Black holes in the limiting curvature theory of gravity

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"Two-dimensional black holes in the limiting curvature theory of gravity",
V. F. and A. Zelnikov. May 26, 2021.
arXiv : 2105.12808

Famous Penrose and Hawking theorems on singularities inside black holes imply that the General Relativity does not give consistent description of the spacetime in the BH interior.

Stationary BH solutions of the Einstein equations have a curvature singularity in their interior.

For Schwarzschild BH $\mathcal{K} = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} = \dfrac{48 M^2}{r^6}.$

In 1982 Markov formulated a limiting curvature principle. Following his arguments we can propose the following limiting curvature condition: there should exist a fundamental length scale ℓ such that $R \le B\ell^{-2}$.

Here R is a scalar invariant describing spacetime curvature. We also require that dimensionless constant B is universal, that is it depends only on the theory, but it is independent of a particular choice of a solution.

- One or more new universes formation inside a BH;
- Bouncing solutions in cosmology;
- Solution of the mass inflation problem.

Limiting curvature gravity (LCG) theory.

Main idea: Let $L(q,\dot{q})$ be a Lagrangian and $Q(q,\dot{q})\le 0$ be an inequality constraint. Denote

$$L(q,\dot{q},\chi,\zeta)=L(q,\dot{q})+\chi[Q(q,\dot{q})+\zeta^2].$$

Variation over the Lagrange multipliers χ and ζ gives $Q(q,\dot{q})+\zeta^2=0, \quad \chi\,\zeta=0.$

Subcritcal regime: $Q<0\to\zeta^2=-Q,\ \chi=0;$

Supercritcal regime: $Q=0\to\zeta=0,\ \chi\neq 0.$

$2D$ dilaton gravity model.

$$I_{DG} = \frac{1}{2}\int d^2x\,|\,g\,|^{1/2}\,(\psi^2 R + 4(\nabla\psi)^2 + 4\lambda^2\psi^2).$$

C. Callan et al, Nucl. Phys. B262 (1985) 593;

E. Witten, Phys. Rev. D44 (1991) 314;

G. Mandal, A. M. Sengupta and S. R. Wadia, Mod. Phys. Lett. A6 (1991) 1685;

S. Elitzur, A. Forge and E. Rabinovici, Nucl. Phys. B 359 (1991) 581;

V.F., Phys. Rev. D46 (1992) 5383;

C. Callan, Jr., S. Giddings, J. Harvey and A. Strominger,

Phys. Rev. D45 (1992) R1005.

2D limiting curvature gravity (LCG) model

$$I_{LCG} = I_{DG} + I_{\chi}, \quad \tilde{\chi} = \chi + \psi^2.$$

$$I_{\chi} = \frac{1}{2}\int_M d^2x\,|g|^{1/2}\,\tilde{\chi}(R - \Lambda + \zeta^2),$$

$$I_{LCG} = \frac{1}{2}\int d^2x|g|^{1/2}\,[\chi R + 4(\nabla\psi)^2 + 4\lambda^2\psi^2$$

$$+ (\psi^2 - \chi)(\Lambda - \zeta^2)].$$

Field equations

Constraint equations:

$$(\chi - \psi^2)\zeta = 0, \quad R - \Lambda + \zeta^2 = 0;$$

Dilaton eqn: $\Box\psi - (\lambda^2 + \frac{1}{4}(\Lambda - \zeta^2))\psi = 0,$

"Gravity" eqns: $\chi_{;\alpha\beta} + \frac{1}{2} g_{\alpha\beta} R\chi = Q_{\alpha\beta},$

$$Q_{\alpha\beta} = 4\psi_{;\alpha}\psi_{;\beta} + \frac{1}{2} g_{\alpha\beta}[-4\psi_{;\epsilon}\psi^{;\epsilon} + (4\lambda^2 + R)\psi^2].$$

Subcritical regime

$$\chi = 0, \quad \zeta^2 = \Lambda - R,$$

$$\Box\psi - (\lambda^2 + \frac{1}{4}R)\psi = 0,$$

$$\psi = \exp(-\phi), \quad \phi_{;\alpha\beta} = -\frac{1}{4}g_{\alpha\beta}R,$$

Killing vector: $\xi^\alpha = -\varepsilon^{\alpha\beta}\phi_{;\beta}$

A static 2D black hole:

Subcritical domain

$$ds^2 = -fdt^2 + f^{-1}dr^2, \quad f = 1 - \frac{m}{\lambda} e^{-2\lambda r},$$

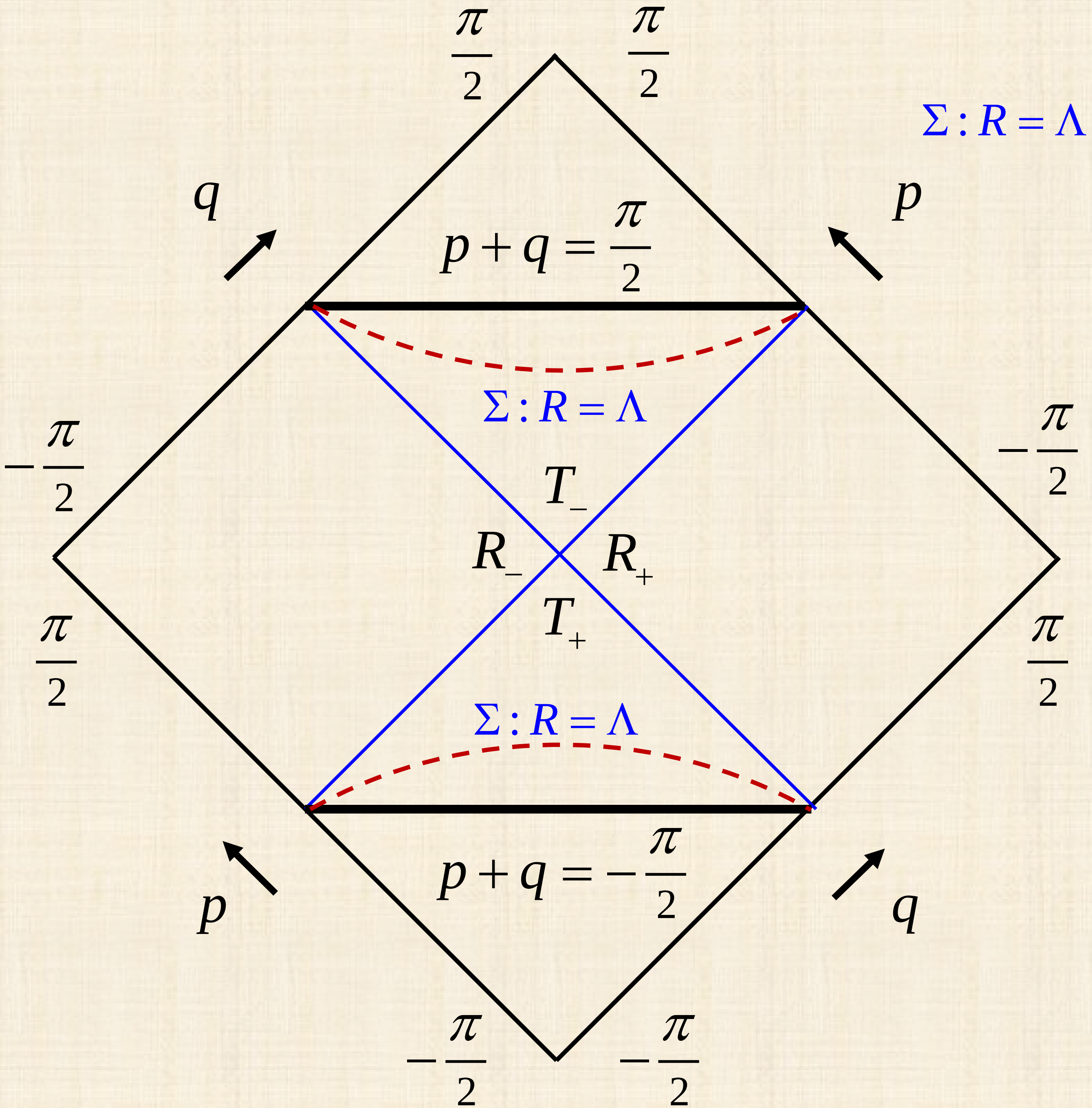
$$\psi = e^{\lambda r}, \quad \phi = -\lambda r; \ r_H = \frac{1}{2\lambda} \ln \frac{m}{\lambda}.$$

$$ds^2 = -\Omega\, dp\, dq, \quad -\pi/2 < p, q < \pi/2;$$

$$\Omega = \frac{1}{\lambda^2 \cos p \, \cos q \, \cos(p+q)}.$$

$$\Psi_1 : p \to q, \quad q \to p;$$

$$\Psi_2 : p \to -q, \quad q \to -p.$$


$\frac{\pi}{2}$
$\frac{\pi}{2}$
$\Sigma : R = \Lambda$
q
p
$p + q = \frac{\pi}{2}$
$\Sigma : R = \Lambda$
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$
T_-
R_-
R_+
T_+
$\frac{\pi}{2}$
$\frac{\pi}{2}$
$\Sigma : R = \Lambda$
$p + q = -\frac{\pi}{2}$
p
q
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$

Metric and dilaton field in the black hole interior

$$ds^2 = -d\tau_-^2 + a_-^2(\tau_-)dt^2, t \in (-\infty, \infty),$$

$$a_-(\tau_-) = \tan(\lambda\tau_-),$$

$$r = \frac{1}{2\lambda}\ln\left(\frac{m\cos^2(\lambda\tau_-)}{\lambda}\right).$$

$$R = \frac{4\lambda^2}{\cos^2(\lambda\tau_-)}. \quad \tau_- \in (0, \frac{\pi}{2\lambda}).$$

$$\psi = \sqrt{\frac{m}{\lambda}}\cos(\lambda\tau_-).$$

Boundary values on junction surface Σ

$$r_{-,\Lambda} - r_H = 12\lambda \ln \beta, \quad \beta = \frac{4\lambda^2}{\Lambda} < 1.$$

$$\tau_{-,\Lambda} = \frac{1}{\lambda} \arccos \sqrt{\beta}.$$

$$a_{-,\Lambda} = \sqrt{\frac{1-\beta}{\beta}}. \quad H_{-,\Lambda} = \frac{\dot{a}_{-,\Lambda}}{a_{-,\Lambda}} = \frac{\lambda}{\sqrt{\beta(1-\beta)}}.$$

$$\psi_\Lambda = \sqrt{\frac{m\beta}{\lambda}} = 2\sqrt{\frac{m\lambda}{\Lambda}}, \quad \dot{\psi}_\Lambda = -\sqrt{m\lambda(1-\beta)}.$$

Supercritical regime: Metric

$$ds^2 = -d\tau_+^2 + a(\tau_+)^2 dt^2. \quad \xi = \partial / \partial t,$$

$$\tilde{\tau} = \sqrt{\frac{\Lambda}{2}}\,\tau_+. \quad R = 2\frac{\ddot{a}}{a} = \Lambda\frac{a''}{a} = \Lambda.$$

Case $0 < \beta < 1/2;$

$$a = A_1 \cosh(\tilde{\tau}); \quad H_k = \sqrt{\frac{\Lambda}{2}} \tanh(\tilde{\tau})$$

Conformal diagram

$$ds_+^2 = -d\tau^2 + a^2 dt^2, \quad a = A_1 \cosh(\tilde{\tau}),$$

$$A_1 = \sqrt{\frac{1}{2\beta} - 1}, \quad \tilde{\tau} = \sqrt{\frac{\Lambda}{2}}(\tau + \Delta\tau),$$

$$\Delta\tilde{\tau} = \sqrt{\frac{2}{\beta}} \arccos\sqrt{\beta} - \operatorname{arccoth}(\sqrt{2(1-\beta)}),$$

$$T = T_0 + \frac{2}{\alpha}\arctan[\exp(\tilde{\tau})], \quad \alpha = \sqrt{\frac{\Lambda}{2}} A_1;$$

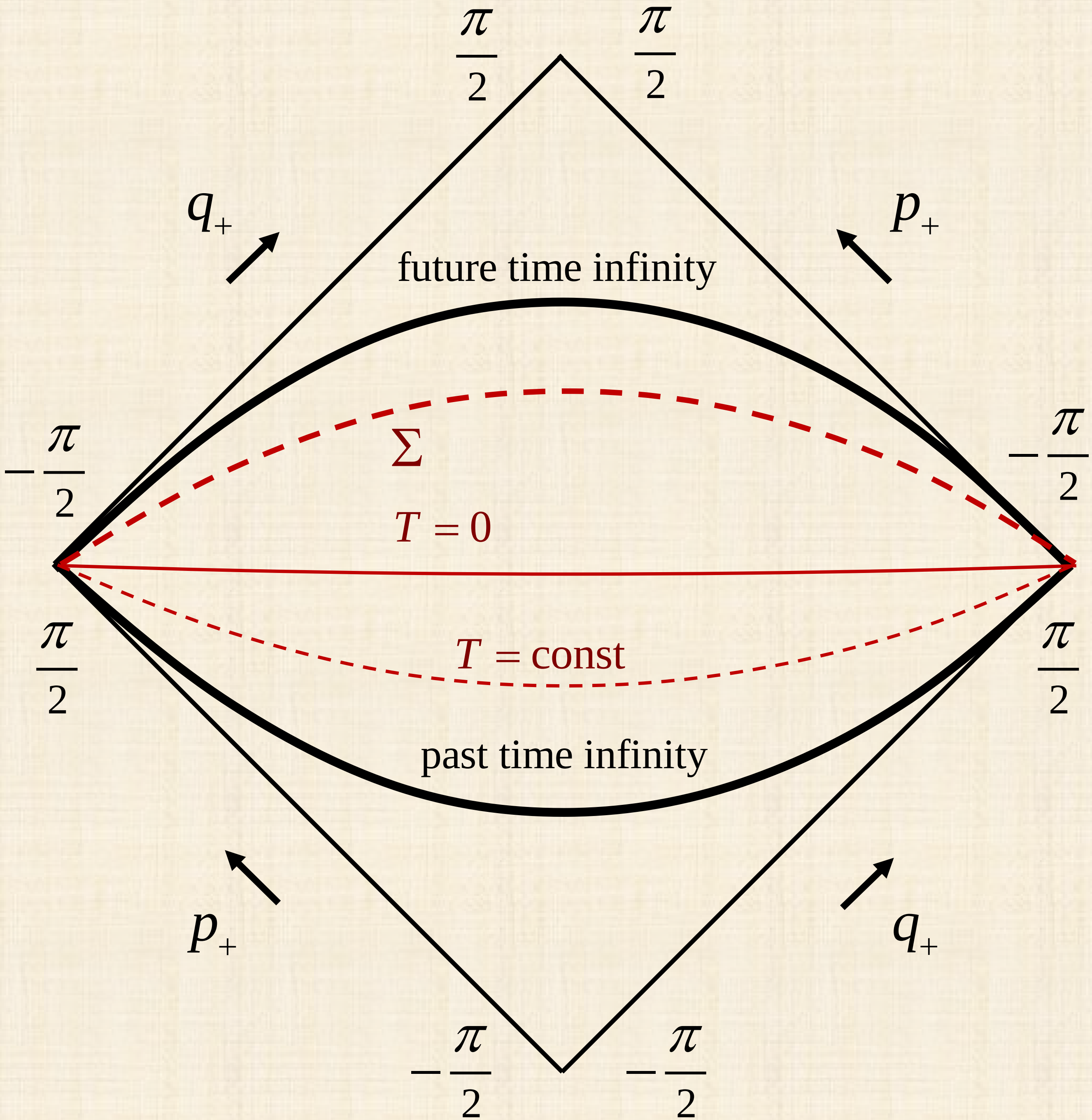
$$ds_+^2 = \frac{A_1^2}{\cos^2(\alpha T)}(-dT^2 + dt^2).$$

For the construction of the conformal diagram an additional conformal transformation which brings the spatial infinity to a "finite coordinate distance" is required. This can be achieved by introducing new null coordinates $\tan(p_+) = T - t, \tan(q_+) = T + t$ that span the interval $-\pi/2 < p_+, q_+ < \pi/2.$

$$ds_+^2 = -\Omega_+ dp_+ dq_+ ,$$

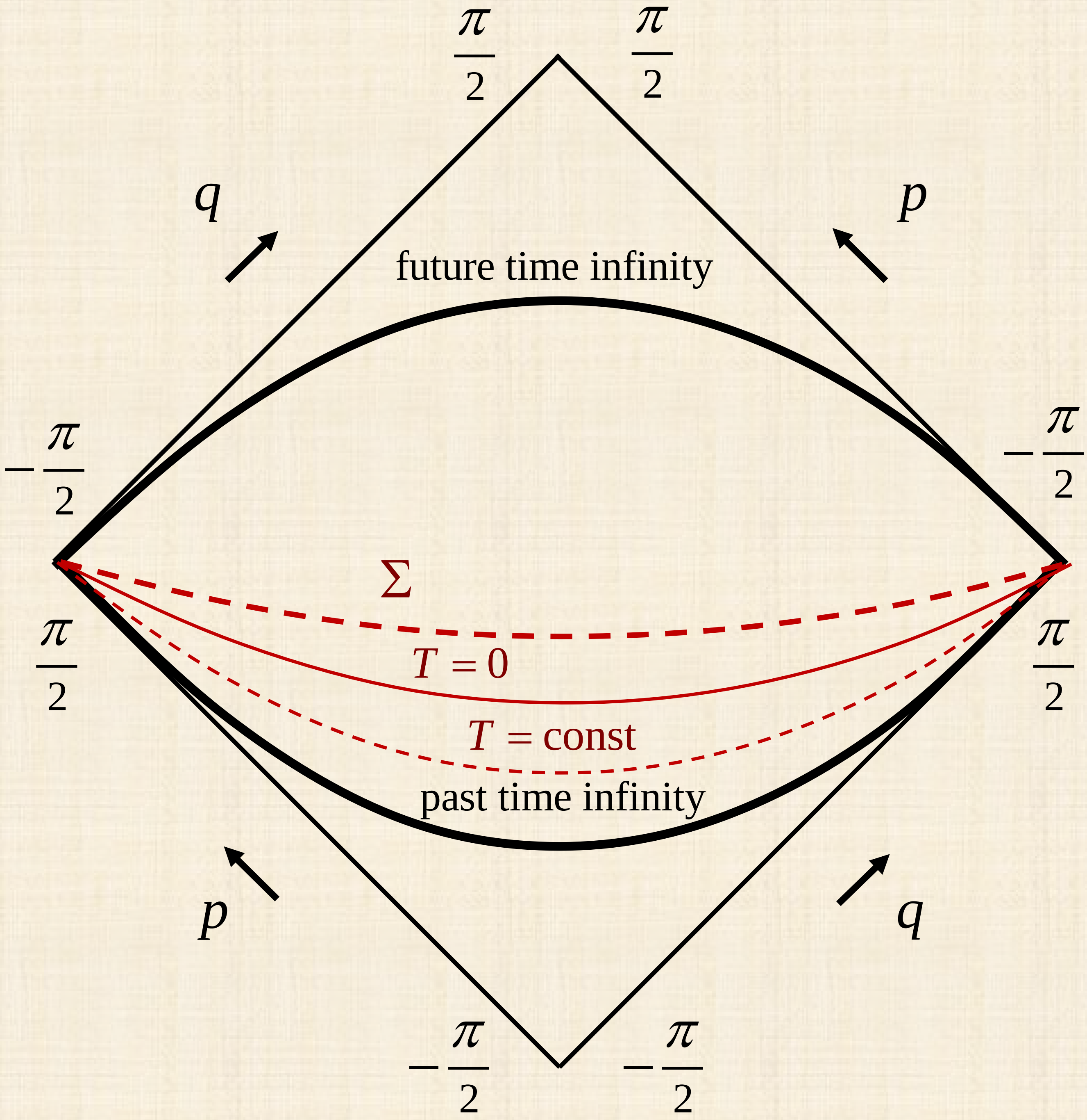
$$\Omega_+ = \frac{A_1^2}{\cos^2(\alpha T)\cos^2 p_+ \cos^2 q_+},$$

$$T = \frac{\sin(p_+ + q_+)}{2\cos p_+ \cos q_+}.$$



$\frac{\pi}{2}$
$\frac{\pi}{2}$
q_+
p_+
future time infinity
$-\frac{\pi}{2}$
Σ
$-\frac{\pi}{2}$
$T = 0$
$\frac{\pi}{2}$
$T = \text{const}$
$\frac{\pi}{2}$
past time infinity
p_+
q_+
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$

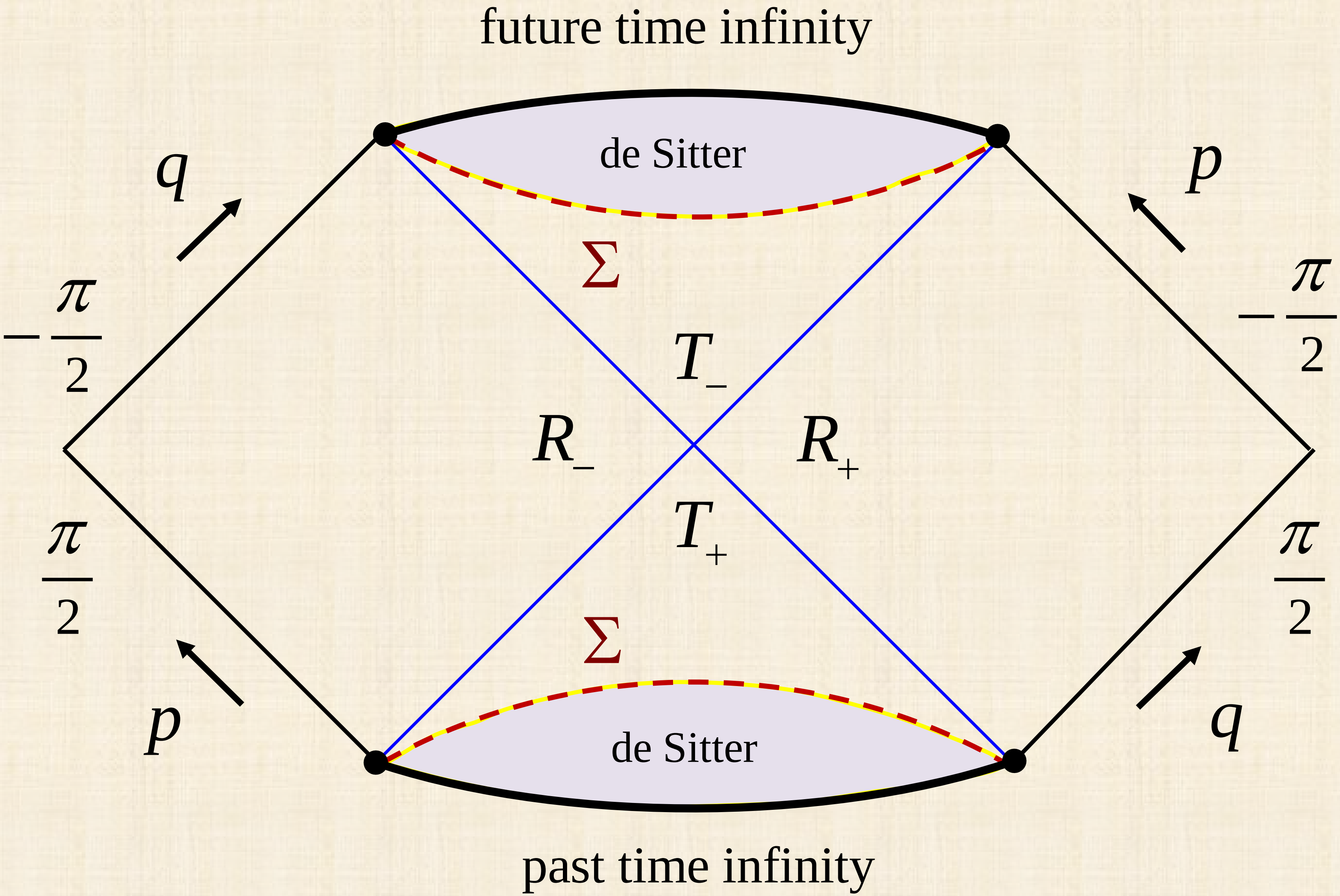
The form of the metric is preserved under the following change of the null coordinates $p = P(p_+), \quad q = Q(q_+)$. We assume that both functions are monotonic smooth functions of their arguments and they are chosen so that $P(\pm\pi/2) = \pm\pi/2, \quad Q(\pm\pi/2) = \pm\pi/2.$



$\frac{\pi}{2}$
$\frac{\pi}{2}$
q
p
future time infinity
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$
Σ
$\frac{\pi}{2}$
$\frac{\pi}{2}$
$T = 0$
$T = \text{const}$
past time infinity
p
q
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$

Matching conditions on Σ:

$$[a]=0,[H]=0,[\psi]=0,[\dot{\psi}]=0,[\chi]=0.$$


future time infinity
de Sitter
q
p
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$
Σ
T_-
R_-
R_+
T_+
$\frac{\pi}{2}$
$\frac{\pi}{2}$
Σ
p
q
de Sitter
past time infinity

Supercritical regime: Dilaton field

$$\Box\psi - \lambda^2(1+\frac{1}{\beta})\psi = 0,$$

$$z = \begin{cases} \sinh\tilde{\tau} \\ \cosh\tilde{\tau}, \\ \exp\tilde{\tau} \end{cases} \quad \nu = \frac{1}{2}\sqrt{1+2\beta}.$$

$$\psi_1 = \frac{P^{i\nu}_{-\frac{1}{2}}\left(\frac{z}{\sqrt{z^2+1}}\right) + P^{-i\nu}_{-\frac{1}{2}}\left(\frac{z}{\sqrt{z^2+1}}\right)}{(z^2+1)^{\frac{1}{4}}},$$

$$\psi_2 = \frac{Q^{i\nu}_{-\frac{1}{2}}\left(\frac{z}{\sqrt{z^2+1}}\right)}{(z^2+1)^{\frac{1}{4}}}.$$

At large time $\tilde{\tau}$ the dilaton field has oscillatory behavior with exponentially descreasing amplitude.

Supercritical regime: χ field

$$\chi = z\{\frac{\psi^2(z_0)}{z_0} - \int_{z_0}^{z} \frac{dz}{z^2}(2(z^2+k)(\partial_z\psi)^2 + (\beta+1)\psi^2)\}.$$

Can a solution leave the supercritical regime?

Answer: No

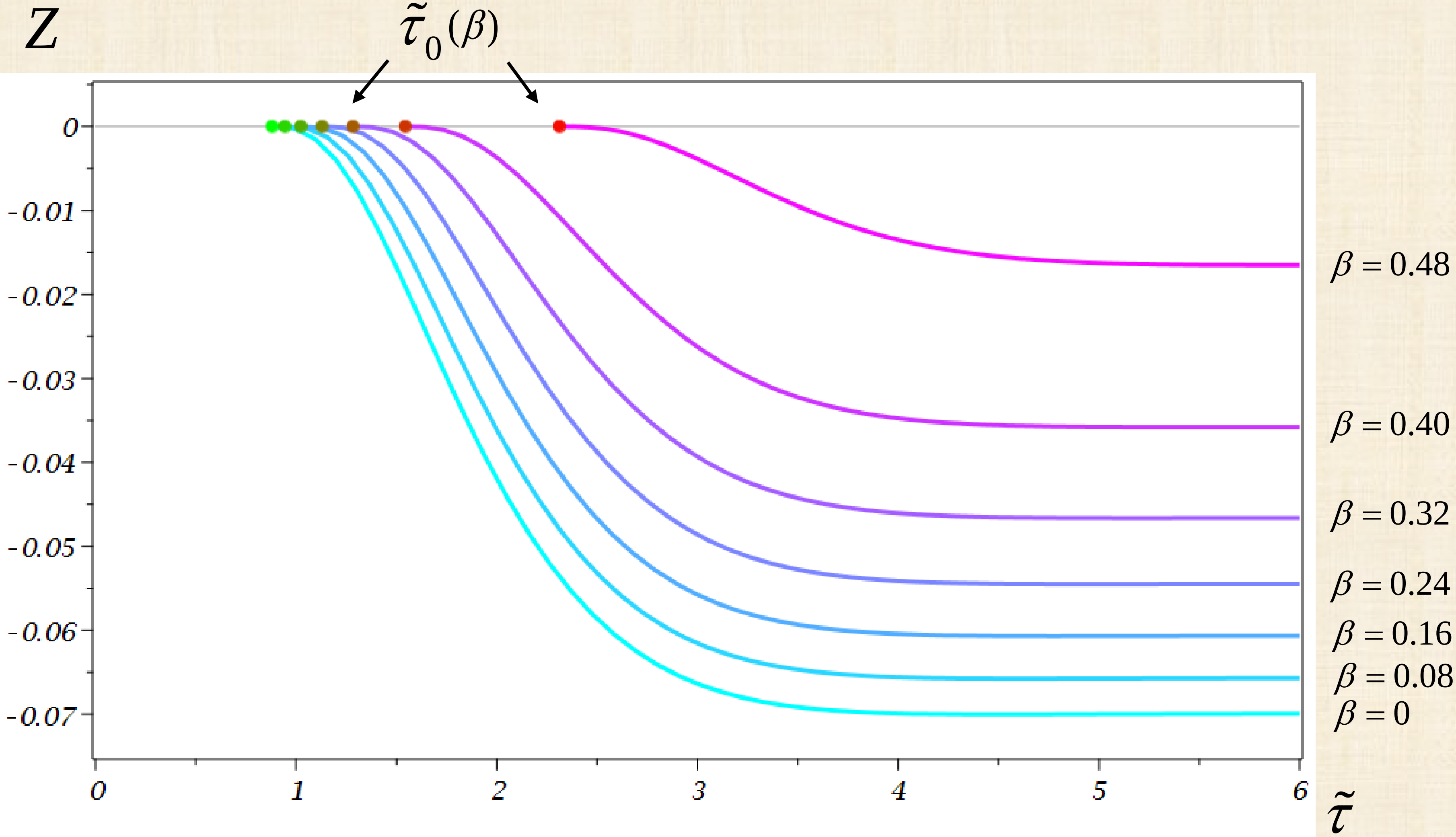
$$\Psi = \frac{1}{2}\sqrt{\frac{\Lambda}{m\lambda}}\,\psi, \quad J = \frac{1}{2}\sqrt{\frac{\Lambda}{m\lambda}}\,\psi', \quad \tilde{Z} = \frac{\Lambda}{4m\lambda}\left(\frac{\chi - \psi^2}{a'}\right).$$

$$\Psi' = J,$$

$$J' = -\frac{a'}{a}J - \frac{1+\beta}{2}\Psi,$$

$$\tilde{Z}' = -\frac{2a}{a'^2}\left(J^2 + \frac{\beta}{2}\Psi^2 + \frac{a'}{a}\Psi J\right),$$

$$\tilde{Z} = 0, \quad \Psi = 1, \quad J = -1\sqrt{2}\sqrt{1-\beta}.$$



$\tilde{Z}$
$\tilde{\tau}_0(\beta)$
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
0
1
2
3
4
5
6
$\tilde{\tau}$
$\beta = 0.48$
$\beta = 0.40$
$\beta = 0.32$
$\beta = 0.24$
$\beta = 0.16$
$\beta = 0.08$
$\beta = 0$

Formation and evaporation of a black hole

Vaidya-type generalization of the static solution

$$ds^2 = -fdv^2 + 2dvdr, \quad f = 1 - \frac{m(v)}{\lambda}\exp(-2\lambda r).$$

$$\Sigma : \exp(-2\lambda r_\Lambda) = \frac{\Lambda}{4\lambda m(v)}.$$

$$r_{-,\Lambda} - r_H = \frac{1}{2\lambda}\ln\beta.$$

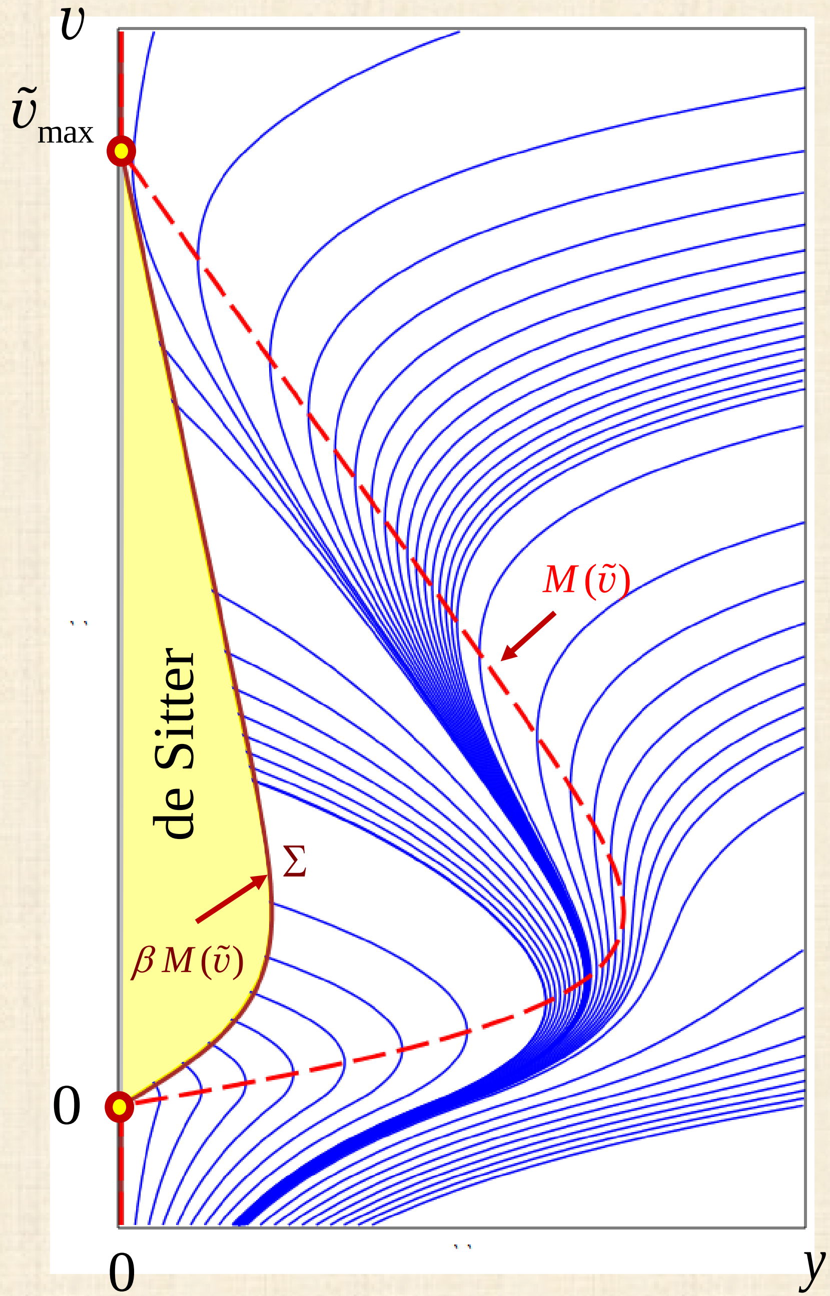
$$f_\Lambda = f(r_\Lambda) = 1 - \beta^{-1}, \quad \beta = \frac{4\lambda^2}{\Lambda} < 1.$$

$$y = \exp(2\lambda r), \quad \tilde{v} = \lambda v,$$

$$M(\tilde{v}) = m(v)/\lambda, \quad dS^2 = \lambda^2 ds^2.$$

$$dS^2 = -f d\tilde{v}^2 + \frac{1}{y} d\tilde{v} dy, \quad f = 1 - \frac{M(\tilde{v})}{y}.$$

$$y_H = M(\tilde{v}), \quad y_\Lambda = \beta y_H = \beta M(\tilde{v}).$$



$\tilde{v}$
$\tilde{v}_{\max}$
$M(\tilde{v})$
de Sitter
Σ
$\beta M(\tilde{v})$
0
0
y

$$\boxed{-\frac{M'}{M} > \frac{1}{\beta} - 1.}$$

Summary

- Limiting curvature gravity theory: Covariant action which implies inequalities constraining curvature invariants;
- Sub- and supercritical regimes;
- LCG version of 2D dilaton gravity;
- An eternal black hole has two deSitter cores;

- If subcritical solution enters the supercritical regime it never leaves it again;
- Formation and evaporation of 2D BH in LCG: one deSitter core (expanding deSitter iniverse indide BH);
- DeSitter core becomes visible at the last stage of BH evaporation;
- Further work: 4D LCG models of cosmology and BHs.