c_4 there is a hairy black hole solution for some

$$r_H \quad \text{and} \quad U_H$$

then for the dual set of parameters

$$\tilde{\eta} = \pi/2 - \eta, \qquad \tilde{c}_3 = 3 - c_3, \qquad \tilde{c}_4 = 4c_3 + c_4 - 6$$

there is the dual solution characterized by

$$\tilde{r}_H = U_H \quad \text{and} \quad \tilde{U}_H = r_H$$

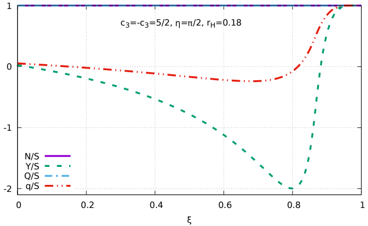
Duality changes the black hole size: if $r_H < 0.86$ then $\tilde{r}_H > 0.86$.

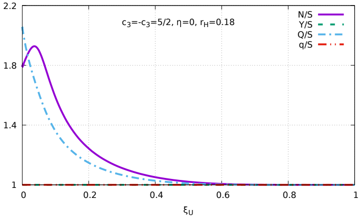
Hairy Schwarzschild

Particularly interesting are hairy Schwarzschild solutions for $\kappa_1 = 0$ or $\kappa_2 = 0$ when one of the metric becomes exactly Schwarzschild but the other remains hairy, because

$$G^{\mu}_{\nu}(g) = \kappa_1 \, T^{\mu}_{\nu}(g, f), \qquad G^{\mu}_{\nu}(f) = \kappa_2 \, \mathcal{T}^{\mu}_{\nu}(g, f)$$

For $\kappa_1 \sim 10^{-34}$ hary black holes must be very close to the hairy Schwarzschild.





Left: $\kappa_1 = 0$, $\kappa_2 = 1$, g-metric is Schwarzschild, f-metric is hairy.
Right: $\kappa_2 = 0$, $\kappa_1 = 1$, f-metric is Schwarzschild.

Stability analysis

Time-dependent fields spherically symmetric fields

$$
\begin{aligned}
ds_g^2 &= -Q^2 dt^2 + \frac{dr^2}{N^2} + r^2 d\Omega^2, \\
ds_f^2 &= -\big(q^2 - \alpha^2 Q^2 N^2\big) dt^2 + \left(\frac{U'^2}{Y^2} - \alpha^2\right) dr^2 + U^2 d\Omega^2 \\
&\quad - 2\alpha\left(q + \frac{QNU'}{Y}\right) dtdr
\end{aligned}
$$

where, expanding around a static background

$$
\begin{aligned}
Q(r,t) &= \overset{(0)}{Q}(r) + \delta Q(r,t), \qquad q(r,t) = \overset{(0)}{q}(r) + \delta q(r,t), \\
N(r,t) &= \overset{(0)}{N}(r) + \delta N(r,t), \qquad Y(r,t) = \overset{(0)}{Y}(r) + \delta Y(r,t), \\
U(r,t) &= \overset{(0)}{U}(r) + \delta U(r,t), \qquad \alpha(r,t) = \delta\alpha(r,t)
\end{aligned}
$$

Master equation

Assuming the harmonic time-dependence for all amplitudes

$$\delta Q(r,t) = e^{i\omega t}\delta Q(r), \quad \delta N(r,t) = e^{i\omega t}\delta N(r), \ldots$$

the temporal variable separates and the linearized field equations reduce (after heavy transformations) to a one-channel equation

$$\frac{d^2\Psi}{dr_*^2} + \big(\omega^2 - V(r)\big)\Psi = 0 \qquad (\spadesuit)$$

describing the scalar polarization of the massive graviton. Here Ψ is a linear combination of perturbations, $V(r)$ is made of the background amplitudes, and the tortoise coordinate

$$r_* \in (-\infty, \infty) \quad \text{when} \quad r \in [r_H, \infty)$$

Negative modes of ($\spadesuit$), that is bound state solutions with $\omega^2 < 0$, correspond to unstable modes.

Unstable solutions

All solutions with $\kappa_1 = \kappa_2 = 1/2$ ($\eta = \pi/4$) previously found by the Portuguese group are unstable.

A puzzle: what they decay to ? Not to Schwarzschild, because it is also unstable. Can they radiate away all their energy such that the horizon disappear ? In GR such a process is classically impossible. What about bigravity ?

Stable solutions

Hairy black holes become stable in the physically interesting case of small $\kappa_1 = \cos^2 \eta$!

Parameter space

Parameters

Solutions depend in 4 parameters r_H, c_3, c_4, η. A complete classification is very difficult.

Strategy: choose representative values

$$c_3 = -c_4 = 5/2$$

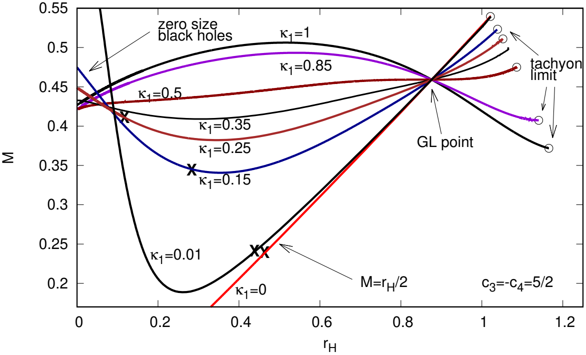
and classify all solutions depending on $r_H, \kappa_1 = \cos^2\eta$.

Perform the duality to obtain

$$c_3 = 1/2, \qquad c_4 = 3/2$$

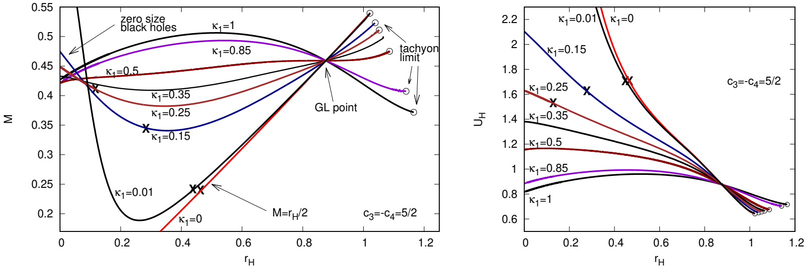
and classify again.

ADM mass $M(r_H)$ for different η



r_H in units of $1/m$ and M in units of $(\mathbf{M}_{\rm Pl}/m)\,\mathbf{M}_{\rm Pl}$. GL point: $r_H = 0.86 = 2M$. The mass M is the same with respect to each metric and almost always close to the mass of Schwarzschild black hole of size $\sim 1/m$. For this not to be too large one should choose $1/m \sim 10^6$ km and $\kappa_1 = 10^{-34} \Rightarrow$ the maximal mass of hairy black holes is $\sim 10^6 M_\odot$ = typical mass of supermassive black holes.

ADM mass



$$M_{ADM} = \frac{r_H}{2} + \kappa_1 \int_{r_H}^{\infty} r^2 T_{00} dr \equiv M_{\rm bare} + M_{\rm hair}$$

the hair mass $M_{\rm hair}$ can be positive or negative. The physical value $\kappa_1 = 10^{-34}$ is very close to $\kappa_1 = 0 \Rightarrow M(r_H)$ is very close to $r_H/2$ and can be small. For $r_H \to 0$ the g-metric develops a naked singularity but the ADM mass remains finite. For $r_H \to r_H^{\rm max}$ solutions disappear via fusion of roots and show regions where the effective graviton mass becomes imaginary.

Parameter space for $c_3 = -c_4 = 5/2$

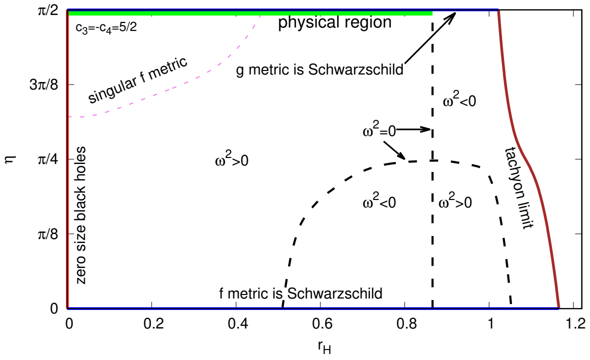


Figure: $\kappa_1 = \cos^2\eta$ and $\kappa_2 = \sin^2\eta$. The dashed black $\omega^2 = 0$ lines separate stable and unstable sectors. The upper left corner contains solutions with a singular f-metric, but their g-geometry is regular.

Physical solutions

$$G^{\mu}_{\nu}(g) = \kappa_1 \, T^{\mu}_{\nu}(g,f), \qquad G^{\mu}_{\nu}(f) = \kappa_2 \, \mathcal{T}^{\mu}_{\nu}(g,f),$$

where $\kappa_1 = 10^{-34}$ $(\kappa_2 = 1 - \kappa_1 \approx 1) \Rightarrow$ the physical g-metric is extremely close to Schwarzschild, all the hair is contained in the f-metric which is directly invisible.

Normally the hairy black holes cannot be distinguished from Schwarzschild. However, in violant processes like black hole collisions the source $T^{\mu}_{\nu}(g,f)$ can become strong enough to overcome the 10^{34} suppression $\Rightarrow$ the hairy features can become visible.

It is possible that "hairy signatures" are contained in GW signals from black hole mergers. These signatures should be stronger for small black holes.

Approximation for small κ_1

$$G_{\mu\nu}(g) = \kappa_1 \, T_{\mu\nu}(g,f), \qquad G_{\mu\nu}(f) = \kappa_2 \, \mathcal{T}_{\mu\nu}(g,f),$$

1. Set $\kappa_1 = 0$ then g=Schwarzschild of radius r_H.
2. Solve $G_{\mu\nu}(f) = \mathcal{T}_{\mu\nu}(g,f)$ to find f as function of r_H.
3. Use the solution to compute the source $T_{\mu\nu}(g,f)$.
4. Return to $\kappa_1 \neq 0$ can compute the mass

$$M_{ADM} \approx \frac{r_H}{2} + \kappa_1 \int_{r_H}^{\infty} r^2 \, T_{00}(g,f) dr$$

Steps 2 and 3 give interesting result: the components of $f_{\mu\nu}$ and $T_{00}(g,f)$ become very large as $r_H \to 0$.

Solution for $f_{\mu\nu}$ on Schwarzschild background

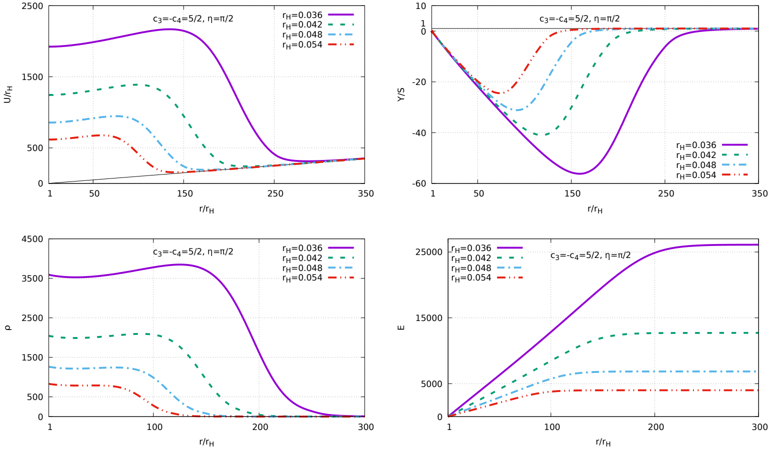


Figure: The amplitudes U, Y in the f-metric, the "hair energy density" $T_{00}(g,f)$ and its integral $E(r)$ become very large for small r_H.

Approximation for M for small κ_1

$$M(r_H) \approx \frac{r_H}{2} + \kappa_1 \frac{0.005}{(r_H)^{4.6}}$$

Assuming $\kappa_1 = \gamma^2 \times 10^{-34}$ (with $\gamma \in [0,1]$) the minimum is

$$(\mathbf{r}_H)_{\rm min} \approx 0.52\,\gamma^{1.35}\ \mathrm{km}, \qquad \mathbf{M}_{\rm min} \approx 0.2\,\gamma^{1.35} \times \mathrm{M}_\odot$$

$\Rightarrow$ size and mass of the lightest hairy black hole. As for the heaviest,

$$(r_H)_{\rm max} \sim 10^6\ \mathrm{km}, \quad \mathrm{M}_{\rm min} \sim 3 \times 10^6 \mathrm{M}_\odot$$

Summary of results

The ghost-free bigravity theory admits indeed stationary and asymptotically flat solutions describing black holes with a massive graviton hair.

For them not to be unphysically heavy, one is bound to assume the graviton mass to be $1/\mathrm{m} = \gamma \times 10^6$ km with $\gamma \in [0, 1]$. The agreement with the cosmological data is then achieved by assuming that $\kappa_1 = \gamma^2 \times (\mathrm{M_{ew}}/\mathrm{M_{Pl}})^2 = \gamma^2 \times 10^{-34}$.

Stable hairy black holes are described by a g-metric which is extremely close to Schwarzschild, but their f-metric is different. Their mass ranges from $0.2\,\gamma^{1.35} \times \mathrm{M}_\odot$ to $0.3 \times 10^6\,\gamma^{1.35} \times \mathrm{M}_\odot$. Yet heavier black holes in the theory should be "bald".

If the bigravity theory indeed applies to describe physics, the astrophysical black holes cannot be bald Schwarzschild because it is unstable. They should be hairy Schwarzschild hiding hair in the f-metric.

Summary of results

The f-metric is not coupled to matter and cannot be directly probed, while the deviation of the “visible” g-metric from Schwarzschild is suppressed by the factor of 10^{-34}. Therefore, in usual conditions hairy black holes should be undistinguishable from the usual GR black holes.

In black hole collisions the interaction between the two metrics may produce $T_{\mu\nu}(g, f)$ strong enough to overcome the 10^{-34} suppression in $G_{\mu\nu}(g) = \kappa_1\, T_{\mu\nu}(g, f)$. Then the deviation from GR may become visible, perhaps it is contained in signals from black hole mergers detected by LIGO/VIRGO.

The effect should be larger for small black holes because the $f_{\mu\nu}$ components are very large for small r_H thus enhancing $T_{\mu\nu}(g, f)$. It is possible that hair imprints will be visible when smaller mass mergers are detected. The observed bound $m < 10^{-23}$ eV is also presumably because only massless GW have been seen so far.