How Inflationary Gravitons Affect Gravitational Radiation & the Force of Gravity

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Quantum Gravity & Cosmology

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Diversity in Physics
Researchers from 3 Continents

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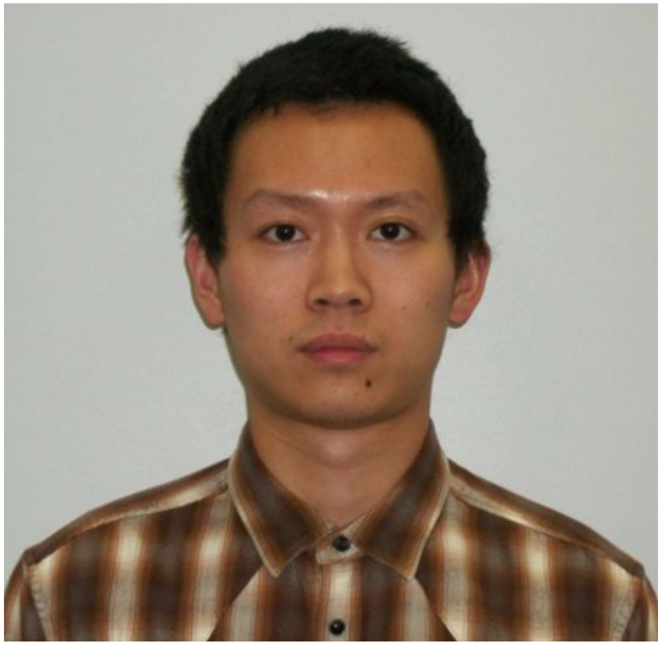

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Inflation Rips Virtual Gravitons from the Vacuum

- This is what caused the tensor power spectrum

$$\Delta_h^2(k) \cong \frac{\hbar G H^2(t_k)}{\pi c^5}$$ (NB a tree order QG effect)

- The occupation numbers are staggering

$$N(t,k) = \frac{\pi \Delta_h^2(k)}{64 G k^2} \times a^2(t)$$ (Setting ħ and c back to one)

- These gravitons must change particle kinematics & the force of gravity
- It even happens on flat space background
  - E.g., detecting gravitational radiation using pulsar timing
  - E.g., QG corrections to the Newtonian potential

$$\Psi(r) = -\frac{GM}{r}\left\{1 + \frac{41}{10\pi}\frac{\hbar G}{c^3 r^2} + \cdots\right\}$$

How Does One Study These Effects?

1. Compute the graviton self-energy
   - $g_{\mu\nu}(x) \equiv a^2\big[\eta_{\mu\nu} + \kappa\, h_{\mu\nu}(x)\big]$ $\kappa^2 \equiv 16\pi G$
   - $-i[{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$ the 1PI 2-point function for $h_{\mu\nu}$
2. Use it to quantum-correct the linearized Einstein Equation
   - $\mathcal{L}^{\mu\nu\rho\sigma} h_{\rho\sigma}(x) - \int d^4x' [{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x') h_{\rho\sigma}(x') = \tfrac{1}{2}\kappa\, T^{\mu\nu}_{lin}(x)$
3. Solve the equation for gravitons & for the response to a point mass
   - $T^{\mu\nu}_{lin}(x) = 0$ ➔ $h_{\mu\nu}(x) = \epsilon_{\mu\nu}\, u(t,k) e^{i\vec{k}\cdot\vec{x}}$ with $\epsilon_{\mu 0} = 0 = k_i \epsilon_{ij} = \epsilon_{ii}$
   - $T^{\mu\nu}_{lin}(x) = -M a(t) \delta^3(\vec{x}) \delta^\mu_0 \delta^\nu_0$ ➔ $\kappa h_{00} = -2\Psi(t,r)$ & $\kappa h_{ij} = -2\Phi(t,r)$

$-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$ is a bi-tensor density ➔ How do we represent its tensor structure?

- Exploit the symmetries of cosmology
  - $ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x}$ ➔ homogeneity & isotropy
- Special tensors are
  - δ^μ_0 (time is special)
  - $\overline{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta^\mu_0 \delta^\nu_0$ (spatial metric)
  - $\bar{\partial}^\mu \equiv \partial^\mu + \delta^\mu_0 \partial_0$ (spatial derivative)
- Initial Representation

  $-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x') = \sum_{i=1}^{21} [^{\mu\nu}\mathcal{D}_i^{\rho\sigma}](x;x') \times T^i(x;x')$ (5 on flat space)
- Reflection Invariance ➔ 21-7 = 14 independent $T^i(x;x')$'s
  - $[^{\mu\nu}\mathcal{D}_3^{\rho\sigma}] = \overline{\eta}^{\mu\nu}\delta^\rho_0\delta^\sigma_0$ & $[^{\mu\nu}\mathcal{D}_4^{\rho\sigma}] = \delta^\mu_0\delta^\nu_0\overline{\eta}^{\rho\sigma}$ ➔ $T^4(x;x') = T^3(x';x)$

$$-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x') = \sum_{i=1}^{21}\left[{}^{\mu\nu}\mathcal{D}_i^{\rho\sigma}\right] \times T^i(x;x')$$

- The $T^i(x;x')$ depend on
  $a\ ,\ a'\ ,\ i\delta^4(x-x')\ \&\ \Delta x^2 \equiv (x-x')^2$
- E.g., $T^2(x;x') = \frac{\kappa^2\ln(a)}{16\pi^2}\Big[\frac{438}{15}\partial^4 + \cdots + \frac{184}{3}a^2a'^2H^4\Big]i\delta^4(x-x') + \frac{\kappa^2}{64\ \ {}^4}\Big\{\Big[\frac{61}{120}\partial^6 + \cdots\Big]\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] + \cdots + a^3a'^3H^6\left[11\partial_0^2 + \partial^2\right]\left[\ln(H^2\Delta x^2)\right]^2\Big\}$
- Inherited from flat space
- Strongest de Sitter effects

i	$[{}^{\mu\nu}\mathcal{D}_i^{\rho\sigma}]$	i	$[{}^{\mu\nu}\mathcal{D}_i^{\rho\sigma}]$	i	$[{}^{\mu\nu}\mathcal{D}_i^{\rho\sigma}]$
1	$\bar\eta^{\mu\nu}\bar\eta^{\rho\sigma}$	8	$\bar\partial^\mu\bar\partial^\nu\bar\eta^{\rho\sigma}$	15	$\delta^{(\mu}_{\ 0}\bar\partial^{\nu)}\delta^\rho_{\ 0}\delta^\sigma_{\ 0}$
2	$\bar\eta^{\mu(\rho}\bar\eta^{\sigma)\nu}$	9	$\delta^{(\mu}_{\ 0}\bar\eta^{\nu)(\rho}\delta^{\sigma)}_{\ 0}$	16	$\delta^\mu_{\ 0}\delta^\nu_{\ 0}\bar\partial^\rho\bar\partial^\sigma$
3	$\bar\eta^{\mu\nu}\delta^\rho_{\ 0}\delta^\sigma_{\ 0}$	10	$\delta^{(\mu}_{\ 0}\bar\eta^{\nu)(\rho}\bar\partial^{\sigma)}$	17	$\bar\partial^\mu\bar\partial^\nu\delta^\rho_{\ 0}\delta^\sigma_{\ 0}$
4	$\delta^\mu_{\ 0}\delta^\nu_{\ 0}\bar\eta^{\rho\sigma}$	11	$\bar\partial^{(\mu}\bar\eta^{\nu)(\rho}\delta^{\sigma)}_{\ 0}$	18	$\delta^{(\mu}_{\ 0}\bar\partial^{\nu)}\delta^{(\rho}_{\ 0}\bar\partial^{\sigma)}$
5	$\bar\eta^{\mu\nu}\delta^{(\rho}_{\ 0}\bar\partial^{\sigma)}$	12	$\bar\partial^{(\mu}\bar\eta^{\nu)(\rho}\bar\partial^{\sigma)}$	19	$\delta^{(\mu}_{\ 0}\bar\partial^{\nu)}\bar\partial^\rho\bar\partial^\sigma$
6	$\delta^{(\mu}_{\ 0}\bar\partial^{\nu)}\bar\eta^{\rho\sigma}$	13	$\delta^\mu_{\ 0}\delta^\nu_{\ 0}\delta^\rho_{\ 0}\delta^\sigma_{\ 0}$	20	$\bar\partial^\mu\bar\partial^\nu\delta^{(\rho}_{\ 0}\bar\partial^{\sigma)}$
7	$\bar\eta^{\mu\nu}\bar\partial^\rho\bar\partial^\sigma$	14	$\delta^\mu_{\ 0}\delta^\nu_{\ 0}\delta^{(\rho}_{\ 0}\bar\partial^{\sigma)}$	21	$\bar\partial^\mu\bar\partial^\nu\bar\partial^\rho\bar\partial^\sigma$

Constraints on the $T^i(x;x')$ involve divergences

- Divergence on one point

$$D_\nu \times -i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x') = \sum_{i=1}^{10}[^{\mu}\mathcal{D}_i^{\rho\sigma}] \times S^i(x;x')$$

- Divergence on each point

$$D_\nu D'_\sigma \times -i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x') = \delta_0^\mu \delta_0^\rho \times R^1(x;x') \\ +\bar{\eta}^{\mu\rho} \times R^2(x;x') + \delta_0^\mu \bar{\partial}^\rho \times R^3(x;x') \\ +\bar{\partial}^\mu \delta_0^\rho \times R^4(x;x') + \bar{\partial}^\mu \bar{\partial}^\rho \times R^5(x;x')$$

i	$[^{\mu}\mathcal{D}_i^{\rho\sigma}]$	i	$[^{\mu}\mathcal{D}_i^{\rho\sigma}]$
1	$\delta^\mu{}_0 \delta^\rho{}_0 \delta^\sigma{}_0$	6	$2\overline{\eta}^{\mu(\rho}\delta^{\sigma)}{}_0$
2	$2\delta^\mu{}_0 \delta^{(\rho}{}_0 \overline{\partial}^{\sigma)}$	7	$2\overline{\partial}^\mu \overline{\partial}^{(\rho}\delta^{\sigma)}{}_0$
3	$\delta^\mu{}_0 \overline{\eta}^{\rho\sigma}$	8	$2\overline{\eta}^{\mu(\rho}\overline{\partial}^{\sigma)}$
4	$\delta^\mu{}_0 \overline{\partial}^\rho \overline{\partial}^\sigma$	9	$\overline{\partial}^\mu \overline{\eta}^{\rho\sigma}$
5	$\overline{\partial}^\mu \delta^\rho{}_0 \delta^\sigma{}_0$	10	$\overline{\partial}^\mu \overline{\partial}^\rho \overline{\partial}^\sigma$

Expressing the $S^i(x;x')$ in terms of the $T^i(x;x')$ and the $R^i(x;x')$ in terms of the $S^i(x;x')$

$S^i(x;x')$	Expansion in $T^j = T^j(x;x')$ and $T^{jR} = T^j(x';x)$
S^1	$(D-1)aHT^3 + (\partial_0 - aH)T^{13} - \frac{1}{2}\nabla^2 T^{14R} + aH\nabla^2 T^{16R}$
S^2	$(\frac{D-1}{2})aHT^5 + \frac{1}{4}T^9 - \frac{1}{2}aHT^{10R}$ $+\frac{1}{2}(\partial_0 - aH)T^{14} + \frac{1}{4}\nabla^2 T^{18} - \frac{1}{2}aH\nabla^2 T^{19R}$
S^3	$(D-1)aHT^1 + aHT^2 + (\partial_0 - aH)T^{3R} - \frac{1}{2}\nabla^2 T^{5R} + aH\nabla^2 T^{7R}$
S^4	$(D-1)aHT^7 + \frac{1}{2}T^{10} + aHT^{12}$ $+(\partial_0 - aH)T^{16} + \frac{1}{2}\nabla^2 T^{19} + aH\nabla^2 T^{21}$
S^5	$T^3 - \frac{1}{2}\partial_0 T^{14R} + \nabla^2 T^{16R}$
S^6	$\frac{1}{4}\partial_0 T^9 - \frac{1}{4}\nabla^2 T^{10R}$
S^7	$\frac{1}{2}T^5 - \frac{1}{4}T^{10R} + \frac{1}{4}\partial_0 T^{18} - \frac{1}{2}\nabla^2 T^{19R}$
S^8	$\frac{1}{2}T^2 + \frac{1}{4}\partial_0 T^{10} + \frac{1}{4}\nabla^2 T^{12}$
S^9	$T^1 - \frac{1}{2}\partial_0 T^{5R} + \nabla^2 T^{7R}$
S^{10}	$T^7 + \frac{1}{2}T^{12} + \frac{1}{2}\partial_0 T^{19} + \nabla^2 T^{21}$

$R^i(x;x')$	Expansion in $S^j = S^j(x;x')$
R^1	$(\partial_0' - a'H')S^1 - \nabla^2 S^2 + (D-1)a'H'S^3 + a'H'\nabla^2 S^4$
R^2	$\partial_0' S^6 - \nabla^2 S^8$
R^3	$\partial_0' S^2 - S^3 - \nabla^2 S^4$
R^4	$(\partial_0' - a'H')S^5 - S^6 - \nabla^2 S^7 + 2a'H'S^8 + (D-1)a'H'S^9 + a'H'\nabla^2 S^{10}$
R^5	$\partial_0' S^7 - S^8 - S^9 - \nabla^2 S^{10}$

Graviton loops more complex than matter loops

- Each matter vertex is conserved

$$D_\mu \times -i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x') = 0$$

$$D'_\rho \times -i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x') = 0$$

➔ 4 Structure functions (2 on flat space)

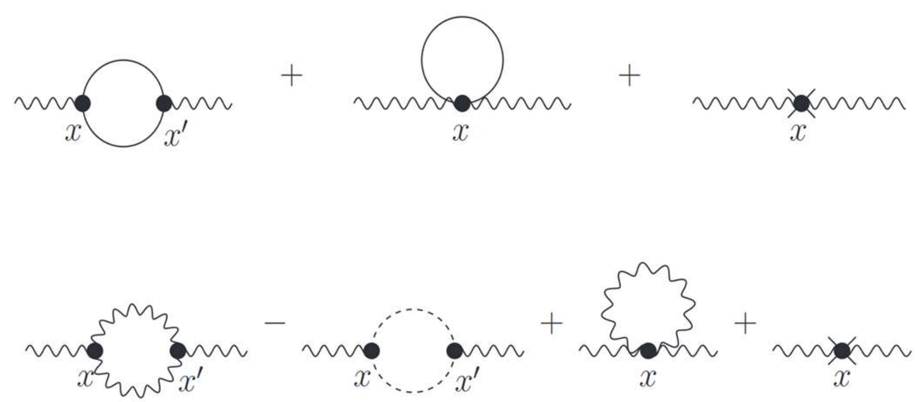
T^{12} , T^{16} , T^{18} & T^{19}

- Graviton vertices are not conserved,
Only the double-divergence vanishes

$$D_\mu \times D'_\rho \times -i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x') = 0$$

➔ 9 Structure functions (3 on flat space)

T^{12}, T^{16} , T^{18} , T^{19} ,
S^2 , S^4 , S^7 , S^8 & S^{10}



How $-i[{}^{\mu\nu}\Sigma^{\rho\sigma}]$ depends on $T^{12}(x;x')$ & $S^4(x;x')$

- Ricci and Weyl Operators
  - $\Pi_A^{\mu\nu} \equiv \bar{\partial}^\mu \bar{\partial}^\nu - \nabla^2 \overline{\eta}^{\mu\nu}$ (spatial transverse projector)
  - $\mathcal{R}_A^{\mu\nu} \equiv \Pi_A^{\mu\nu} + \frac{(D-2)\nabla^2}{\partial_0 - Ha} Ha \delta_0^\mu \delta_0^\nu$
  - $\mathcal{C}_{AA}^{\mu\nu\rho\sigma} \equiv \Pi_A^{\mu(\rho} \Pi_A^{\sigma)\nu} - \frac{1}{D-2} \Pi_A^{\mu\nu} \Pi_A^{\rho\sigma}$
- T^{12} Dependence (annihilated by a single divergence)
  - $-½\mathcal{C}_{AA}^{\mu\nu\rho\sigma} \frac{1}{\nabla^2} T^{12}(x;x')$
- S^4 Dependence (only annihilated by double divergence)
  - $\frac{1}{\partial_0 - Ha} \delta_0^\mu \delta_0^\nu \times \mathcal{R}_A^{\rho\sigma}(x') \times S^4(x;x') + \mathcal{R}_A^{\mu\nu}(x) \times \frac{1}{\partial_0' - H'a'} \delta_0^\rho \delta_0^\sigma \times S^4(x';x)$

The Lichnerowicz Operator $\mathcal{L}^{\mu\nu\rho\sigma}$ on de Sitter

- On Gravitons

  $\mathcal{L}^{\mu\nu\rho\sigma}\left[u(t,k)e^{i\vec{k}\cdot\vec{x}}\epsilon_{\rho\sigma}\right] = -½a^2\left[\partial_0^2 + 2aH\partial_0 + k^2\right]u(t,k) \times e^{i\vec{k}\cdot\vec{x}}\epsilon^{\mu\nu}$

- On Potentials

  $\mathcal{L}^{\mu\nu\rho\sigma}\left[-2\delta_\rho^0\delta_\sigma^0\Psi(t,r) - 2\bar{\eta}_{\rho\sigma}\Phi(t,r)\right]$

  - $\mu = 0, \nu = 0$ ➔ $a^2\left[6a^2H^2\Psi - 2\left(\nabla^2 - 3aH\partial_0\right)\Phi\right]$
  - $\mu = 0, \nu = j$ ➔ $a^2\partial^j[2aH\Psi + 2\partial_0\Phi]$
  - $\mu = i, \nu = j$ ➔ $a^2\partial^i\partial^j[\Psi - \Phi] + a^2\delta^{ij}\left[-\left(\nabla^2 + 2aH\partial_0 + 6a^2H^2\right)\Psi + \left(\nabla^2 - 4aH\partial_0 - 2\partial_0^2\right)\Phi\right]$
  - Only need 2 equations ➔ use $a^2[2aH\Psi + 2\partial_0\Phi]$ & $a^2[\Psi - \Phi]$

Tree order solutions

- For gravitons
  - $u_0(t,k) = \frac{H}{\sqrt{2k^3}}\left[1 - \frac{ik}{Ha}\right]\exp\left[\frac{ik}{Ha}\right] \rightarrow \frac{H}{\sqrt{2k^3}}\left[1 + \frac{k^2}{2H^2a^2} + \cdots\right]$
  - This is what causes the tensor power spectrum
- Response to a point mass
  - $\Psi_0(t,r) = \Phi_0(t,r) = -\frac{GM}{ar}$
  - Just a de Sitter version of the Newtonian potential of flat space
  - Gravitational "slip" $\Psi - \Phi = 0$ in classical general relativity, but not in modified gravity or for quantum gravity

The $\int d^4x' [^{\mu\nu}\Sigma^{\rho\sigma}](x;x') h_{\rho\sigma}(x')$ side of the equation

- On gravitons

$$-\tfrac{1}{2}a^2\left[\partial_0^2 + 2Ha\partial_0 + k^2\right]u_1(t,k) = \int d^4x'\, iT^2(x;x')u_0(t',k)e^{-i\vec{k}\cdot\Delta\vec{x}}$$

- Response to a point mass

$$4H^3\Psi_1 + 4a^2\partial_0\Phi_1 = 2\int d^4x' \begin{Bmatrix} iT^{14}(x';x)\Psi_0(x') + \\ \left[3iT^5(x';x) - iT^{10}(x;x') - iT^{19}(x;x')\nabla'^2\right]\Phi_0(x') \end{Bmatrix}$$

$$a^2\Psi_1 - a^2\Phi_1 = -2\int d^4x' \left\{iT^{16}(x';x)\Psi_0(x') + \left[3iT^7(x';x) + iT^{12}(x;x')\right]\Phi_0(x')\right\}$$

What about reality & causality?
➔ Schwinger-Keldysh Formalism

- Work in conformal coordinates
  - $d^2s = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x} = a^2\left[-d\eta^2 + d\vec{x}\cdot d\vec{x}\right]$
- Replace each $T^i(x;x')$ by $T^i_{++}(x;x') + T^i_{+-}(x;x')$
  - They have a relative minus sign
  - And each Δx^2 is replaced by $\Delta x^2_{+\pm}$
- The Schwinger-Keldysh intervals
  - $\Delta x^2_{++} \equiv \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\epsilon)^2$
  - $\Delta x^2_{+-} \equiv \|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' + i\epsilon)^2$ (NB they cancel for $\eta < \eta'$)
- $\ln\left(H^2\Delta x^2_{++}\right) - \ln\left(H^2\Delta x^2_{+-}\right) = 2\pi i\theta(\eta - \eta' - \|\vec{x} - \vec{x}'\|)$
  - This makes $iT^i(x;x')$ real and vanish outside the past light-cone

Results from a matter contribution (MMCφ) arXiv:1101.5805 & 1510.03352

- For gravitons
  - $u_1(t,k) = 0$ (also true on flat space)
- Response to a point mass
  - $\Psi(t,r) = -\frac{GM}{ar}\left\{1 + \frac{G}{20\pi a^2 r^2} - \frac{GH^2}{30\pi}[\ln(a) + 9\ln(aHr)] + \cdots\right\}$
  - $\Psi - \Phi = -\frac{GM}{ar}\left\{0 + \frac{G}{30\pi a^2 r^2} - \frac{GH^2}{30} \times 20aHr + \cdots\right\}$
  - Fractional $\frac{G}{a^2 r^2}$ corrections just de Sitter versions of known flat space effects
  - Fractional GH^2 corrections unique to de Sitter
  - Perturbation theory breaks down at large distances & late times!
  - Gravitational slip persists to spatial infinity

Inflationary gravitons can affect gravitons (hep-ph/9602317 & arXiv:2103.08547)

- $-\frac{1}{2}a^2\left[\partial_0^2 + 2aH\partial_0 + k^2\right]u_1(t,k) =$ Source
  - Source $\sim H^2a^4\ln(a)$ ➔ $u_1 \to -\frac{1}{3}[\ln(a)]^2$ Source $\sim H^2a^4$ ➔ $u_1 \to -\frac{2}{3}\ln(a)$
  - Source $\sim H^2a^3\ln(a)$ ➔ $u_1 \to \frac{\ln(a)}{a}$ Source $\sim H^2a^3$ ➔ $u_1 \to \frac{1}{a}$
  - Source $\sim H^2a^2\ln(a)$ ➔ $u_1 \to \frac{\ln(a)}{a^2}$ Source $\sim H^2a^2$ ➔ $u_1 \to \frac{1}{a^2}$
- Leading parts of $T^2(x;x')$ give
  - $\frac{\kappa^2\ln(a)}{16\pi^2} \times \frac{184}{3}a^2a'^2H^4 \times i\delta^4(x-x')$ ➔ Source $= -\frac{23\kappa^2H^2}{6\pi^2} \times u_0(t,k) \times H^2a^4\ln(a)$
  - $\frac{\kappa^2}{64\pi^4} \times a^3a'^3H^6\left[11\partial_0^2 + 2\partial^2\right]\left[\ln\left(H^2\Delta x^2\right)\right]^2$ ➔ Source $\to \frac{27\ {}^2H^2}{2\pi^2} \times u_0(\infty,k) \times H^2a^4\ln(a)$
- Physical interpretation
  - Spin-spin interaction (absent for scalars) remains effective as graviton red shifts
  - Fractional correction to $\Delta_h^2(k)$ of $GH^2 \times N^2$ might eventually be resolved by 21cm data
  - Note again the breakdown of perturbation theory

Wait a minute! Isn't quantum gravity nonrenormalizable?

- Use QG as an effective field theory in the sense of Donoghue
  - Inflation produces IR gravitons ➔ these are IR effects
  - Leading IR effects from nonlocal and UV finite parts of $-i[{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$
- Extract derivatives to make primitive contributions integrable $\int d^4x'$
  - $\frac{a^2 a'^2 \kappa^2 H^4}{\Delta x^{2D-4}} = \frac{a^2 a'^2 \kappa^2 H^4 \partial^2}{2(D-3)(D-4)} \left[\frac{1}{\Delta x^{2D-6}}\right]$ (could take $D = 4$ except for factor of $\frac{1}{D-4}$)
- Add $\quad 0 = \frac{4\pi^{D/2}}{\Gamma\left(\frac{D}{2}-1\right)} i\delta^D(x-x') - \partial^2 \left[\frac{\mu^{D-4}}{\Delta x^{D-2}}\right]$
  - $\frac{a^2 a'^2 \kappa^2 H^4}{\Delta x^{2D-4}} = \frac{4\pi^{D/2}\, a^2 a'^2 \kappa^2 H^4 i\delta^D(x-x')}{2(D-3)(D-4)\Gamma\left(\frac{D}{2}-1\right)} + \frac{a^2 a'^2 \kappa^2 H^4 \partial^2}{2(D-3)(D-4)} \left[\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}}\right]$
- Expand last term around $D = 4$
  - $\frac{a^2 a'^2 \kappa^2 H^4}{\Delta x^{2D-4}} = \frac{4\pi^{D/2}\, a^4 \kappa^2 H^4 i\delta^D(x-x')}{2(D-3)(D-4)\Gamma\left(\frac{D}{2}-1\right)} - \frac{1}{4} a^2 a'^2 \kappa^2 H^4 \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2}\right] + (D-4)$

Now renormalize with a local D-dimensional counterterm

- All have a factor of
  - $\sqrt{-g} = a^D = a^4 \times a^{D-4} = a^4[1 + (D-4)\ln(a) + \cdots]$
- Primitive contribution plus counterterm
  - $\frac{a^2 a'^2 \kappa^2 H^4}{\Delta x^{2D-4}} - \frac{4\pi^{\frac{D}{2}} a^D \kappa^2 H^4 i\delta^D(x-x')}{2(D-3)(D-4)\Gamma\left(\frac{D}{2}-1\right)}$

  $= -2\pi^2\kappa^2 H^4 a^4 \ln(a) i\delta^4(x-x') - \frac{1}{4} a^2 a'^2 \kappa^2 H^4 \partial^2 \left[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right] + O(D-4)$
  - Coefficient of logarithms a unique prediction of quantum gravity
- Finite part of counterterm controlled by μ gives
  - $-2\pi^2 a^4 \kappa^2 H^4 i\delta^4(x-x')$ ➔ weaker by a factor of $\ln(a)$

What happens when perturbation theory breaks down? ➔ sum up the leading logarithms

- Two sources of large logarithms require two techniques
  - Stochastic method fails (arXiv:0803.2377), as does RG (arXiv:0805.3089)
- Propagators have a "normal" part and a "tail" part
  - $D = 4$ ➔ $i\Delta(x; x') = \frac{1}{4\pi^2}\frac{1}{aa'\Delta x^2} - \frac{H^2}{8\pi^2}\ln\left(\frac{1}{4}H^2\Delta x^2\right)$
- Some large logarithms come from the tail part
  - Recall $\frac{\kappa^2}{64\ \ {}^4} \times a^3 a'^3 H^6 \left[11\partial_0^2 + 2\partial^2\right]\left[\ln\left(H^2\Delta x^2\right)\right]^2$
  - Can probably sum these using a variant of Starobinsky's stochastic technique
- Some large logarithms associated with UV divergences
  - Recall $\frac{\kappa^2\ln(a)}{16\pi^2} \times \frac{184}{3} a^2 a'^2 H^4 \times i\delta^4(x - x')$
  - Can probably sum these using a variant of the Renormalization Group

Model using nonlocal effective actions

- Factors of $\ln(a)$ on de Sitter consistent with $\frac{1}{\Box} R$
  - $\Box f(t) = -\frac{1}{a^3}\frac{d}{dt}\left[a^3 \dot{f}\right]$ ➔ $\frac{1}{\Box} R = -\int_0^t dt' \frac{1}{a^3(t')} \int_0^{t'} dt''\, a^3(t'') \times 12H^2 = -4\ln(a) + \cdots$
- NB such factors build up and freeze in
- Hence they can remain today on largest scales
  - Could affect vacuum energy
    - Perhaps inflation is begun by a large $\Lambda > 0$ and gradually ended by $E_{int} < 0$ of inflationary gravitons?
    - Perhaps there is no Dark Energy but rather a residual effect caused by the onset of matter domination?
  - Could affect the force of gravity
    - Perhaps there is no Dark Matter but rather a modification of gravitational force?

The Gauge Issue

- Graviton propagator is gauge dependent
  - On flat space $-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$ inherits this gauge dependence
  - Leading de Sitter contributions probably also
- We must eliminate this!
  - But it's important to keep a sense of perspective
  - Just because something is gauge dependent does not mean it vanishes!
- Inflationary gravitons SHOULD modify gravity
  - MMC scalars do & there is no gauge issue
  - Purging gauge dependence likely only changes numerical coefficients
  - We have already done this on flat space

Our Program: Short-circuit Donoghue's path to low energy QG effects

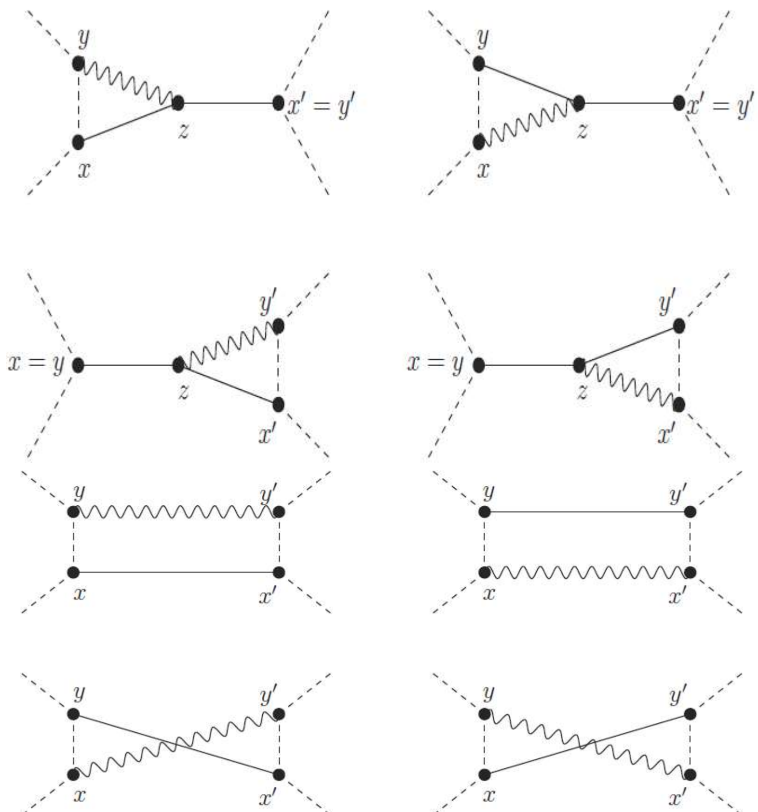
- Basic Setup ➔ Scatter 2 massive particles with some massless field
  - Then add QG corrections
- Donoghue (gr-qc/9405057)
  - Use inverse scattering to infer QG corrections to exchange potential
  - Compute amplitudes in Fourier momentum space
  - Isolate nonlocal, nonanalytic contributions
- Our variation
  - View amplitudes as correcting effective field equation for massless field
  - Work locally in position space
  - Isolate same nonlocal, nonanalytic contributions
    - This makes amplitudes resemble effective field equation

Apply Donoghue Identities to extract QG corrections to a massless scalar on flat space

- $i\Delta_m(x;y)\, i\Delta(x;x')i\Delta(y;x')$
$$\rightarrow \frac{i\delta^D(x-y)}{2m^2}[i\Delta(x;x')]^2$$
- $m^2\left(\partial_x+\partial_y\right)^2[i\Delta_m(x;y)\, i\Delta_m(x';y')i\Delta(x;x')i\Delta(y;y')]$
$$\rightarrow -\delta^D(x-y)\delta^D(x'-y')[i\Delta(x;x')]^2$$
- $m^2\left(\partial_x+\partial_y\right)^2[i\Delta_m(x;y)\, i\Delta_m(x';y')\, i\Delta(x;y')\, i\Delta(y;x')]$
$$\rightarrow +\delta^D(x-y)\delta^D(x'-y')[i\Delta(x;x')]^2$$
- Capture the nonanalytic parts which give low energy QG effects
  - Lifted from gr-qc/9405057 and hep-th/9602121
  - We only translated them to position space!

All the gauge dependence cancels

Some of the many diagrams



Each class gives the same spacetime form times a different $C_i(a, b)$

i	1	a	$\frac{1}{b-2}$	$\frac{(a-3)}{(b-2)^2}$
0	$+\frac{3}{4}$	$-\frac{3}{4}$	$-\frac{3}{2}$	$+\frac{3}{4}$
1	0	0	0	$+1$
2	0	0	0	0
3	0	0	$+3$	-2
4	$+\frac{17}{4}$	$-\frac{3}{4}$	0	$-\frac{1}{4}$
5	-2	$+\frac{3}{2}$	$-\frac{3}{2}$	$+\frac{1}{2}$
Total	$+3$	0	0	0

Conclusions

- The graviton self-energy quantum-corrects the linearized Einstein Eqn

$$\mathcal{L}^{\mu\nu\rho\sigma} h_{\rho\sigma}(x) - \int d^4x' \, [{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x') h_{\rho\sigma}(x') = ½\, \kappa T^{\mu\nu}(x)$$

- Strength of effects depends on occupation number
  - Flat Space: $N(t,k) = 0$ ➔ Not much happens
  - Inflation: $N(t,k) = \frac{\pi \Delta_h^2(k)}{64 G k^2} \times a^2(t)$ ➔ Large distance & late time growth
- Graviton contributions are more complicated than those from matter
  - 9 Structure Functions versus only 4 for matter (3 versus 2 on flat space)
  - Also stronger: $u_1 = 0$ for MMCS but $u_1(t,k) \rightarrow GH^2[\ln(a)]^2 \times u_0(t,k)$ for gravitons
- Large space & time logarithms cause breakdown of perturbation theory
  - Sum "tail" logs with Starobinsky formalism & UV logs with RG
  - Model using Nonlocal Effective Action ➔ likely gives modified gravity at late times
- More work needed to eliminate gauge dependence
  - Use Donoghue identities to view amplitude as modification of $-i[{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$
  - But this will only change numerical coefficients of corrections