QUARKS ONLINE WORKSHOPS – 2021
"Modification of Gravity: Theories and Observations"

Perturbations of black holes in shift-symmetric scalar-tensor theories

Tsutomu Kobayashi

Rikkyo University

Based on work with

Keitaro Tomikawa (Rikkyo)

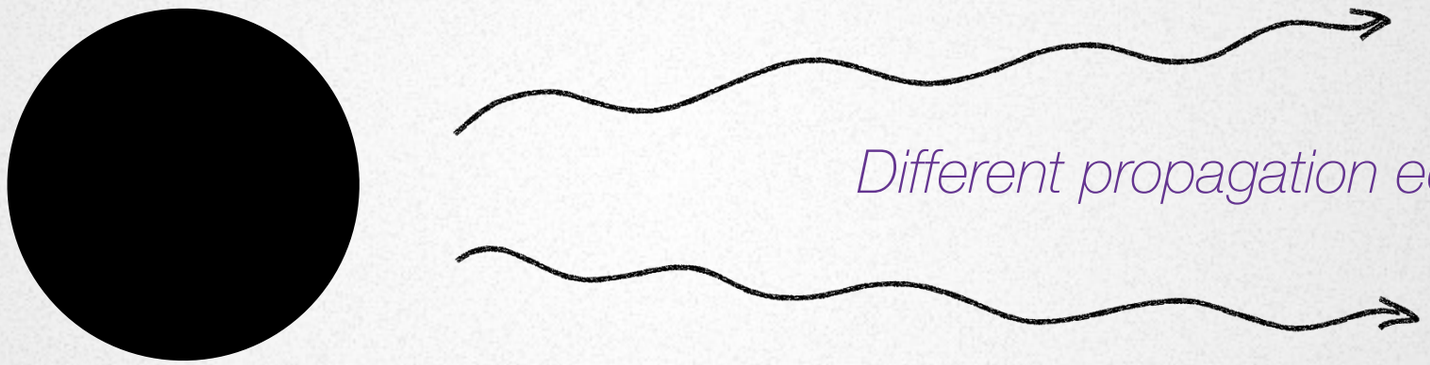
Phys.Rev.D 103 (2021) 8, 084041 [2101.03790]

Introduction

- Gravitational waves (GWs) from black holes (BHs): important tool for **testing gravity** in strong field regime

Hair?

Different geometry from GR?



Different propagation equation from GR?

- This work: Odd-parity (axial) perturbations (~ GWs) of a BH with time-dependent scalar hair in general (shift-symmetric) scalar-tensor theories
  —— *General form of the Lagrangian for odd modes?*

Introduction

- EFT approach is useful for static and spherically symmetric background in scalar-tensor theories

$$g_{\mu\nu} = g_{\mu\nu}(r), \quad \phi = \phi(r)$$

Kase *et al.* (2014); Franciolini *et al.* (2019); Kuntz *et al.* (2020)

- Our background:

$$g_{\mu\nu} = g_{\mu\nu}(r), \quad \phi = \mu t + \psi(r)$$

configuration allowed in shift-symmetric theories:

$$\mathcal{L}(\cancel{\phi}, \partial_\mu\phi, \cdots)$$

  - Time dependence breaks the usual assumption of EFT
  - Better to start from covariant action

General scalar-tensor theories

Horndeski theory

- GR

The most general scalar-tensor theory with 2nd-order EOMs

- free from Ostrogradsky instability by construction

Horndeski (1974)

$$\mathcal{L} = F_0(\phi, X) + F_1(\phi, X)\Box\phi + F_2(\phi, X)R + F_{2,X}\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right] + \cdots$$

$$X = -\frac{1}{2}g^{\mu\nu}\phi_\mu\phi_\nu, \quad \phi_\mu = \nabla_\mu\phi \qquad \phi_{\mu\nu} = \nabla_\mu\nabla_\nu\phi$$

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shift symmetry

$$X = -\frac{1}{2}g^{\mu\nu}\phi_\mu\phi_\nu, \quad \phi_\mu = \nabla_\mu\phi \qquad \phi_{\mu\nu} = \nabla_\mu\nabla_\nu\phi$$

General scalar-tensor theories

Horndeski theory

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DHOST

(**D**egenerate **H**igher-**O**rder **S**calar-**T**ensor Theory)

Healthy theories beyond Horndeski

Langlois & Noui (2015);
Crisostomi *et al.* (2016);
Ben Achour *et al.* (2016)

- Higher-order EOMs, but degenerate

metric + single scalar, Ostrogradsky-stable

2 + **1** DOFs

DHOST theories

Langlois & Noui (2015);
Crisostomi *et al.* (2016);
Ben Achour *et al.* (2016)

$$\mathcal{L} = F_0(X) + F_1(X)\Box\phi$$

all possible terms of the form $(\nabla\nabla\phi)^2$ ↓

$$+ F_2(X)R + A_1(X)\phi_{\mu\nu}\phi^{\mu\nu} + \cdots + A_5(X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2$$

$$+ F_3(X)G_{\mu\nu}\phi^{\mu\nu} + B_1(X)(\Box\phi)^3 + \cdots + B_{10}(X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^3$$

↑ all possible terms of the form $(\nabla\nabla\phi)^3$

Degeneracy conditions:
algebraic relations among functions to eliminate Ostrogradsky ghost

DHOST theories

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Degeneracy conditions:
algebraic relations among functions to eliminate Ostrogradsky ghost

⇕

Extensions of DHOST theories: degeneracy conditions may be relaxed

- U-DHOST:
  degenerate only in unitary gauge → no propagating ghost — De Felice *et al.* (2018)
- Scordatura (detuned DHOST):
  not problematic from EFT viewpoint — Motohashi & Mukohyama (2020)

Background examples

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right), \quad \phi = \mu t + \psi(r)$$

- Stealth solutions in Horndeski and DHOST theories

$$A = B = 1 - \frac{r_h}{r}, \quad X = \mathrm{const.}$$

Mukohyama (2005); Babichev & Charmousis (2014); TK & Tanahashi (2014); Babichev *et al*. (2016); Ben Achour& Liu (2019); Motohashi & Minamitsuji (2019); Minamitsuji & Edholm (2019); …

- Non-stealth, $X = X(r) \neq \mathrm{const.}, \ldots$

Rinaldi (2012); Babichev *et al*. (2017); Minamitsuji & Edholm (2020); …

BH perturbations in scalar–tensor theories

- Horndeski, $\phi = \phi(r)$ TK, Motohashi, Suyama (2012, 2014)
- Horndeski, $\phi = \mu t + \psi(r)$

  Ogawa, TK, Suyama (2016); Takahashi & Suyama (2017); Khoury *et al*. (2020)

- Quadratic DHOST, $\phi = \mu t + \psi(r)$

  Takahashi, Motohashi, Minamitsuji (2019); de Rham & Zhang (2019); Langlois, Noui, Roussille 2103.14750

- EFT, $\phi = \phi(r)$ Kase *et al*. (2014); Franciolini *et al*. (2019); Kuntz *et al*. (2020)
- This work: Odd modes, cubic ~~DHOST~~ + matter, $\phi = \mu t + \psi(r)$
  - Odd modes (~GWs) are healthy even without degeneracy (Degeneracy removes ghost in “scalar” sector)
  - Can be applied to U-DHOST and scordatura theories

  Tomikawa & TK, Phys.Rev.D 103 (2021) 8, 084041 [2101.03790]

Odd-parity perturbations in Regge-Wheeler gauge

$g_{\mu\nu} = \bar{g}_{\mu\nu}(r) + h_{\mu\nu}$, where

$$h_{t\theta} = -\frac{1}{\sin\theta}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} h_0^{(\ell m)}(t,r)\partial_\varphi Y_{\ell m}, \quad h_{t\varphi} = \cdots h_0^{(\ell m)} \cdots ,$$

$$h_{r\theta} = \cdots h_1^{(\ell m)} \cdots , \quad h_{r\varphi} = \sin\theta\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} h_1^{(\ell m)}(t,r)\partial_\theta Y_{\ell m},$$

$h_{\theta\theta} = h_{\theta\varphi} = h_{\varphi\varphi} = 0$ ← gauge

$$\mathcal{L} = \mathcal{L}^{\mathrm{DHOST}} + \frac{1}{2}h^{\mu\nu}T_{\mu\nu}$$

minimally-coupled matter

Source term (energy-momentum tensor):

$$T_{t\theta} = -\frac{1}{\sin\theta}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} S_0^{(\ell m)}(t,r)\partial_\varphi Y_{\ell m}, \quad T_{t\varphi} = \cdots S_0^{(\ell m)} \cdots ,$$

$T_{r\theta}, T_{r\varphi} \supset S_1^{(\ell m)}(t,r), \quad T_{\theta\theta}, T_{\theta\varphi}, T_{\varphi\varphi} \supset S_2^{(\ell m)}(t,r)$

General form of Lagrangian

$$S = \sum_{\ell=2}^{m} \sum_{m=-\ell}^{\ell} \int \mathrm{d}t\mathrm{d}r \left[\mathcal{L}^{(2)}_{\ell m} + \mathcal{L}^{\mathrm{source}}_{\ell m} + \mathrm{c.c.} \right], \quad \text{where}$$

$$\mathcal{L}^{(2)}_{\ell m} = \frac{1}{2r^2} \left\{ \left[2(r a_3)' + a_1 \right] |h_0|^2 - a_2 |h_1|^2 + r^2 a_3 \left(|\dot{h}_1|^2 - 2\dot{h}_1^* h_0' + |h_0'|^2 + \frac{4}{r} \dot{h}_1^* h_0 \right) + 2 a_4 h_1^* h_0 \right\}$$

$$\mathcal{L}^{\mathrm{source}}_{\ell m} = -\frac{\ell(\ell+1)}{2} \left(\frac{h_0^* S_0}{\sqrt{AB}} - \sqrt{AB} h_1^* S_1 \right)$$

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New

$$\mathcal{L}^{\mathrm{source}}_{\ell m} = -\frac{\ell(\ell+1)}{2} \left(\frac{h_0^* S_0}{\sqrt{AB}} - \sqrt{AB} h_1^* S_1 \right)$$

$$a_4 \propto \mu$$

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$$a_4 \propto \mu$$

$$a_{1,2,3,4} = a_{1,2,3,4}(r) \supset F_2, F_3, A_1, B_2, B_3, B_6$$

Coefficients of terms built out of curvature tensors, $\phi_{\mu\nu}\phi^{\mu\nu}$, and $\phi^{\nu}_{\mu}\phi^{\rho}_{\nu}\phi^{\mu}_{\rho}$

Master variable

Introduce an auxiliary field $\chi = \chi^{(\ell m)}(t, r)$ and rewrite the Lagrangian as

$$\mathcal{L}^{(2)}_{\ell m} = \frac{1}{2r^2}\left[a_1|h_0|^2 - a_2|h_1|^2 + a_4 h_1^* h_0 + 2r^2 a_3 \chi^* \left(-\frac{1}{2}\chi + \dot{h}_1 - h_0' + \frac{2}{r}h_0\right)\right]$$

$\delta h_0, \delta h_1$ ⇒ Constraint eqs. $h_0 = \cdots, \quad h_1 = \cdots$

⇒

$$\mathcal{L}^{(2)}_{\ell m} = \frac{r^2}{4(\ell-1)(\ell+2)}\left\{b_1|\dot{\chi}|^2 - b_2|\chi'|^2 + 2b_3\dot{\chi}^*\chi' - \left[\frac{\ell(\ell+1)}{r^2}a_3 + (\cdots)\right]|\chi|^2\right\}$$

$$\mathcal{L}^{\rm source}_{\ell m} = -(\cdots)\chi^* S_{\rm odd}, \quad S_{\rm odd} \supset S_0^{(\ell m)}, S_1^{(\ell m)}$$

Stability

$a_3 > 0$

⇒ Generalized Regge-Wheeler equation for χ

RW equation & effective metric

RW equation can be written in the form

$$Z^{\mu\nu} D_\mu D_\nu \left(\frac{\widetilde{\chi}}{r}\right) - V\frac{\widetilde{\chi}}{r} = (\cdots) S_{\rm odd} \qquad \frac{\widetilde{\chi}}{r} \propto \frac{a_3}{(a_1 a_2 + a_4^2)^{1/4}}\chi$$

Effective metric for gravitons — in general, $Z_{\mu\nu} \neq \bar{g}_{\mu\nu}$

$a_4 \propto \mu$ ↙ ↑ gravitons ↖ photons

$$Z_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \frac{-a_2 \mathrm{d}t^2 - 2a_4 \mathrm{d}t\mathrm{d}r + a_1 \mathrm{d}r^2}{(\ell-1)(\ell+2)a_3} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2)$$

$Z_{\mu\nu} = \bar{g}_{\mu\nu}$ in theories with $c_{\rm GW} = 1$ for cosmological tensor modes

Effective metric & stability

Effective metric can be put into diagonal form in terms of $\mathrm{d}\tau = \mathrm{d}t + \dfrac{a_4}{a_2}\mathrm{d}r$

$$\Rightarrow \quad Z_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \frac{1}{(\ell-1)(\ell+2)a_3}\left[-a_2\mathrm{d}\tau^2 + \frac{(a_1a_2+a_4^2)}{a_2}\mathrm{d}r^2\right] + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

Horizon for gravitons: $\quad a_2(r_g) = 0$

Horizon for photons: $\quad A(r_h) = B(r_h) = 0$

Stability condition proposed by Takahashi, Motohashi, Minamitsuji (2019),

$a_2 > 0$, is in fact not related to stability

$a_2 = 0$: inner boundary

Example 1: Stealth Schwarzschild in quadratic DHOST theory

Motohashi & Minamitsuji (2019); Takahashi, Motohashi, Minamitsuji (2019)

$$A = B = 1 - \frac{r_h}{r}, \quad X = X_0 = \text{const.}$$

Effective metric for gravitons (~ Schwarzschild with $r_g \neq r_h$):

$$Z_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\frac{1 - r_g/r}{1+\mathcal{A}}\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{1 - r_g/r} + r^2\mathrm{d}\Omega^2$$

where

$$r_g := (1+\mathcal{A})r_h, \quad \mathcal{A} := \frac{2X_0 A_1(X_0)}{F_2(X_0)} \ (= \text{const.})$$

Potential: $\widetilde{V} = \dfrac{1 - r_g/r}{1+\mathcal{A}}\left[\dfrac{\ell(\ell+1)}{r^2} - \dfrac{3r_g}{r^3}\right]$ ~ RW in GR with $r_g \neq r_h$

See also recent work: Langlois, Noui, Roussille 2103.14750

Example 2: Stealth Schwarzschild in cubic DHOST theory

Minamitsuji & Edholm (2019)

$$A = B = 1 - \frac{r_h}{r}, \quad X = X_0 = \text{const.}$$

Effective metric for gravitons:

($A_1(X_0) = 0$ is assumed)

$$Z_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -f(r)\mathrm{d}\tau^2 + \frac{g(r)}{f(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

where

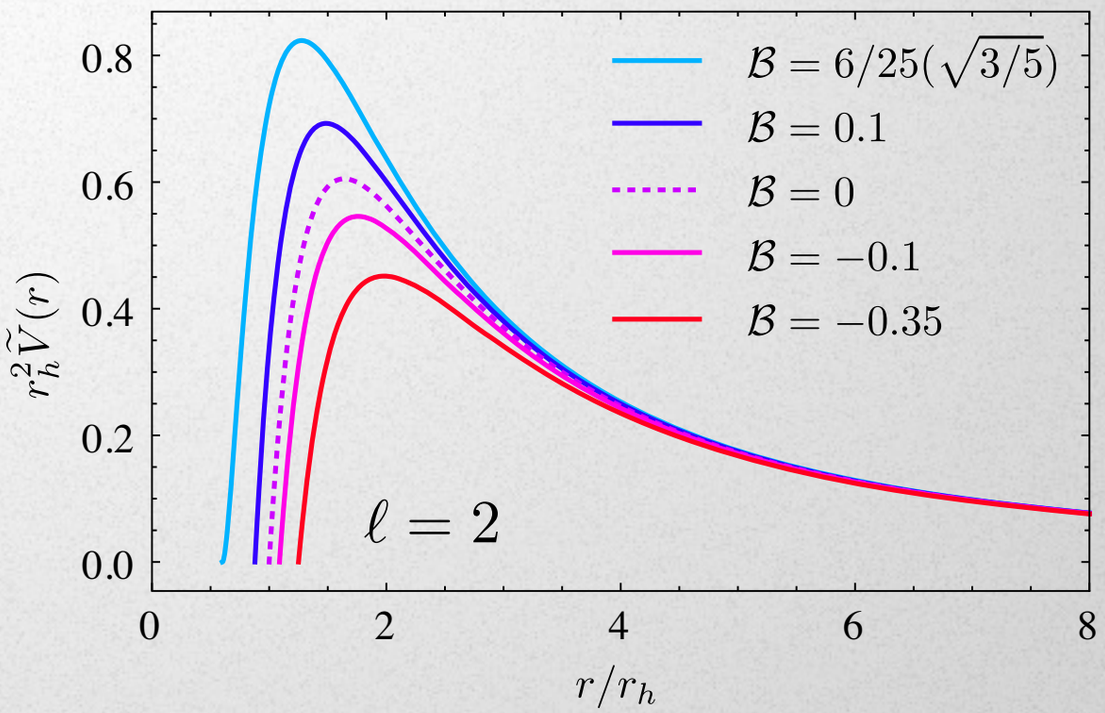
$$f(r) = 1 - \frac{r_h}{r} + \mathcal{B}\left(\frac{r_h}{r}\right)^{5/2},$$

$$g(r) = 1 - \mathcal{B}\left(\frac{r_h}{r}\right)^{3/2},$$

$$\mathcal{B} = \frac{81}{2}\frac{\mu^3}{r_h}\frac{B_1(X_0)}{F_2(X_0)} \ (= \text{const.})$$

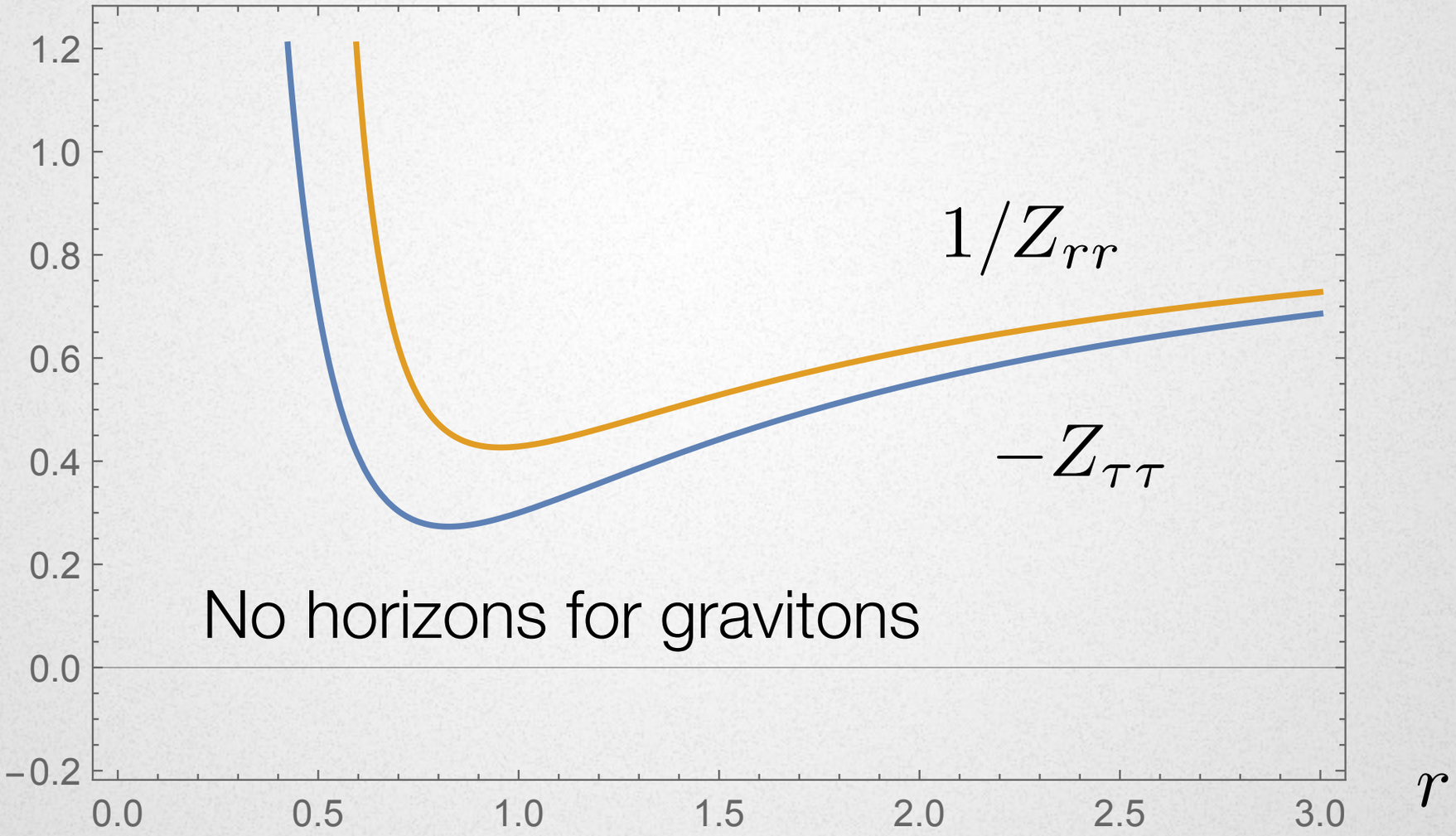
$\tau \to t, \quad Z_{\mu\nu} \to \overline{g}^{\text{Sch.}}_{\mu\nu}$ for $r \gg r_h$

Potential:



Geometry of Minamitsuji–Edholm solution for gravitons

$$\mathcal{B} > \frac{6}{25}\sqrt{\frac{3}{5}} \simeq 0.186$$



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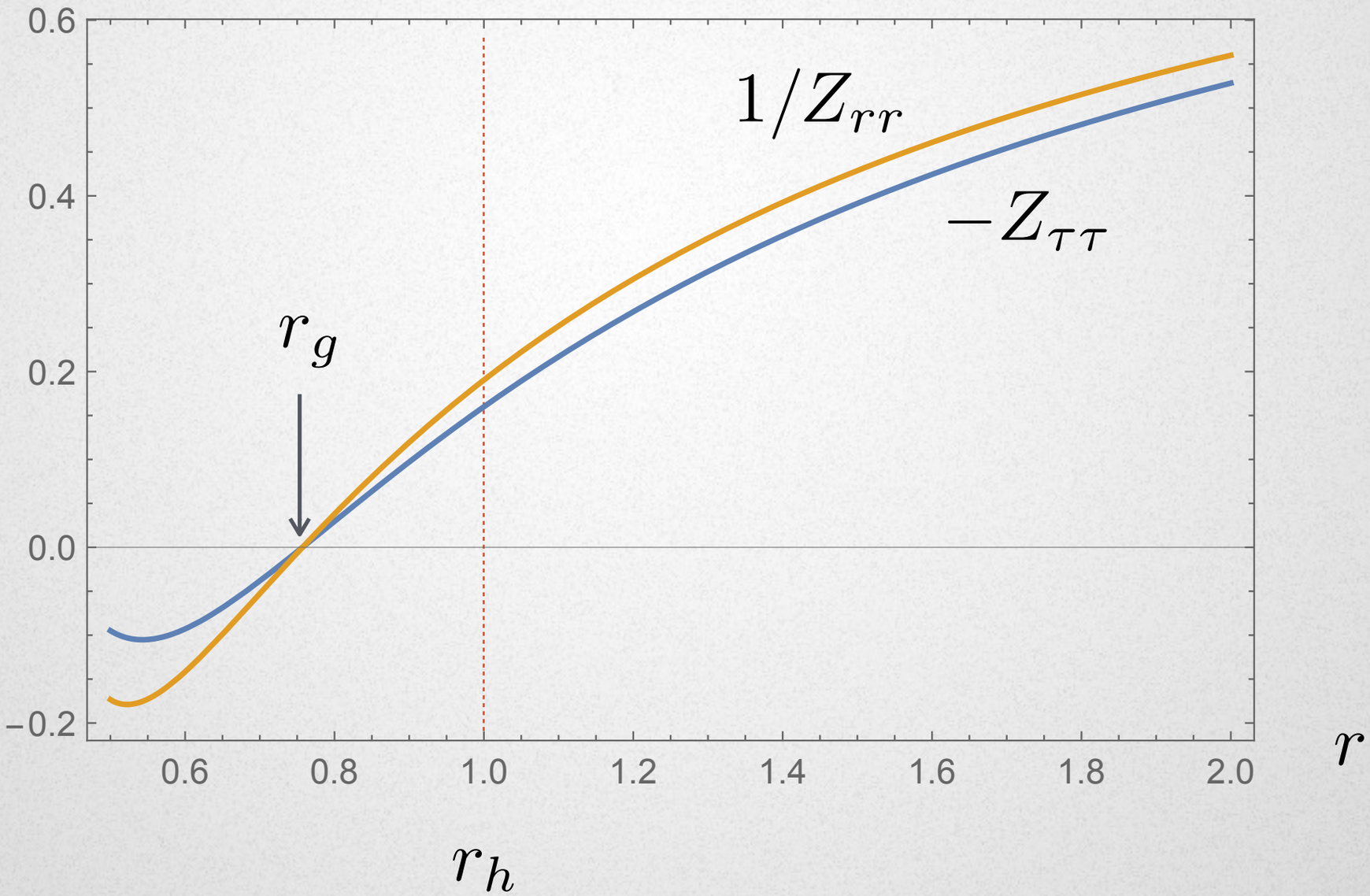
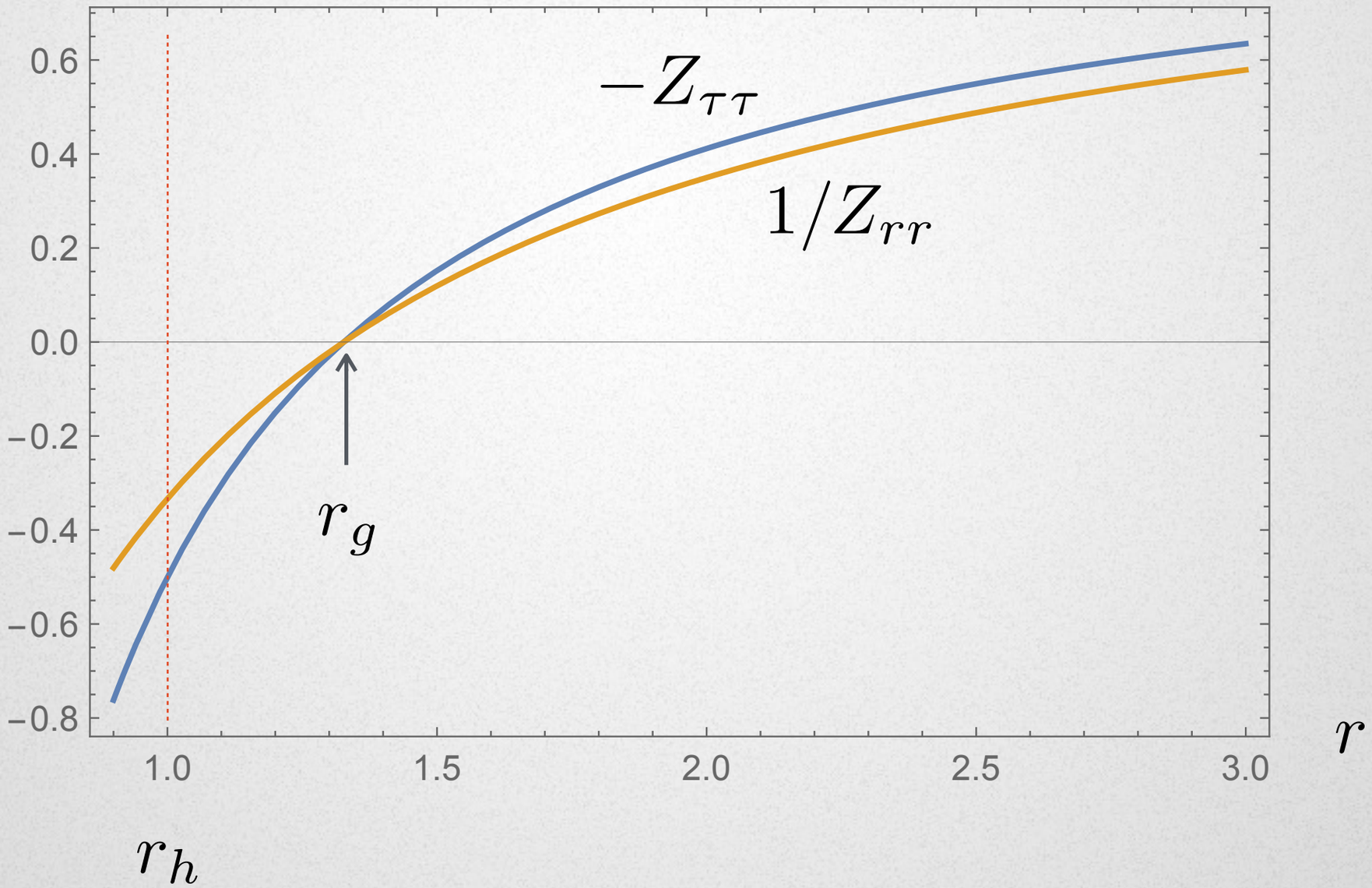
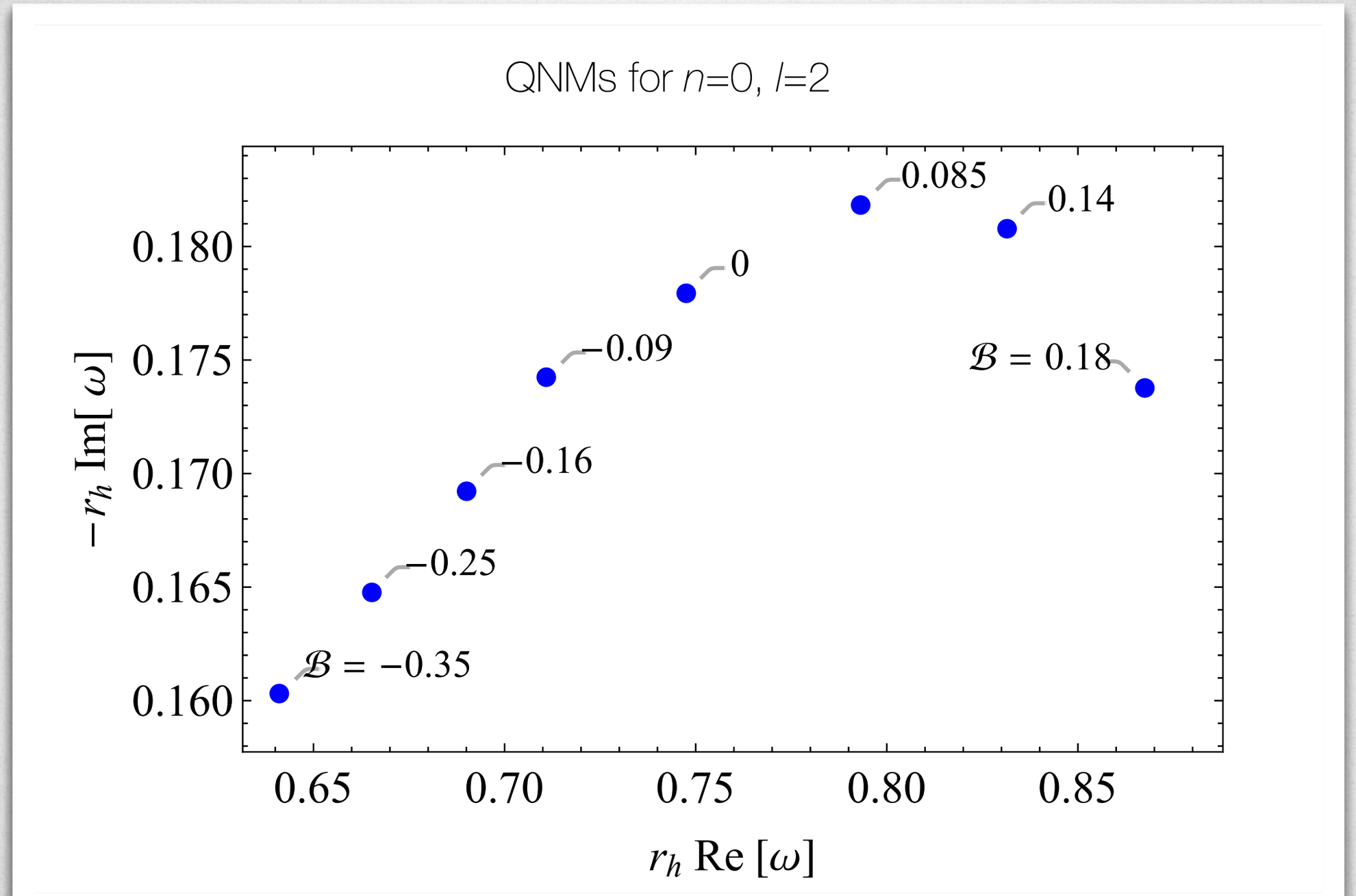


Geometry of Minamitsuji–Edholm solution for gravitons

$$\mathcal{B} < 0$$



Quasi-normal modes



Conclusions

- Odd-parity perturbations of static, spherically symmetric BHs with linearly time-dependent scalar hair in general shift-symmetric scalar-tensor theories
  - General form of **quadratic Lagrangian** and **effective metric** for odd modes are determined
- Can be applied to cubic DHOST, U-DHOST, and scordatura (detuned DHOST) theories
- Gravitons see different BH geometry
- Refined stability conditions, QNMs, …