Quantizing Analytic Infinite Derivative (AID) gravity theories: propagator and unitarity

Alexey Koshelev

Universidade da Beira Interior, Covilhã, Portugal

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Mainly based on recent papers
with Sravan Kumar, Alexei Starobinsky and Anna Tokareva,

and a work in progress

Action to study

We start straight with [arxiv:1602.08475, arXiv:1606.01250]

$$S=\int d^Dx\sqrt{-g}\left(\frac{M_P^2 R}{2}-\Lambda\right.$$

$$\left.+\frac{\lambda}{2}\left(R\mathcal{F}_R(\Box)R+L_{\mu\nu}\mathcal{F}_L(\Box)L^{\mu\nu}+W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\right)\right)$$

Here $\mathcal{F}_X(\Box)=\sum_{n\geq 0}f_{X_n}\Box^n$ and $L_{\mu\nu}=R_{\mu\nu}-\frac{1}{D}Rg_{\mu\nu}$

This is the most general action to study linear perturbations around MSS.

Thanks to the Bianchi identities one can further achieve $\mathcal{F}_L(\Box)=0$ in $D=4$ and $\mathcal{F}_L(\Box)=\mathrm{const}$ in $D>4$.

Pure gravity arguments why infinite derivatives appear

We start with

$$S = \int d^D x \sqrt{-g} \left(\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\hat{\mathcal{O}}_{iI} \mathcal{Q}_{iI}) \right)$$

Here $\mathcal{P}$ and $\mathcal{Q}$ depend on curvatures and $\mathcal{O}$ are operators made of covariant derivatives.

Everywhere the respective dependence is *analytic* in IR.

Let's name it *general analytic gravity*

Excluding all the terms which vanish around MSS and massaging others we arrive to the action on the previous slide.

Spin-2 on MSS:

$$S_2 = \frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\, h^{\perp}_{\nu\mu} \left(\bar{\Box} - \frac{\bar{R}}{6}\right) \left[\mathcal{P}(\bar{\Box})\right] h^{\perp\,\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda \mathcal{F}_W \left(\bar{\Box} + \frac{\bar{R}}{3}\right)\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

The Stelle's case corresponds to $\mathcal{F}_W = 1$ such that

$$\mathcal{P}(\bar{\Box})_{Stelle} = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda \cdot 1 \cdot \left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

This is an obvious second pole which will be the ghost.

Spin-0 on MSS:

$$S_0 = -\frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\ \phi(3\bar{\Box} + \bar{R})\left[\mathcal{S}(\bar{\Box})\right]\phi$$

$$\mathcal{S}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} - \frac{2}{M_P^2}\lambda \mathcal{F}_R(\bar{\Box})(3\bar{\Box} + \bar{R})$$

This *is* the ghost in Einstein-Hilbert case $\mathcal{F}_R = 0$, but it is constrained and is not physical.

Thus, $\mathcal{S}(\bar{\Box})$ *can* have one root as a function of $\bar{\Box}$ and as such generate one more pole in the propagator and it will be not a ghost. That is like, $\mathcal{F}(\bar{\Box}) = \text{const}$

This would be exactly the scalar mode of a local $f(R)$ gravity.

What else can AID quadratic action serve for?

- If we just start with the initially proposed quadratic in curvature action it can accommodate many interesting solutions without requiring any other more general gravity model.
- For example, any conformally flat metric which satisfies $\Box R = r_1 R$ with constant r_1 is a solution.
- In particular, Starobinsky inflation is an exact solution here.
- Solution representing a ghost-free bouncing scenarios also were found.

We put forward the idea that the quadratic in curvatures AID action is enough to attack quantization of gravity!

Physical propagators around FRW:

$$\Phi \mathcal{O}_s \Phi \to \mathcal{O}_s = \frac{(6\lambda\Box\mathcal{F}(\Box) - 1)(2\lambda\Box\mathcal{F}_W(\Box) + 1)}{2\lambda(\mathcal{F}(\Box) + \frac{1}{3}\mathcal{F}_W(\Box))}$$

$$h_{ij}\mathcal{O}_t h^{ij} \to \mathcal{O}_t = \Box(2\lambda\Box\mathcal{F}_W(\Box) + 1)$$

We want no ghosts in the tensor sector which implies there is a canonical graviton only and also no ghosts in the scalar sector which means at most a scalaron.

Physical excitations

Effectively we modify the propagators as follows

$$\Box - m^2 \to \mathcal{G}(\Box)$$

Recall, in $D = 4$ in $(- + ++)$

$$L = \frac{1}{2}\phi(\Box - m^2)\phi \text{ – good field}$$

$-\Box$ gives a ghost, $+m^2$ gives a tachyon (for real m).

Consider

$$L = \frac{1}{2}\phi(\Box - m^2)(\Box - \mu^2)\phi$$

This Lagrangian describes 2 physical excitations and the second one is a ghost. The higher degree polynomial in $\Box$ will just produce more ghosts.

Analytic Infinite Derivative (AID) way around

To preserve the physics we demand

$$\mathcal{G}(\Box) = (\Box - m^2)e^{2\sigma(\Box)}$$

where $\sigma(\Box)$ must be an *entire* function resulting in the fact that the exponent of it has no roots.

Thus

$$L = \frac{1}{2}\phi(\Box - m^2)e^{2\sigma(\Box)}\phi$$

So, yes, we can incorporate infinite number of derivatives by employing properties of entire functions.

FRW continued:

$$\mathcal{O}_s = \frac{(6\lambda\Box\mathcal{F}(\Box) - 1)(2\lambda\Box\mathcal{F}_W(\Box) + 1)}{2\lambda(\mathcal{F}(\Box) + \frac{1}{3}\mathcal{F}_W(\Box))} = (\Box - \mu^2)e^{2\sigma_0(\Box)}$$

$$\mathcal{O}_t = \Box(2\lambda\Box\mathcal{F}_W(\Box) + 1) = \Box e^{2\sigma(\Box)}$$

Then, avoiding all odds:

$$\mathcal{F}_W(\Box) = \frac{e^{2\sigma(\Box)} - 1}{2\lambda\Box}$$

$$\mathcal{F}(\Box) = \frac{1}{6\lambda\mu^2} + \frac{1}{3\mu^2}(\Box - \mu^2)\mathcal{F}_W(\Box)$$

Non-local scalar field [arxiv:2103.01945]

Consider AID scalar field action:

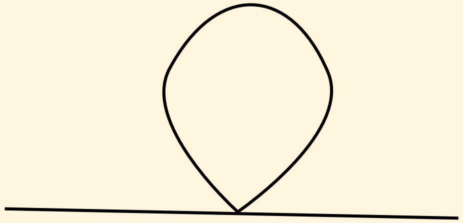
$$L = -\frac{1}{2}\phi(\Box - m^2)f^{-1}(\Box)\phi - \frac{\lambda}{4!}\phi^4$$

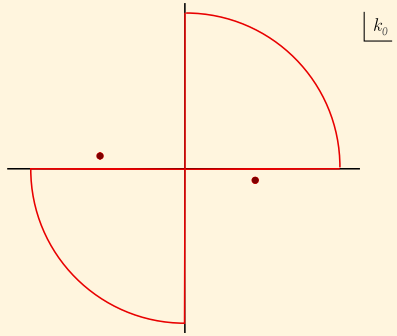
and we use here $(+ - --)$ signature.

Again, we demand the form-factor to be an exponent of an entire function. We also normalize it as $f(0) = f(m^2) = 1$ to preserve the local answers in the IR limit.

We can adjust the fall rate for large momenta by choosing the form-factor. Power-counting convergence requires the fall faster than $\sim 1/p^2$.

Tadpole and fate of the Wick rotation





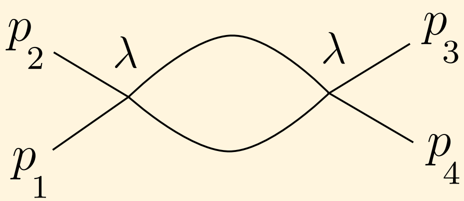
$$\mathcal{A} = \int \frac{d^4k f(k_0^2 - \vec{k}^2)}{k_0^2 - \vec{k}^2 - m^2}$$

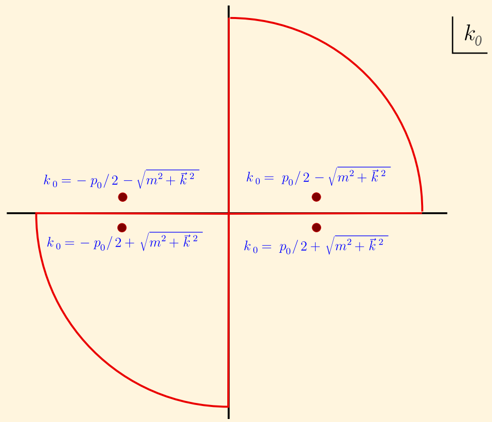
$$\mathcal{A}_E = -i \int \frac{d^4k_E f(-k_{0E}^2 - \vec{k}^2)}{k_{0E}^2 + \vec{k}^2 + m^2}$$

where $k_{0E} = -ik_0$.

$$\mathcal{A}_{\mathcal{C}}^{\infty} = \int_r^R \frac{f(0)(i+1) - f(z^2) - if(-z^2)}{z^2}$$

Fish and one-loop unitarity





As a matter of definition we write amplitudes in Euclidean signature and analytically continue the result to Minkowski values of external momenta. **[Pius,Sen,arXiv:1604.01783]**

$$\mathcal{M} = -i\frac{\lambda^2}{32\pi^4} I(p)$$

We compute the integral with euclidean internal momentum k **and also account for poles shown above.**

Result for the fish graph with $f(\Box) = e^{\alpha\Box}$

$$I(p) = -\pi^3 + \frac{2i\pi^2}{\alpha p^2}\left[e^{\alpha p^2} - \alpha p^2 \mathrm{Ei}(\alpha p^2) - e^{\alpha p^2/2} + \frac{1}{2}\alpha p^2 \mathrm{Ei}(\alpha p^2/2)\right]$$

For $\alpha \to 0$ we restore the logarithmic singularity common in the cut-off regularization using the fact that for small values of the argument

$$\mathrm{Ei}(z) \approx \gamma + \log z + z$$

.

$$\mathcal{M}_{total} = -i\frac{\lambda^2}{32\pi^4}(I(\sqrt{s}) + I(\sqrt{t}) + I(\sqrt{u}))$$

Result for the fish graph with $f(k^2) = f(-k^2)$

$$\mathcal{M}(p) = -\frac{\lambda^2}{64\pi^3 p}\int_0^\infty J_1(px)J_1(kx)J_1(qx)f(k^2)f(q^2)dkdqdx + i\frac{\lambda^2\pi}{32} + \frac{\lambda^2}{32p^2}\int_{-p^2}^{p^2} f(z)dz$$

If $f(z)$ is an integrable function than the last term gives an apparently universal $\sim 1/p^2$ contribution for any even form-factor.

We can show numerically that the model remains weakly coupled in contrast to $f(p^2) = e^{-\alpha p^2}$

Examples used were $f = e^{-p^4}$ and $f = e^{-\Gamma(0,p^4)-\gamma-\log(p^4)}$

Non-local Higgs inflation as a toy model [arxiv:2006.06641]

The bottom-line AID modified action is as follows:

$$L = \frac{1}{2}M_P^2 R_E + \frac{1}{2}\phi\Box e^{2\sigma(\Box)}\phi - V(\phi)$$

$\sigma(\Box)$ is an entire function

and we return here to $(-+++)$ signature.

We can make $\phi = 0, \infty$ to be ghost-free vacua but all the way in between effective new modes appear. Namely, this depends on algebraic roots of an equation

$$\Box\, e^{2\sigma(\Box)} = \frac{\partial^2 V(\phi)}{\partial\phi^2}$$

Choosing the potential we may have several points where its second derivative vanishes. For all other values ϕ we have infinitely many new effective modes.

What are these new modes? – Half of them are ghosts!

- As long as the second derivative of the potential is non-zero there is an infinite number of new modes with complex conjugate masses squared and all are heavy with $|m| > M_P$
- The following condition

$$(\mathrm{Im}(m^2))^2 < 9H^2\mathrm{Re}(m^2)$$

guarantees no classical growing behavior for these new effective modes in an (A)dS space-time characterized by the Hubble rate H.

- It is important to understand that values of m are governed mainly by the shape of the entire function and also by the value of H originating from the potential while the restriction which excludes growing classical behavior does not depend on the entire function.

Conclusions and Outlook

- **A class of analytic infinite derivative (AID) theories has been considered targeting the goal of constructing a UV complete and unitary gravity. These models have clear connection with SFT.**
- **It features many nice properties, like native embedding of the Starobinsky inflation, finite Newtonian potential at the origin, presence of a non-singular bounce, healing of non-renormalizable models including Higgs inflation, etc.**
- **We provide an explicit computation showing that the physical propagator depends on just one entire function despite previous studies where two independent functions were considered.**
- **We describe how unitarity is maintained in AID field theories and perform certain explicit checks including the Optical Theorem verification.**

Thank you for listening!