Semiclassical $\mathcal{S}$–matrix & black hole entropy in dilaton gravity

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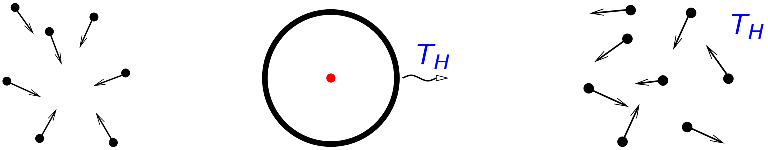
M. Fitkevich, DL, Y. Zenkevich, `arXiv: 2004.13745`

M. Fitkevich, DL, S. Sibiryakov, `arXiv: 2006.03606`

Information paradox = apparent unitarity loss

$\hbar = c = k_B = M_{pl} = 1$

Black holes evaporate, $T_H = (8\pi M)^{-1}$ *Hawking '71*



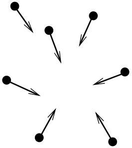
$\Psi_{\rm in}$ $\rightarrow$ BH $\rightarrow$ $\hat{\rho}_{\rm out} = {\rm e}^{-\hat{H}/T_H} \neq |\Psi_{\rm out}\rangle\langle\Psi_{\rm out}|$

Non–unitary evolution?

BUT

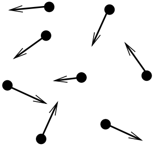
- AdS/CFT: QGRA on $AdS_5 \times S_5$ = unitary CFT *Maldacena '01*
  black holes = thermal states
  unitary $\Leftarrow$ unitary
- New entanglement entropy: $\Sigma = -{\rm tr}(\hat{\rho}_{\rm out} \ln \hat{\rho}_{\rm out})$
  "purification" of states $\Rightarrow$ unitarity! *Penington '19*

see also Almheiri, Hartman, Maldcena, Marolf, Stanford, Shenker, . . . '19-20



Ψ_{in}

$$\Psi_{\text{in}} \to \Psi_{\text{out}} = \hat{\mathcal{S}}\Psi_{\text{in}}$$



Ψ_{out}

This is a scattering process!

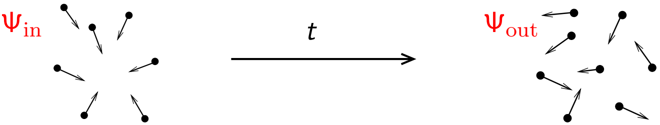
$\mathcal{Q}$: $\hat{\mathcal{S}}$–matrix for this process?

Unitarity test: $\hat{\mathcal{S}}^\dagger \hat{\mathcal{S}} = \hat{I}$

't Hooft '99

Bezrukov, DL, Sibiryakov '15

Direct semiclassical method



$$\langle \Psi_{\rm out} | \hat{\mathcal{S}}_{\rm tot} | \Psi_{\rm in} \rangle = \int \underbrace{\mathcal{D}\Phi}_{g_{\mu\nu},\, f,\, \dots} \; \mathrm{e}^{iS_{\rm tot}[\Phi]}$$

$\leftarrow$ includes $\Psi_{\rm in}$, $\Psi_{\rm out}$

$\leftarrow$ flat $\to$ flat!

$S_{\rm tot} \gg 1$: Saddle–point method!

- $\Phi_s(x)$: $\delta S_{\rm tot}/\delta\Phi = 0$ — saddle–point configuration (classical solution)
- Low E: flat $\to$ flat — fixed topology
- $\langle \Psi_{\rm out} | \hat{\mathcal{S}} | \Psi_{\rm in} \rangle = \not{\mathcal{A}}\, \mathrm{e}^{iS_{\rm tot}[\Phi_s]}$ — ответ

BUT collapsing solutions: flat $\to$ black hole

incorrect asymptotics!

Complex saddle–point configurations?

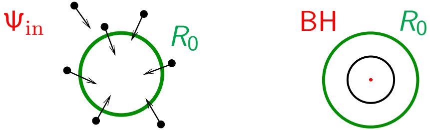
How to find them?

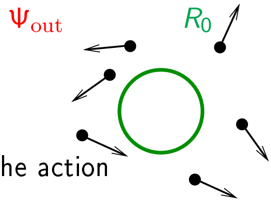
Bezrukov, DL, Sibiryakov, 1503.07181

Imagine that we can do nasty things

DL, Panin, Sibiryakov '07
Bezrukov, DL, Sibiryakov '15

Consider functional $T_{\rm int}[\Phi] \sim \int \mathbf{Mass}_{R_0}\, dt$





Add a constraint $T_{\rm int}[\Phi] = T_0$ to the action

$\Downarrow$

Correct asymptotics of Φ_s!

(but incorrect equations)

Properties of $T_{\rm int}[\Phi]$

- ≥ 0 & rep-invariant
- $= +\infty$ if $\Phi \to$ black hole
- Finite for scattering

Faddeev–Popov unity

$$1 = \int_0^\infty dT_0\, \delta(T_0 - T_{\rm int}[\Phi]) = \int_0^\infty dT_0 \int_{-i\infty}^{i\infty} d\epsilon\; e^{\epsilon(T_0 - T_{\rm int}[\Phi])}$$

$$\langle \Psi_{\rm in} | \hat{S} | \Psi_{\rm out} \rangle = \int \mathcal{D}\Phi \;\cdot\; e^{iS_{\rm tot}[\Phi]}$$

Bezrukov, DL '04

DL, Panin, Sibiryakov '07

Strategy:

1. **First, integrate over Φ**
   - $\Rightarrow$ New action: $S_\epsilon = S_{\rm tot} + i\epsilon T_{\rm int}$
   - $\Rightarrow \Phi_\epsilon \Rightarrow \Phi_\epsilon[x]$ — saddle–point of : flat at $t \to \pm\infty$
   - $\Rightarrow \mathrm{Re}\,\epsilon > 0$ — integral converges!
2. **Second, integrate over ϵ и T_0**
   - $\Rightarrow T_0 = T_{\rm int}[\Phi_\epsilon]$ & $\epsilon \to +0$

$$\langle \Psi_{\rm in} | \hat{S} | \Psi_{out} \rangle \approx \lim_{\epsilon \to +0} e^{iS_\epsilon[\Phi_\epsilon]}$$

In 4D gravity: *Bezrukov, DL, Sibiryakov, 1503.07181*

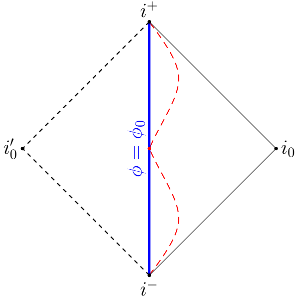
$$\mathcal{S} = \int\limits_{\phi<\phi_0} d^2x\sqrt{|g|}e^{-2\phi}\left[R + 4(\nabla\phi)^2 + 4\lambda^2\right] \leftarrow \text{CGHS: } \phi,\ g_{\mu\nu}$$

↑ mass scale

Callan, Giddings, Harvey, Strominger '92

$$+ \int\limits_{\phi=\phi_0} \mathrm{e}^{-2\phi_0}(2K + 4\lambda) \leftarrow \text{boundary GH term}$$

$$-m \int\limits_{\text{traj}} d\tau \leftarrow \text{matter: quantum particle}$$



$\phi < \phi_0 \Rightarrow \boxed{r > r_0}$

Properties

- Exact solutions
- Stable vacuum
- Black holes:

  $T_H = \lambda/2\pi$ — temperature

  $r_h = \ln(M/2\lambda)/2\lambda$ — Schw. radius

Chung, Verlinde '93

Fitkevich, DL, Zenkevich '17

- **Quantize the model:** *Russo, Susskind, Thorlacius '92*

$$Z = \int \underbrace{\mathcal{D}\Phi}_{\text{Strominger '92}} \; e^{iS_{\rm CGHS} - c\int \phi R + \text{matter}}$$

$\uparrow$ *RST counterterm*

- **Perform singular Weyl transformation:**

$$\boxed{\hat{g}_{ab} = e^{-2\phi} g_{ab}\,, \quad \hat{\phi} = e^{-2\phi} + c\phi} \leftarrow \underline{\text{non–invertable at } \phi(x) = \phi_{cr}!}$$

- **Result:** $Z = \int \underbrace{\mathcal{D}\hat{\Phi}\; e^{i\int \left(\hat{\mathcal{R}}\hat{\phi} + 4\lambda^2\right) + \text{материя}}}_{\text{flat JT gravity } (\Lambda \to 0)}$ *Fitkevich, DL, Zenkevich '20*

- **BUT flat JT is unitary** $\leftarrow$ and no black holes

Dubovsky, Gorbenko, Mirbabayi '17

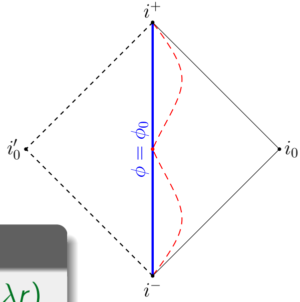
- $\Rightarrow$ **CGHS is sick at $\phi = \phi_{cr}$!**

Solution: $\phi < \phi_0$ — cutoff the singularity!

Reflections from the boundary

$$\boxed{\langle p|\hat{S}|-p\rangle = \int \mathcal{D}\Phi\; e^{iS_{tot}[\Phi]}}$$ — refl. amplitude

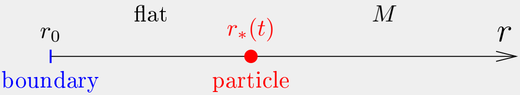
(energy M)



(Semi)classical solutions

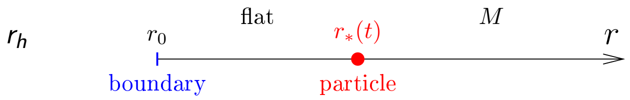
- **Schwarzschild coordinates:** $(r,\ t) \leftarrow (\phi = -\lambda r)$
- **"Birkhoff" theorem:**

  empty space = flat or BH(M)



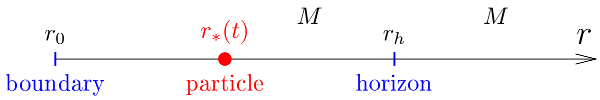
- **Sewing at r_*:** eq. for particle $r = r_*(t)$

- Low M



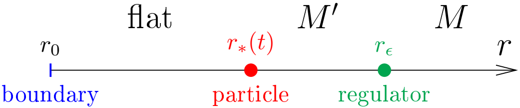
Trivial reflection from the boundary

- High M $\Leftrightarrow$ $r_h > r_0$



Black hole formation at $M > M_{cr}$

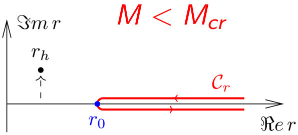
$$i\epsilon T_{\rm int} = i\epsilon \int d^2x \sqrt{-g}\; \underbrace{\delta(\phi - \phi_\epsilon)}_{r_\epsilon = -\phi_\epsilon/\lambda}\; \underbrace{\left[(\nabla\phi)^2 - \lambda^2\right]^2}_{\text{zero for flat}} \geq 0$$

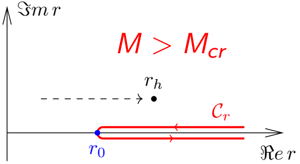


- Eqs. at r_ϵ: $\boxed{M' = M + i\epsilon'}$ $\epsilon' > 0$
- Complex motion: $\mathrm{Im}\, r_h > 0$!

Properties of $T_{\rm int}$

- rep.–invariant
- ≥ 0
- finite: $\Phi \to$ плоское
- $+\infty$: $\Phi \to$ BH



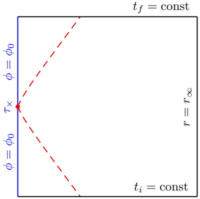


$\Rightarrow$ **Complex solutions with correct asymptotics!**

Classical action: $\langle p|\hat{S}|-p\rangle = \exp(iS_\epsilon)$

$$S_\epsilon = \underbrace{S_{\rm CGHS}}_{\text{gravity}} + \underbrace{S_{{\rm GH},\,\phi_0}}_{\text{boundary}} + \underbrace{S_{{\rm GH},\,\infty}}_{\text{infinity}} + \underbrace{S_m}_{\text{particle}} \underbrace{- i\ln\Psi^*_{\rm out} - i\ln\Psi_{\rm in}}_{\text{states: } \pm ipr}$$

$$\underbrace{-\; S_{\rm free}}_{\text{free action}} + \underbrace{i\epsilon(T_{\rm int} - \tau_0)}_{\text{regulator}} \leftarrow \boxed{\epsilon \to +0}$$



$$S_\epsilon = -\frac{M - M_{\rm cr}}{\lambda}\ln\left(1 - \frac{M + i\epsilon'}{M_{\rm cr}}\right) + \frac{p}{\lambda}\left(1 - \ln\frac{M_{\rm cr}}{2\lambda}\right)$$

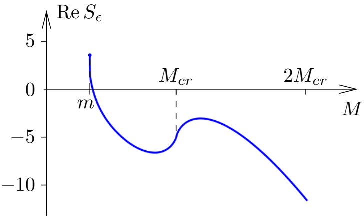
$$-\frac{p}{\lambda}\ln\left(\frac{1}{2} + \frac{Mm^2}{8M_{\rm cr}p^2} + \frac{p_\times}{2p}\right) + \frac{2M_{\rm cr}}{\lambda}\ln\left(\frac{4M_{\rm cr}(p_\times + M) + m^2}{4M_{\rm cr}(p_\times + M) - m^2}\right)$$

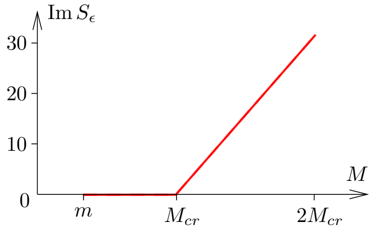
$$+\frac{M}{\lambda}\ln\left(\frac{4M^3 - 3m^2M + (4M^2 - m^2)p_\times}{(p+M)^3} + \frac{m^2(4M^2 + m^2)}{4M_{\rm cr}(p+M)^3}\right)$$

where $p^2 = M^2 - m^2$, $\quad p_\times^2 = (M + m^2/4M_{\rm cr})^2 - m^2$, $\quad M_{cr} = 2\lambda e^{-2\phi_0}$

Result: $\langle p|\hat{S}|-p\rangle = \exp(iS_\epsilon)$

- **Amplitude:** $\mathrm{Im}\, S_\epsilon = \frac{\pi}{\lambda}(M - M_{\rm cr})\,\theta(M - M_{\rm cr})$





- **Probability** $\mathcal{P} \approx \mathrm{e}^{-2\mathrm{Im}\, S_\epsilon} = \begin{cases} 1\,, & M < M_{cr} \\ \mathrm{e}^{-\pi(M-M_{cr})/\lambda}\,, & M > M_{cr} \end{cases}$

- **High energies**: particle → BH → particle

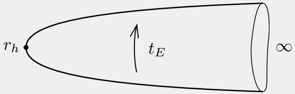
$$\Rightarrow \mathcal{P} \approx N_{BH}^{-1} = \mathrm{e}^{-\Sigma_{BH}}$$

- $\boxed{\Sigma_{BH} = \frac{M - M_{cr}}{T_H} = \frac{2\pi}{\lambda}(M - M_{cr})}$ — BH entropy!

Agrees with Fiola, Preskill, Strominger, Trivedi '94; Myers '94; Hayward '94

Last remark: Euclidean calculation of entropy

Gibbons–Hawking instanton



No points with $\phi = \phi_0$!

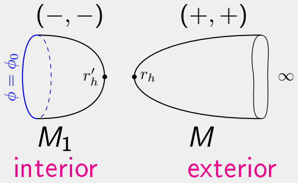
- $\Sigma_{\rm BH} = \frac{2\pi}{\lambda} M$ — gives this instanton
- $\Sigma_{\rm BH} = \frac{2\pi}{\lambda} (M - 2\lambda e^{-2\phi_0})$ — our result

Something wrong with Euclidean calculation?

This solution does not give contribution!

Require that the boundary $\phi = \phi_0$ is present

New instanton



- Minimum of S_E at $M_1 = 2\lambda e^{-2\phi_0}$
- $\Sigma_{\mathrm{BH}} = \frac{2\pi}{\lambda}\left(M - 2\lambda e^{-2\phi_0}\right)$ — agrees!

$\Rightarrow$ **We should require** that the boundary is present

Conclusions

- We have a method to calculate $\langle \Psi_{\rm out} | \hat{\mathcal{S}} | \Psi_{\rm in} \rangle$!
- Collapsing solutions do not contribute into $\hat{\mathcal{S}}$.
- But complex ones do!
- Application to shells in 4D gravity: *Bezrukov, DL, Sibiryakov '15*
- Can we test unitarity of $\hat{\mathcal{S}}$?
  Coherent states:
  $$\langle a | \hat{I} | b \rangle = \mathrm{e}^{\int dk\, a_k^* b_k} = \sum_c \underbrace{\langle a | \hat{\mathcal{S}}^\dagger | c \rangle \, \langle c | \hat{\mathcal{S}} | b \rangle}_{\text{calculate semiclassically!}}$$
  BUT: need solutions in field theory!
- Any connection to replica instantons? $\leftarrow \rho_{\rm out}^n = (\Psi_{out}^\dagger \Psi_{\rm out})^n$
  $$\text{replica inst} = 2n \text{ complex solutions?}$$

Thanks for attention!