Non-singular cosmological models with strong gravity in the past.

Valery Rubakov

Inst. Nuclear Research, Russian Acad. Sci.

Dept. of Particle Physics and Cosmology
Physics Faculty, Moscow State University

+ Yu. Ageeva, P. Petrov

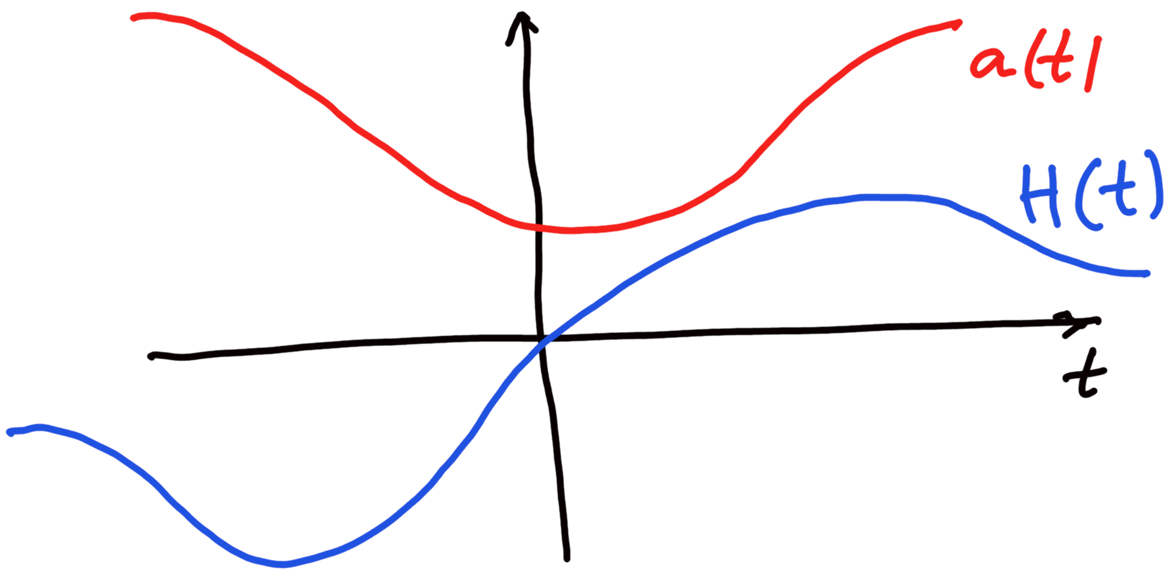
Introduction

Focus of this talk: homogeneous isotropic, spatially flat Universe.

Non-singular models:

Bounce

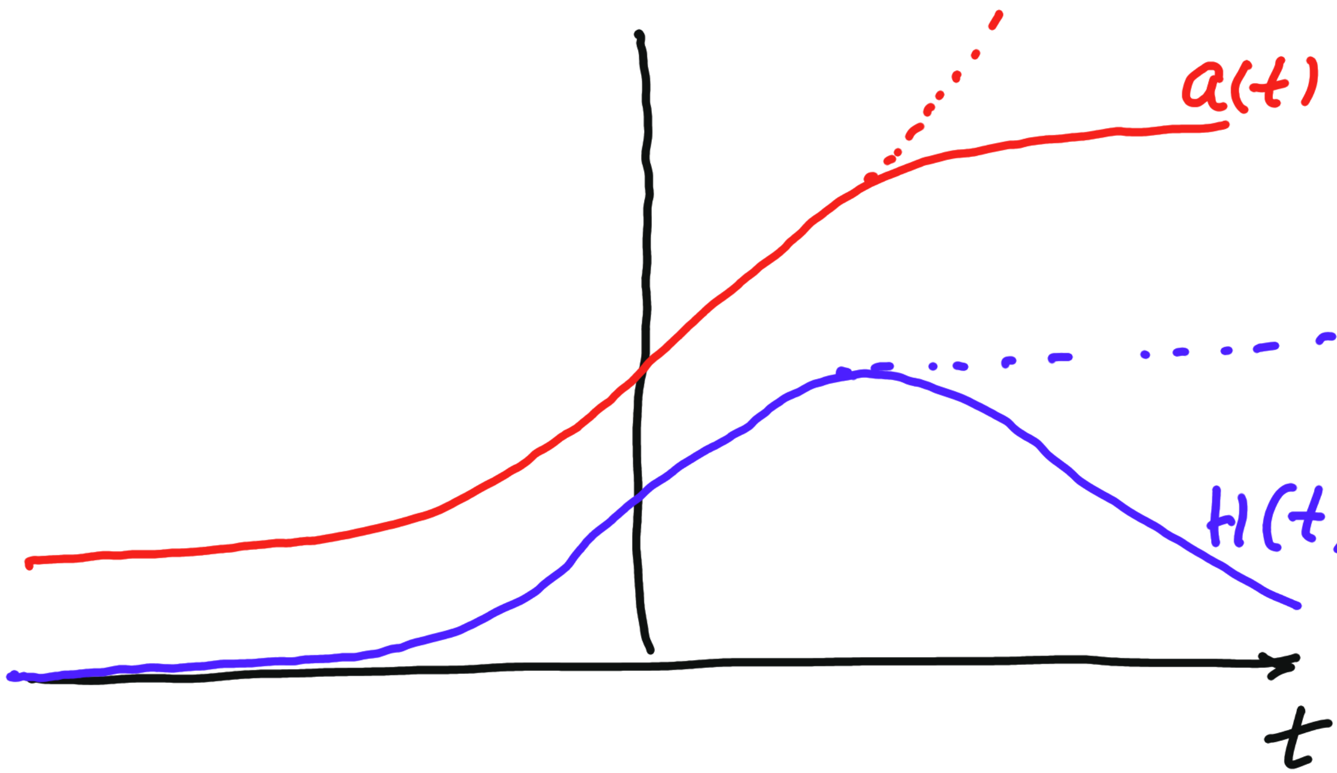
Start from slowly contracting Universe ($H < 0$), then contraction rate increases, energy density builds up, at some moment of time contraction terminates (bounce), Universe starts to expand ($H > 0$). At some point conventional hot epoch (or inflation) begins.



Genesis

Creminelli, Nicolis, Trincherini' 2010

Start from Minkowski, empty space ($H = 0$), then energy density builds up, Universe starts to expand ($H > 0$), expansion accelerates. At some point conventional hot epoch (or inflation) begins.



Motivation

- Curiuosity. Always good to have alternatives even to compelling scenarios like inflation.
- No initial singularity.
- Horizon, curvature problems “solved” by assumption about initial state.
- Very long prehistory without matter energy density $\Longrightarrow$ useful for relaxing the cosmological constant

V.R. ’99;
Mukohyama, Randall ’2003

DRAWBACK

Generation of (nearly) flat power power spectrum of scalar perturbations not so automatic as compared to in inflation

Obstacle in classical GR (if spatial curvature negligible): both bounce and Genesis need exotic matter which violates the Null Energy Condition,
i.e. has $p < -\rho$; where $\rho = T_0^0$, energy density; $p = T_1^1 = T_2^2 = T_s^3$, effective pressure.

- If the NEC holds: a combination of Einstein equations (spatially flat):

$$\frac{dH}{dt} = -4\pi G(\rho + p)$$

Hubble parameter always decreases. No bounce, no Genesis.

Penrose theorem for expanding Universe: there was a singularity in the past, $H = \infty$.

NEC is not violated in conventional field theories

with Lagrangians involving first derivatives only.

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

Buniy, Hsu, Murray' 2006

NEC-violation in theories of this sort:

- Either ghosts: both kinetic an gradient terms have wrong sign. Hyperbolic equation of motion, but negative energies $\Longleftrightarrow$ ghosts: $E = -\sqrt{p^2 + m^2}$ Catastrophic vacuum instability

- Or gradient instabilities: only gradient term has wrong sign. Elliptic equation of motion $\Longrightarrow$ gradient instability

$$E^2 = -(p^2 + m^2) \quad \Longrightarrow \quad \delta\pi \propto \mathrm{e}^{|E|t}$$

Also catastrophic

Horndeski and $p < -\rho$

Twist: scalar-tensor theories with second derivatives in the Lagrangian.

Danger: higher order equations of motion $\Longrightarrow$ extra degrees of freedom = Ostrogradsky ghosts

Not necessarily!

- Emphasis of this talk: Horndeski Horndeski' 1974
  aka Euler hierarchies, aka generalized Galileons, aka KGB, aka generalized Fab Four
  - Second derivatives in Lagrangian, second order field equations
  - Simplest case: Creminelli, Nicolis, Trincherini' 10, Deffayet, Pujolas, Sawicki, Vikman' 10, Kobayashi, Yamaguchi, Yokoyama' 10

$$L = -\frac{1}{16\pi G}R + F(\pi, X) - K(\pi, X)\Box\pi$$

  where again $X = (\partial\pi)^2$.
  - Explicit examples of stable NEC-violation.

No-Go

However, things are not so simple.

"Complete cosmologies": $-\infty < t < +\infty$

Explicit examples of Genesis (or bounce) with Horndeski:
either Big Rip singularity in future, $\pi = \infty$, $H = \infty$ at $t < \infty$

Creminelli, Nicolis, Trincherini '2010

or gradient/ghost instability

Cai, Easson, Brandenberger '2012; Koehn, Lehners, Ovrut '2014;
Pirtskhalava, Santoni, Trincherini, Uttayarat '2014; Qiu, Wang '2015;
Kobayashi, Yamaguchi, Yokoyama '2015; Sosnovikov '2015

Can one avoid instability?

No-go in Horndeski! Libanov, Mironov, V.R.' 16; Kobayashi' 16

General Horndeski theory

Require: both "Einstein" equations and π-field equation second order

Four arbitrary functions of π and X: $F \equiv G_2$; $K \equiv G_3$; G_4; G_5

Horndeski' 1974; Deffayet, Esposito-Farese, Vikman' 09

$$L = F(\pi,X) - K(\pi,X)\Box\pi$$
$$+ G_4(\pi,X)R + G_{4,X}\cdot\left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2\right]$$
$$+ G_5\cdot G^{\mu\nu}\nabla_\mu\nabla_\nu\pi - \frac{1}{6}G_{5,X}\cdot\left[(\Box\pi)^3 - 3\Box\pi\cdot(\nabla_\mu\nabla_\nu\pi)^2 + 2(\nabla_\mu\nabla_\nu\pi)^3\right]$$

- Modified gravity (scalar-tensor).
- NB: always in Jordan frame.

No-go theorem for Genesis in Horndeski: gradient/ghost instability at some stage (which may be quite late)

Libanov, Mironov, V.R.' 16; Kobayashi' 16

Choose unitary gauge $\delta\pi = 0$.

$$ds^2 = N^2 dt^2 - a^2 \mathrm{e}^{2\zeta}(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj})(N^i dt + dx^i)(N^j dt + dx^j)$$

Dynamical variables: transverse traceless h_{ij} and ζ (lapse δN and shift N^i are not dynamical, as usual).

Upon solving for constraints, find quadratic Lagrangians for perturbations

$$L_S = \mathscr{G}_\mathscr{S} \dot{\zeta}^2 - a^{-2}\mathscr{F}_\mathscr{S}(\partial_i \zeta)^2 \,, \quad L_T = \mathscr{G}_\mathscr{T} \dot{h_{ij}}^2 - a^{-2}\mathscr{F}_\mathscr{T}(\partial_k h_{ij})^2$$

NB: $\mathscr{G}_\mathscr{T}$, $\mathscr{F}_\mathscr{T}$ = effective M_{Pl}^2.

Stable background $\Longleftrightarrow \mathscr{G}_\mathscr{T}, \mathscr{F}_\mathscr{T}, \mathscr{G}_\mathscr{S}, \mathscr{F}_\mathscr{S} > 0$.

To simplify formulas (but not outcome): $G_5 = 0$. Tensor sector:

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4G_{4X}X,$$
$$\mathcal{F}_{\mathcal{T}} = 2G_4$$

Scalar sector:

$$\mathcal{G}_{\mathcal{S}} = \frac{\Sigma \mathcal{G}_{\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_{\mathcal{T}},$$
$$\mathcal{F}_{\mathcal{S}} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_{\mathcal{T}},$$
$$\xi = \frac{a\mathcal{G}_{\mathcal{T}}^2}{\Theta}.$$

Where

$$\Theta = -K_X X\dot{\pi} + 2G_4 H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi}$$
$$\Sigma = F_X X + 2F_{XX}X^2 + 12HK_X X\dot{\pi} + 6HK_{XX}X^2\dot{\pi} - K_\pi X - K_{\pi X}X^2$$
$$-6H^2G_4 + 42H^2G_{4X}X + 96H^2G_{4XX}X^2 + 24H^2G_{4XXX}X^3 - 6HG_{4\pi}\dot{\pi}$$
$$-30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi}$$

Key relation (by explicit calculation in general Horndeski)

$$\frac{d\xi}{dt} = a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

$$\xi = -\frac{a(t)\mathscr{G}_{\mathscr{T}}^{2}(t)}{\Theta(t)}$$

where $\Theta(t) = -2HG_4 + \dot{\pi}XK_X + \dots$, a complicated expression.

Main property: ξ never crosses zero ($\Theta = \infty$ is a singularity).

$$\xi(t_f) - \xi(t_i) = \int_{t_i}^{t_f} dt \; a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

Impossible for $\mathscr{F}_{\mathscr{S}} > 0$, $\mathscr{F}_{\mathscr{T}} > 0$, and

$$\int_{-\infty}^{t_f} dt \; a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) = \infty \, , \quad \int_{t_i}^{+\infty} dt \; a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) = \infty$$

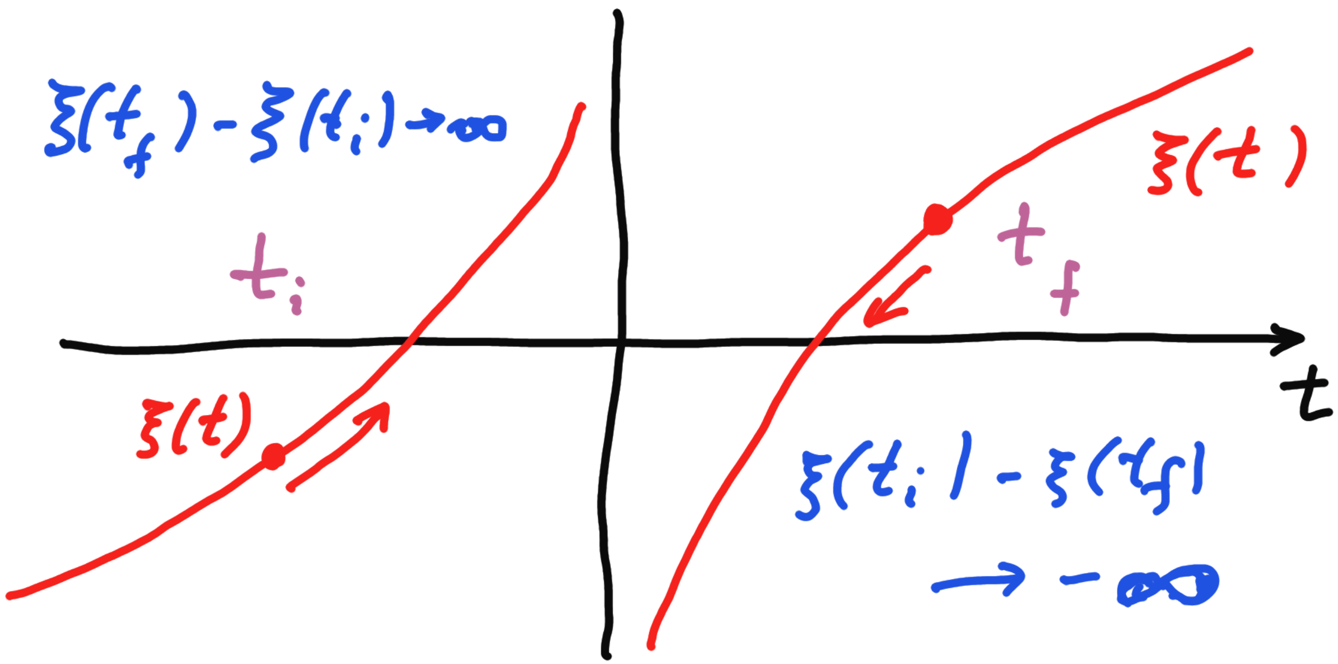
Recall that $a(t) \to \infty / \text{const}$ as $t \to -\infty$ and $a(t) \to \infty$ as $t \to +\infty$ for bounce/Genesis No-go

$$\xi(t)-\xi(0)=\int_0^t dt\ a(t)(\mathscr{F}_{\mathscr{S}}+\mathscr{F}_{\mathscr{T}}) \implies \xi(t)\to+\infty \text{ as } t\to+\infty$$

$$\xi(0)-\xi(t)=\int_t^0 dt\ a(t)(\mathscr{F}_{\mathscr{S}}+\mathscr{F}_{\mathscr{T}}) \implies \xi(t)\to-\infty \text{ as } t\to-\infty$$

Thus, $\xi(t)$ crosses zero, QED.

Even if $\Theta = 0$ at some time $\Longleftrightarrow$ $\xi = \infty$, there is necessarily ξ-crossing:



Side remark: Θ-crossing $\Theta = 0$ at some t is not a problem by itself. $\mathscr{F}_{\mathscr{S}}, \mathscr{G}_{\mathscr{S}} = \infty$, but solutions for ζ remain finite. Also: no singularity in equations in Newtonian gauge

Ijjas' 17; Mironov, V.R., Volkova' 18

- Argument intact in presence of extra matter (obeying NEC) which interacts with Horndeski sector only gravitationally
- Extends to Horndeski theory with multiple (Horndeski or conventional) scalars

Kolevatov, Mironov '2016

Akama, Kobayashi '2017

Ways out

Go beyond Horndeski theory (not this talk)

Effective field theory: Cai et.al.' 2016, Creminelli et.al.'2016
Covariant formalism: Kolevatov et.al.' 2017, Cai, Piao' 2017;
Complete cosmologies: Mironov, V.R. Volkova' 2018, 2019

Has its own problems.

Within Horndeski theory, classical stability (absence of gradient instabilities and ghosts) requires

$$\int_{-\infty}^{t} dt \; a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) < \infty \, .$$

OPTIONS:

- Option 1. Start from de Sitter rather than Minkowski, $a(t) = \mathrm{e}^{H_i t}$, $H_i > 0$.

  Pirtskhalava, Santoni, Trincherini, Uttayarat '2014

  Claim: even small H_i does the job at expence of fine-tuning (?).
  Interesting, albeit not quite Genesis.

- Option 2: modified Genesis. Power law behavior with $a(t) \to 0$ as $t \to -\infty$, so that

$$\int_{-\infty}^{t} a(t)\, dt < \infty$$

Say

$$a = \frac{1}{|t|^{\alpha}}\,, \quad \alpha > 1$$

Hubble parameter and its derivatives vanish as $t \to -\infty$.

Libanov, Mironov, V.R.' 16

Naively: space in nearly Minkowskian as $t \to -\infty$.

Does not work: past geodesic incompleteness.

[Recall: we are in Jordan frame.]

Strong gravity in the past

Yet another way out, still in Horndeski.
Example: bounce, $a(t) \to \infty$ as $t \to -\infty$>

$\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} \to 0$ as $t \to -\infty$, so that

Kobayashi '2016; Ijjas, Steinhardt '2016

$$\int_{-\infty}^{t_f} dt \; a(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}}) < \infty$$

No-go theorem does not work.

But gravity tricky as $t \to -\infty$: effective Planck mass vanishes.

Strong coupling?

Examples:

$$\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} = \frac{1}{(-t)^{2\mu}} \quad \text{as} \quad t \to -\infty.$$

Can one trust classical field theory treatment of cosmological evolution?

Energy scale of classical evolution $E_{class} = H$, $\dot{H}/H = (-t)^{-1} \to 0$

How does it compare with strong coupling scales E_{strong} inferred from interactions of ζ and h_{ij}?

Classical treatment of evolution legitimate
for $E_{strong} >> E_{class}$ as $t \to -\infty$.

Example (part of the story): tensor sector up to cubic terms.
At given moment of time rescale spatial cordinates to set $a = 1$
(equivalently, work in terms of physical spatial momenta $\vec{p} = \vec{k}/a$).
Then (note that $\mathscr{G}_{\mathscr{T}} = \mathscr{F}_{\mathscr{T}}$)

$$S_{hh}^{(2,3)} = \int d^4x \left(\mathscr{F}_{\mathscr{T}} \dot{h}_{ij}^{\,2} - \mathscr{F}_{\mathscr{T}} (\partial_k h_{ij})^2 + \frac{\mathscr{F}_{\mathscr{T}}}{4} \left(h_{ik} h_{jl} - \frac{1}{2} h_{ij} h_{kl} \right) \partial_k \partial_l h_{ij} \right)$$

To figure out strong coupling energy scale, canonically normalize

$$h_{ij} = h_{ij}^{(c)} / \sqrt{\mathscr{F}_{\mathscr{T}}}$$

$$S^{(2,3)}_{hh} = \int d^4x \left(\dot{h}^{(c)\,2}_{ij} - (\partial_k h^{(c)}_{ij})^2 + \frac{1}{4\sqrt{\mathscr{F}_{\mathscr{T}}}} (h^{(c)}_{ik} h^{(c)}_{jl} - \frac{1}{2} h^{(c)}_{ij} h^{(c)}_{kl}) \partial_k \partial_l h^{(c)}_{ij} \right)$$

Dimension-5 operator "suppressed" by $1/\sqrt{\mathscr{F}_{\mathscr{T}}}$ $\Longleftrightarrow$
quantum strong coupling energy scale $E_{strong} = \sqrt{\mathscr{F}_{\mathscr{T}}} \propto (-t)^{-\mu}$

$E_{strong} \to 0$ as $t \to -\infty$, but $E_{strong} \gg E_{class} = (-t)^{-1}$ for $\mu < 1$.

Healthy early bounce stage within classcal field theory at weak coupling.

- This extends to scalar plus tensor sectors and all orders in perturbation theory.

Ageeva, Evseev, Melichev, V.R.' 18, 20;
Ageeva, V.R., Petrov' 20, 21

- "Calculate" action for δN, N^i, h_{ij}, ζ order by order in perturbation theory
- Solve constraint equations for δN, N^i, plug back $\Longrightarrow$ obtain unconstrained action for h_{ij}, ζ
- Canonically normalize $h^{(c)}_{ij} = t^{\mu} h_{ij}$, $\zeta^{(c)} = t^{\mu} \zeta$

Structure of interaction term in Lagrangian for perturbations

$$(\sqrt{-g}\mathscr{L})_{(pq)} = \sum_l \Lambda_l(t) \cdot (\partial)^{c_l} \cdot [\zeta^{(c)}]^p \cdot [h_{ij}^{(c)}]^q$$

Strong coupling scale “on dimensional grounds”

$$E_l(t) = [\Lambda_l(t)]^{-\frac{1}{c_l+p+q-4}}$$

Outcome:

Lowest strong coupling energy scale comes from above dimension-5 operator in tensor sector and dimension-6 operators in scalar sector, e.g.

$$(-t)^{1-2\mu} \cdot \dot{\zeta}(\partial_i \zeta)^2 = (-t)^{1+\mu} \cdot \dot{\zeta}^{(c)}(\partial_i \zeta^{(c)})^2$$

This gives

$$E_{strong} = (-t)^{-\frac{1+\mu}{2}}$$

which is higher than $E_{class} = t^{-1}$ again for $\mu < 1$.

In a large region of parameter space, classical field theory treatment of cosmological evolution is legitimate, even though "effective Planck masses squared" $\mathscr{G}_{\mathscr{T}}, \mathscr{F}_{\mathscr{T}}, \mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} \to 0$ as $t \to -\infty$. Viable scenario.

Overall picture: Universe starts at very low quantum gravity scale $E_{strong} \propto |t|^{-\alpha}$ but expands so slowly that $E_{class} \ll E_{strong}$. Standard Model scales are above E_{strong}. Gravity is the strongest force.

Similar construction works for Genesis

NB. Often said: geodesic incompleteness of tensor/scalar modes for

$$\int_{-\infty}^{t} dt \; a(t) \; \mathscr{G}_{\mathscr{T},\mathscr{S}} < \infty$$

But: geodesic incompleteness is not well defined notion for massless excitations. Upon field redefinition to canonically normalized field

$$L_T = \mathscr{G}_{\mathscr{T}} \dot{h}^2 - a^{-2} \mathscr{F}_{\mathscr{T}} (\partial_k h)^2 \Longrightarrow \dot{h}_{(c)}^2 - a^{-2} (\partial_k h_{(c)})^2 + \text{non-derivative terms}$$

i.e. $\mathscr{G}_{\mathscr{T}} \Longrightarrow 1$.

This observation does not apply to masive particles: proper time is measured in units of m^{-1}, where m becomes time-dependent after field redefinition.

Complete cosmologies

Intelligent design: proof by example

Dubbed "Inverse method" by Ijjas, Steinhardt' 2016

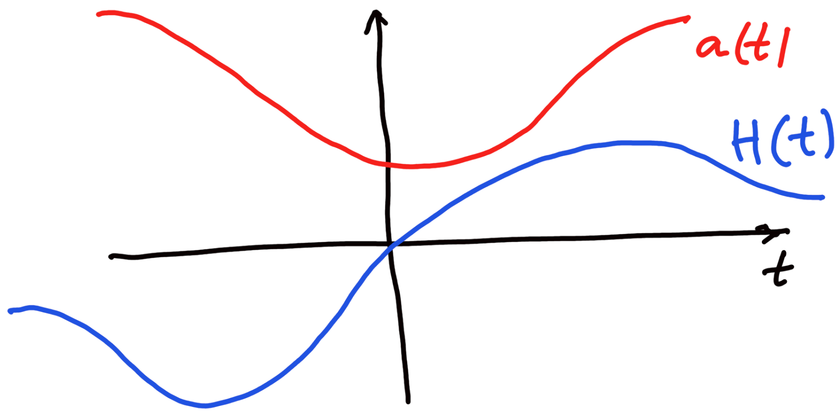
- Choose background $\pi(t) = t$, no loss of generality (field redefinition).

  Then $X = (\partial\pi)^2 = 1$.

  Field equations and stability conditions involve Lagrangian functions F, K, G_4 and their X-derivatives F_X, F_{XX}, etc, all at $\pi(t) = t$, $X = 1$.

  These are yet undetermined independent functions of time $f_0(t) = F(\pi(t), X = 1)$, $f_1(t) = F_X(\pi(t), X = 1)$, etc..

- Choose your favorite $H(t)$.



In particular, theory at late times becomes GR + conventional massless scalar field $\phi = (2/3)^{1/2} \log \pi$ ("kination"), i.e., at late times $\phi = \sqrt{\frac{2}{3}} \log\, t$, $H = \frac{1}{3t}$ and

$$L = -\frac{1}{2}R + \frac{1}{3}\left(\frac{\partial\pi}{\pi}\right)^2 \Longleftrightarrow G_4 = -\frac{1}{2}\,, \quad F(\pi, X) = \frac{1}{3}\frac{X}{\pi^2}\,, \quad K = F_4 = 0\,.$$

- Cook up Lagrangian functions in such a way that
  - Field equations are satisfied
  - Stability conditions are satisfied at all times
  - Classical field theory description of background is reliable at all times, including $t \to -\infty$

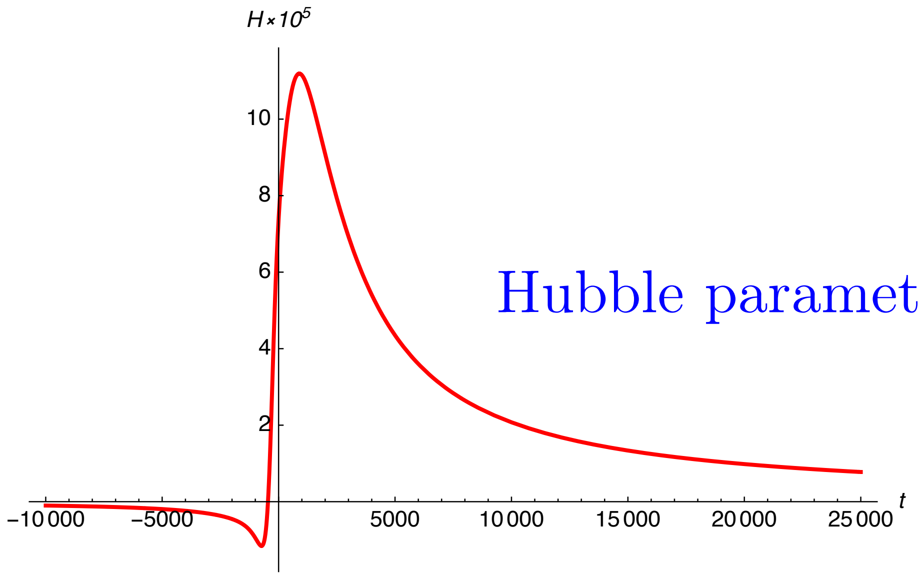
All this can be done for bounce (and also Genesis)

Ageeva, Petrov, V.R.' 2021

Moreover, one can design a model in such a way that

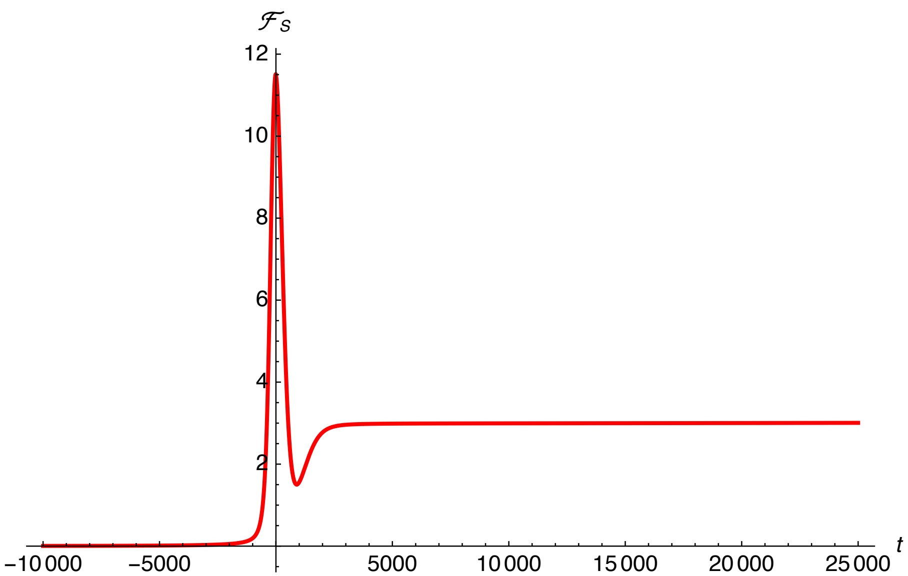
- Tensor and scalar perturbations are subluminal at all times (or luminal, if one wishes so)

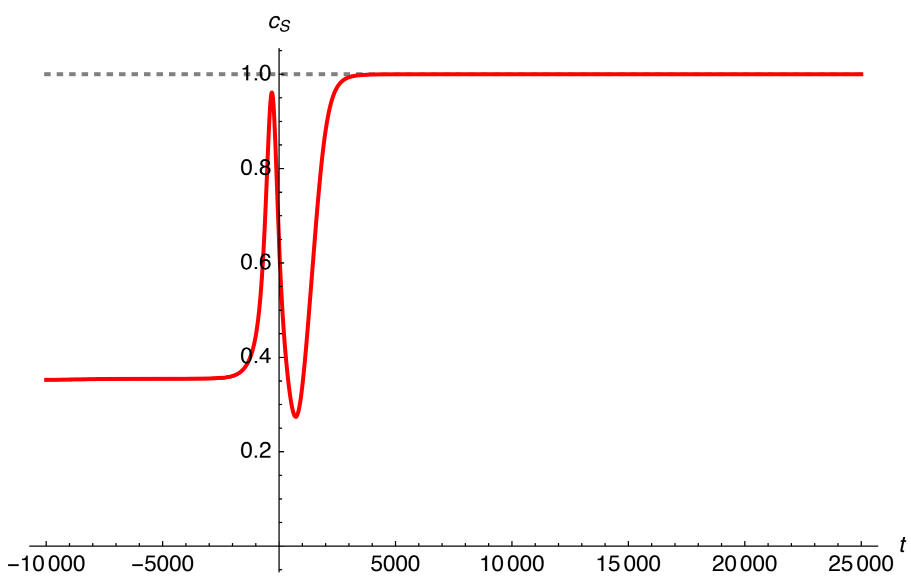
Bounce to kination



Hubble parameter $\boldsymbol{H}$

t

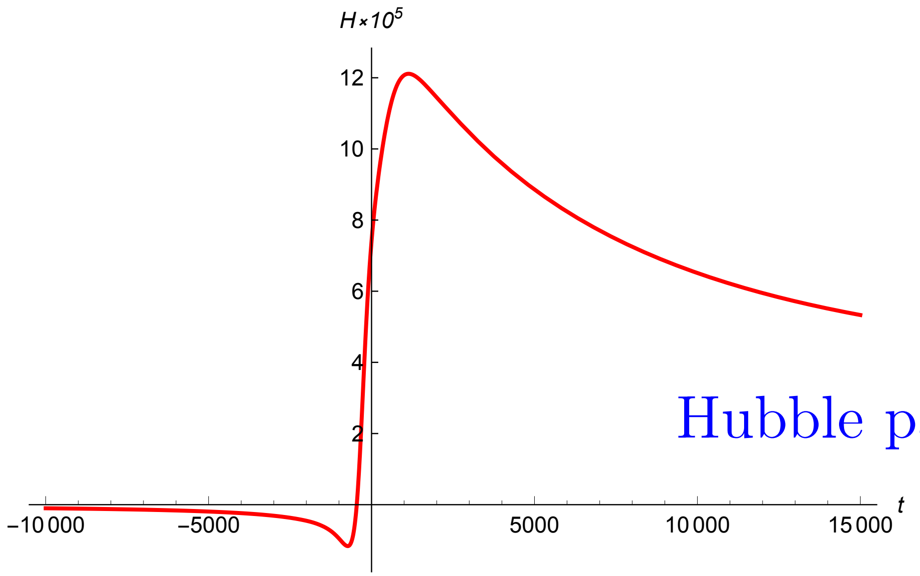




$\mathscr{F}_{\mathscr{S}}$

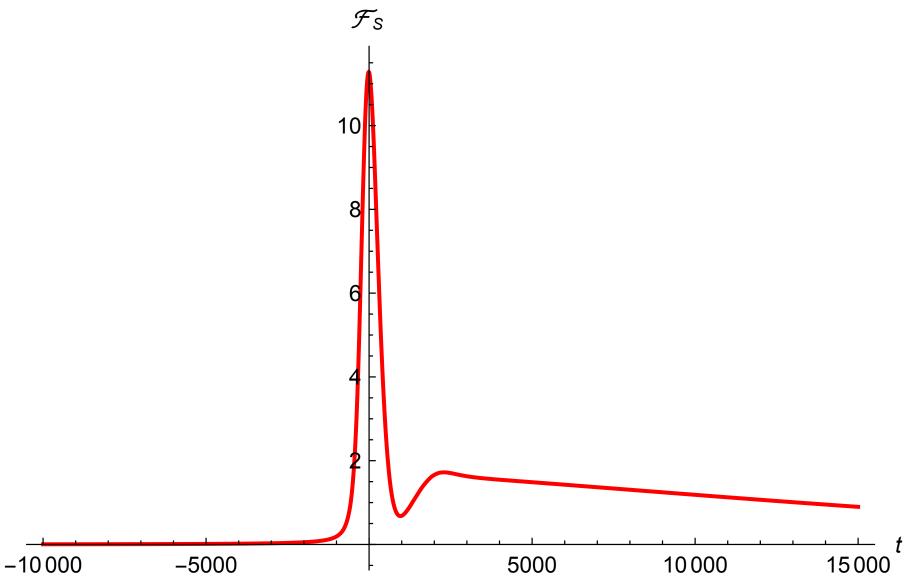
speed of scalar prturbations

Bounce to inflation

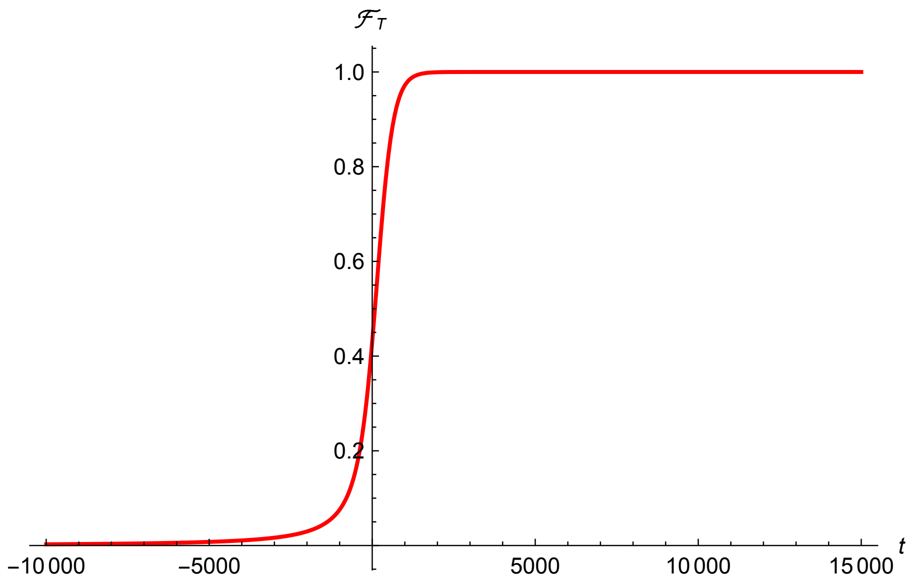


Hubble parameter $\boldsymbol{H}$

t



$\mathscr{F}_{\mathscr{S}}$



$\mathscr{F}_{\mathscr{T}}$

To conclude

- Constructing complete ($-\infty < t < +\infty$) non-singular cosmology (bounce, Genesis) is difficult.
  - Within scalar-tensor gravity: non-trivial kinetic/gradient terms
  - bounce epoch, early Genesis per se not so prolematic
  - however, almost all complete cosmologies plagued with instability ("No-go")
- Possible way out (not the only one): strong gravity in the past; effective Planck mass tends to 0 as $t \to -\infty$. "Gravity as the weakest force".
  - Classical field theory treatment of background evolution can be rendered legitimate, nevertheless.
- Healthy bounce and Genesis cosmologies have been constructed in this framework
- Whether realistic scalar (and tensor) perturbations may be generated without inflation, remains to be seen.

Backup

Equivalent ADM formulation of Horndeski theory

Gleyzes, Langlois, Piazza, Vernizzi '2014

- Make use of ADM form of metric (perturbations included to all orders)

$$ds^2 = N^2 dt^2 - \gamma_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

- Choose unitary gauge with $\pi = t$

- Field variables are all in metric
- Horndeski action reads ($G_5 = 0$ for brevity)

$$S = \int \sqrt{-g} d^4x \, [\mathscr{A}_2(t,N) + \mathscr{A}_3(t,N)\mathscr{K} + \mathscr{A}_4(t,N)(\mathscr{K}^2 - \mathscr{K}^i_j \mathscr{K}^j_i - {}^{(3)}R)]$$

where $\mathscr{K} = \gamma^{ij}\mathscr{K}_{ij}$,

$$\mathscr{K}_{ij} \equiv \frac{1}{2N}\left(\dot{\gamma}_{ij} - {}^{(3)}\nabla_i N_j - {}^{(3)}\nabla_j N_i\right)$$

is extrinsic curvature to hypersurfaces $t = const.$

$F(\pi, X)$, $K(\pi, X)$, $G_4(\pi, X)$ $\Longleftrightarrow$ $\mathscr{A}_2(t,N)$, $\mathscr{A}_3(t,N)$, $\mathscr{A}_4(t,N)$

- Homogeneous field eqs. are simple

$$(N\mathscr{A}_2)_N + 3N\mathscr{A}_{3N}H + 6N^2(N^{-1}\mathscr{A}_4)_N H^2 = 0 \, ,$$

$$\mathscr{A}_2 - 6\mathscr{A}_4 H^2 - \frac{1}{N}\frac{d}{dt}(\mathscr{A}_3 + 4\mathscr{A}_4 H) = 0$$

Concrete example of bounce ($1 > \mu > 1/2$): as $t \to -\infty$

$$\mathscr{A}_2 = (-t)^{-2\mu-2} \cdot \left(-\frac{a_1}{N^2} + \frac{a_2}{N^4}\right)$$

$$\mathscr{A}_3 = (-t)^{-2\mu-1} \cdot \frac{a_3}{N^3}$$

$$\mathscr{A}_4 = -\frac{1}{2}(-t)^{-2\mu}$$

When converted to covariant Lagrangian formalism it becomes the early bounce model with

$$F = c_1 X \cdot \mathrm{e}^{2\mu\pi} + c_2 X^2 \cdot \mathrm{e}^{(2\mu-2)\pi} + 4\mu^2 \cdot X \cdot \ln X \cdot \mathrm{e}^{2\mu\pi},$$

$$K = c_3 X \cdot \mathrm{e}^{(2\mu-2)\pi} + 2\mu \mathrm{e}^{2\mu\pi} + \mu \cdot \ln X \cdot \mathrm{e}^{2\mu\pi},$$

$$G_4 = \frac{1}{2}\mathrm{e}^{2\mu\pi}.$$

CHANGING GEERS

Horndeski theory: exotic version of Genesis only.

Effective “Planck masses” tend to zero as $t \to -\infty$,

gravity is the strongest force at early Genesis stage

Can Genesis be less exotic?

Horndeski is not the most general scalar-tensor theory with tensor + one scalar modes $\Longrightarrow$ No Ostrogradsky ghost

- Variation of action may give higher order field equations, but they may combine in such a way that the resulting equations are second order.

  Degenerate Higher-Order Scalar Tensor theories, DHOST

  Langlois, Noui' 16; Crisostomi, Koyama, Tasinato' 16

  Quadratic in second derivatives action

$$S = \int d^4x \sqrt{-g} \left(F(\pi, X) - K(\pi, X)\Box\pi + G_4(\pi, X)R + \sum_{i=1}^{5} A_i(\pi, X) L_i \right),$$

  with

$$L_1 = \pi_{;\mu\nu}\pi^{;\mu\nu}, \quad L_2 = (\Box\pi)^2,$$

$$L_3 = \pi^{,\mu}\pi_{;\mu\nu}\pi^{,\nu}\Box\pi, \quad L_4 = \pi^{,\mu}\pi_{;\mu\nu}\pi^{;\nu\rho}\pi_{,\rho}, \quad L_5 = \left(\pi^{,\mu}\pi_{;\mu\nu}\pi^{,\nu}\right)^2,$$

  DHOST: in general, non-linear relations between $G_4, A_1, \ldots, A_5$.

- Relatively simple subclass: "beyond Horndeski" theories

  Zumalacárregui, Gacia-Bellido' 2014; Gleyzes, Langlois, Piazza, Vernizzi' 2014

  Linear relations:
  $A_3 = -A_4 \equiv 2F_4,\ A_1 = -A_2 = -G_{4X} + XF_4;\ A_5 = 0 \Longrightarrow$

$$
\begin{aligned}
L = & F(\pi, X) - K(\pi, X)\Box\pi \\
& + G_4(\pi, X)R + G_{4,X} \cdot \left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2\right] \\
& + F_4(\pi, X)\epsilon^{\mu\nu\lambda\rho}\epsilon^{\mu'\nu'\lambda'}{}_{\rho}\ \partial_\mu\pi \cdot \partial_{\mu'}\pi \cdot \nabla_\nu\nabla_{\nu'}\pi \cdot \nabla_\lambda\nabla_{\lambda'}\pi
\end{aligned}
$$

- Way to understand (sometimes): disformal transformation

$$g_{\mu\nu} \to \Omega(\pi, X)g_{\mu\nu} + \Lambda(\pi, X)\partial_\mu\pi\partial_\nu\pi$$

  Horndeski $\to$ beyond Horndeski

  NB: This is formal trick: Ω, Λ may be singular.
  And in Genesis case (also bounce) they are!

No-go theorem for Genesis no longer holds

Effective field theory: Cai et.al.' 2016, Creminelli et.al.'2016

Covariant formalism: Kolevatov et.al.' 2017, Cai, Piao' 2017

One again has

$$\frac{d\xi}{dt} = a(t)(\mathscr{F}_{\mathscr{S}} + \mathscr{F}_{\mathscr{T}})$$

but now

$$\xi = -\frac{a(t)\mathscr{G}_{\mathscr{T}}(\mathscr{G}_{\mathscr{T}} + 2F_4X^2)}{\Theta(t)}$$

can cross zero.

Recall: $\Theta(t) = 0$ at some t not a problem

Ijjas' 17;

Mironov, V.R., Volkova' 18

In fact, Θ-crossing does occur in known examples.

Θ-crossing: why is it harmless?

$$L_S = \mathscr{G}_{\mathscr{S}}\dot{\zeta}^2 - a^{-2}\mathscr{F}_{\mathscr{S}}(\partial_i\zeta)^2$$

with

$$\mathscr{G}_{\mathscr{S}} = \frac{\Sigma \mathscr{G}_{\mathscr{T}}{}^2}{\Theta^2} + 3\mathscr{G}_{\mathscr{T}},$$

$$\mathscr{F}_{\mathscr{S}} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathscr{F}_{\mathscr{T}},$$

$$\xi = \frac{a\left(\mathscr{G}_{\mathscr{T}} + 2F_4X^2\right)\mathscr{G}_{\mathscr{T}}}{\Theta}.$$

Thus, at crossing, $\Theta \propto (t - t_\times)$ we have $\mathscr{G}_{\mathscr{S}}, \mathscr{F}_{\mathscr{S}} \propto (t - t_\times)^{-2}$.
Equation of motion (spatial Fourier)

$$\frac{\partial}{\partial t}(\mathscr{G}_{\mathscr{S}}\dot{\zeta}) + k^2\mathscr{F}_{\mathscr{S}}\zeta = 0 \implies \ddot{\zeta} - \frac{2}{(t-t_\times)^2}\dot{\zeta} + k^2\zeta = 0$$

Solution $\zeta = c_1(1 + O[(t-t_\times)^2]) + c_2(t-t_\times)^3$, smooth.
$\delta N \propto \dot{\zeta}/(t - t_\times)$ and N^i also smooth. Q.E.D.

Intelligent design: proof by example

Dubbed "Inverse method" by Ijjas, Steinhardt' 2016

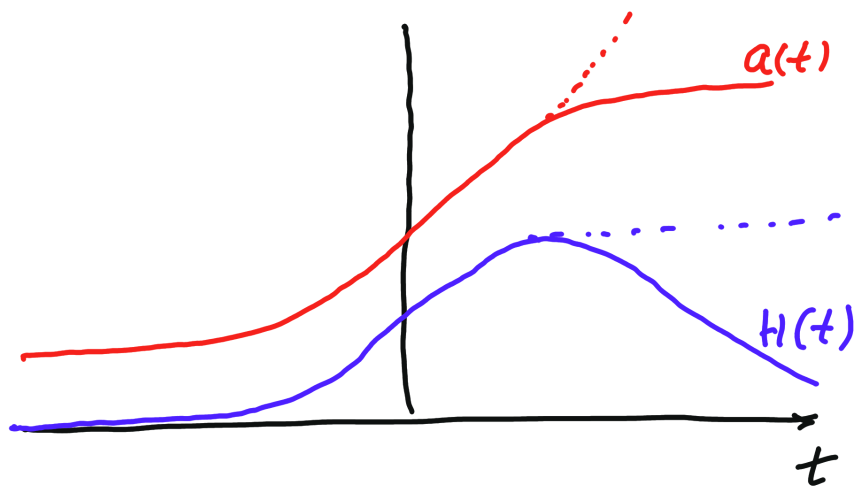
- Choose background $\pi(t) = t$, no loss of generality (field redefinition).

  Then $X = (\partial\pi)^2 = 1$.

  Field equations and stability conditions involve Lagrangian functions F, K, G_4 and F_4 and their X-derivatives F_X, F_{XX}, etc, all at $\pi(t) = t$, $X = 1$.

  These are yet undetermined independent functions of time $f_0(t) = F(\pi(t), X = 1)$, $f_1(t) = F_X(\pi(t), X = 1)$, etc..

- Choose your favorite $H(t)$.



In particular, theory at late times may become GR + conventional massless scalar field $\phi = (2/3)^{1/2} \log \pi$, i.e., at late times $\phi = \sqrt{\frac{2}{3}} \log\ t$, $H = \frac{1}{3t}$ and

$$L = -\frac{1}{2}R + \frac{1}{3}\left(\frac{\partial\pi}{\pi}\right)^2 \Longleftrightarrow G_4 = -\frac{1}{2}\,, \quad F(\pi, X) = \frac{1}{3}\frac{X}{\pi^2}\,, \quad K = F_4 = 0\,.$$

- Cook up Lagrangian functions in such a way that
  - Field equations are satisfied
  - Stability conditions are satisfied at all times

All this can be done for Genesis (and also bounce)

Mironov, V.R., Volkova' 2018, 2019

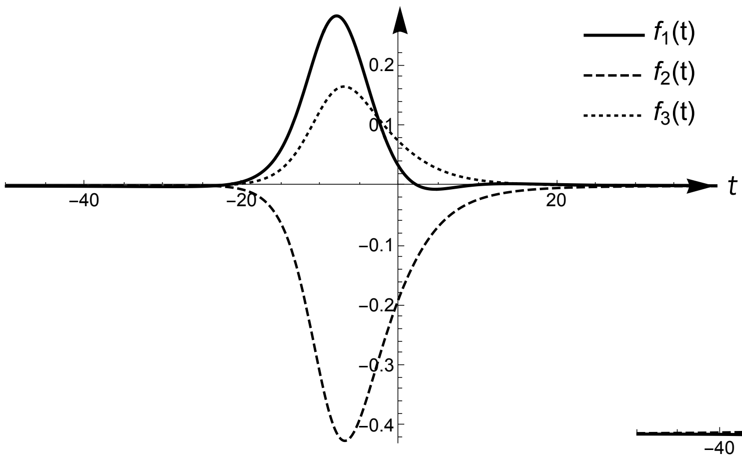
Moreover, one can design a model in such a way that

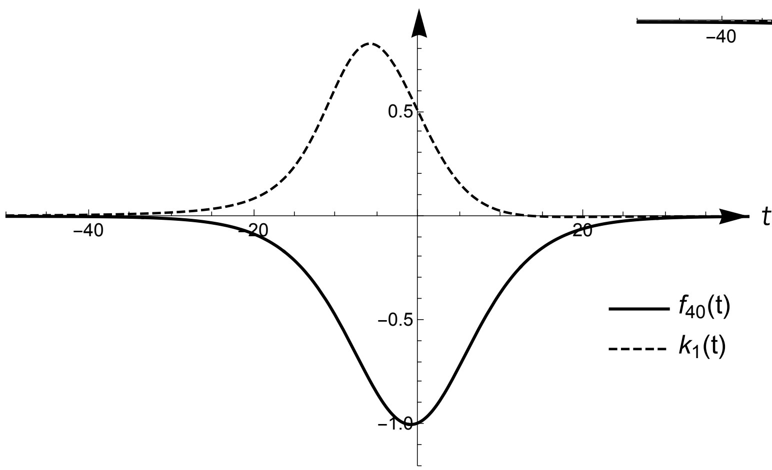
- Tensor perturbations are subluminal at all times (or luminal, if one wishes so)
- Scalar perturbations are subluminal at all times

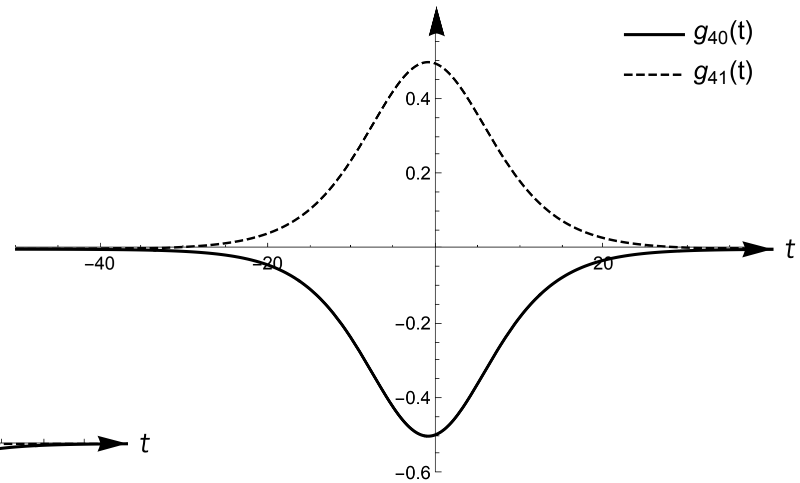
Nothing sophisticated:

$$F(\pi,X) = f_1(\pi)\cdot X + f_2(\pi)\cdot X^2 + f_3(\pi)\cdot X^3\ ,\quad K(\pi,X) = k_1(\pi)\cdot X,$$

$$G_4(\pi,X) = \frac{1}{2} + g_{40}(\pi) + g_{41}(\pi)\cdot X\ ,\quad F_4(\pi,X) = f_{40}(\pi).$$



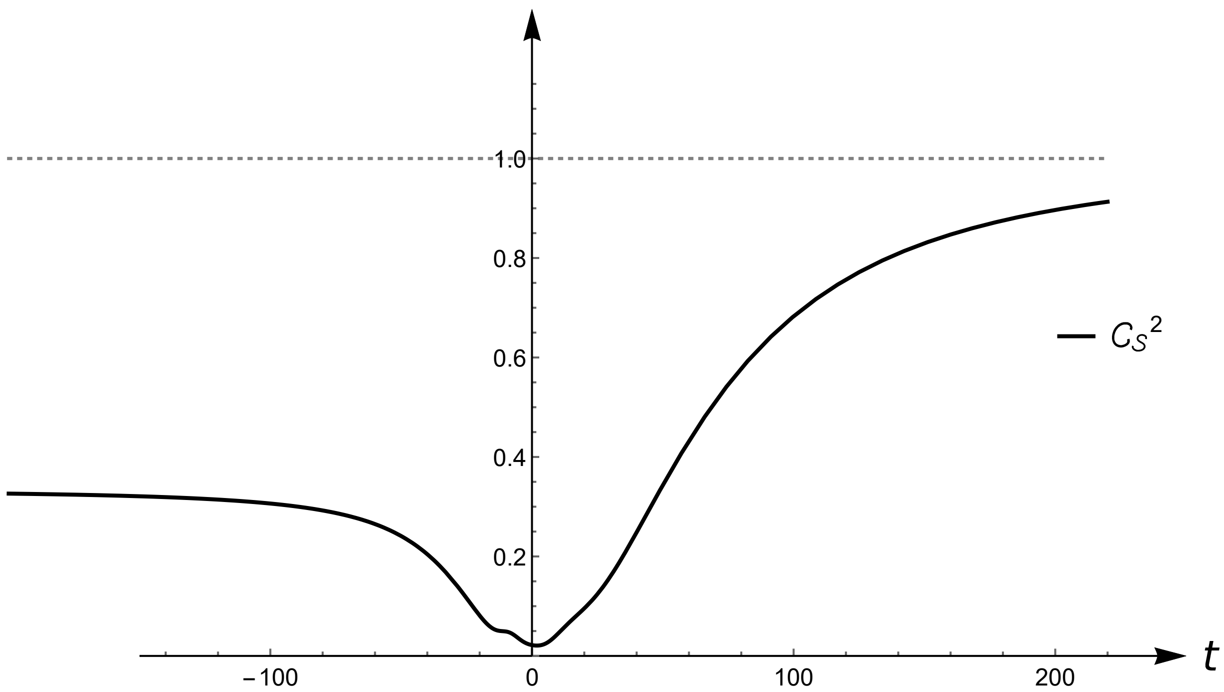




Sound speeds:

$$c_{\mathscr{T}}^2 = \frac{\mathscr{F}_{\mathscr{T}}}{\mathscr{G}_{\mathscr{T}}} = 1$$

$$c_{\mathscr{S}}^2 = \frac{\mathscr{F}_{\mathscr{S}}}{\mathscr{G}_{\mathscr{S}}} < 1$$



Healthy Genesis with GR asymptotics at $t \to +\infty$

However, there is still an issue to worry about: superluminality.

Theory with superluminal excitations cannot descend from healthy Lorentz-invariant UV-complete theory

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '2006

Not an issue in DHOST theories per se.

But things change once one allows for extra field(s) ("matter")

NB: consistency issue unrelated to Genesis.

DHOST Ia with additional scalar field

Additional minimally coupled scalar: $L_\chi = (\partial\chi)^2$

NB. Naively: luminal propagation.

Consider DHOST + rolling χ background, $\dot{\chi}_c \neq 0$

New featue: DHOST pertutbations kinetically mix with $\delta\chi$

Langlois, Mancarella, Noui, Vernizzi '2017

Perturbations about FLRW background with DHOST and rolling χ_c:
in scalar sector parametrized by u^A, $A = 1, 2$, $u^1 = \zeta$, $u^2 = \delta\chi$.

Quadratic Lagrangian (modulo terms with less than two derivatives)

$$L^{(2)\,scalar}_{\pi+\chi} = G_{AB}\,\dot{u}^A\dot{u}^B - \frac{1}{a^2}F_{AB}\,\partial_i u^A \partial_i u^B$$

$$G_{AB} = \begin{pmatrix} \mathcal{G}_{\mathcal{S}} & \dot{\chi}_c g \\ \dot{\chi}_c g & 1 \end{pmatrix}, \qquad F_{AB} = \begin{pmatrix} \mathcal{F}_{\mathcal{S}} & \dot{\chi}_c f \\ \dot{\chi}_c f & 1 \end{pmatrix},$$

$g, f(\pi, X)$ = combinations of functions in DHOST Lagrangian,
vanish in Horndeski limit

Sound speeds squared c^2 are determined by

$$\det\left(F_{AB} - c^2 G_{AB}\right) = 0$$

If $c_{\mathscr{S}}^2 \equiv \frac{\mathscr{F}_{\mathscr{S}}}{\mathscr{G}_{\mathscr{S}}} < 1$ (subluminal DHOST), then one of the sound speeds

$$c^2 = 1 + \frac{\dot{\chi}_c^2 (f-g)^2}{\mathscr{G}_{\mathscr{S}}(1-c_{\mathscr{S}}^2)} + \mathscr{O}(\dot{\chi}_c^4)$$

For $c_{\mathscr{S}}^2 \equiv \frac{\mathscr{F}_{\mathscr{S}}}{\mathscr{G}_{\mathscr{S}}} = 1$ (luminal DHOST), one of the sound speeds

$$c^2 = 1 + \left(\frac{\dot{\chi}_c^2 (f-g)^2}{\mathscr{G}_{\mathscr{S}}}\right)^{1/2} + \mathscr{O}(\dot{\chi}_c^2)$$

In both cases one of the modes superluminal unless $g = f$

Mironov, V.R., Volkova '2020

Beyond Horndeski:

$$f - g = 2F_4 X$$

precisely the combination used to evade no-go for Genesis.

Beyond Horndeski does not marry conventional scalars

and other perfect fluids with luminal excitations, e.g., luminal k-essence.

Imposing $g = f \Longrightarrow$ Very special DHOST theory.

$$S = \int d^4x\sqrt{-g}\left(F(\pi,X) + K(\pi,X)\Box\pi + G_4(\pi,X)R + \sum_{i=1}^{5} A_i(\pi,X)L_i\right)$$

DHOST Ia:

$$A_2 = -A_1$$

$$\begin{aligned} 8(G_4 - XA_1)^2 \cdot A_4 = & -16XA_1^3 + 4(3G_4 + 16XG_{4X})A_1^2 - X^2G_4A_3^2 \\ & - (16X^2G_{4X} - 12XG_4)A_3A_1 - 16G_{4X}(3G_4 + 4XG_{4X})A_1 \\ & + 8G_4(XG_{4X} - G_4)A_3 + 48G_4G_{4X}^2 \end{aligned}$$

$$8(G_4 - XA_1)^2 \cdot A_5 = (4G_{4X} - 2A_1 + XA_3)\left(-2A_1^2 - 3XA_1A_3 + 4G_{4X}A_1 + 4G_4A_3\right)$$

Extra condition $f = g$:

$$A_3 = \frac{2(A_1 - 2G_{4X})(A_1X - 2G_4)}{X(3A_1X - 4G_4)}$$

Still unknown whether this theory admits healthy Genesis.

To summarize

- Construcing Genesis (an also bounce) cosmology, thus avoiding classical singularity, does not appear impossible.
- This requires unusual fields with complicated Lagrangians involving second derivatives.
  - Absence of Ostrogradsky ghost, catastrophic instabilities and superluminality imposes strong (non-linear!) constraints on functions in Lagrangian.
- Is the price too high —maybe!

Other issues

- Transition to hot epoch. Does not appear problematic, similar to *k*-inflation.

  Armendariz-Picon, Damour, Mukhanov' 99

- Generation of density perturbations. Need a separate mechanism to generate nearly flat power spectrum.

  To name a few:

  - Matter bounce

    Finelli, Brandenberger' 2001

    Wands' 98

  - Conformal mechanism

    V R' 2009

    Creminelli, Nicolis, Trincherini' 2010

    Hinterbichler, Khouri' 2011, ...

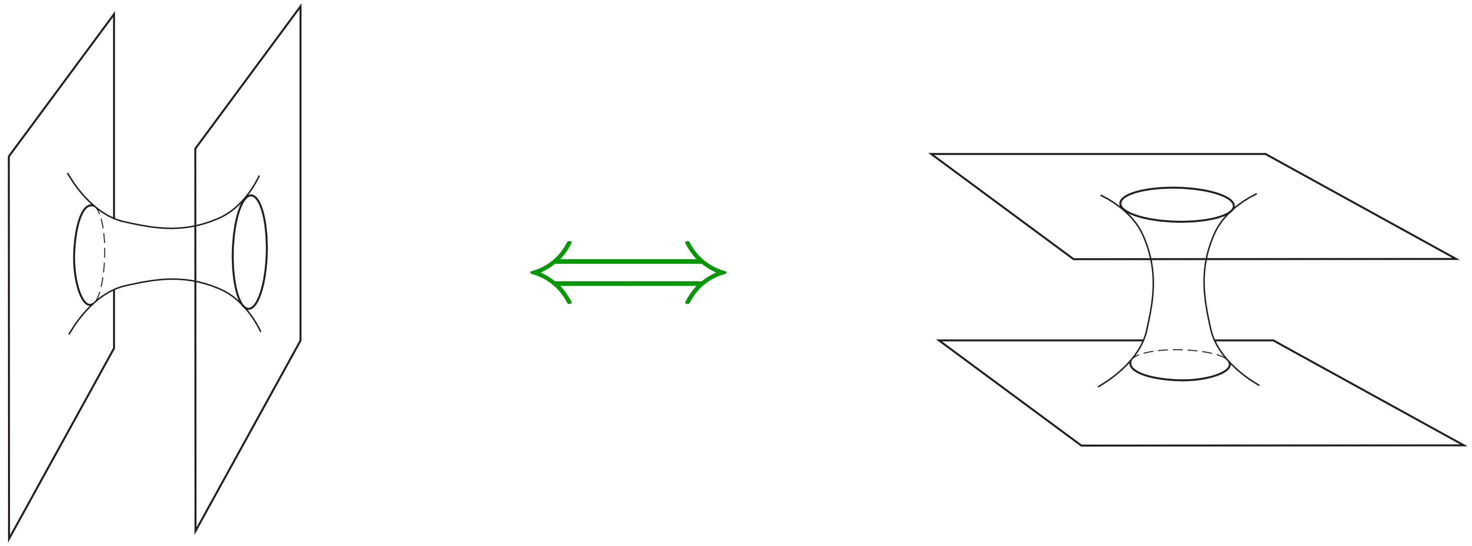
- Possible way to generate tensor perturbations (gravity waves) with blue or peaked power spectrum (cf. NANOGrav)

  Tahara, Kobayashi' 20

Instead of conclusion: where else DHOST may be instrumental?

- Lorentzian wormholes

Static wormhole $\Longleftrightarrow$ Bouncing Universe



No-go in NEC-preserving theories

No-go in Horndeski: no stable, static, spherically symmetric wormholes: always ghosts.

V.R.' 16; Evseev, Melichev' 18

Not obviously impossible in DHOST

Mironov, V.R., Volkova' 18; Francolini et. al.' 18

Studying stability HUGELY difficult.

- Creation of a universe in the laboratory
  - Question raised in mid-80's, right after invention of inflationary theory

    Berezin, Kuzmin, Tkachev' 1984; Guth, Farhi' 1986

    Idea: create, in a finite region of space, inflationary initial conditions $\Longrightarrow$ this region will inflate to enormous size and in the end will look like our Universe.

  - Do not need much energy: pour little more than Planckian energy into little more than Planckian volume.

    If NEC holds, no way: initial singularity

    Guth, Farhi' 1986; Berezin, Kuzmin, Tkachev' 1987

    How about DHOST theories?

Amazingly, many questions of principle still not answered.
Ahead: more to understand.

Backup slides

No-go theorem for theories with Lagrangians involving first derivatives of fields only (and minimal coupling to gravity)

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

Buniy, Hsu, Murray' 2006

$$L = F(X^{IJ}, \pi^I)$$

with $X^{IJ} = \partial_\mu \pi^I \partial^\mu \pi^J \Longrightarrow$

$$T_{\mu\nu} = 2\frac{\partial F}{\partial X^{IJ}} \partial_\mu \pi^I \partial_\nu \pi^J - g_{\mu\nu} F$$

In homogeneous background

$$T_{00} \equiv \rho = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} - F$$

$$T_{11} = T_{22} = T_{33} \equiv p = F$$

and

$$\rho + p = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} = 2\frac{\partial F}{\partial X^{IJ}} \dot{\pi}^I \dot{\pi}^J$$

NEC-violation: matrix $\partial F/\partial X_c^{IJ}$ non-positive definite. But

Lagrangian for perturbations $\pi^I = \pi_c^I + \delta\pi^I$

$$L_{\delta\pi} = A_{IJ}\ \partial_t \delta\pi^I \cdot \partial_t \delta\pi^J - \frac{\partial F}{\partial X_c^{IJ}}\ \partial_i \delta\pi^I \cdot \partial_i \delta\pi^J + \dots$$

Gradient instabilities and/or ghosts

NB. Loophole: $\partial F/\partial X_c^{IJ}$ degenerate.

Higher derivative terms (understood in effective field theory sense) become important and help.
Ghost condensate

Formulas in beyond Horndeski

To simplify: $G_5 = F_5 = 0$
NB: Horndeski is restored for $F_4 = 0$

Tensor sector:

$$\mathscr{G}_{\mathscr{T}} = 2G_4 - 4G_{4X}X - 2F_4X^2,$$
$$\mathscr{F}_{\mathscr{T}} = 2G_4$$

Scalar sector:

$$\mathscr{G}_{\mathscr{S}} = \frac{\Sigma \mathscr{G}_{\mathscr{T}}^2}{\Theta^2} + 3\mathscr{G}_{\mathscr{T}},$$
$$\mathscr{F}_{\mathscr{S}} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathscr{F}_{\mathscr{T}},$$
$$\xi = \frac{a\left(\mathscr{G}_{\mathscr{T}} + 2F_4X^2\right)\mathscr{G}_{\mathscr{T}}}{\Theta}.$$

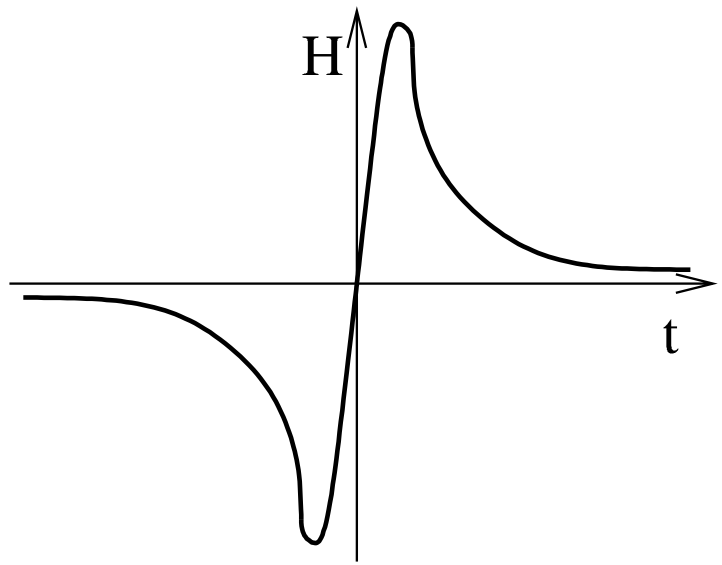
Where

$$\Theta = -K_X X\dot{\pi} + 2G_4 H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi}$$
$$-10HF_4X^2 - 4HF_{4X}X^3,$$

$$\Sigma = F_X X + 2F_{XX}X^2 + 12HK_X X\dot{\pi} + 6HK_{XX}X^2\dot{\pi} - K_\pi X - K_{\pi X}X^2$$
$$-6H^2G_4 + 42H^2G_{4X}X + 96H^2G_{4XX}X^2 + 24H^2G_{4XXX}X^3 - 6HG_{4\pi}\dot{\pi}$$
$$-30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi} + 90H^2F_4X^2 + 78H^2F_{4X}X^3 + 12H^2F_{4XX}X^4$$

Bounce by intelligent design

- Choose your favorite $H(t)$ such that $H(t) \to \frac{1}{3t}$ as $|t| \to \infty$
  GR + Galileon = conventional massless scalar.

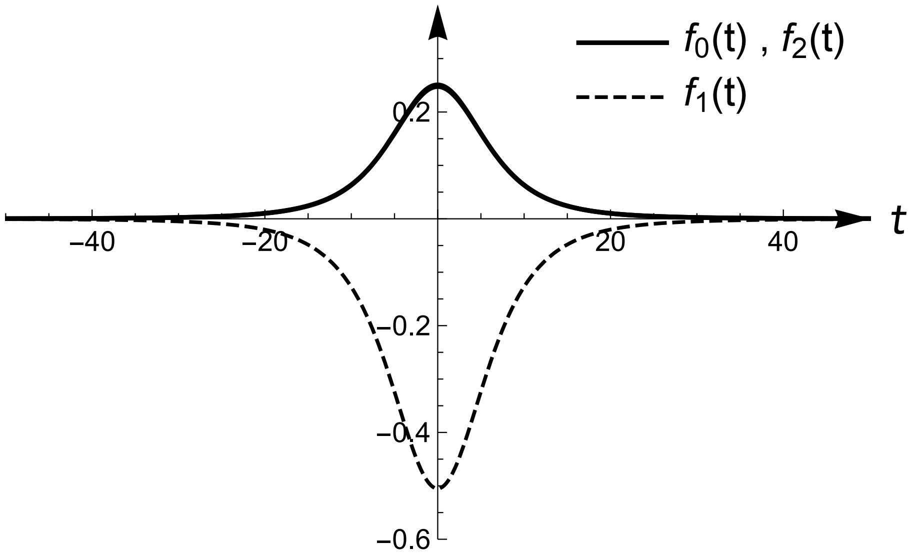


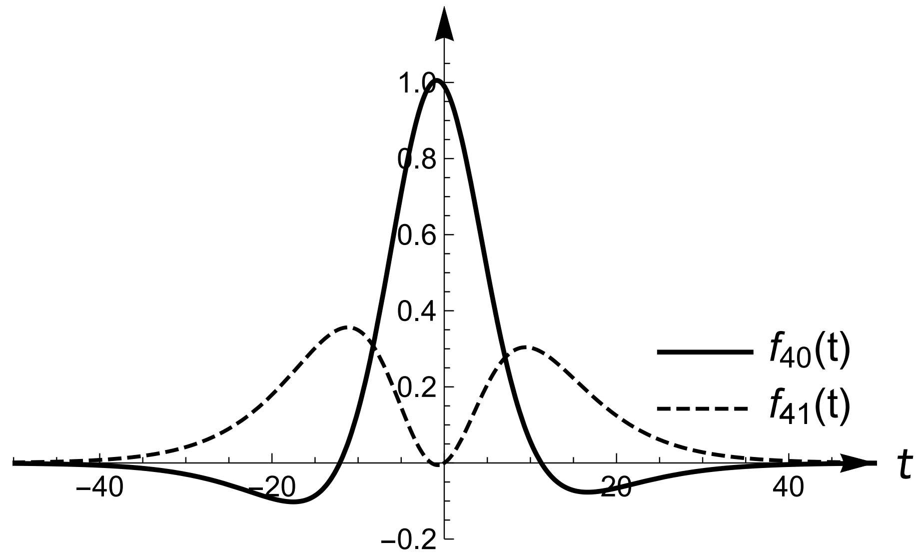
- Asymptotics of Lagrangian functions as $|t| \to \infty$:

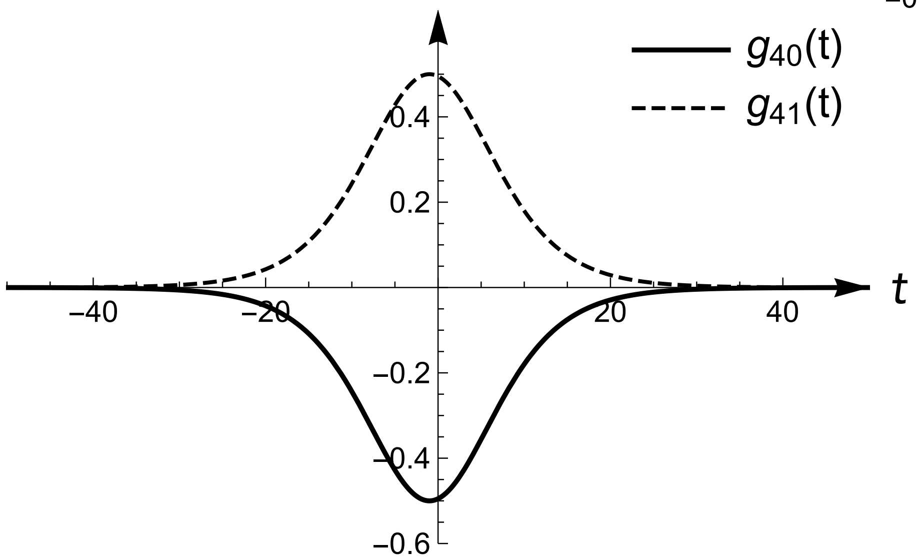
$$F(t) = \frac{1}{t^2}, \quad F_X(t) = \frac{1}{t^2} \Longrightarrow F = \frac{(\partial\pi)^2}{\pi^2} = (\partial \log \pi)^2$$

$$G_4 = \frac{M_{Pl}^2}{16\pi}, \qquad K = F_4 = 0$$

- Cook up Lagrangian functions in such a way that
  - Field equation are satisfied
  - Stability conditions are satisfied at all times

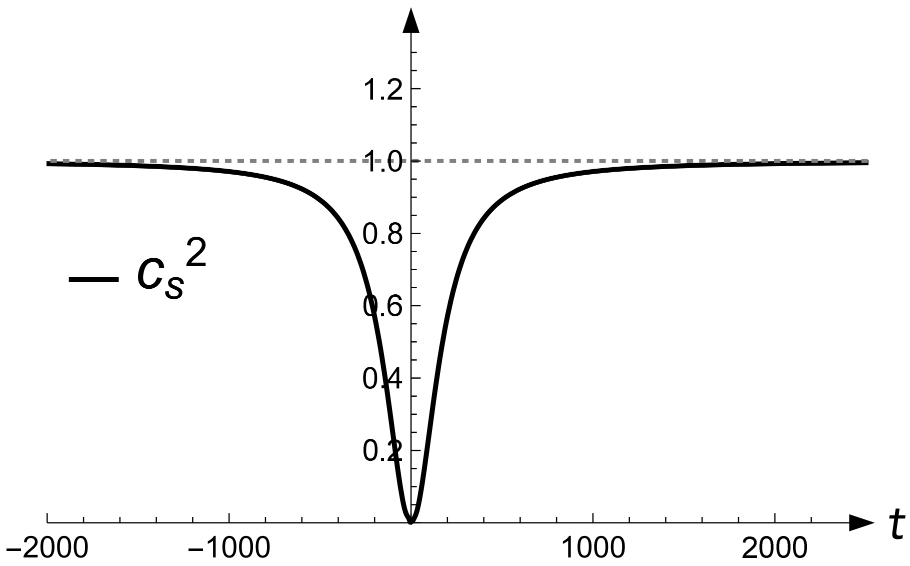


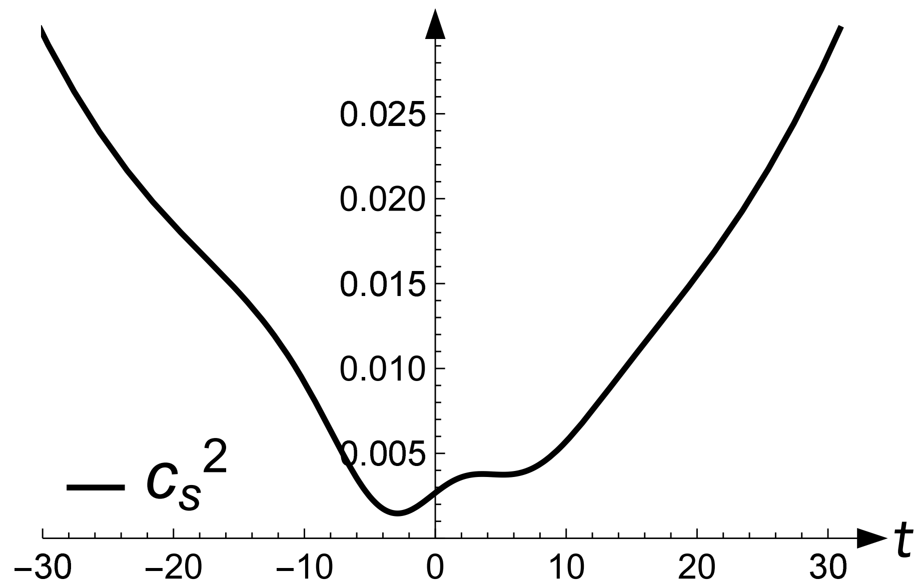




No kidding: speed of gravity waves is always 1.

Speed of scalar perturbation $0 < c_s^2 \leq 1$





Completely stable bounce