Quantum gravity, renormalization group and recurrence pole relations

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Outline of the talk

- Motivations/goals
- 't Hooft's renormalization group equations
- RG equations for metric
- Higher-curvature couplings
- Complete set of RG equations
- Solving RG equations: some lower order examples
- Some general statements (any loop order)
- Conclusions

- Think in terms of entire perturbation series
- Find relations between single pole $1/(d-4)$ and higher poles $1/(d-4)^n\,,\ n>1$
- How the higher poles vanish on-shell ($R_{ij}=0$)?
- What is the complete perturbation theory and its lower energy theory?

Three remarks before I start

1. A parallel activity: to make sense of non-renormalizable theories

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2. Only pure quantum gravity will be considered ($\Lambda = 0$, no matter)

$$L_{gr} = -\frac{1}{G_N}\int R\sqrt{g}\,d^4x = \frac{L_0}{G_N}$$

3. Quadratic gravity: $R + R^2$:

- has modified propagator with a ghost
- renormalizable Stelle '77
- renormalization of metric is multiplicative Kalmykov-Kazakov '97, Kalmykov '98, Kalmykov, Kazakov, Pronin, Stepanyantz '98 and possibly others
- belongs to a different class than quantum GR
- it will not be considered here

The bare coupling constant ($\epsilon = d - 4$)

$$\lambda_B = \mu^\epsilon(\lambda_R + \sum_{k=1} \frac{a_k(\lambda_R)}{\epsilon^k})$$

The renormalized coupling λ_R is a function of scale μ such that

$$\mu\partial_\mu\lambda_R = -\epsilon\lambda_R + \beta(\lambda_R)$$

The bare coupling is supposed to be independent of μ so that $\mu\partial_\mu\lambda_B = 0$,

$$\epsilon \sum_{k=1} \frac{a_k}{\epsilon^k} + \beta(\lambda_R) + (-\epsilon\lambda_R + \beta(\lambda_R)) \sum_{k=1} \frac{a'_k(\lambda_R)}{\epsilon^k} = 0\,,$$

where $a'_k(\lambda_R) \equiv \partial_{\lambda_R} a_k(\lambda_R)$ and the terms linear in ϵ cancel out.

$$\epsilon^0 : \quad \beta(\lambda_R) = a_1(\lambda_R) - \lambda_R a'_1(\lambda_R)\,.$$

So that the beta function expresses in terms of a single pole a_1.

$$1/\epsilon^k : \quad a_{k+1}(\lambda_R) - \lambda_R a'_{k+1}(\lambda_R) = \beta(\lambda_R) a'_k(\lambda_R)\,, \quad k \geq 1$$

It is a recurrence relation.

't Hooft '73

Renormalizable field theory:

$$\lambda_B = \lambda_B(\mu, \lambda_R, m_R, \epsilon)$$
$$\phi_B = \phi_B(\mu, \phi_R, \lambda_R, m_R, \epsilon)$$
$$m_B = m_B(\mu, m_R, \lambda_R, \epsilon)$$

Field renormalization is multiplicative.

UV divergences are hidden in renormalization of ϕ, m, λ in the bare action

$$W_B(\lambda_B, \phi_B, m_B) = W_Q(\lambda_R, \phi_R, m_R, \epsilon)$$

The idea now is to generalize 't Hooft's RG approach to Quantum Gravity.

The important point here is that one needs to renormalize metric $g_{B,ij} = g_{B,ij}(\mu, \epsilon, g_R)$.

The metric renormalization is non-multiplicative (theory is non-renormalizable)

This idea was on the surface since 't Hooft and Veltman '74:

$$g_{ij} \to g_{ij} + \frac{G_N}{\epsilon}(a\, R_{ij} + b\, g_{ij} R)$$

it removes 1-loop divergences

More generally

$$g_{ij} \to g_{ij} + X_{ij}$$

may remove a UV term $\int G^{ij} X_{ij}$

How do it consistently? We need to use Renormalization group

In any $d \geq 4$ we keep the dimensionality of Newton constant $[G_N] = 2$ (same as in $d = 4$)

The bare metric has dimensionality $[g_{B,ij}] = -(d-4)$

$$g_{B,ij} = \mu^{-\epsilon}(g_{R,ij} + \sum_{k=1} \epsilon^{-k}\, h_{k,ij}(g_R))\,,$$

$h_{k,ij}(g_R)$ are local covariant functions of the renormalized metric g_R.
One has that

$$\mu\partial_\mu g_{R,ij} = \epsilon g_{R,ij} + \beta_{ij}(g_R)\,,$$

where the beta function $\beta_{ij}(g_R)$ is a local function of g_R.

Note: These equations are similar to renormalization of the target metric in the $d = 2$ sigma-models Fridan '80, AlvarezGaume, Freedman, Mukhi '81

However, the target metric in a sigma model represents an infinite set of couplings while here we deal with a field renormalization.

The bare metric $g_{B,ij}$ is independent of the scale μ so that $\mu\partial_\mu g_{B,ij} = 0$

$$-\epsilon \sum_{k=1} \frac{h_k}{\epsilon^k} + \beta(g_R) + \sum_{k=1} \frac{1}{\epsilon^k} h'_k(g_R) \times (\epsilon g_R + \beta(g_R)) = 0\,,$$

where we define

$$(h' \times f)_{ij} \equiv \frac{d}{dt} h_{ij}(g + t\, f)|_{t=0}\,.$$

$$\epsilon^0 : \quad \beta(g_R) = h_1(g_R) - h'_1(g_R) \times g_R\,.$$

$$1/\epsilon^k : \quad h_{k+1} - h'_{k+1} \times g_R = h'_k \times \beta(g_R)\,,\ k \geq 1\,.$$

This is a recurrence relation for the higher pole residues in terms of the lower pole residues. It was used as a consistency condition in 2d σ-models.

As always in the case of the RG equations, the complete information about the renormalization is contained in a single pole h_1

In the perturbative expansion with respect to the Newton constant each term can be expressed as a power series in G_N,

$$h_k = \sum_{l=k} G_N^l \, h_{k,l} \ , \ \ \beta_{ij} = \sum_{l=1} G_N^l \beta_{l,ij} \, ,$$

where $h_{k,l}$ is a local polynomial in curvature of degree l (we count two covariant derivatives acting on a curvature to have same degree 1 as the curvature itself). Notice that $h'_g(g)g$ is a transformation of $h(g)$ under the rescaling of the metric, $g \to \lambda g$. So that one finds that $h'_{k,l}(g) \times g = (1-l)h_{k,l}$.

$$\beta_l = l \, h_{1,l} \, , \quad h_{k+1,l} = \frac{1}{l} \sum_{p=1}^{l-1} h'_{k,l-p} \times \beta_p = \frac{1}{l} \sum_{p=1}^{l-1} p \, h'_{k,l-p} \times h_{1,p} \, , \quad k \geq 1 \, .$$

This is first set of the RG equations that we will deal with.

We have to include the higher curvature terms (polynomial in Riemann tensor) with the couplings $\{\lambda_l\}$

$$\frac{1}{G_N} L_0 + \sum_{l=2} W_l\,, \qquad W_l = \Lambda_l P_l\,, \quad \Lambda_l = G_N^{l-1} \lambda_l$$

P_l is a set of invariants of degree $l+1$ constructed from Riemann tensor

$l = 1$: no independent invariant due to Gauss-Bonnet

$l = 2: \quad P_2 = R_{ab}{}^{cd} R_{cdmn} R^{mnab}$

$l > 2$: No general classification theorem for possible invariants

These terms will modify the Feynman diagrams and produce new UV divergent terms
Expanding metric over Minkowski spacetime, $g_{ij} = \eta_{ij} + \sqrt{G}\phi_{ij}$, where ϕ_{ij} is a perturbation, one finds that the terms

$$W_l \sim \Lambda_l(\sqrt{G})^{l+1} \int d^4x(\partial\partial\phi)^{l+1}, \quad l \geq 2$$

where $\Lambda_l = G^{l-1}\lambda_l$.
It does not modify graviton propagator but adds new $(l+1)$-point vertices
A Feynman diagram with $\Lambda_{p_1}, \Lambda_{p_2}, \ldots, \Lambda_{p_n}$ vertices, r GR internal lines and m GR vertices such that $p_1 + \cdots + p_n - n + m = l + 1$ produces a UV divergent term of $(l+1)$-th order in curvature,

$$\Lambda_{p_1} \ldots \Lambda_{p_n} G^{(n+r)} \int d^4x\mathcal{R}^{l+1}, \quad p_1 + \cdots + p_n = (l-1) - r\,.$$

The counter-term has to have dimension zero, so that one gets a condition for p_i as above. Since $r \geq 0$ one has a condition on the values of p_i:

$$p_k \geq 2\,, \qquad p_1 + \cdots + p_n \leq l - 1$$

Thus, the lowest parameter $v_{1,2}$ (or the counter-term $V_{1,2}$) is independent of any λ.
$V_{k,l}$ and $L_{k,l}$, $l \geq 3$ can be polynomial functions of λ_p, $p \leq l-1$. For instance, counter-terms $V_{k,3}$ and $L_{k,3}$ can be at most linear in λ_2.
When the higher curvature terms are present, the lower loops may give contributions to the curvature terms that appear at the GR loop order l. For convenience, we will still refer to l as a loop order.

I. Renormalization of higher curvature couplings:

$$\lambda_l^B = \mu^{-l\epsilon}\left(\lambda_l^R + \sum_{k=1}\epsilon^{-k}a_{k,l}(\lambda^R)\right)\ , \quad l \geq 2$$

$$\mu\partial_\mu\lambda_l^R = \epsilon\, l\, \lambda_l^R + \hat{\beta}_l\ ,$$

$$\hat{\beta}_l = l a_{1,l} - \sum_{p=2}^{l-1} p\lambda_p\partial_{\lambda_p}a_{1,l}\ ,$$

$$\left(1 - \frac{1}{l}\sum_{p=2}^{l-1} p\lambda_p\partial_{\lambda_p}\right)a_{k+1,l} = \sum_{p=2}^{l-1}\hat{\beta}_p\partial_{\lambda_p}a_{k,l}\ .$$

II. Renormalization of metric:

$$\beta_1 = h_{1,1}\,, \quad \beta_l = l\, h_{1,l} - \sum_{p=2}^{l-1} p\lambda_p \partial_{\lambda_p} h_{1,l}\,, \;\; l \geq 2$$

$$\left(1 - \frac{1}{l}\sum_{p=2}^{l-1} p\lambda_p\partial_{\lambda_p}\right) h_{k+1,l} = \frac{1}{l}\sum_{p=1}^{l-1} h'_{k,l-p} \times \beta_p + \frac{1}{l}\sum_{p=2}^{l-1} \hat{\beta}_p \partial_{\lambda_p} h_{k,l}\,, \;\; k \leq l-1$$

RG equations modified due to the higher curvature couplings

III. The quantum gravitational action

$$L_Q(g_R(\mu),\lambda_l^R(\mu))=\mu^{-\epsilon}(\frac{1}{G}L_0(g_R)+W(g_R,\lambda_R)+\sum_{k=1}\frac{1}{\epsilon^k}(L_k(g_R,\lambda_R)+V_k(g_R,\lambda_R)))\,.$$

L_k are terms vanishing on-shell, i.e. contain at least one power of Ricci tensor R_{ij}
$V_k=\sum_{l\geq k}G_N^{l-1}V_{k,l},\ V_{k,l}=v_{k,l}P_l$
($v_{1,2}$ was computed by Goroff and Sagnotti '86 and van de Venn '92)

$\mu\partial_\mu L_Q=0$ leads to equation

$$-\epsilon(\frac{1}{G_N}L_0+\sum_{k=1}\frac{1}{\epsilon^k}(L_k+V_k))+\frac{1}{G_N}L_0'\cdot(\epsilon g+\beta)+\sum_{k=1}\frac{1}{\epsilon^k}(L_k'+V_k')\cdot(\epsilon g+\beta)$$
$$+\sum_{l=2}G_N^{l-1}[-\epsilon\lambda_l P_l+\lambda_l P_l'\cdot(\epsilon g+\beta)+P_l(\epsilon\, l\,\lambda_l+\hat{\beta}_l)]$$
$$+\sum_{k=1}\frac{1}{\epsilon^k}\sum_{p=2}\partial_{\lambda_p}(L_k+V_k)(\epsilon p\lambda_p+\hat{\beta}_p)=0$$

We defined

$$L'\cdot\beta\equiv\frac{d}{dt}L[g+t\beta]|_{t=0}=\int d^4x\sqrt{g}\,(L')^{ij}\beta_{ij}\,.$$

$(L_0')^{ij}=(R^{ij}-\frac{1}{2}g^{ij}R)=G^{ij}$ is the Einstein tensor.

Solving RG equations

Basic data is in the single pole terms in the quantum action: $L_{1,l}$ and $V_{1,l}$, $l \geq 1$

- at order ϵ^0 one has

$$lL_{1,l} - L'_0 \cdot \beta_l - P_l\,(\hat{\beta}_l - lv_{1,l}) - \sum_{m=2}^{l-1} \lambda_m P'_m \cdot \beta_{l-m} - \sum_{p=2}^{l-1} p\lambda_p \partial_{\lambda_p}(L_{1,l} + V_{1,l}) = 0\,,$$

Solving this equation one finds beta functions for higher curvature couplings $\hat{\beta}_l$ and for metric $\beta_{l,ij}$.

- at order $1/\epsilon^k$, $k \geq 1$ one has

$$(1 - \frac{1}{l}\sum_{p=2}^{l-1} p\lambda_p \partial_{\lambda_p})(L_{k+1,l} + V_{k+1,l})$$

$$= \frac{1}{l}\sum_{p=1}^{l-k}(L'_{k,l-p} + V'_{k,l-p}) \cdot \beta_p + \frac{1}{l}\sum_{p=2}^{l-1} \hat{\beta}_p \partial_{\lambda_p}(L_{k,l} + V_{k,l})\,, \quad l \geq k+1$$

This is recurrence equation for the higher poles: $L_{k,l}$ and $V_{k,l}$, $1 < k \leq l$.

$$\sum_{k=1} \frac{1}{\epsilon^k} \sum_{l=k} G_N^{l-1}(L_{k,l} + V_{k,l}) = \sum_{l=1} G_N^{l-1} \sum_{k=1}^{l} \frac{1}{\epsilon^k}(L_{k,l} + V_{k,l})$$

in l-th loop order term $k = 1$ is a single pole while term $k = l$ is the highest pole

$\underline{l=1}$

$$-L_{1,1} + L_0' \cdot \beta_1 = 0$$

In one loop, the quantum effective action contains terms quadratic in the Ricci scalar and in the Ricci tensor, $V_{1,1} = 0$
We represent

$$L_{1,1} = \int d^4x\sqrt{g}(aG_{ij}^2 + bG^2) = \int d^4x\sqrt{g}\, G^{ij}X_{ij}^{(1)}\, ,\; X_{ij}^{(1)} = a\, G_{ij} + b\, g_{ij} G\, ,$$

Values of a and b are available in the literature and are known to depend on the gauge. The equation for the beta function β_1 is

$$L_0' \cdot X^{(1)} = L_0' \cdot \beta_1\, .$$

A solution of this equation is

$$(\beta_1)_{ij} = X_{ij}^{(1)} = a\, G_{ij} + b\, g_{ij}\, G\, .$$

up to usual ambiguity

$$\beta_{ij} \to \beta_{ij} + \nabla_i \xi_j + \nabla_j \xi_i$$

Beta functions

$\underline{l=2}$

$$-2L_{1,2} + L_0' \cdot \beta_2 + P_2\,(\hat{\beta}_2 - 2v_{1,2}) = 0$$

$$P_2 = \int d^4x \sqrt{g} R_{ijkl} R^{klmn} R_{mn}{}^{ij}$$

By definition, $L_{1,2}$ vanishes on-shell and hence can be presented in the form

$$L_{1,2} = L_0' \cdot X^{(2)} = \int d^4x \sqrt{g} G^{ij} X^{(2)}_{ij}\,,$$

where $X^{(2)}$ is quadratic in curvature (necessarily vanish on-shell)
First two terms vanish on-shell and the last term does not vanish. It gives the two loop beta functions for the metric and for the cubic coupling λ_2,

$$(\beta_2)_{ij} = 2X^{(2)}_{ij}\,, \quad \hat{\beta}_2 = 2v_{1,2}$$

Tensor $X^{(2)}_{ij}$ is a local tensor quadratic in curvature or its covariant derivatives. Its general form

$$X^{(2)}_{ij} = c_0\, g_{ij} R_{nklm} R^{nklm} + G_{kl} Y^{kl}_{ij}$$

does not vanish on-shell in general. Note: c_0 is non-zero in Goroff and Sagnotti '86

$\underline{l=3}$:

$$-3L_{1,3} + L_0' \cdot \beta_3 + P_3\,(\hat{\beta}_3 - 3v_{1,3}) + \lambda_2\,P_2' \cdot \beta_1 + 2\lambda_2\partial_{\lambda_2}(L_{1,3} + v_{1,3}P_3) = 0$$

In this equation the terms containing P_3 do not vanish on-shell. Hence the sum of these terms has to vanish separately from the other terms. This gives us a relation for the beta function for λ_3 in front of the quartic power of the Riemann tensor,

$$\hat{\beta}_3 = 3v_{1,3} - 2\lambda_2\partial_{\lambda_2}v_{1,3}$$

The rest can be resolved for a three-loop beta function for the metric

$$(\beta_3)_{ij} = 3X^{(3)}_{ij} - 2\lambda_2\partial_{\lambda_2}X^{(3)}_{ij} - \lambda_2\left(a\,(P_2')_{ij} + b\,g_{ij}g^{kl}(P_2')_{kl}\right)\,,$$

where $L_{1,3} = \int G^{ij}X^{(3)}_{ij}$, a and b are those that appeared in the one-loop equation and

$$(P_2')_{ij} = -3R_i{}^{kln}R_{ln}{}^{mp}R_{mpjk} + \frac{1}{2}g_{ij}R_{klmn}R^{mn}{}_{ab}R^{abkl} - 6\nabla^k\nabla^l(R_{ikmn}R_{jl}{}^{mn})\,,$$

$\underline{l=1}$: no higher poles

$\underline{l=2}$:

$$L_{2,2}+V_{2,2}=\frac{1}{2}L'_{1,1}\cdot\beta_1\,,$$

where we take into account that $V_{1,1}=0$.
$V_{2,2}$ is the only term here that does not contain the Ricci tensor or the Ricci scalar. It has to vanish identically,

$$V_{2,2}=0\ \ (v_{2,2}=0)\,.$$

This 2-loop result was first obtained, using methods different from ours, by 'Chase '82

$$L_{2,2}=\int(a^2R_{injl}G^{nl}G^{ij}+\frac{a^2}{2}G_{ij}\Box G^{ij}-\frac{1}{2}(a^2+6b^2+4ab)G\Box G$$
$$-(\frac{3}{4}a^2+ab)GG^2_{ij}+\frac{1}{4}(ab+a^2)G^3)\,.$$

It is at least quadratic in G_{ij}.

$\underline{l=3}$:
Two values of k are possible: $k=1$ and $k=2$. For $k=1$ the RG equation is

$$(1-\frac{2}{3}\lambda_2\partial_{\lambda_2})(L_{2,3}+V_{2,3})=\frac{1}{3}(L'_{1,2}+V'_{1,2})\cdot\beta_1+\frac{1}{3}L'_{1,1}\cdot\beta_2+\frac{1}{3}\hat{\beta}_2\partial_{\lambda_2}(L_{1,3}+V_{1,3}))$$

while for $k=2$ the RG equation is

$$(1-\frac{2}{3}\lambda_2\partial_{\lambda_2})(L_{3,3}+V_{3,3})=\frac{1}{3}L'_{2,2}\cdot\beta_1+\frac{1}{3}\hat{\beta}_2\partial_{\lambda_2}(L_{2,3}+V_{2,3}))\,,$$

where, as we have shown earlier, $\hat{\beta}_2=2v_{1,2}$. In this order the counter-terms can be at most linear in λ_2 so that $V_{1,3}=V^{(0)}_{1,3}+V^{(2)}_{1,3}\lambda_2$ and the same for the counter-terms $L_{1,3}$.

$$V_{2,3}=\frac{2}{3}v_{1,2}V^{(2)}_{1,3}\,,$$

$$L_{2,3}=\frac{2}{3}v_{1,2}L^{(2)}_{1,3}+\frac{1}{3}(L'_{1,2}+V'_{1,2})\cdot\beta_1+\frac{1}{3}L'_{1,1}\cdot\beta_2\,,$$

$$V_{3,3}=0\ (v_{3,3}=0)\,,\quad L_{3,3}=\frac{1}{3}L'_{2,2}\cdot\beta_1\sim O(G^2)\,.$$

None of them depends on λ_2.

GR counter-terms: some general properties

We define GR counter-terms $\mathcal{L}_{k,l}$ and $\mathcal{V}_{k,l}$ as coming only from GR vertices. They are obtained by taking limit of vanishing λ_p and neglecting derivatives w.r.t λ_p

$$\mathcal{L}_{k+1,l} + \mathcal{V}_{k+1,l} = \frac{1}{l} \sum_{p=1}^{l-k} (\mathcal{L}'_{k,l-p} + \mathcal{V}'_{k,l-p}) \cdot \beta_p^{(0)}\,, \quad l \geq k+1$$

Property 1. In the highest order $k = l$ at any given loop order l

$$\mathcal{V}_{l,l} = 0\,, \quad \mathcal{L}_{l,l} = O(G^2)$$

vanishes at least quadratically in G_{ij}

Property 2. In a higher order $k = l - 1$ at any given loop order l

$$\mathcal{V}_{l-1,l} = 0\,, \quad \mathcal{L}_{l-1,l} = O(G)$$

vanishes linearly in G_{ij}

Property 3. In any loop order $l \geq 4$ in a higher order $k = l - 2$

$$\mathcal{V}_{l-2,l} \neq 0$$

Goroff and Sagnotti '86 computed the higher pole counter-terms $L_{2,2}$ and $V_{2,2}$ in two loops.
Their result is in disagreement with our analysis: their $L_{2,2}$ contains a term $G_{ij}R^{iklm}R^{j}{}_{klm}$ linear in G.

Possible source of disagreement is that they overcount the possible independent curvature invariants:

$$I_1 = R\Box R,\ I_2 = R^3,\ I_3 = R_{ij}\Box R_{ij},\ I_4 = RR_{ij}^2,\ I_5 = R_{ik}R_{jl}R^{ijkl},\ I_6 = R_i{}^jR_j{}^kR_k{}^i,$$
$$I_7 = RR_{ijkl}R^{ijkl},\ I_8 = R_{ij}R^{iklm}R^{j}{}_{klm},\ I_9 = R_{ij}{}^{kl}R_{kl}{}^{mp}R_{mp}^{ij}$$

However in $d = 4$ one has an identity

$$R_i{}^{nkl}R_{jnkl} = 2R_{ikjl}R^{kl} - RR_{ij} + 2R_{ik}R_j{}^k + \frac{1}{4}g_{ij}(R^2_{nklm} - 4R^2_{kl} + R^2)$$

so that I_8 is not an independent invariant and it has to be excluded from the list.

This will change the result for 2-loop higher pole terms.

Quantum gravitational action = Bare gravitational action

$$\frac{1}{G}L_0(g_R) + W(\lambda_R, g_R) + \sum_{k=1} \frac{1}{\epsilon^k}(L_k(g_R, \lambda_R) + V_k(g_R, \lambda_R))$$
$$= \frac{1}{G}L_0(g_B) + W(\lambda_B, g_B)$$

where $W(\lambda, g) = \sum_{l=2} G_N^{l-1}\lambda_l P_l(g)$, P_l is polynomial of degree $l+1$ in Riemann tensor

Notes:

- No Ricci tensor (or Einstein tensor) appears in the renormalized gravitational action accept for L_0

- Low energy gravitational action starts with a cubic term

$$-\frac{1}{G_N}\int R + G_N\lambda_2 \int (R_{ijkl})^3$$

Summary:

- Consistent set of RG equations for pure quantum gravity
- Solving the GR equations one obtains constraints on the possible higher pole counter-terms (could be used to test future higher loop computations in quantum gravity)
- Complete theory contains infinite set of invariants built from the Riemann tensor

Still open questions:

- Whether the complete theory is consistent?
- If not what should we add (or change) to make it consistent?
- Can it be generalized to other non-renormalizable theories?

Technical Challenges:

- Compute higher loops $l > 2$ in quantum GR (it's been 35 years since 2-loop computation was first done by Goroff and Sagnotti and 47 years since 1-loop computation by 't Hooft and Veltman!)