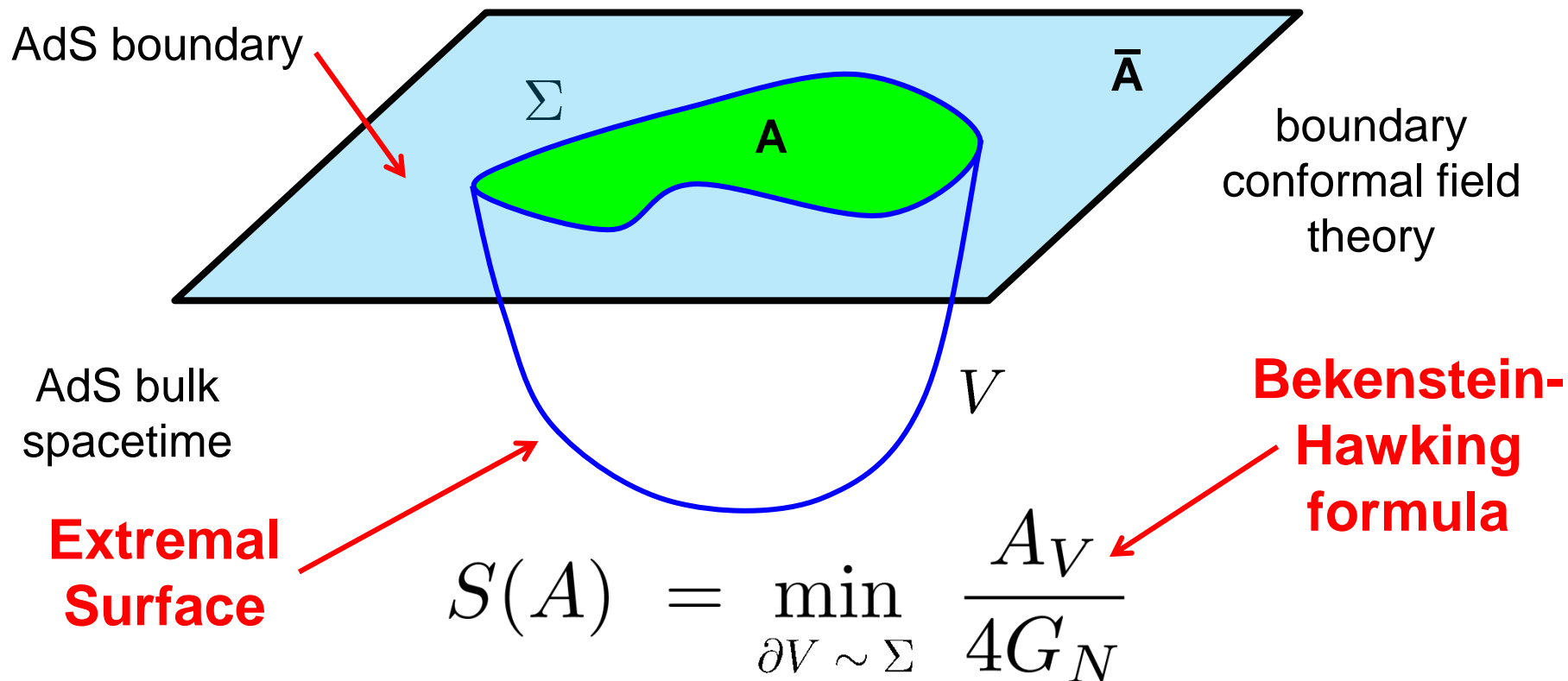




Quantum Extremal Islands Made Easy

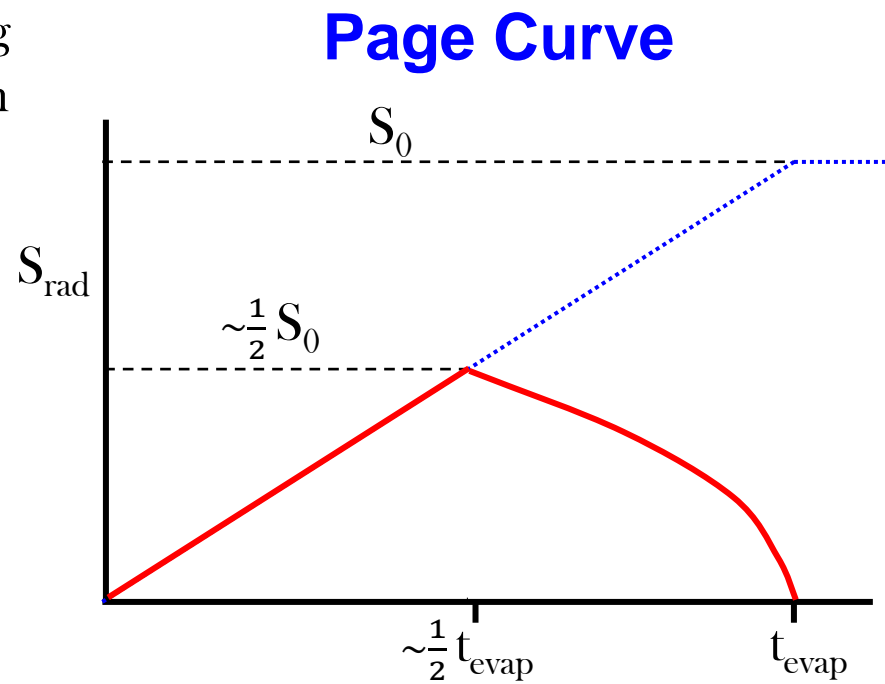
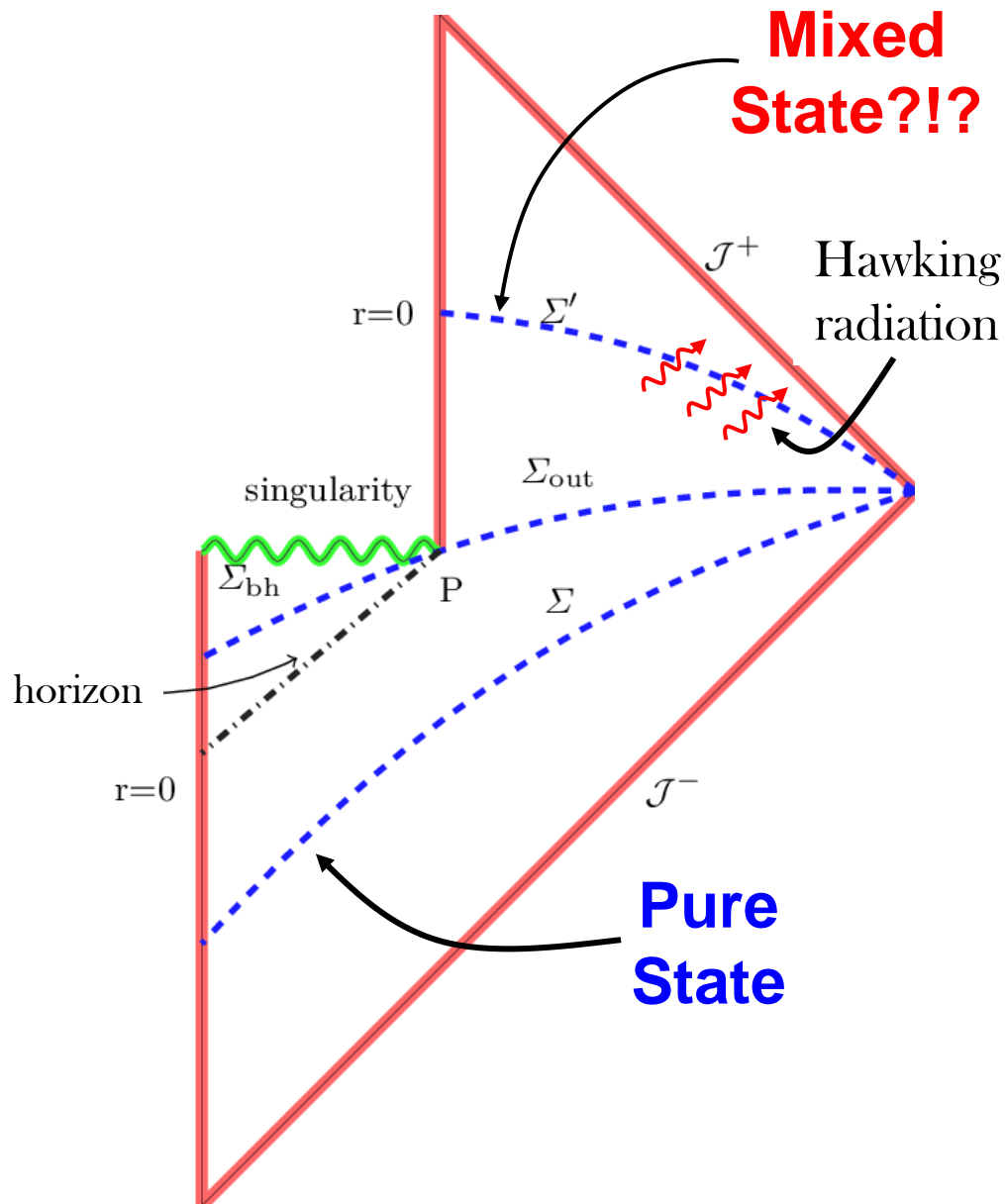
with Chen, Neuenfeld, Reyes & Sandor; Hernandez & Ruan
(2006.04851 & 2010.00018; 2010.16398)

Holographic Entanglement Entropy:



- holographic EE is a fruitful forum for bulk-boundary dialogue:
 - new lessons about quantum field theories
 - new lessons about quantum gravity

Black hole information paradox:



New insights from Holographic EE:

- with recent progress, it is possible to compute the Page curve in a controlled manner!

Penington [arXiv:1905.08255]

Almheiri, Engelhardt, Marolf & Maxfield [arXiv:1905.08762]

* Almheiri, Mahajan, Maldacena & Zhao [arXiv:1908.10996]

➔ Island Rule:

- black hole coupled to an auxiliary **non-gravitational** reservoir (the “bath”), which captures the Hawking radiation
- entropy of the Hawking radiation is given by

$$S_{EE}(\mathbf{R}) = \min \left\{ \text{ext}_{\text{islands}} \left(S_{QFT}(\mathbf{R} \cup \text{islands}) + \frac{A(\partial(\text{islands}))}{4G_N} \right) \right\}$$

- evaluate the (semiclassical) entanglement entropy of quantum fields in the bath region \mathbf{R} combined with various space-like subregions in the gravitating region, ie, islands, which also contribute the usual Bekenstein-Hawking entropy

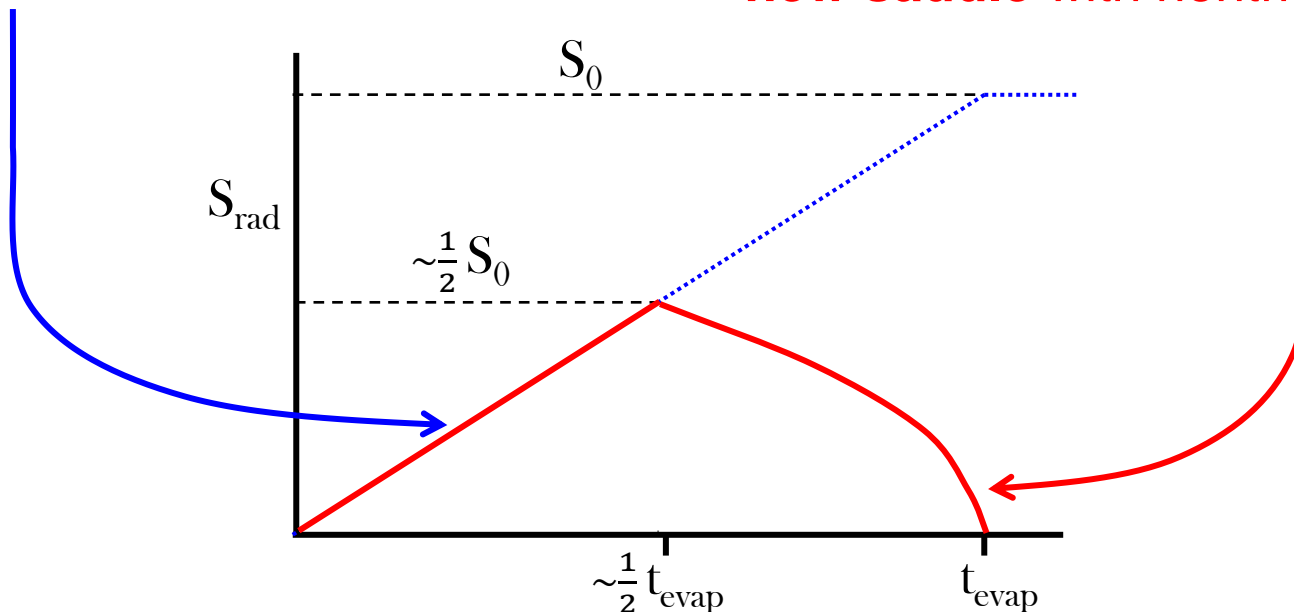
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Early: island is the empty set;
agrees with Hawking's calculation

Late: large entanglement between
radiation and region behind horizon;
new saddle with nontrivial island



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Key ingredients of early calculations:

➔ AdS/CFT **but** absorbing or transparent b.c.

➔ two-dimensional JT gravity

➔ quantum extremal surfaces

extremize geometric/grav. entropy plus quantum S_{EE} of matter fields (Faulkner, Lewkowycz & Maldacena; Engelhardt & Wall)

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Example: Not Evaporating Black Holes!

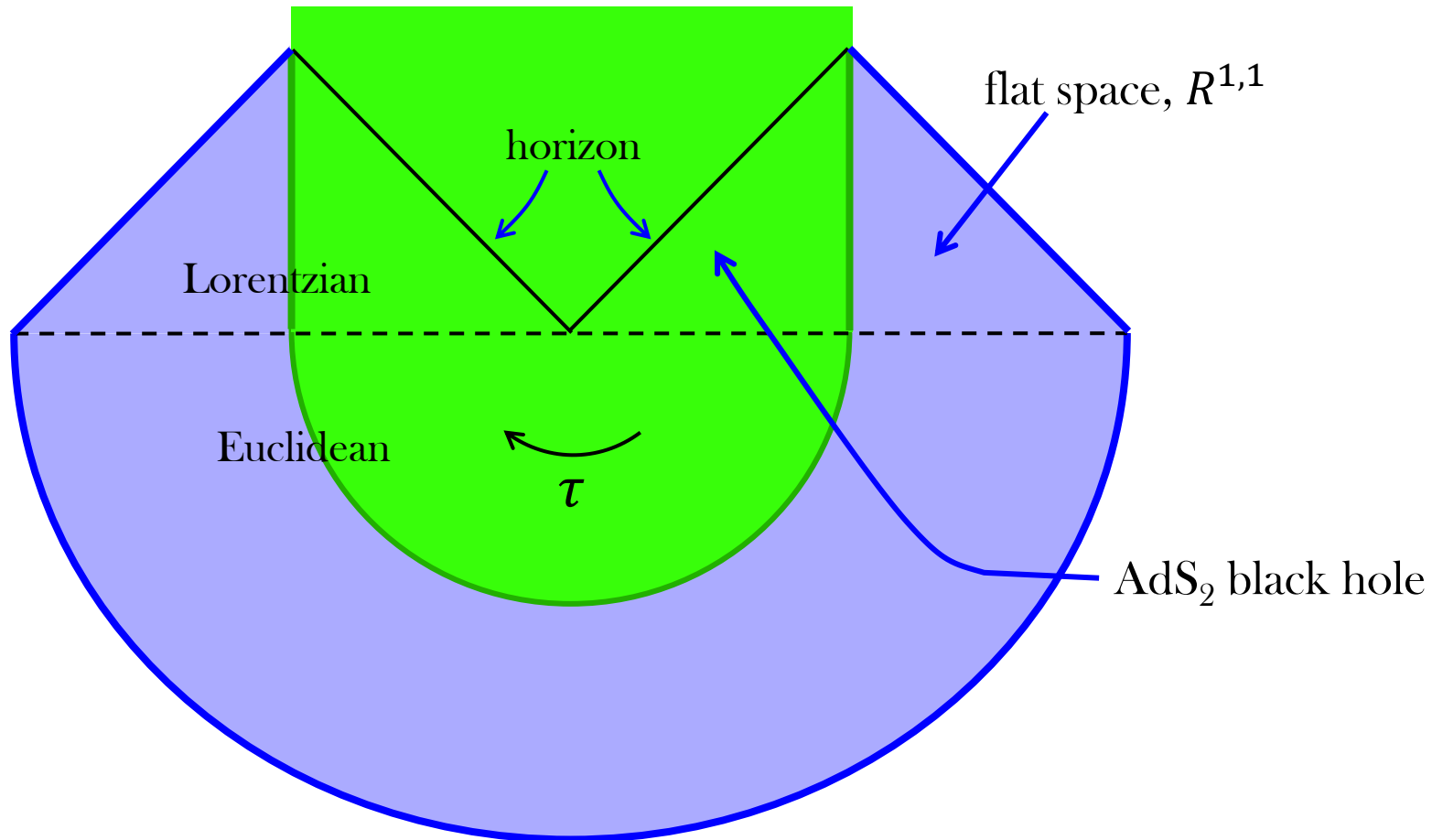
Black Holes in Thermal Equilibrium:

Almheiri, Mahajan & Maldacena

(see also: Rozali, Van Raamsdonk, Waddell & Wakeham)

+ CFT

- simple holographic model: 2d JT gravity $\Lambda = -1$ = 1d quantum mech's
- prepare state with 2d black hole & bath in thermal equilibrium

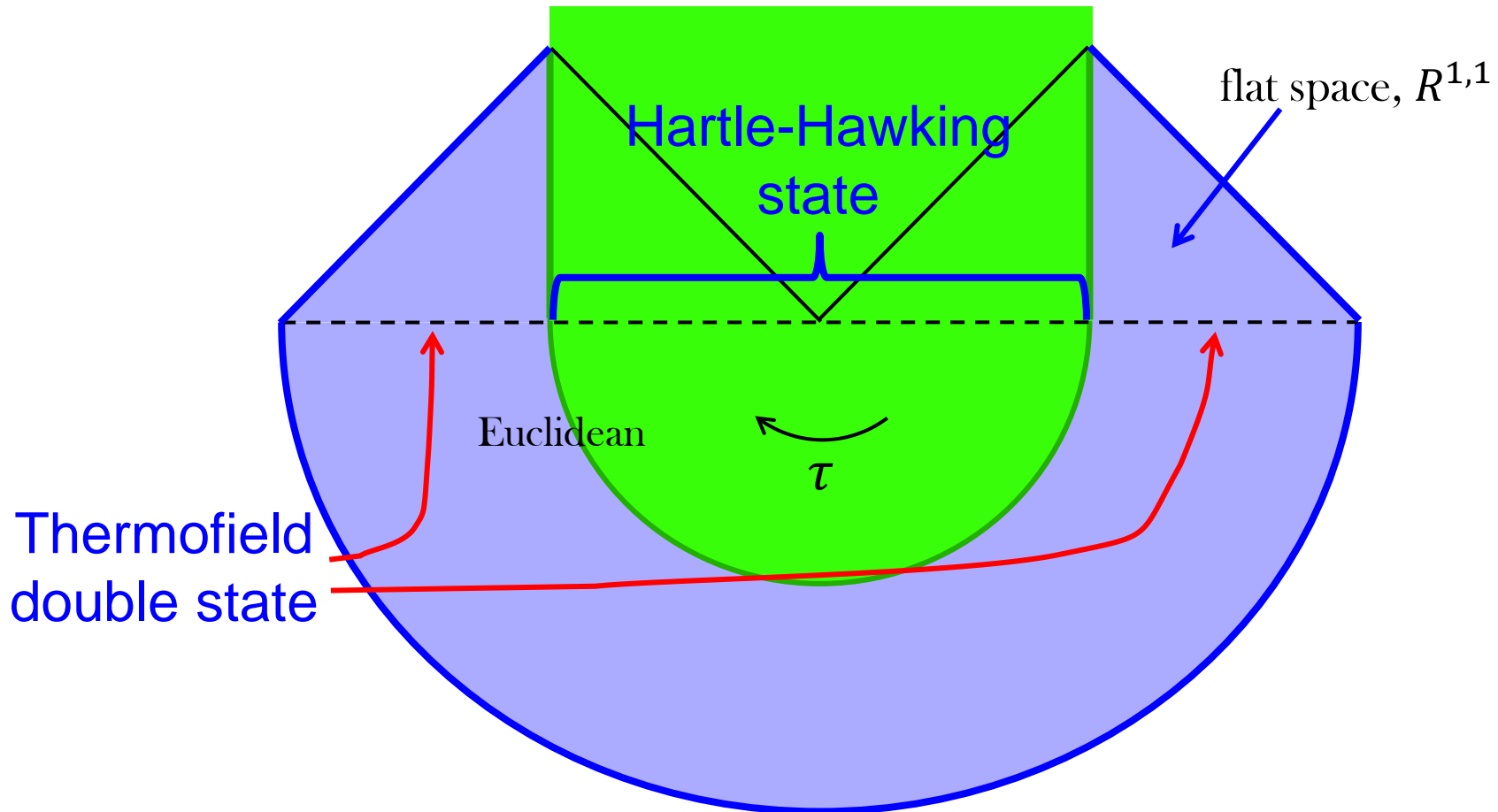


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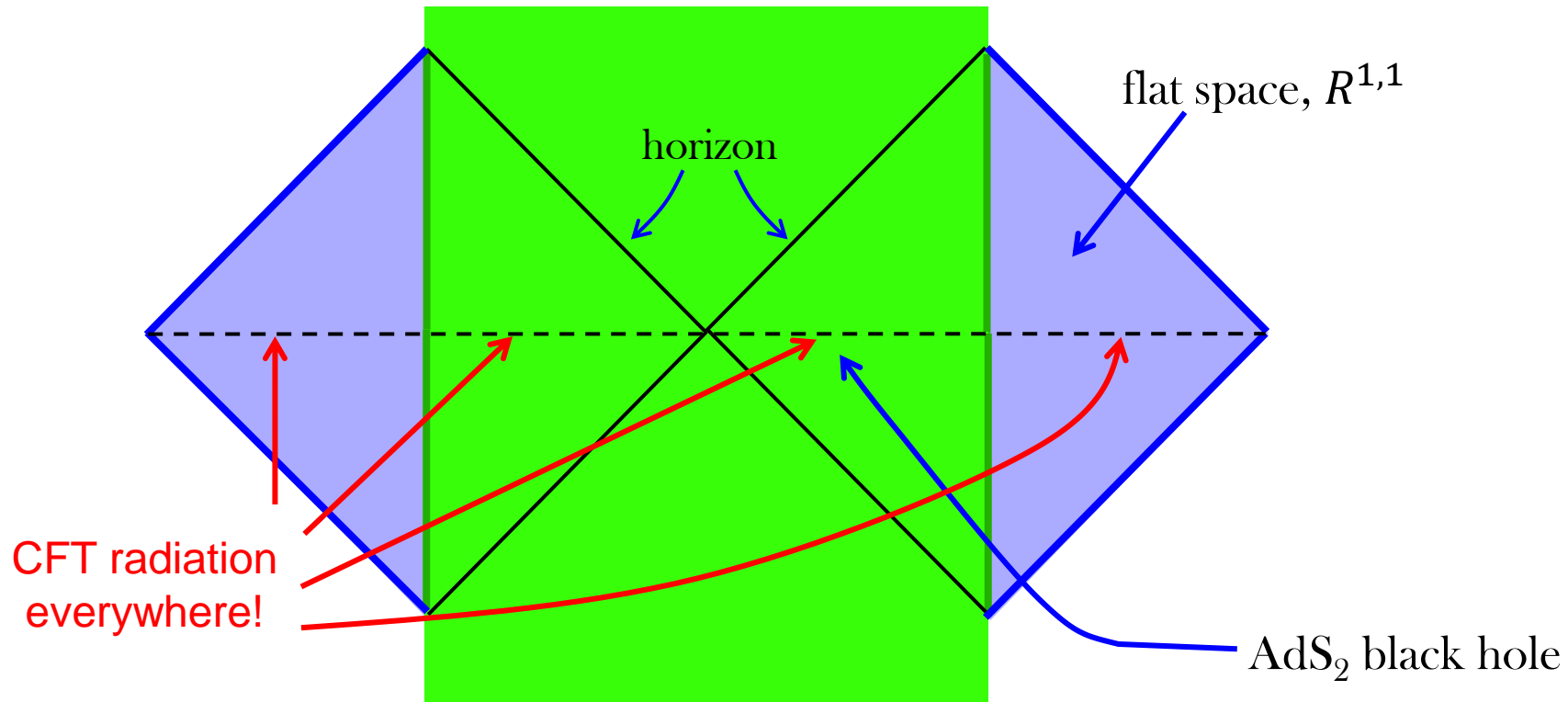


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Thermal equilibrium? No information paradox?

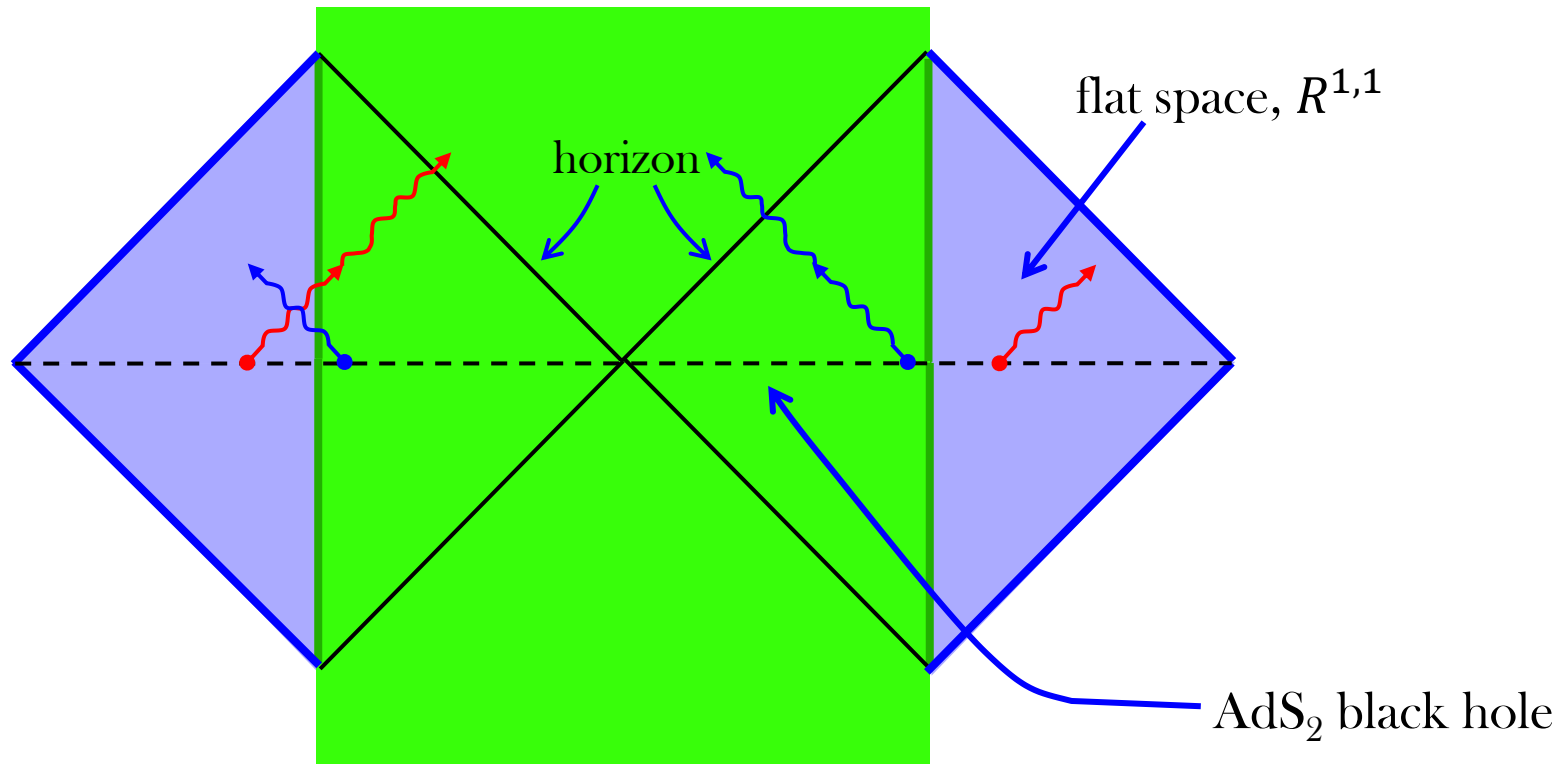
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- eternal BH and bath are continuously exchanging radiation

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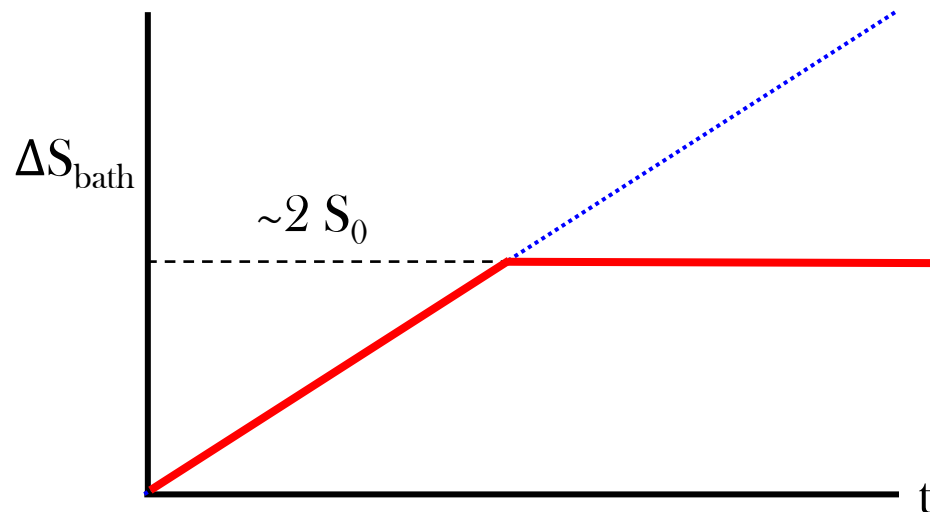
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What does Page curve look like for eternal black hole?

- eternal BH and bath are continuously exchanging radiation



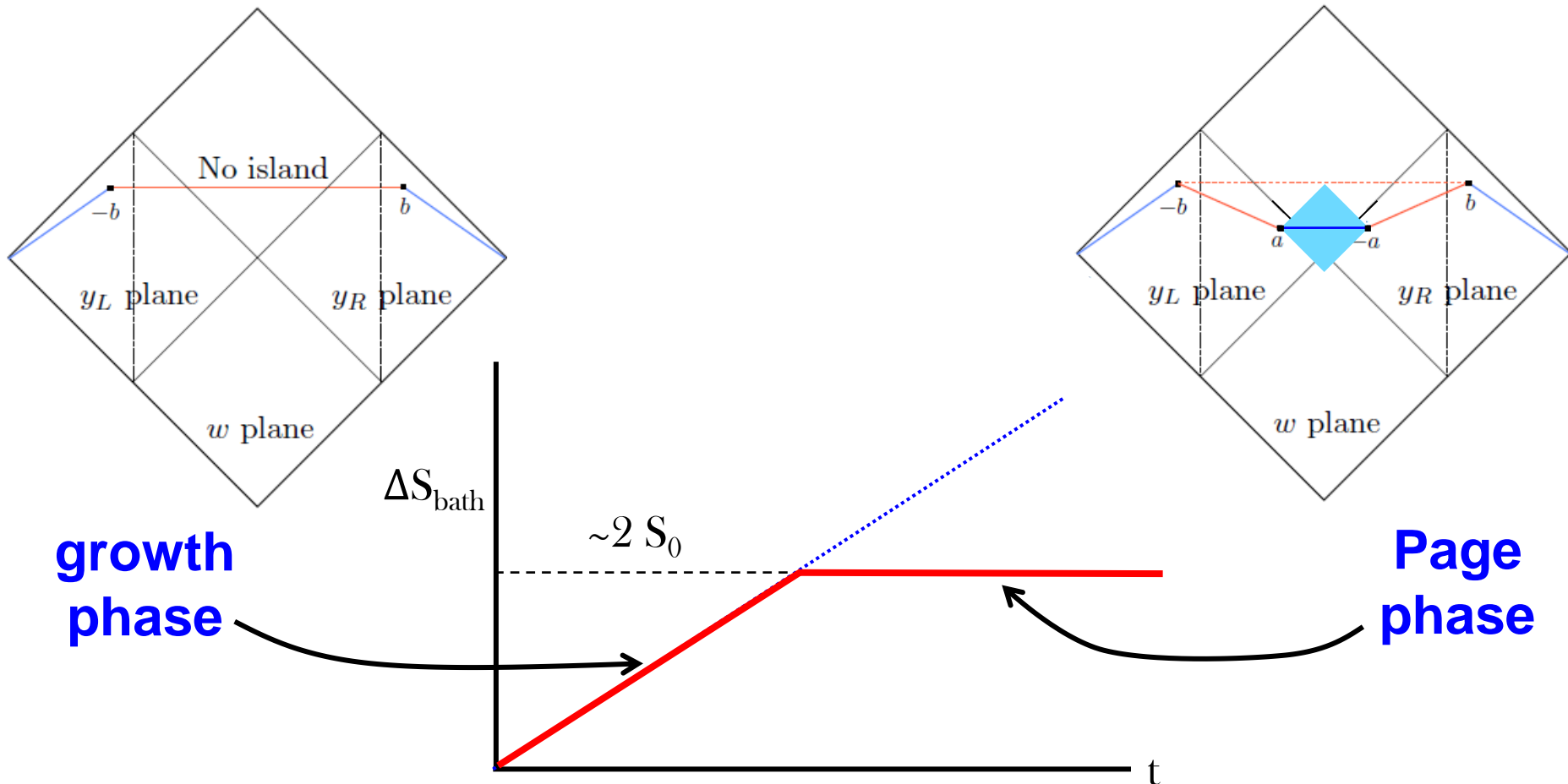
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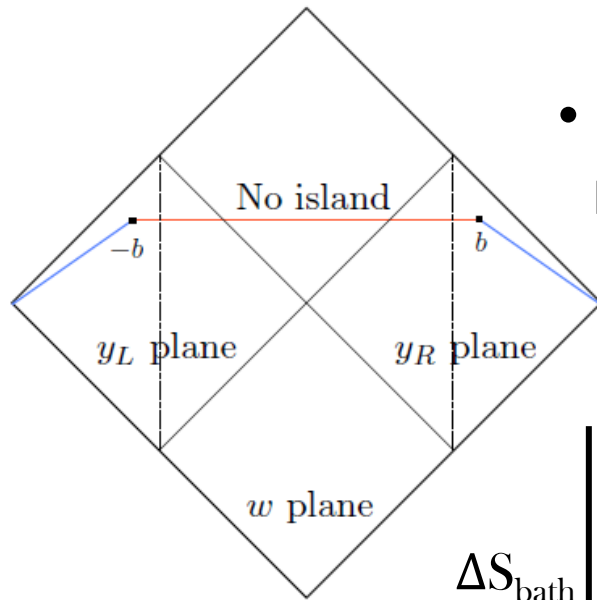


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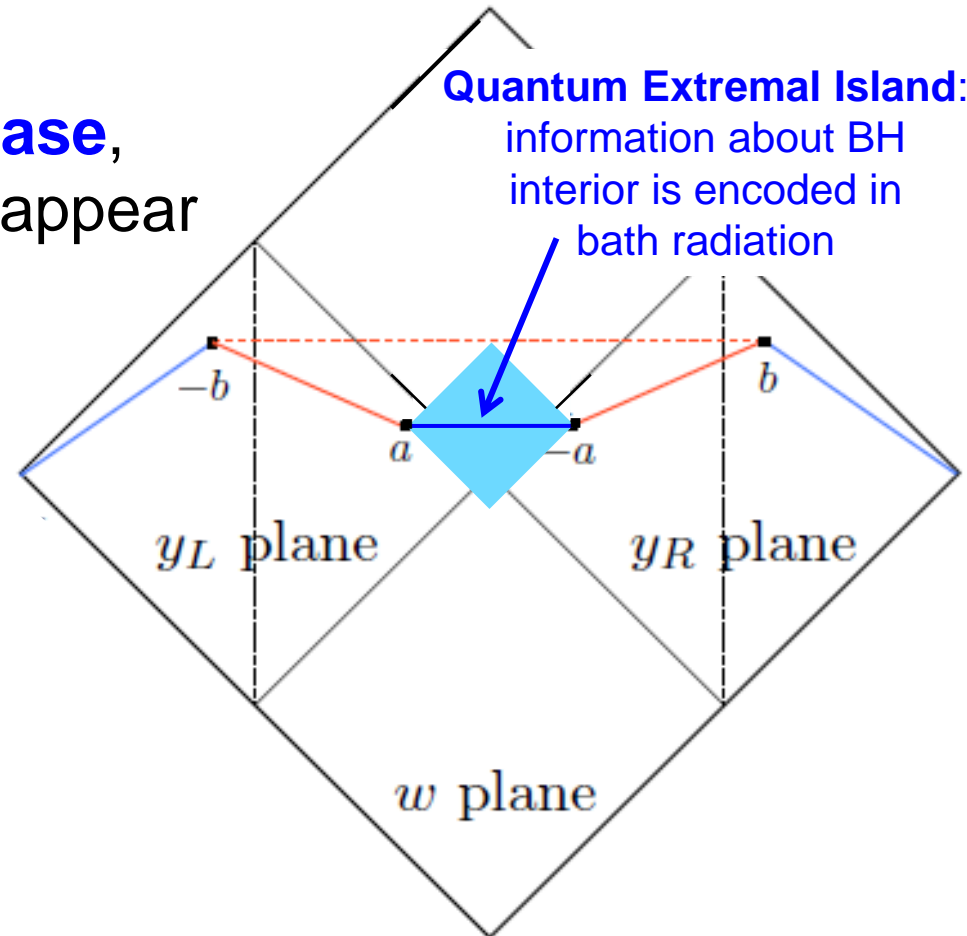
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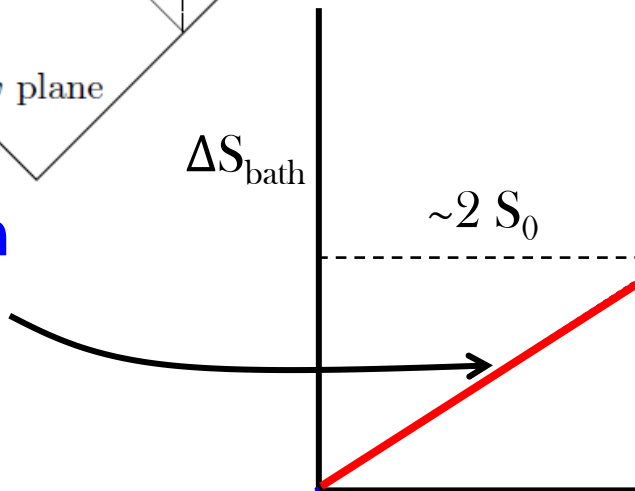
- in **Page phase**, new QESs appear



growth phase

ΔS_{bath}

$\sim 2 S_0$



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Questions, Questions, Questions:

- how important is two dimensions?
- are dof on Planck brane part of boundary or bulk?
- was JT gravity important?
- was ensemble average of SYK model important?
- how was information encoded in Hawking radiation?

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Many of new insights can be understood as familiar properties of holographic entanglement entropy

- how was information encoded in Hawking radiation?

Randall-Sundrum gravity (quick review):

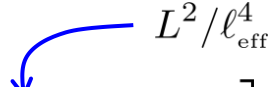
- introduce d-dim. brane in (d+1)-dim. AdS geometry, backreaction creates extra d-dim. graviton mode localized on brane:

$$I_{\text{bulk}} = \frac{1}{16\pi G_{\text{bulk}}} \int d^{d+1}x \sqrt{-g} \left[\frac{d(d-1)}{L^2} + R(g) \right]$$

$$I_{\text{brane}} = -T_0 \int d^d x \sqrt{-\tilde{g}}$$

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with

$$\frac{1}{G_{\text{eff}}} = \frac{2L}{(d-2)G_{\text{bulk}}} ; \quad \frac{1}{\ell_{\text{eff}}^2} \simeq \frac{2}{L^2} \left(1 - \frac{4\pi G_{\text{bulk}} L T_0}{d-1} \right) \ll \frac{1}{L^2}$$

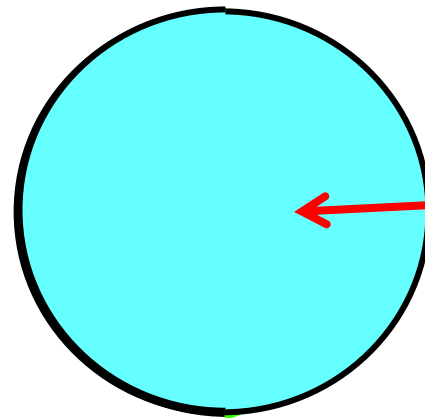
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- “position” of brane can be determined by:
using Israel junction conditions



cross-section
of AdS_{d+1}

$$ds^2 = L^2 [d\rho^2 + \cosh^2 \rho d\Sigma_d^2] \leftarrow \text{AdS}_d$$

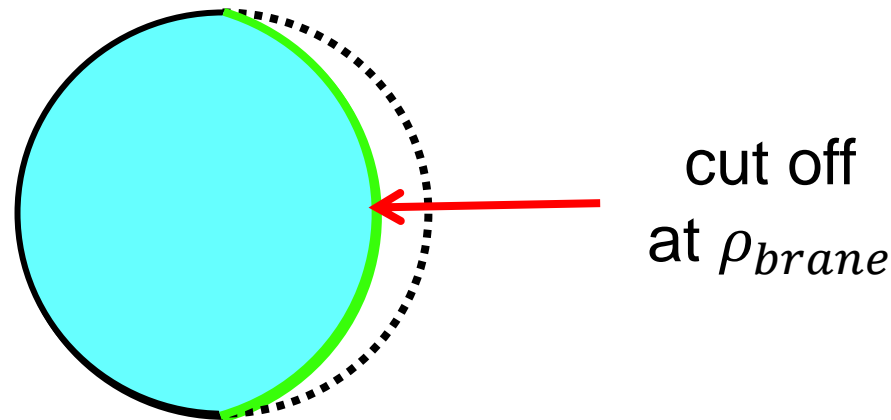
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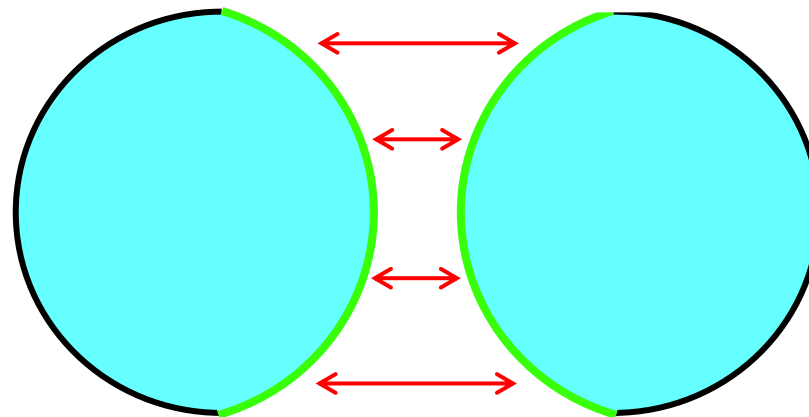
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paste to
2nd copy

$$\begin{aligned} \Delta K_{ij} - \tilde{g}_{ij} \Delta K^\ell_\ell \\ = -8\pi G_{\text{bulk}} T_0 \tilde{g}_{ij} \end{aligned}$$

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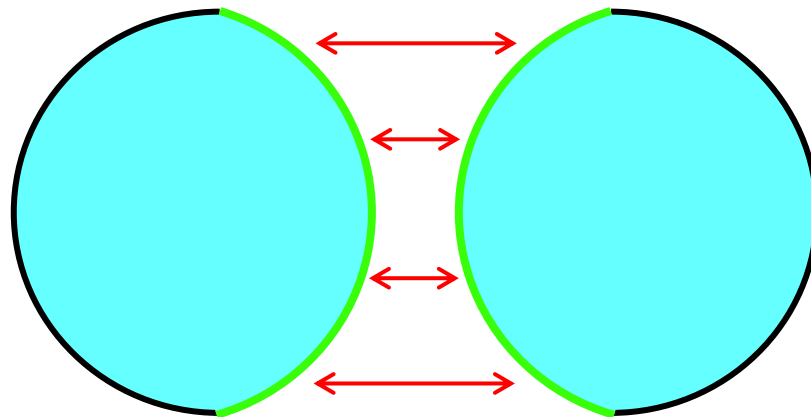
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- “position” of brane can be determined by:
 using Israel junction conditions or solving brane gravity eom

$$\frac{1}{\ell_{\text{eff}}^2} = \frac{1}{\ell_{\text{B}}^2} \left[1 + \frac{1}{4} \frac{L^2}{\ell_{\text{B}}^2} + \dots \right]$$

with $\ell_{\text{B}} = L \cosh \rho_{\text{brane}}$



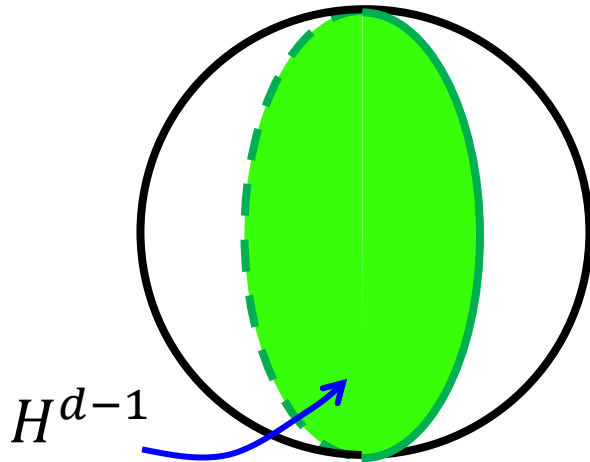
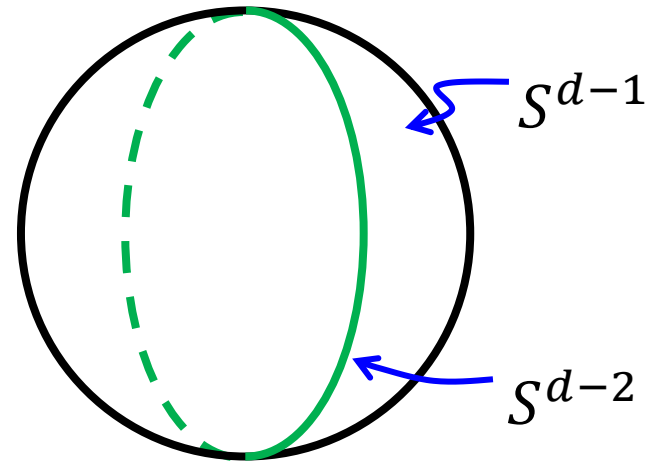
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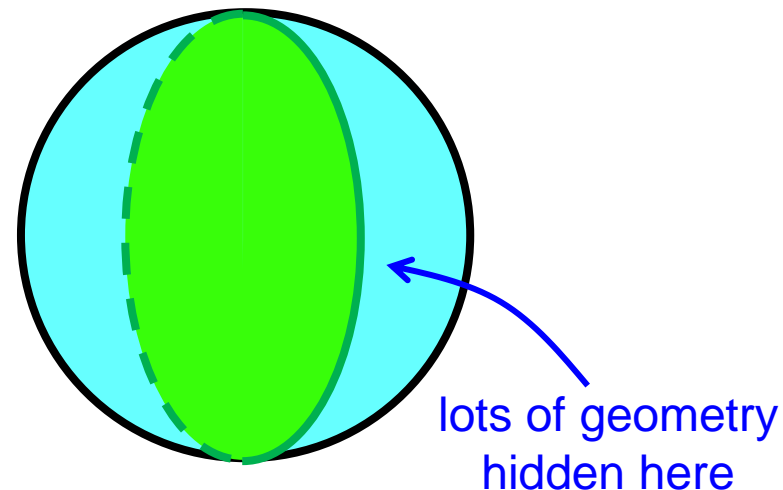
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Randall-Sundrum gravity:

(a) boundary CFT_d coupled to conformal defect (ie, boundary CFT_{d-1})



(b) boundary CFT_d coupled to CFT_d with gravity on AdS_d

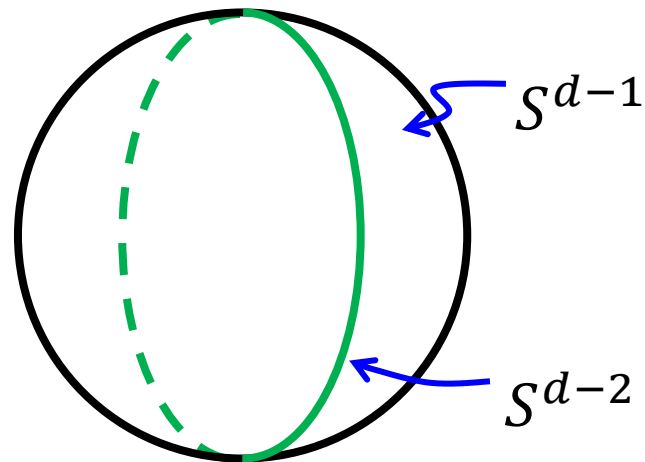


(c) AdS_{d+1} gravity coupled to brane with AdS_d geometry

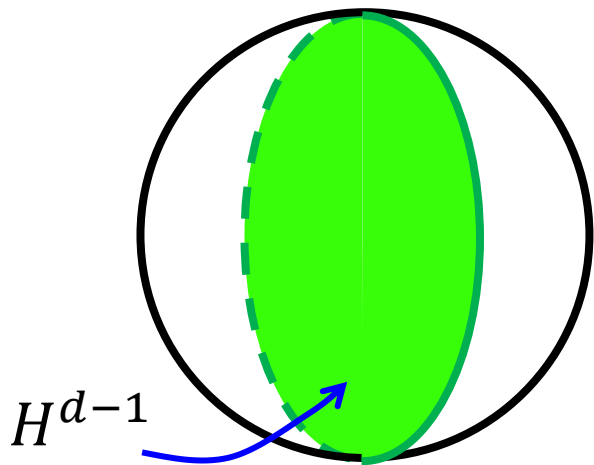
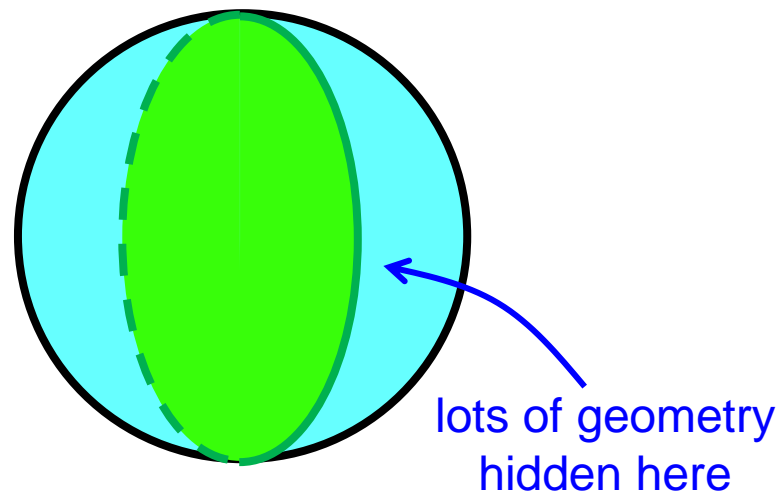
Randall-Sundrum gravity:

Boundary perspective

(a) boundary CFT_d coupled to conformal defect (ie, boundary CFT_{d-1})



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Bulk perspective

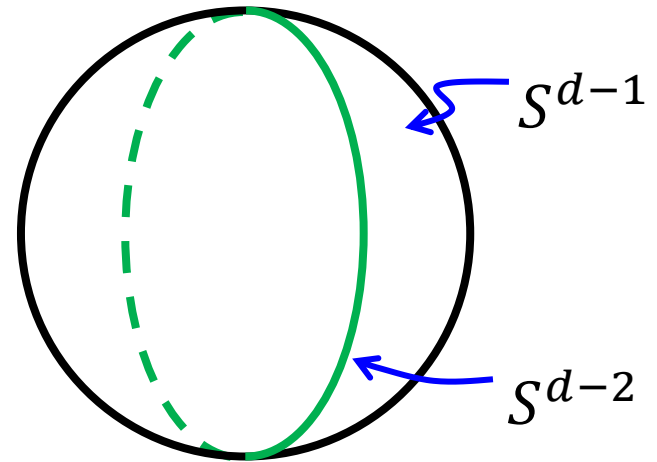
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AdS/CFT correspondence

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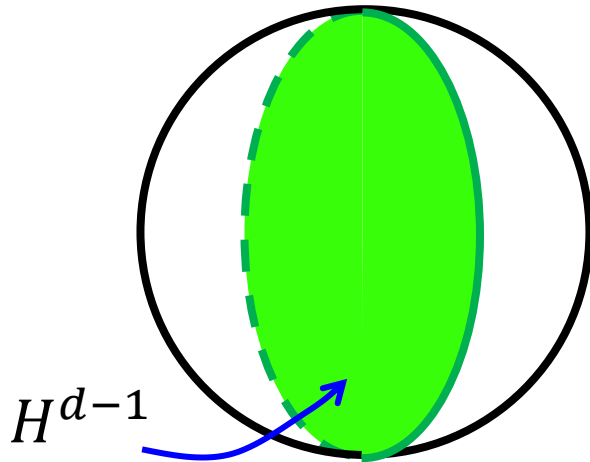
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Brane perspective

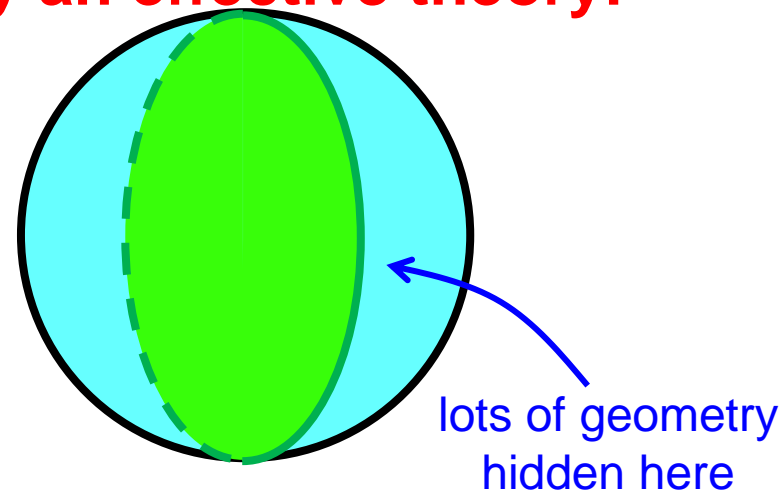
(b) boundary CFT_d coupled to CFT_d with gravity on AdS_d

only an effective theory!



Bulk perspective

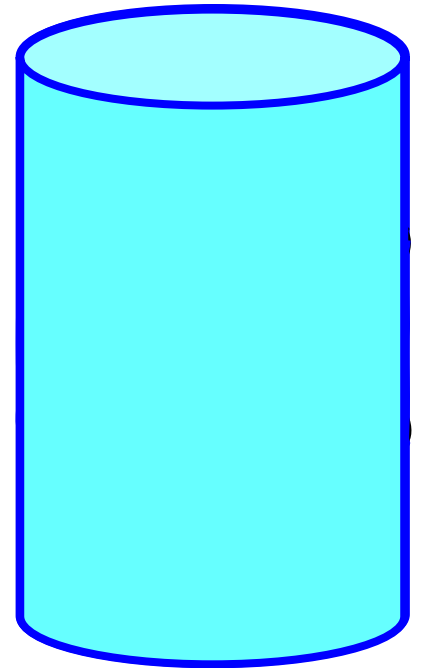
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AdS/CFT correspondence

Black Holes in Thermal Equilibrium:

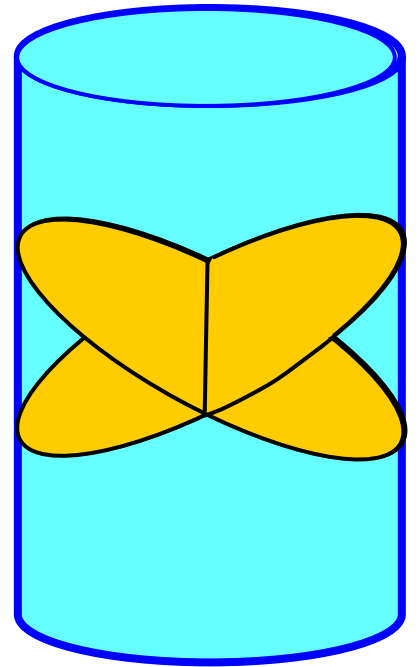
- AdS_{d+1} gravity ~~coupled to brane with AdS_d geometry~~



Black Holes in Thermal Equilibrium:

- ~~AdS_{d+1} gravity coupled to brane with AdS_d geometry~~
- empty AdS_{d+1} space can be described as “hyperbolic” black hole

$$ds^2 = \frac{L^2 d\rho^2}{(\rho^2 - L^2)} - \frac{\rho^2 - L^2}{R^2} dt^2 + \rho^2 d\Sigma_{d-1}^2$$

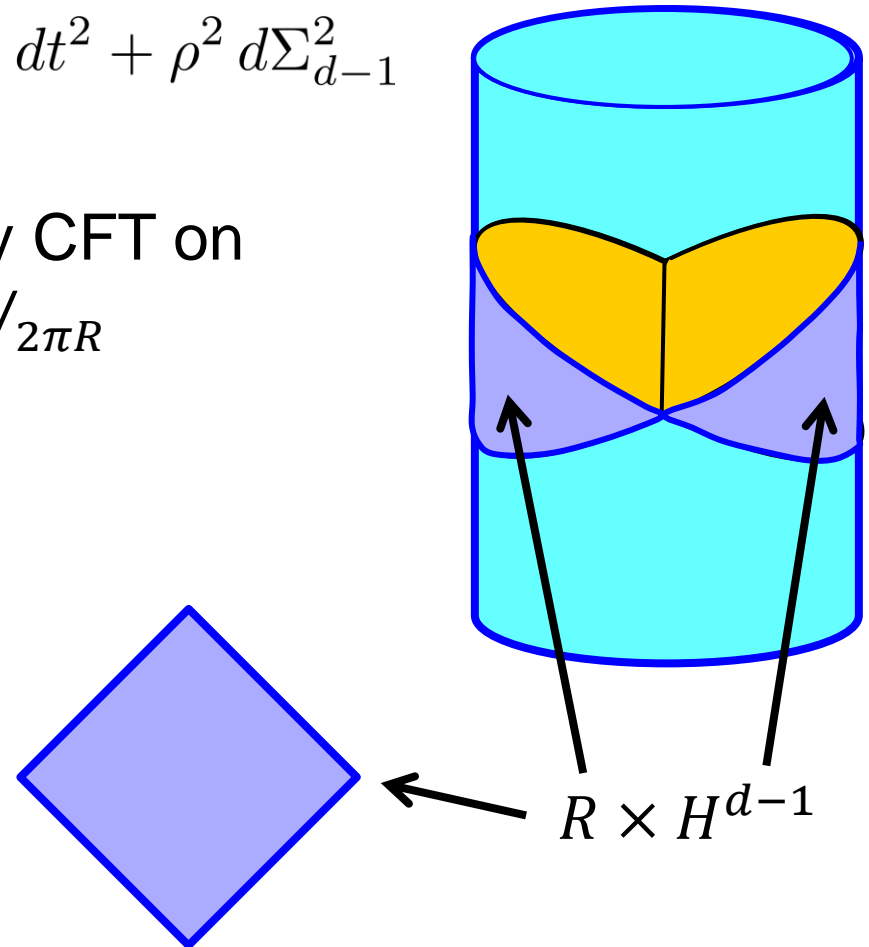


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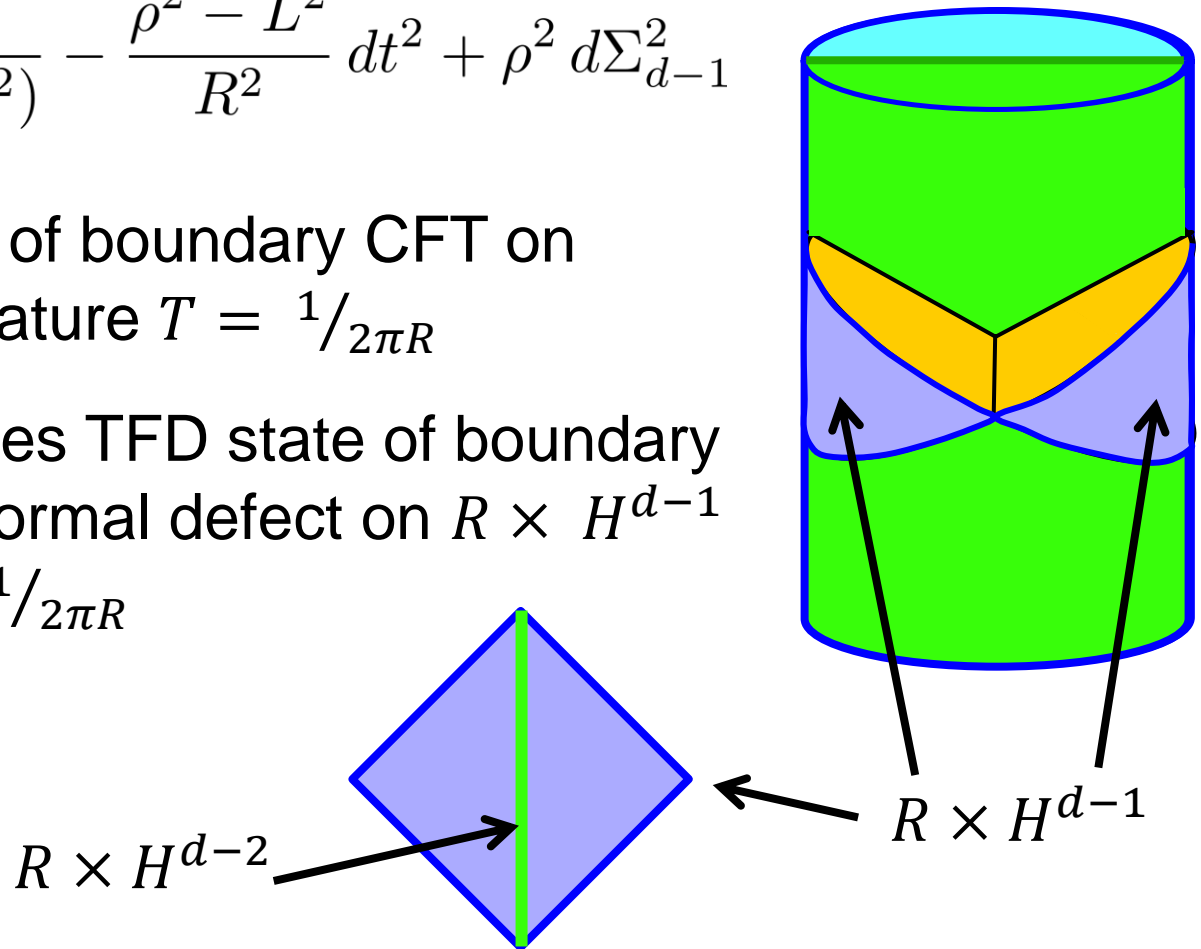


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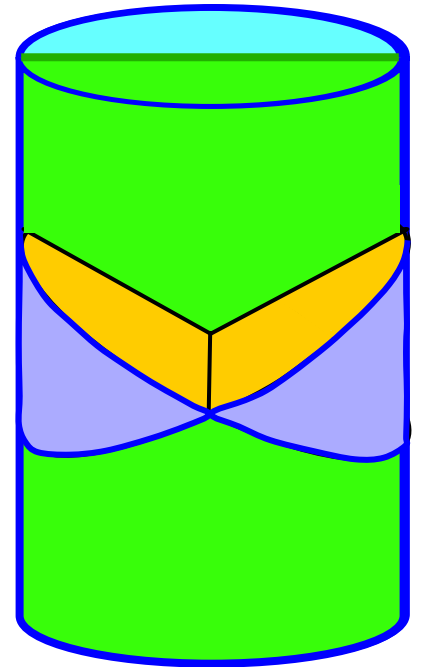


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- insert brane, describes TFD state of boundary CFT coupled to conformal defect on $R \times H^{d-1}$ at temperature $T = 1/2\pi R$
- induced brane metric “inherits” hyperbolic black hole geometry

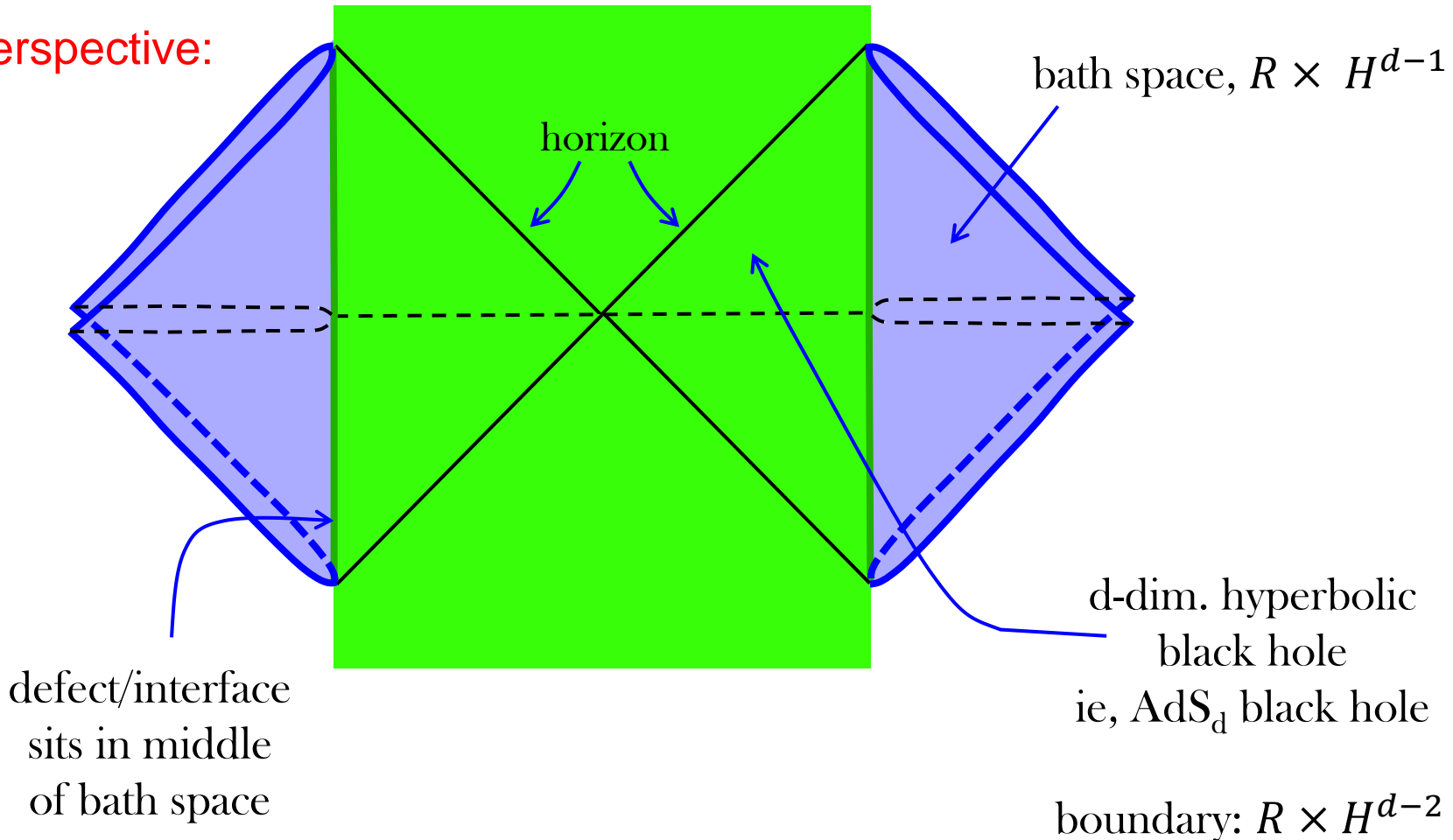


$$ds^2 = \frac{\ell_B^2 d\tilde{\rho}^2}{\tilde{\rho}^2 - \ell_B^2} - \frac{\tilde{\rho}^2 - \ell_B^2}{R^2} dt^2 + \tilde{\rho}^2 d\Sigma_{d-2}^2$$

Black Holes in Thermal Equilibrium:

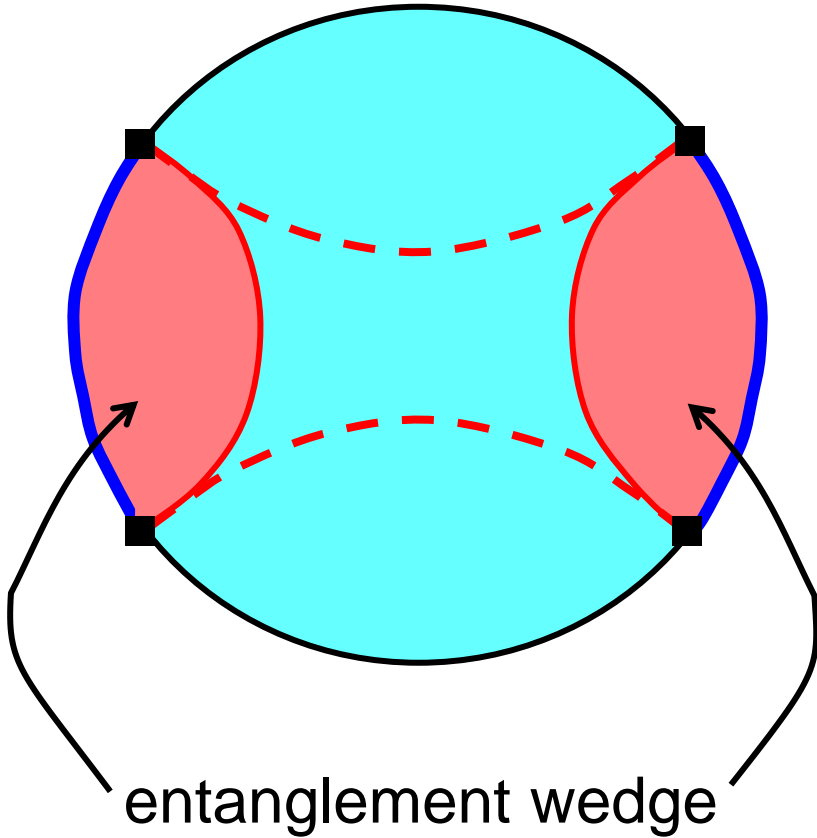
- previous discussion lifts to higher dim'l holographic model with $d=2$ JT gravity \rightarrow induced d -dim. Einstein gravity (& CFT)

Brane perspective:

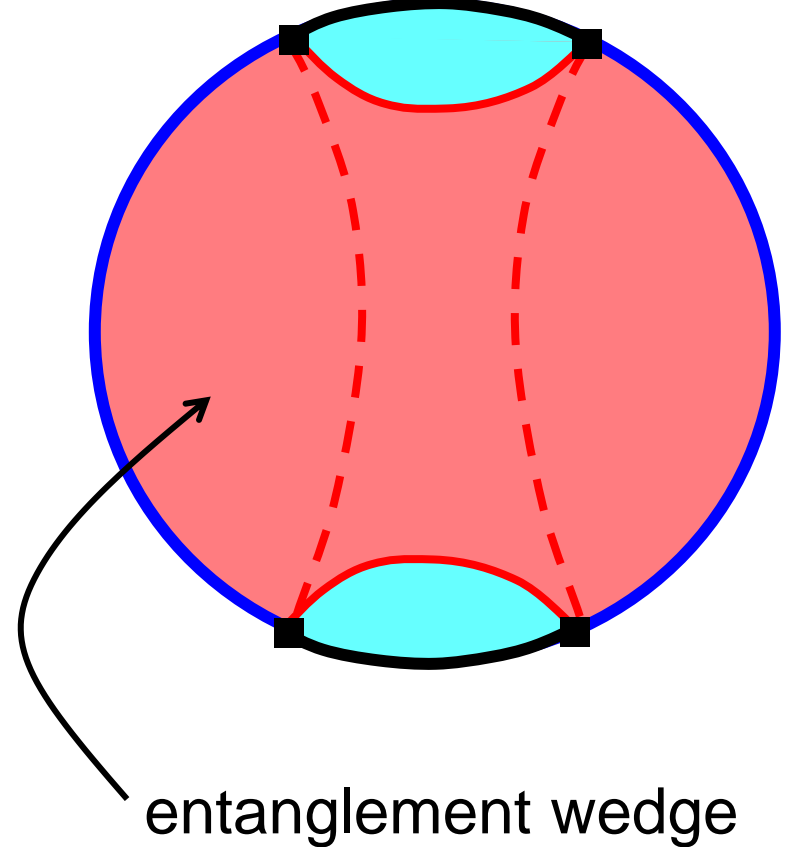


→ recall familiar holographic EE scenario:
two saddles compete to give minimal RT surface

“disconnected phase”

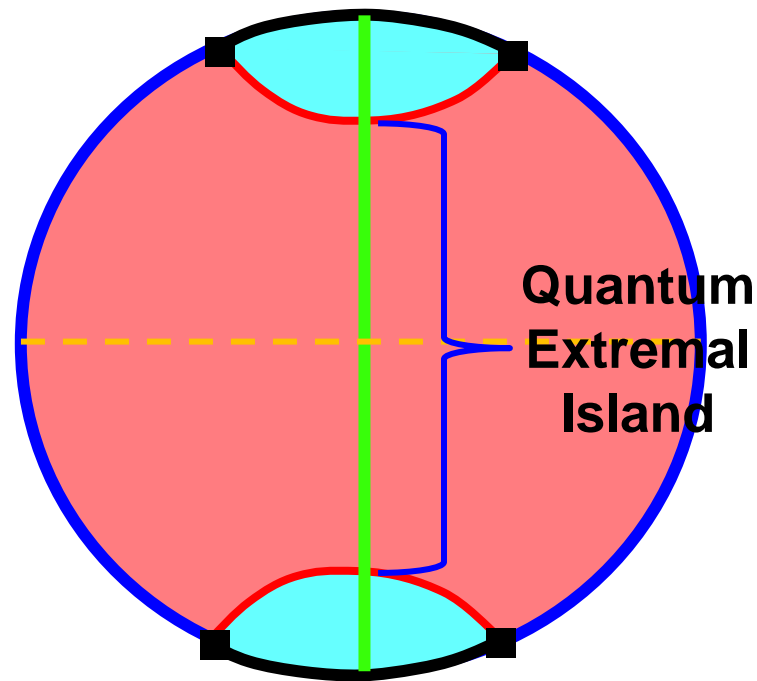
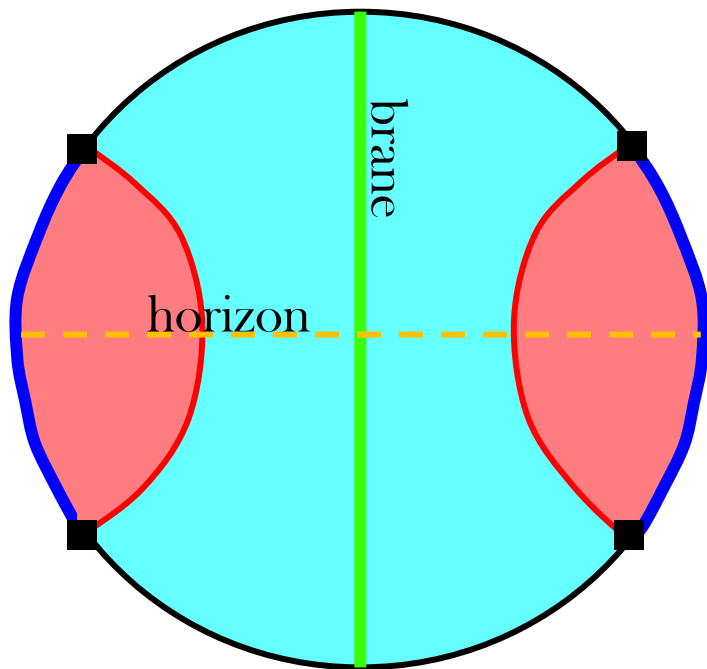


“connected phase”



- entanglement wedge reconstruction: can recover bulk operators (within code subspace) inside entanglement wedge with boundary CFT operators in corresponding boundary subregion

→ recall familiar holographic EE scenario:
two saddles compete to give minimal RT surface



Early times:

- RT surfaces join opposite sides of BH → EE grows with time
- entanglement wedge close to boundary

→ **growth phase**

Late times:

- RT surfaces on single side of BH → EE fixed in time
- entanglement wedge extends through brane → **QE island**

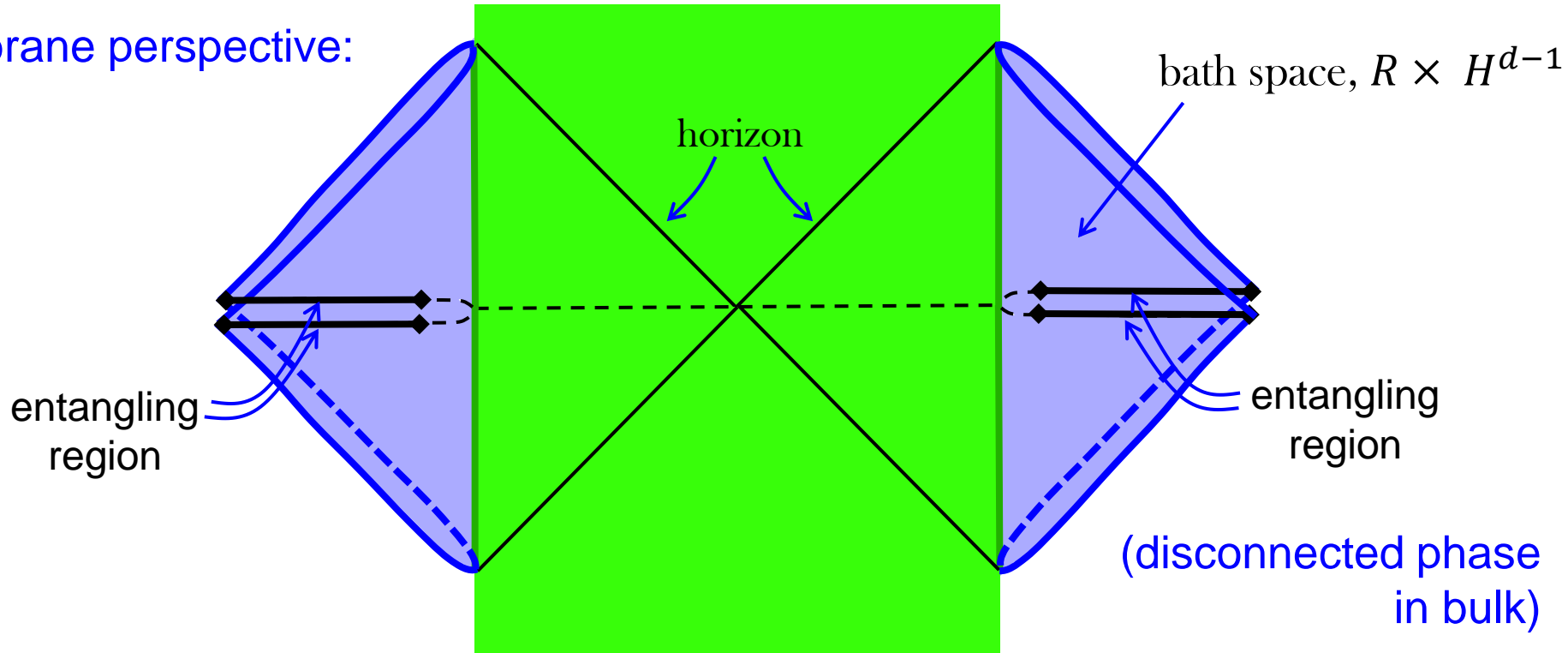
→ **Page phase**

Black Holes in Thermal Equilibrium:

- previous discussion lifts to higher dim'l holographic model with $d=2$ JT gravity \rightarrow induced d -dim. Einstein gravity (& CFT)
- new model reproduces the island formula:

$$S_{EE}(\mathbf{R}) = \min \left\{ \text{ext}_{\text{islands}} \left(S_{QFT}(\mathbf{R} \cup \text{islands}) + \frac{A(\partial(\text{islands}))}{4G_N} \right) \right\}$$

brane perspective:



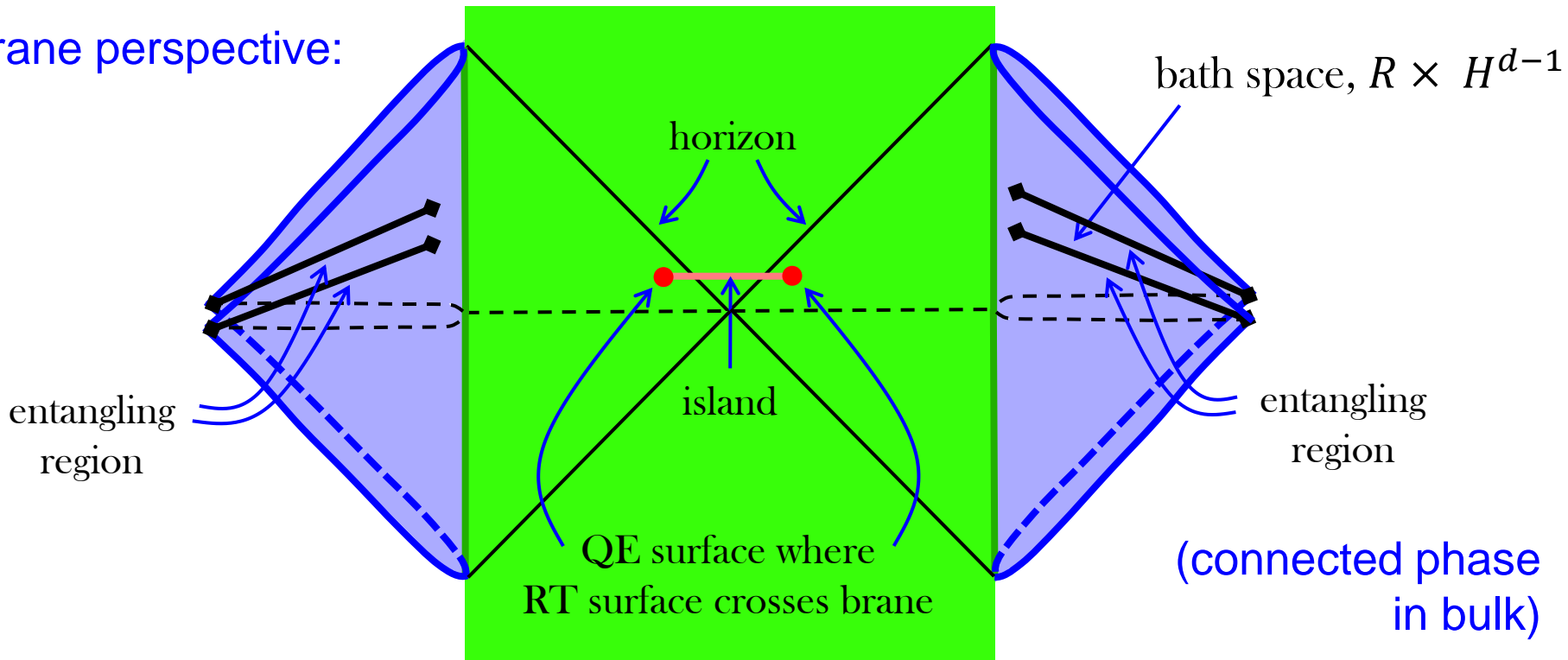
Early times: standard QFT rules apply (no island); EE grows

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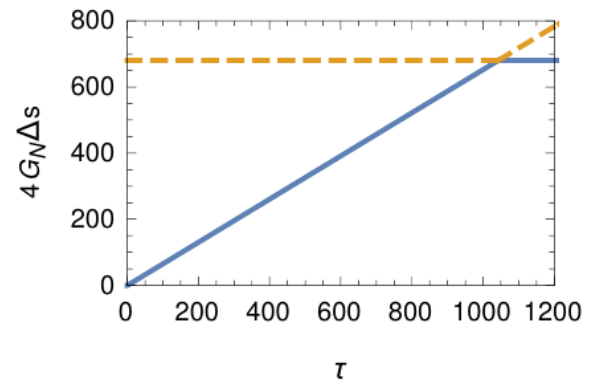
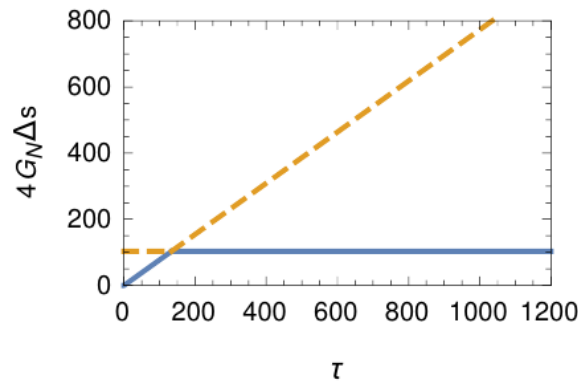
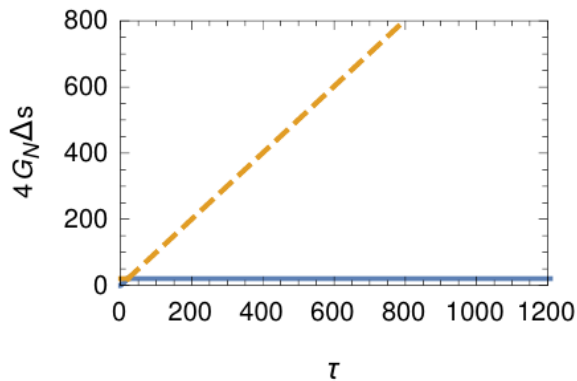
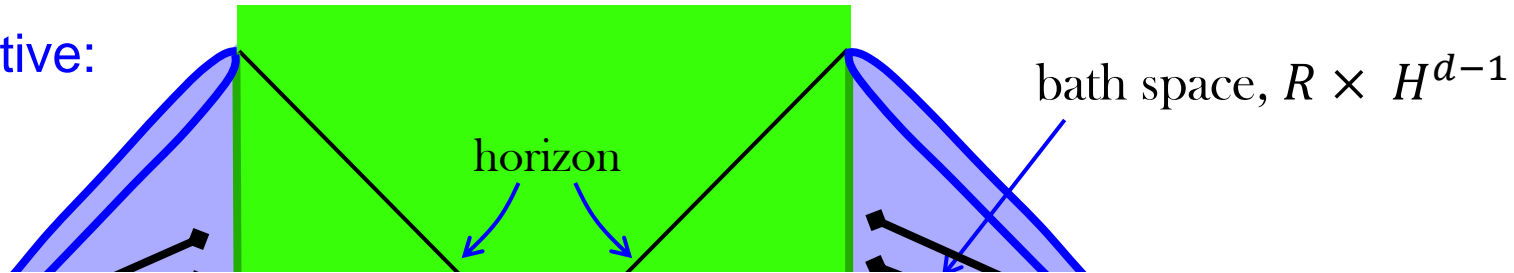
Late times: quantum extremal island forms; EE saturated

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brane perspective:



Late times: quantum extremal island forms; EE saturated

Questions, Questions, Questions:

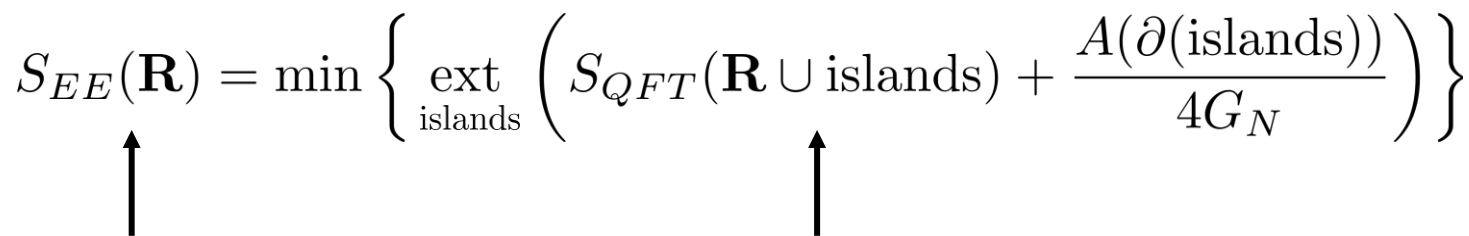
- how important is two dimensions?
 - **not at all**, our construction extends discussion to gravity and black holes in d dimensions
(see also: [Almheiri, Mahajan & Santos](#))
- was JT gravity important?
 - **no**, our construction extends discussion to **Einstein** gravity and black holes in d dimensions
- was ensemble average of SYK model important?
 - **no**, our construction relies on standard rules of AdS/CFT correspondence, ie, do **not** average over couplings in boundary CFT

(Note top-down construction with $D3 \perp D5$ by [Karch & Randall](#))

Questions, Questions, Questions:

- Almheiri, Mahajan & Maldacena distinguish “*full quantum description*” of radiation and “*semiclassical description*” which includes outgoing radiation and purifying partners on QE island (ie, boldface notation)

Island Rule:

$$S_{EE}(\mathbf{R}) = \min \left\{ \underset{\text{islands}}{\text{ext}} \left(S_{QFT}(\mathbf{R} \cup \text{islands}) + \frac{A(\partial(\text{islands}))}{4G_N} \right) \right\}$$


“*full quantum description*”

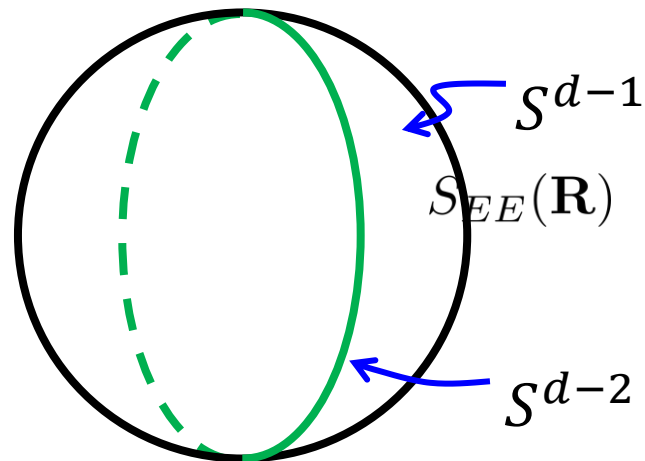
“*semiclassical description*”

- what’s up with that?

Randall-Sundrum gravity:

Boundary perspective

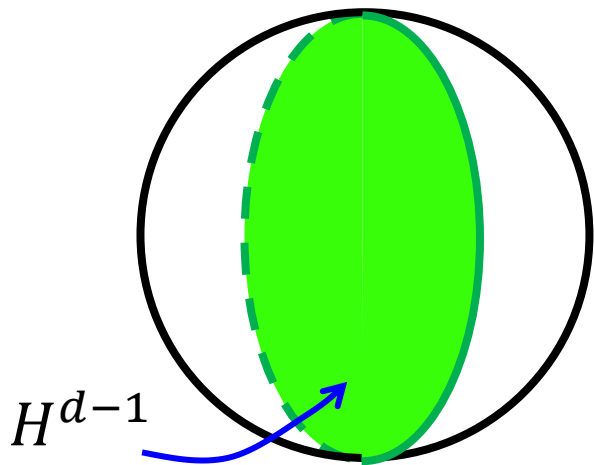
(a) boundary CFT_d coupled to conformal defect (ie, boundary CFT_{d-1})



Brane perspective

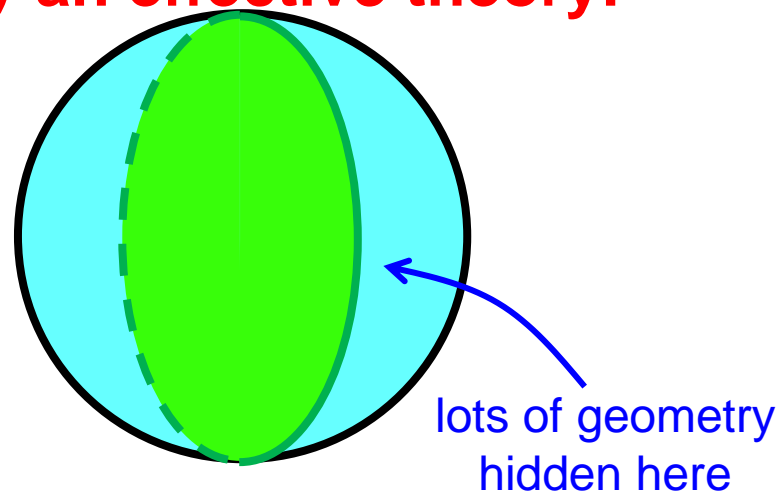
(b) boundary CFT_d coupled to CFT_d with gravity on AdS_d

only an effective theory!



Bulk perspective

(c) AdS_{d+1} gravity coupled to brane with AdS_d geometry



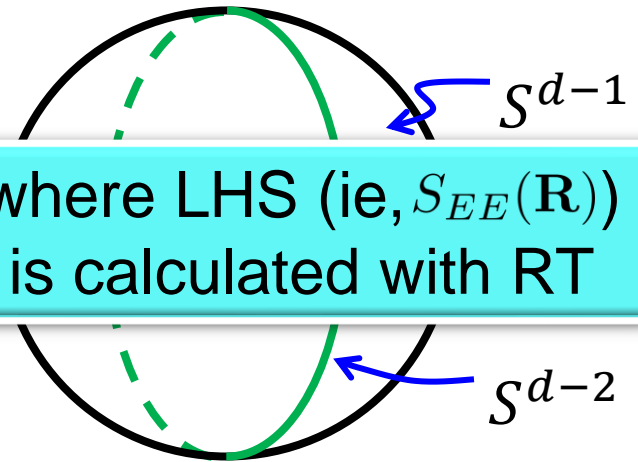
AdS/CFT correspondence

Randall-Sundrum gravity:

Boundary perspective

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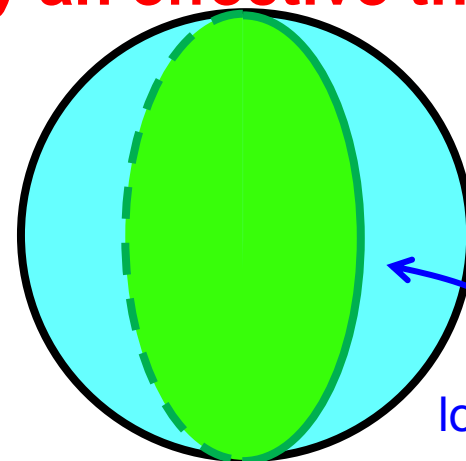
where LHS (ie, $S_{EE}(\mathbf{R})$) is calculated with RT



Brane perspective

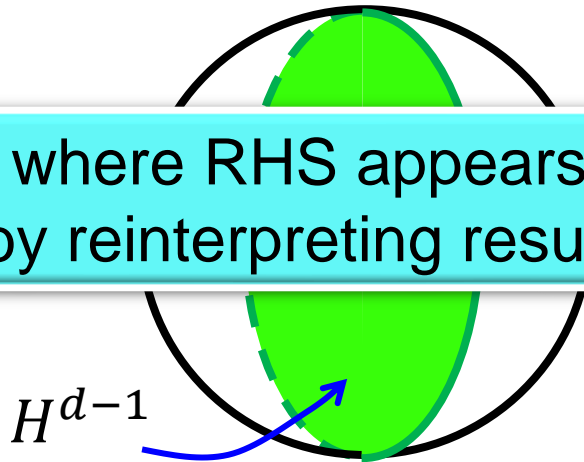
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only an effective theory!



AdS/CFT correspondence

where RHS appears by reinterpreting result



Bulk perspective

(c) AdS_{d+1} gravity coupled to brane with AdS_d geometry

lots of geometry hidden here

Randall-Sundrum gravity:

Boundary perspective

(a) boundary CFT_d coupled to conformal defect (ie, boundary CFT_{d-1})

where LHS (ie, $S_{EE}(\mathbf{R})$) is calculated with RT

Brane perspective

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- provides mnemonic for “effective” gravitational theory
- within this framework, can not reveal “hidden” correlations
compare: Akers, Engelhardt & Harlow

CFT correspondence

H^{d-1}

S^{d-1}

S^{d-2}

Conclusions:

- simple holographic model illustrates the appearance of quantum extremal islands
- new insights viewed as familiar properties of holographic EE
- has information paradox been solved?
NO, not yet!
- Page phase can be described by saddle point without revealing microscopic details with large- N !
→ what/how learn about microstates and information?

Still lots to explore!