Form factors in Quantum Gravity

Frank Saueressig

Institute for Mathematics, Astrophysics and Particle Physics
Radboud University Nijmegen

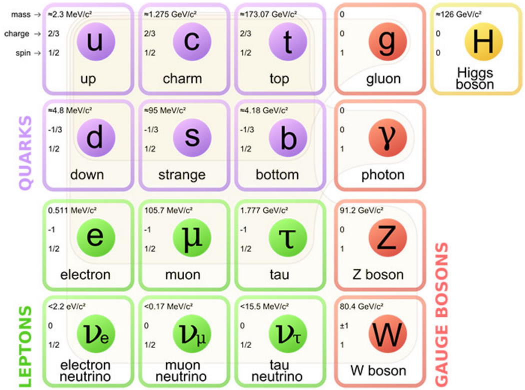
Quarks Online Workshops-2021

Quantum Gravity and Cosmology, July 7^{th}, 2021

Outline

1. motivation
2. gravity as a quantum field theory
3. encoding quantum corrections: the form-factor program
4. form factors for Asymptotic Safety
5. outlook

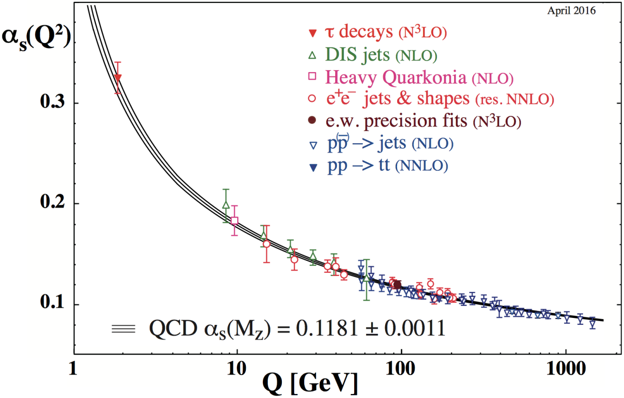
standard model of particle physics



- explains observations very well
- foundation: relativistic quantum field theory

couplings depend on the energy scale

example: the QCD coupling constant is asymptotically free



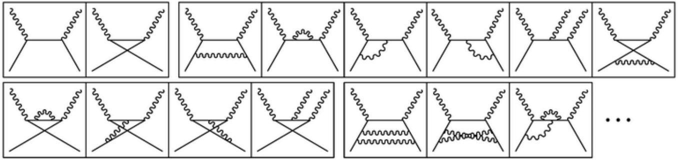
computing observables: Feynman diagrams

start: bare action $S[\text{fields}]$

$\Downarrow$ derive Feynman rules

$\Downarrow$ compute all Feynman diagrams contributing to a process

- expansion in a coupling constant



$\Downarrow$ eliminate infinities

$\Longrightarrow$ obtain cross section

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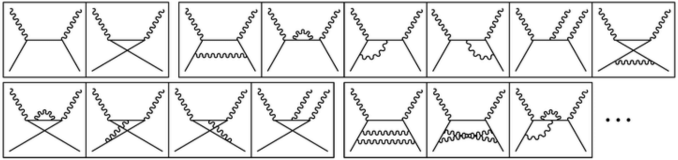

$\Downarrow$ eliminate infinities

$\Longrightarrow$ obtain cross section

drawback: this goes process by process

computing observables: the effective action Γ

start: bare action S[fields]

$\Downarrow$ compute the effective action

$$\Gamma^{1-\text{loop}} = S^{\text{bare}} + \frac{1}{2}\text{Tr}\left[\ln S^{(2)}\right]$$

$\Downarrow$ derive Feynman rules from Γ

- effective propagators/vertices contain quantum corrections

$\Longrightarrow$ obtain cross sections from tree-level diagrams

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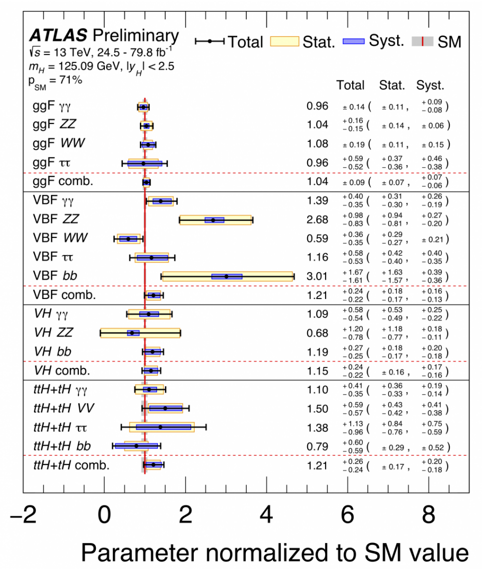
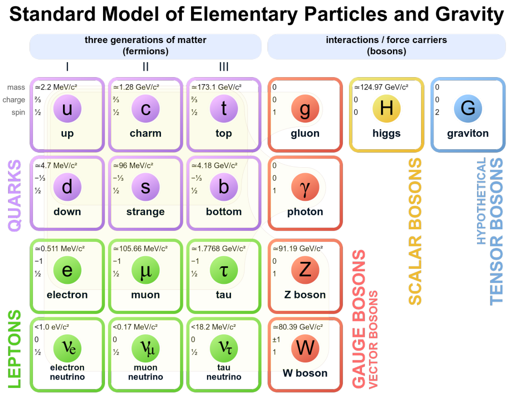
advantage: compute corrections to many processes simultaneously

standard model: experimental confirmation



Towards a new Standard Model



foundation: relativistic quantum field theory

including gravity: general remarks

energy: $\simeq 10^4$ GeV (LHC):

- gravity corrections are weak

  $\Longrightarrow$ scattering in flat background is a good approximation

including gravity: general remarks

energy: $\simeq 10^4$ GeV (LHC):

- gravity corrections are weak

  $\Longrightarrow$ scattering in flat background is a good approximation

energy: $\gtrsim 10^{19}$ GeV (Planck scale):

- Landau pole makes standard model ill-defined
- spacetime fluctuations play an essential role
- UV completion including gravity could add predictive power

particle scattering based on Einstein-Hilbert action

Einstein-Hilbert action with 2 minimally coupled scalar fields:

$$S = -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R + \frac{1}{2} \int d^4x \sqrt{-g} \left[\phi \Delta \phi + \chi \Delta \chi \right]$$

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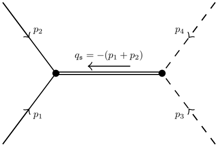
amplitude $\mathcal{A}_s$ for $\phi\phi \to \chi\chi$-scattering in Minkowski space



parameterized by Mandelstam variables:

$$s = (p_1 + p_2)^2 \qquad t = (p_1 + p_3)^2 \qquad u = (p_1 + p_4)^2$$

- s: center-of-mass energy

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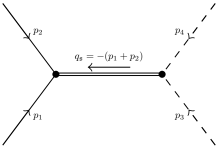
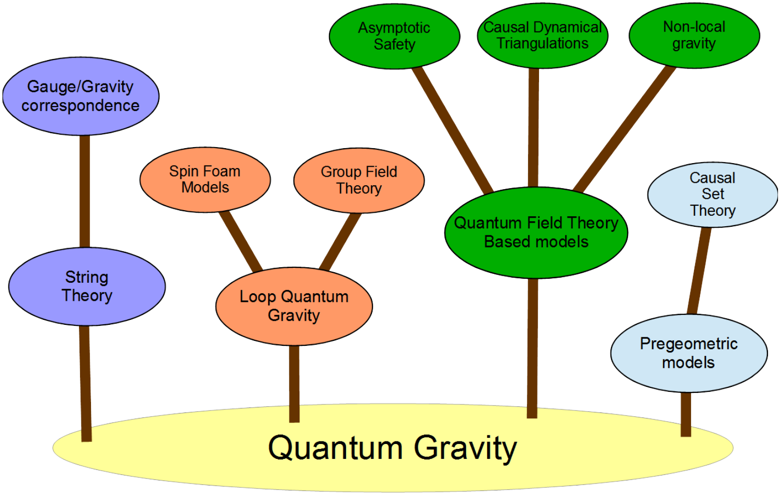


$$\mathcal{A}_s = 8\pi G_N \frac{t\,u}{s} \sim G_N\, s$$

- grows with the center-of-mass energy (unitarity violation)
- perturbative corrections aggravate these divergences

quantum gravity landscape



softening the high-energy scattering

a form-factor program for quantum gravity

form factors for gravity

two form factors at second order in the curvature:

$$\Gamma \simeq \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[-R + \tfrac{1}{2} C_{\mu\nu\rho\sigma} \, W^C(\Delta) \, C^{\mu\nu\rho\sigma} - \tfrac{1}{6} R \, W^R(\Delta) \, R \right]$$

Remarks:

- encode momentum-dependent interactions

  $\Longrightarrow$ generalization of running couplings to curved space

- $W^C(\Delta)$ and $W^R(\Delta)$ fix the graviton propagator

- gravity is a massless field theory

  $\Longrightarrow$ effective action can contain non-local terms!

form factors for gravity: examples

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[K. Stelle, '77]

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effective field theory: $W^C \propto \log(\Delta/\mu^2)$ $W^R \propto \log(\Delta/\mu^2)$

[J.F. Donoghue, B.K. El-Menoufi, '14]

form factors for gravity: examples

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effective field theory: [J.F. Donoghue, B.K. El-Menoufi, '14]	$W^C \propto \log(\Delta/\mu^2)$	$W^R \propto \log(\Delta/\mu^2)$
non-local ghost-free gravity: [T. Biswas, et. al. '11]	$W^C \propto \frac{e^{-\Delta/M^2}-1}{\Delta}$	$W^R \propto \frac{e^{-\Delta/M^2}-1}{\Delta}$

form factors for gravity: examples from phenomenology

two form factors at second order in the curvature:

$$\Gamma \simeq \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[-R + \tfrac{1}{2} C_{\mu\nu\rho\sigma} \, W^C(\Delta) \, C^{\mu\nu\rho\sigma} - \tfrac{1}{6} R \, W^R(\Delta) \, R \right]$$

modifying gravity at cosmic distances:

$$W^C(\Delta) \simeq \frac{m_T^2}{\Delta^2} \, , \qquad W^R(\Delta) \simeq \frac{m_S^2}{\Delta^2}$$

- $W^R(\Delta)$: mimic a cosmological constant (RT-models)

  [E. Belgacem, '20]

- $W^C(\Delta)$: mass terms for gravitational waves

scattering of scalar particles including form factors

consider: scattering of scalars in Minkowski background

building blocks for effective action (includes quantum corrections)

$$\Gamma = \Gamma_{\rm grav} + \Gamma_{\rm gf} + \Gamma_{\rm matter}$$

gravitational part

$$\Gamma_{\rm grav} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[-R + \tfrac{1}{2} C_{\mu\nu\rho\sigma} \, W^C \, C^{\mu\nu\rho\sigma} - \tfrac{1}{6} R \, W^R \, R \right]$$

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supplemented by gauge fixing ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$):

$$\Gamma_{\rm gf} = \frac{1}{32\pi\alpha} \int \sqrt{-\eta} \left(\partial^\mu h_{\mu\nu} - \tfrac{1+\beta}{4} \partial_\nu h \right) \left(\partial_\rho h^{\rho\nu} - \tfrac{1+\beta}{4} \partial^\nu h \right) .$$

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matter sector:

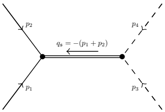
- gravity-matter interactions

$$\Gamma_\phi = \int \sqrt{-g} \Big[\tfrac{1}{2} \phi \, f_{\rm kin}(\Delta) \, \phi + f_{R\phi\phi}(\Delta_1, \Delta_2, \Delta_3) \, R \, \phi \, \phi + f_{Ric\phi\phi}(\Delta_1, \Delta_2, \Delta_3) \, R^{\mu\nu} \, (D_\mu D_\nu \phi) \, \phi \Big]$$

- scalar self-interactions

$$\Gamma_{\phi^2\chi^2} = \tfrac{1}{4} \int \sqrt{-g} \, f_{\phi^2\chi^2}(\Delta_1, \cdots) \, \phi^2 \chi^2$$

scattering of scalar particles including form factors



on-shell amplitude (massless scalars)

$$
\begin{aligned}
\mathcal{A}_s^{\phi\chi} = \tfrac{4\pi G_N}{3} \Big[& - \left(1 + s\, f_{Ric\phi\phi}(s,0,0)\right)\left(1 + s f_{Ric\chi\chi}(s,0,0)\right) \\
& \qquad \times G_2(s) \times \left(t^2 - 4tu + u^2\right) \\
& + \left(s + s^2 f_{Ric\phi\phi}(s,0,0) - 12 s f_{R\phi\phi}(s,0,0)\right) \\
& \times \left(s + s^2 f_{Ric\chi\chi}(s,0,0) - 12 s f_{R\chi\chi}(s,0,0)\right) G_0(s) \Big]
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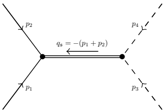
- gauge invariant

scattering of scalar particles including form factors



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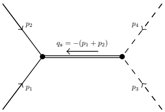
- gauge invariant
- invariant under rescalings of the graviton

scattering of scalar particles including form factors



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\end{aligned}
$$

- gauge invariant
- invariant under rescalings of the graviton
- most general amplitude compatible with quantum field theory

define partial wave amplitudes:

$$a_j^{\phi\chi}(s) \equiv \frac{1}{32\pi} \int_{-1}^{1} \mathrm{d}(\cos\theta)\, P_j(\cos\theta)\, \mathcal{A}_s^{\phi\chi}(s, \cos\theta)\,,$$

- $P_j(x)$: Legendre polynomial of order j

scattering of scalar particles: Stelle gravity

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- $P_j(x)$: Legendre polynomial of order j

example: classical Stelle gravity with minimally coupled scalars

$$\Gamma^{\rm Stelle}_{\rm grav} = \tfrac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[-R + \tfrac{1}{2} C_{\mu\nu\rho\sigma}\, W^C\, C^{\mu\nu\rho\sigma} - \tfrac{1}{6} R\, W^R\, R \right]$$

- non-zero, constant form factors

$$W^R = -c_R G_N\,, \qquad W^C = -c_C\, G_N$$

- partial wave amplitudes

$$a_2^{\text{Stelle}} = -\frac{G_N}{60} s^2 \left[\frac{1}{s} - \frac{1}{s - (c_C\, G_N)^{-1}} \right]$$

scattering of scalar particles: Stelle gravity

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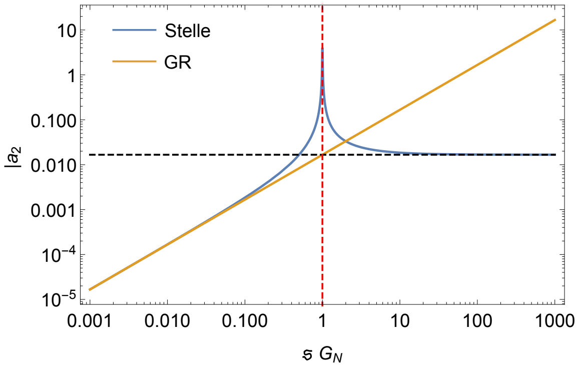


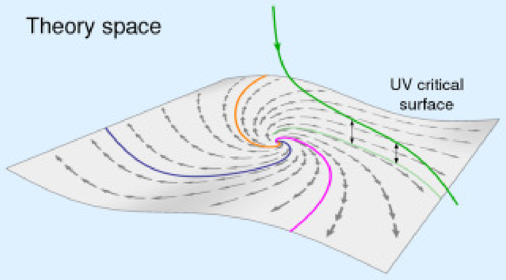
growth of amplitude tamed by massive spin-two poltergeist

Form Factors

implementing Asymptotically Safe Gravity

UV-completion of a quantum field theory

high-energy behavior controlled by a RG fixed point



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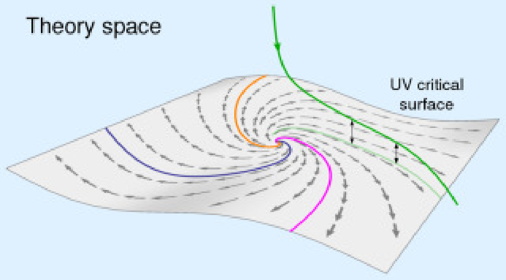


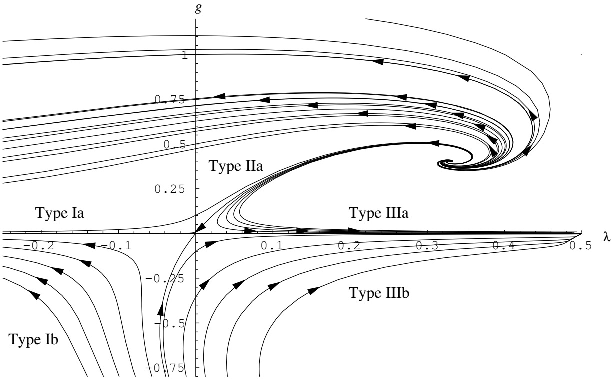
gravitational asymptotic safety program:

gravity in $d = 4$ is controlled by interacting RG fixed point

Weinberg '79
also see: Bonanno, et. al. arXiv:2004.06810

Wilsonian RG flow of the Einstein-Hilbert action

M. Reuter, F.S. '02



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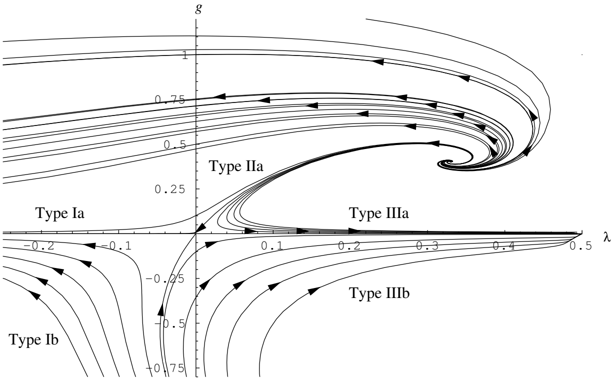

fixed point persists for a wide range of gravity-matter systems

Eichhorn '18

Sakharov's thoughts on Quantum Gravity (1967)

Vacuum quantum fluctuations in curved space and the theory of gravitation

A. D. Sakharov

Dokl. Akad. Nauk SSSR **177**, 70–71 (1967) [Sov. Phys. Dokl. **12**, 1040–1041 (1968). Also S14, pp. 167–169]

Usp. Fiz. Nauk **161**, 64–66 (May 1991)

In Einstein's theory of gravitation one postulates that the action of space-time depends on the curvature (R is the invariant of the Ricci tensor):

$$S(R) = -\frac{1}{16\pi G}\int (dx)\sqrt{-g}R. \qquad (1)$$

The presence of the action (1) leads to a "metrical elasticity" of space, i.e., to generalized forces which oppose the curving of space.

ticles existing in nature, and which he calls "maximons." It is natural to suppose also that the quantity k_0 determines the limit of applicability of present-day notions of space and causality.

Consideration of the density of the vacuum Lagrange function in a simplified "model" of the theory for noninteracting free fields with particles $M \sim k_0$ shows that for fixed ratios of the masses of real particles and "ghost" particles

Asymptotic Safety:

- k_0: scale where spacetime develops self-similar features
- geometry compatible with quantum scale invariance

W. Houthoff, A. Kurov, F.S., JHEP 04 (2020) 099

A. Kurov, F.S., Front. in Phys. 8 (2020) 187

The Asymptotic Safety package

Reuter fixed point (NGFP)

- ensures absence of UV-divergences

NGFP has finite number of relevant parameters (predictivity)

Quantum effective action compatible with observations

- tests of general relativity
  - solar system tests, cosmology, gravitational waves, $\cdots$
- comparible with standard model of particle physics at 1 TeV

structural demands:

- resolution of spacetime singularities?
- unitarity?

blueprint for asymptotically safe amplitudes

requirements

- amplitudes become scale-free at high energy
- no new massive degrees of freedom (ghosts)
- locality at high energy

blueprint for asymptotically safe amplitudes

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form factors for Lorentzian asymptotic safety

- graviton propagators

$$W^R = c_R G_N \tanh(c_R G_N s) \qquad W^C = c_R G_N \tanh(c_C G_N s)$$

- scalar self-interactions $f_{\phi^2\chi^2}$ contributes

$$\mathcal{A}_4^{\phi\chi}(s,t,u) = 4\pi G_N \, G_2(s) \, (t^2 + u^2) f^{\rm int}(s^2 + t^2 + u^2)$$

$f^{\rm int}(x)$ suppresses self-interaction at low energy

blueprint for asymptotically safe amplitudes

remarks and clarifications:

- model has Lorentzian signature
- propagates a massless graviton only
- healthy high-energy behavior

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blueprint for asymptotically safe amplitudes

remarks and clarifications:

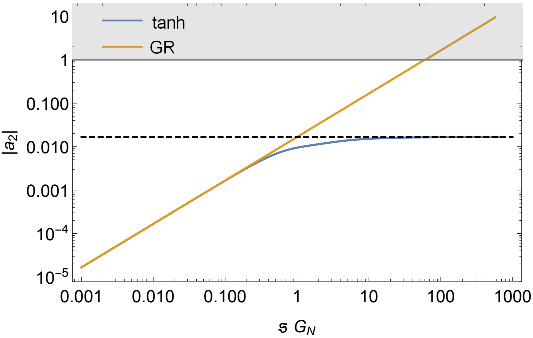
- model has Lorentzian signature
- propagates a massless graviton only
- healthy high-energy behavior

- requires: effective universality

- model is not unique!
- no derivation based on first principles
- analyticity expected from S-matrix theory misses

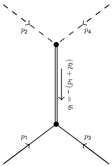
scattering of scalar particles: asymptotic safety

$$a_2 = -\frac{G_N}{60} \frac{s}{1 + c_C\, G_N\, s \tanh(c_C G_N s)}$$

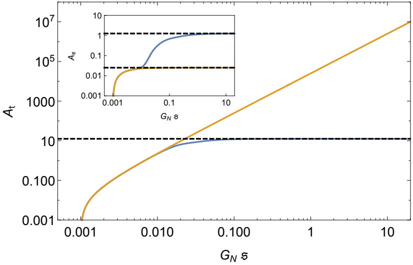


amplitude is scale-free at large center-of-mass energy $G_N s \gg 1$

crossed processes $\phi\phi \to \chi\chi$ (crossing symmetry $s \leftrightarrow t$)



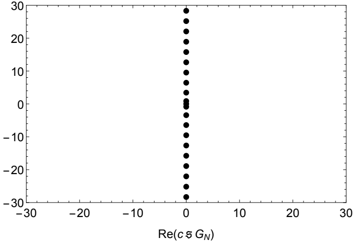
- graviton-contribution to t-channel amplitude $\mathcal{A}_t \sim s^2 G_C(t)$
- divergence canceled by scalar self-interaction



scattering of scalar particles: asymptotic safety

propagators: tower of poles at imaginary squared momentum

$$G_2(s) \propto \frac{1}{s[1 + c_C G_N s \tanh(c_C G_N s)]}$$



poles with vanishing real part:

- avoids causality violation typical for Lee-Wick models summary

form factors for gravity

- inevitably induced by quantum gravity effects
- appear in many approaches to quantum gravity

  [effective field theory, non-local gravity, asymptotic safety, causal dynamical triangulations]
- essential for describing gravity at short distances
- give rise to an interesting phenomenology

  [cosmology, black hole physics, $\cdots$]

summary and perspectives

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  [cosmology, black hole physics, $\cdots$]

gravitational asymptotic safety program

- proof of concept: asymptotically safe amplitudes exist
- gateway for understanding physics of the Reuter fixed point