Quantum Gravity and Cosmology
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Primordial Black Holes

- a couple of recent topics -

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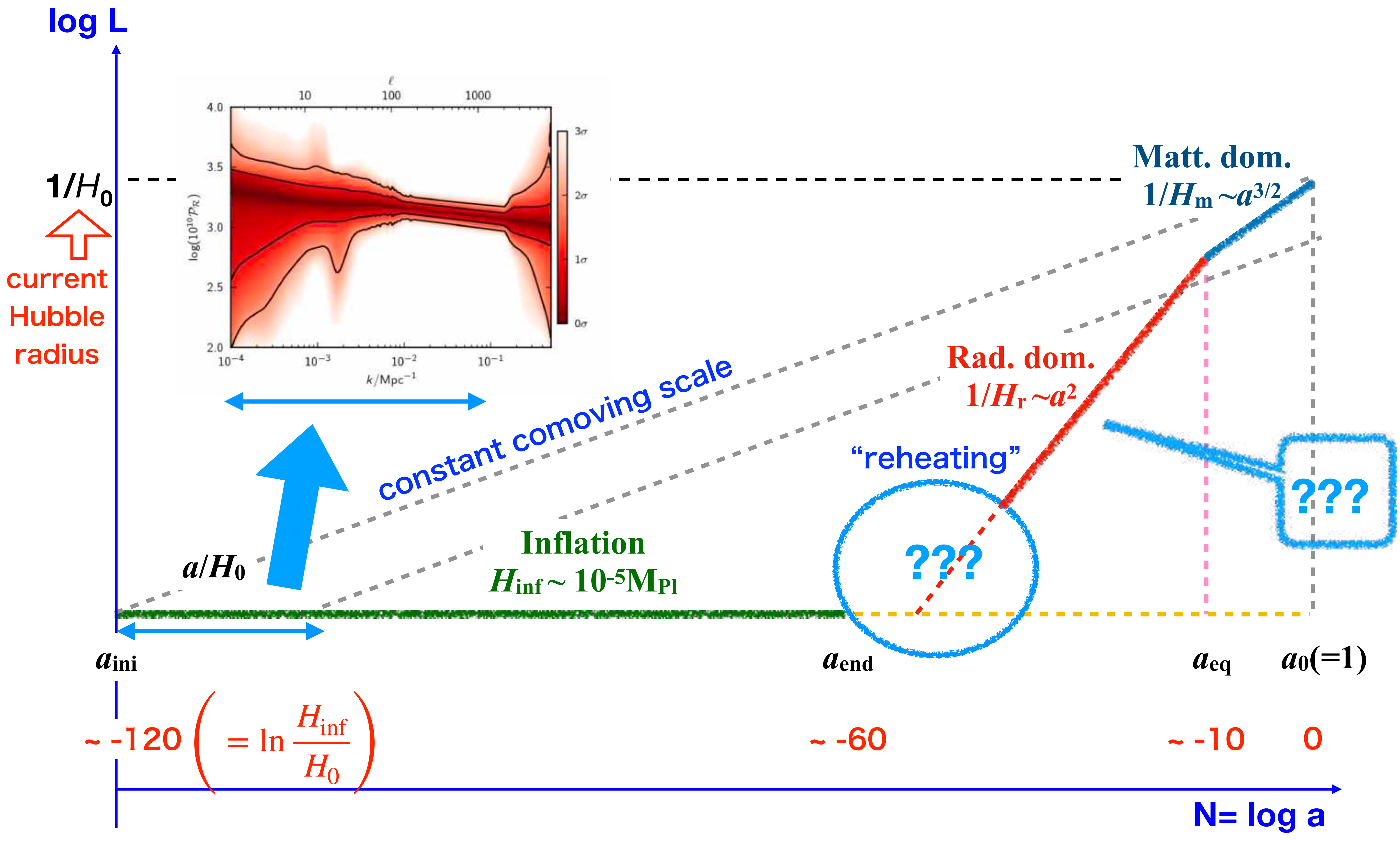
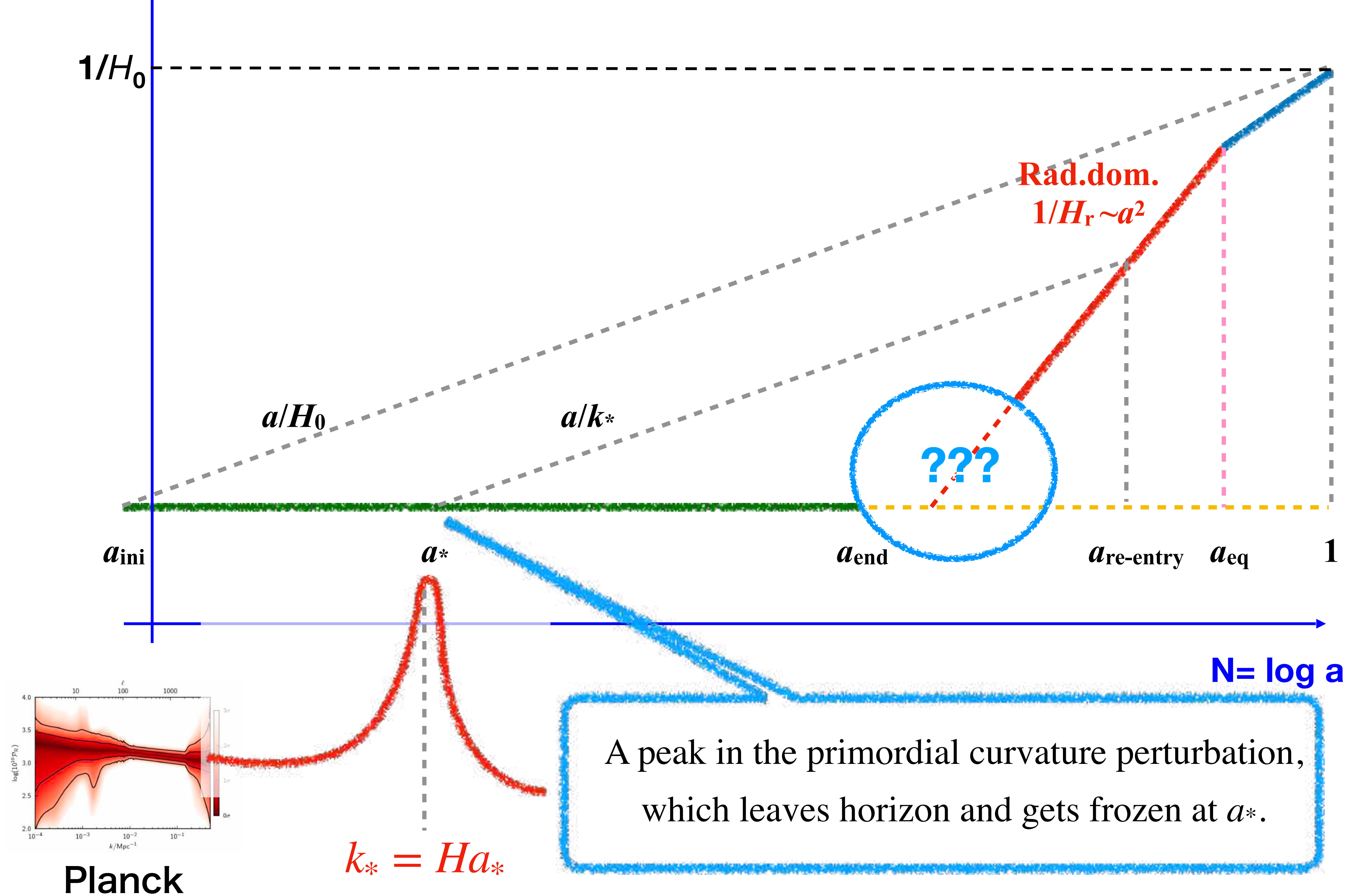
Introduction

curvature perturbation,
formation of PBHs,
and gravitational waves

cosmic spacetime diagram

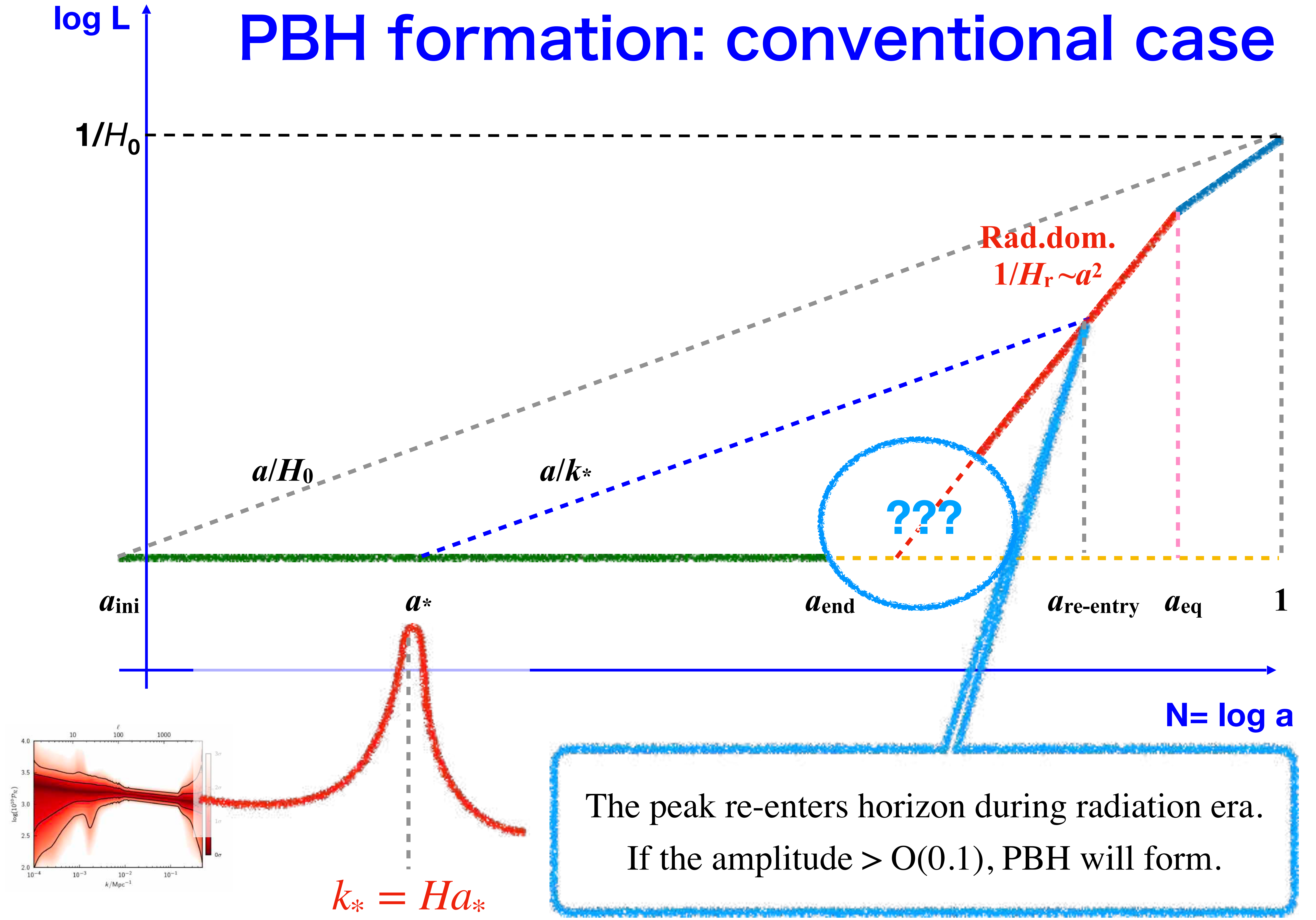


PBH formation: conventional case



A peak in the primordial curvature perturbation, which leaves horizon and gets frozen at a_*.

PBH formation: conventional case



Curvature perturbation to PBH

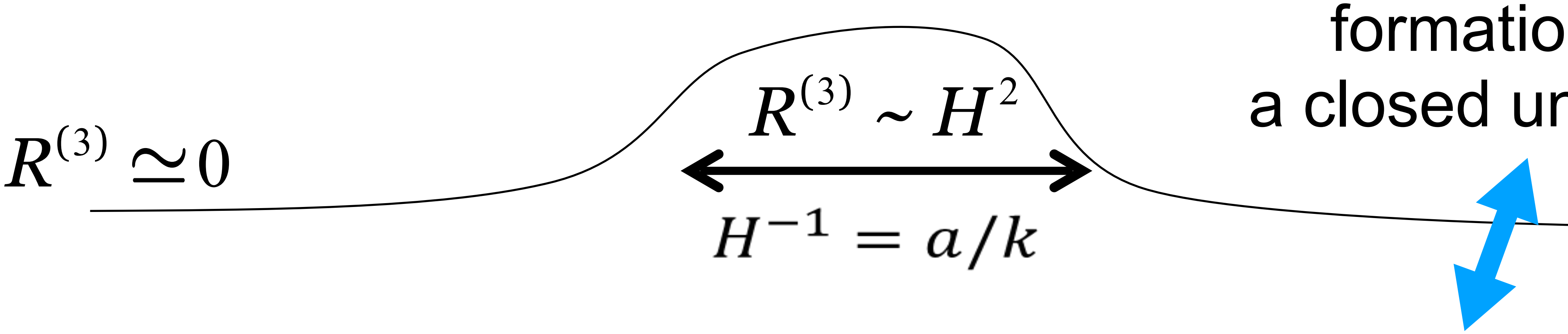
conventional (PBH formation at rad-dominance) case

➢ gradient expansion/separate universe approach

$$6H^2(t,x) + R^{(3)}(t,x) = 16\pi G\rho(t,x) + \cdots$$

Hamiltonian constraint (Friedmann eq.)

$$\Rightarrow \quad R^{(3)} \approx -\frac{4}{a^2}\nabla^2\mathcal{R}_c \approx \frac{8\pi G}{3}\delta\rho_c \quad \Rightarrow \quad \frac{\delta\rho_c}{\rho} \approx \mathcal{R}_c \text{ at } \frac{k^2}{a^2} = H^2$$



formation of a closed universe

➢ If $R^{(3)} \sim H^2$ $(\Leftrightarrow \delta\rho_c/\rho \sim 1)$, it collapses to form BH

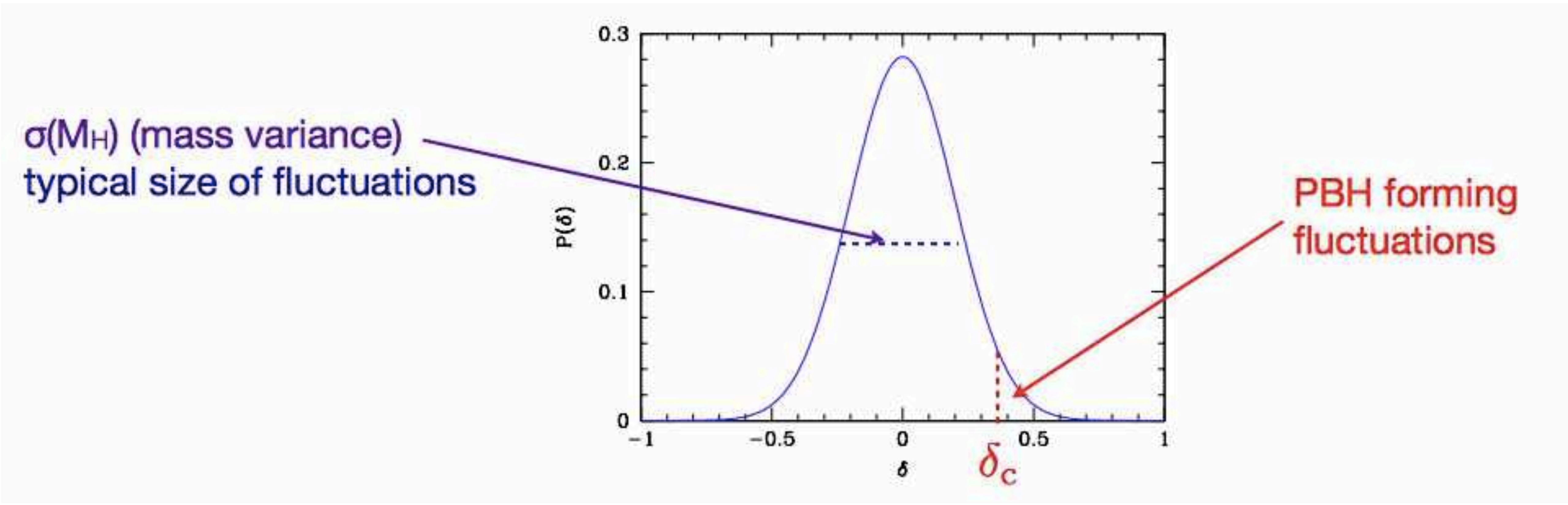
Young, Byrnes & MS '14

➢ Spins of PBHs are expected to be very small

De Luca et al. 1903.01179, …

fraction β that turns into PBHs

for Gaussian probability distribution



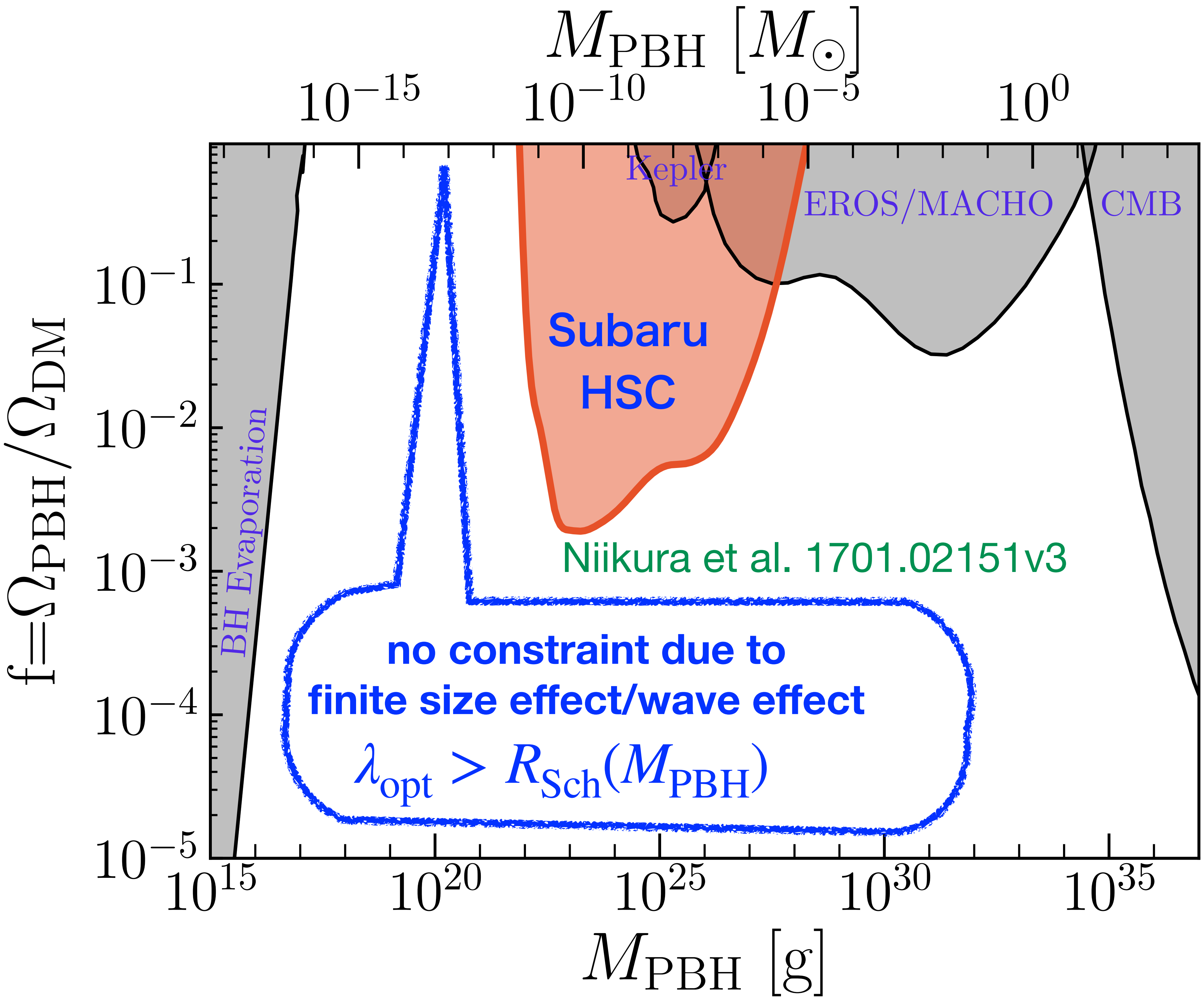
- When $\sigma_M << \delta_c$, β can be approximated by exponential:

$$\beta \approx \sqrt{\frac{2}{\pi}} \frac{\sigma_M}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)$$

$$\delta_c \equiv \left(\frac{\delta\rho_c}{\rho}\right)_{\rm crit} \sim 0.4$$

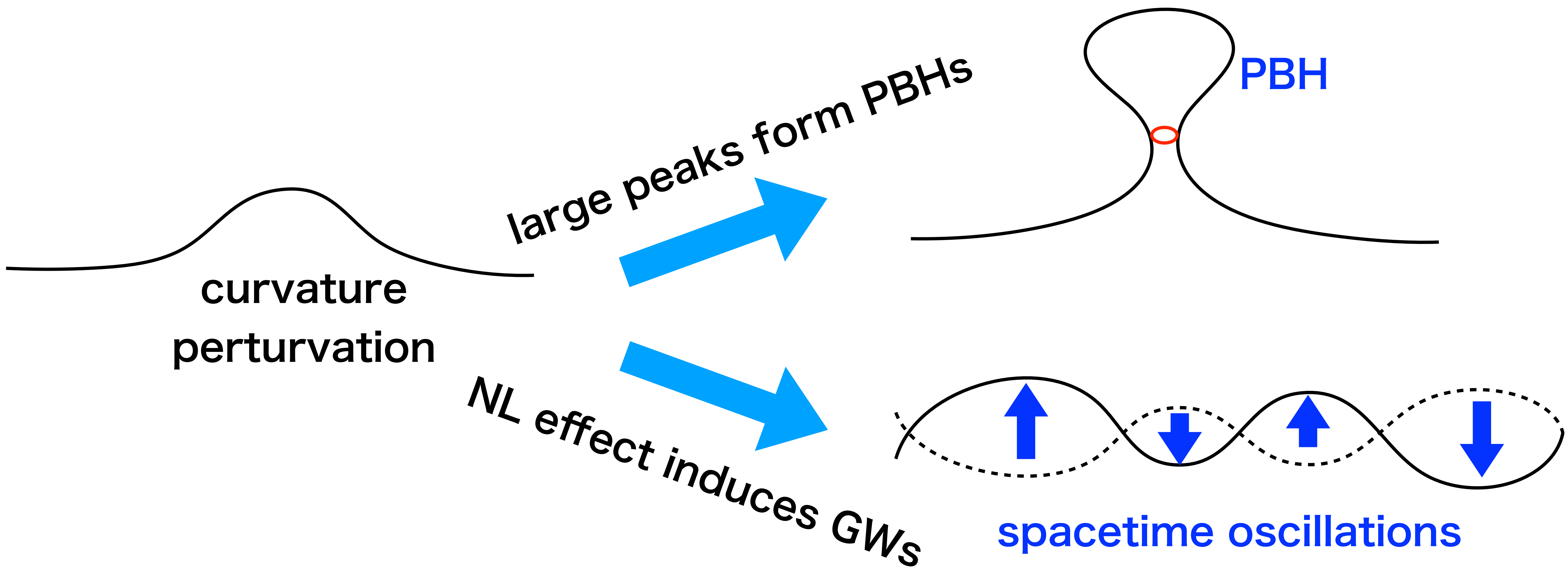
Carr ’75, …

PBH constraints



big window at $M_{\rm PBH} \approx 10^{17} - 10^{22}$ g ⟷ $T_{\text{re-entry}} \sim 10^4$ - 10^8 GeV

GWs can capture PBHs!

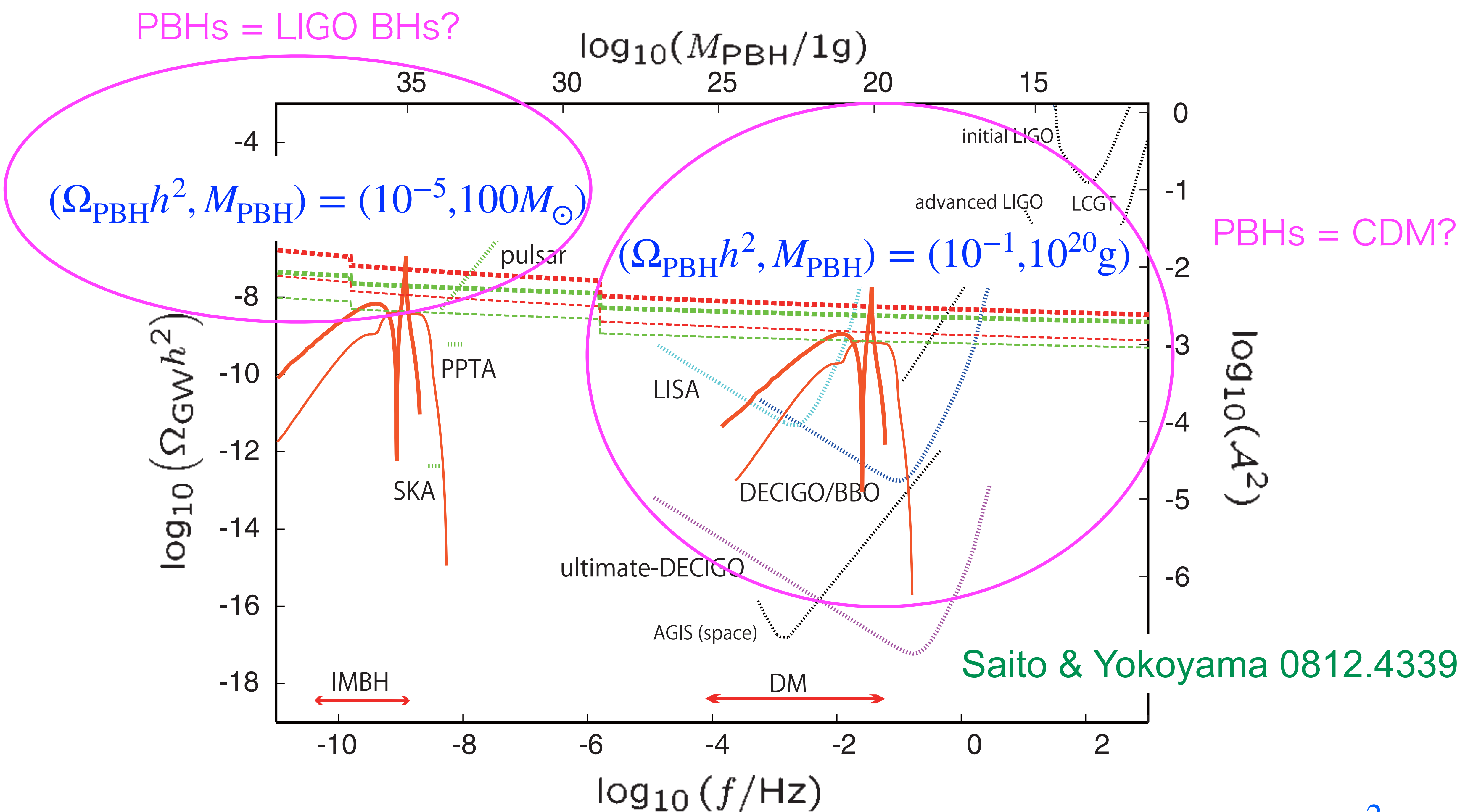


PBHs = CDM with M_{PBH} ~10^{21}g
generates GWs with f~10^{-3} Hz

Background GWs
at LISA band

LIGO-Virgo :10 - 1000 Hz

GWs can test PBH scenario!



Saito & Yokoyama 0812.4339

$$M_{PBH} \sim 0.1 M_\odot \left(\frac{1\mathrm{GeV}}{T}\right)^2 \sim 10 M_\odot \left(\frac{1\mathrm{pc}^{-1}}{k}\right)^2$$

So far, we have focused on PBH formation from primordially adiabatic perturbation

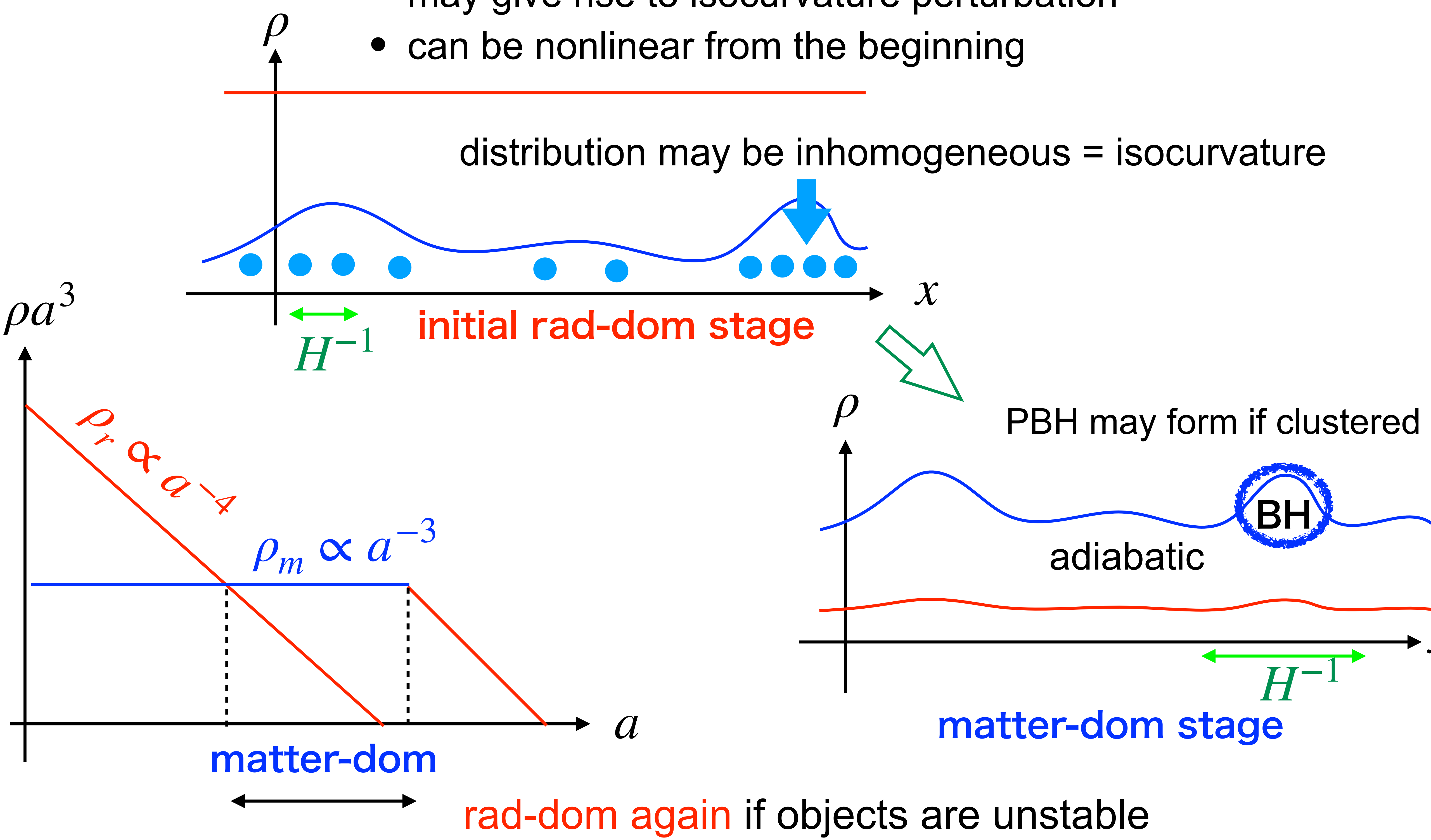
How about primordially isocurvature perturbation?

PBHs from Isocurvature Perturbation

eg, E. Cotner, A. Kusenko, MS & V. Takhistov,1907.10613

non-gravitational formation of compact objects/Q-balls/etc inside horizon.

- may give rise to isocurvature perturbation
- can be nonlinear from the beginning



linear theory

H. Kodama & MS, IJMPA 1 (1986) 265, ibid 2 (1987) 491

matter isocurvature perturbation

$$S \equiv \delta_m - \frac{3}{4}\delta_r \to \delta_m \text{ at } a \to 0$$ (on, say, uniform total density slices)

evolution for $\omega \ll 1$ $\quad \omega \equiv \left(\frac{k}{Ha}\right)_{eq}, \quad R \equiv \frac{a}{a_{eq}}$ modes that are superhorizon at equality

$R \ll 1$ (rad dom)

$$\begin{cases} \mathcal{R}_c = \frac{R}{4} S \quad \left(\Phi = \frac{R}{8} S\right) \\ \delta = \frac{1}{6}\omega^2 R^3 S \end{cases}$$

⇨

$1 \ll R$ (matter dom)

$$\begin{cases} \mathcal{R}_c = \frac{1}{3} S \quad \left(\Phi = \frac{1}{5} S\right) \\ \delta = \frac{4}{15}\omega^2 R\, S \end{cases}$$

$\mathcal{R}_c$:curv pert on comoving slice

Φ : curv pert on Newton slice

horizon crossing: $\omega^2 R = \frac{1}{2}$

⇩

BH formation criterion: $\delta(k = aH) = \frac{2}{15} S > \delta_{\rm cr}\ (\sim 0.5)$ **?**

linear theory continued

evolution for $\omega \gg 1$ (modes that enters horizon before equality)

$$\omega \equiv \left(\frac{k}{Ha}\right)_{eq} \,, \quad R \equiv a/a_{eq}$$

$R \ll \omega^{-1}$ (rad dom)

$$\begin{cases} \mathscr{R}_c = \dfrac{R}{4} S \\ \delta = \dfrac{1}{6}\omega^2 R^3 S \end{cases}$$

⇒

$\omega^{-1} \ll R \ll 1$ (rad dom)

$$\begin{cases} \mathscr{R}_c = \dfrac{3}{4\omega^2 R} S \\ \delta = R\, S \end{cases}$$

⇒

$1 \ll R$ (matter dom)

$$\begin{cases} \mathscr{R}_c = \dfrac{5}{4\omega^2} S \\ \delta = \dfrac{3R}{2} S \end{cases}$$

$$\left(\Phi = \frac{3}{4\omega^2} S\right)$$

conventional growth rate at matter-dom stage

horizon crossing: $\omega R = 1/2$

$$\delta(k = aH) = \frac{1}{2\omega} S \,, \quad \mathscr{R}_c = \frac{3}{2\omega} S$$

$\Phi = O(1)$ implies $S = O(\omega^2) \gg 1$!!

highly nonlinear initial condition

⇑

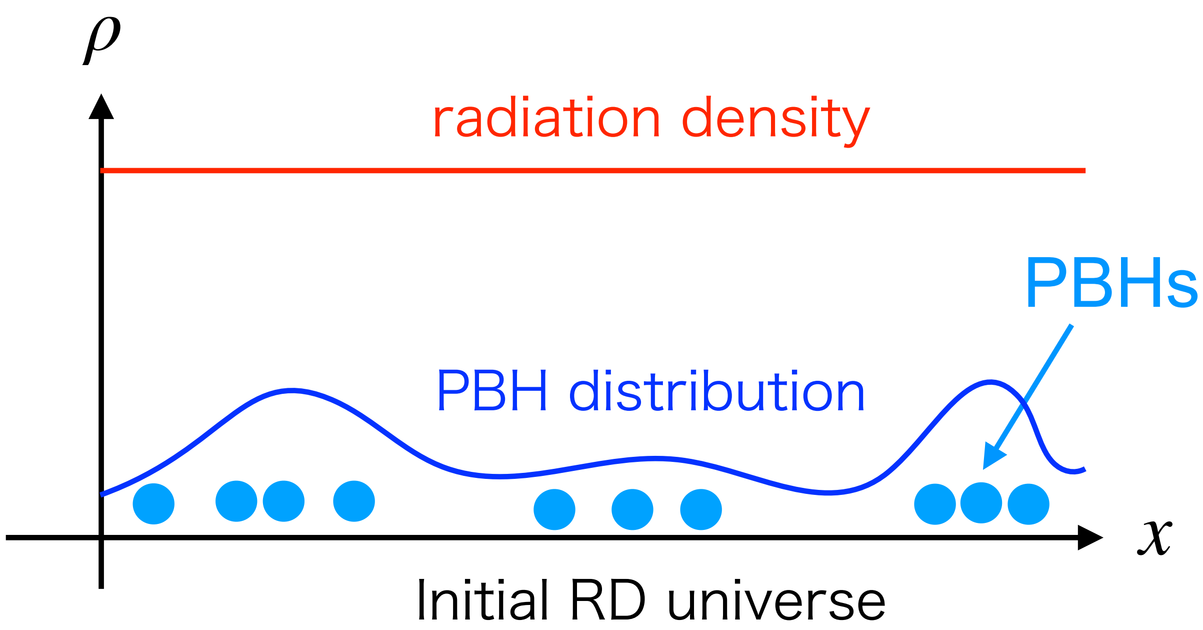
PBH formation criterion?

need more studies!

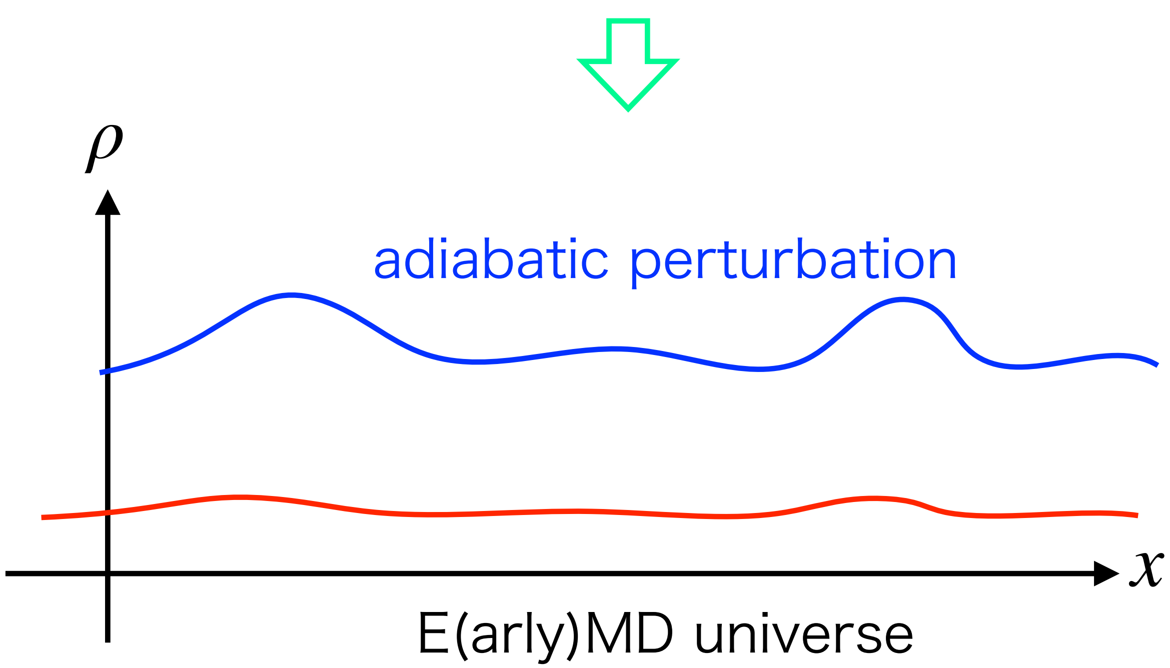
Isocurvature Perturbation due to inhomogeneous PBH distribution

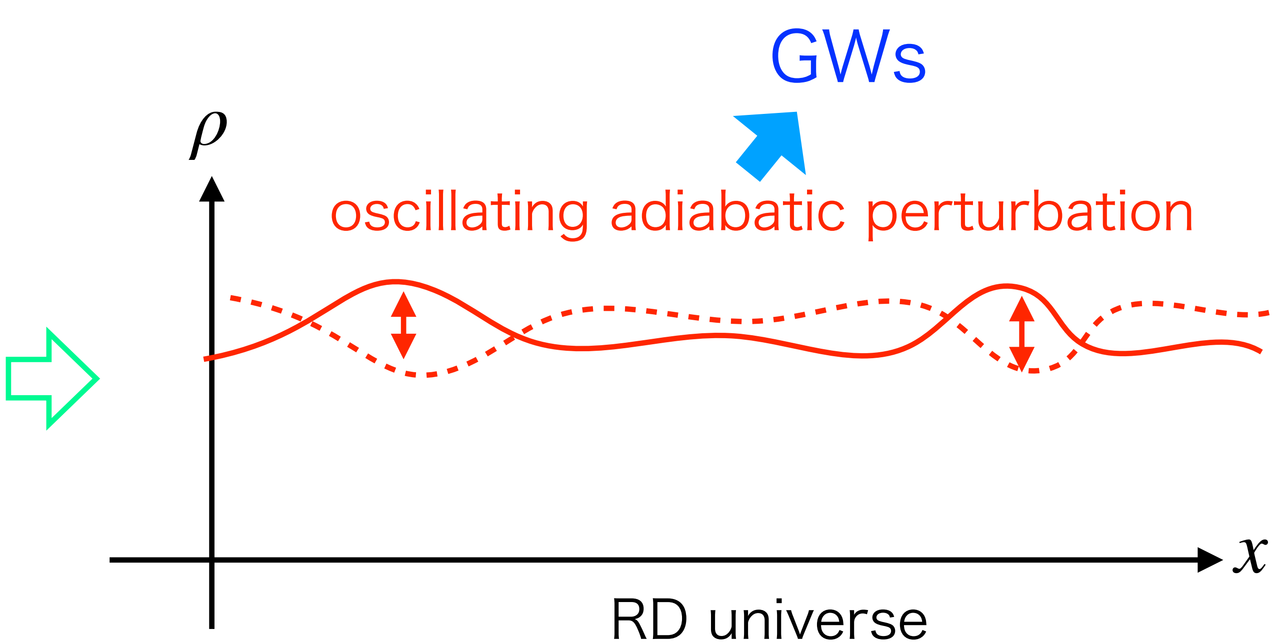
What if formed objects are PBHs?

Papanikolaou et al., arXiv:2010.11573
Domenech, Lin & MS, arXiv:2012.08151



Even if PBHs are unclustered, randomly distributed, the inhomogeneities may induce GWs when the universe is reheated by PBH evaporation





Induced GWs from PBH evaporation

Domenech, Lin & MS, arXiv:2012.0851

- If the transition from EMD to RD is slow ($\Delta t \sim H^{-1}$) as in the case of decaying particles, there will be no significant production of induced GWs.

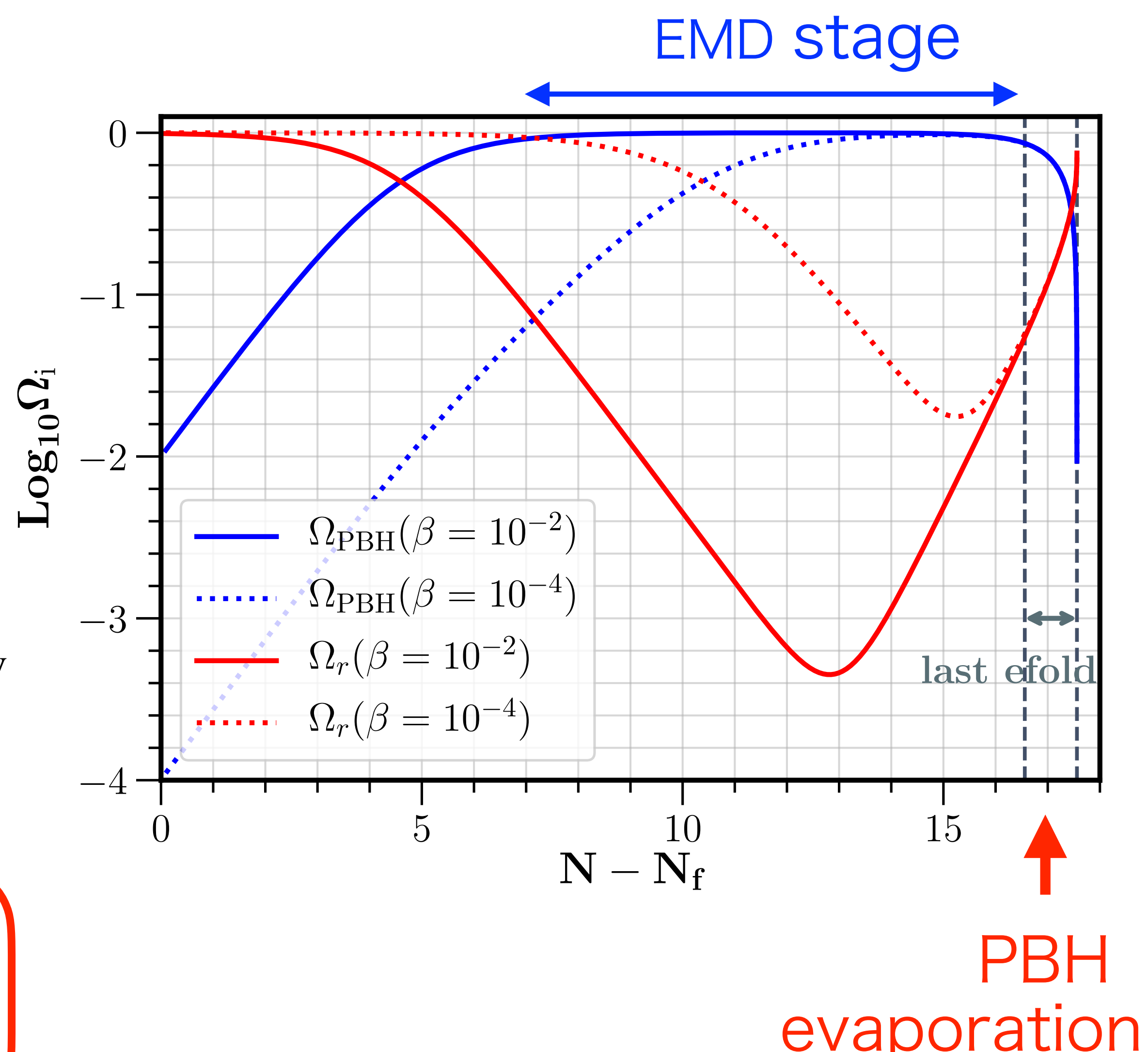
Inomata et al., arXiv:1904.12878

$$Q = Q_0 e^{-\Gamma t} \quad \rightarrow \quad \frac{1}{\Delta t} = \frac{1}{Q}\frac{dQ}{dt} = -\Gamma = const\,.$$

- A fast transition leads to strong enhancement of induced GWs on sub-horizon scales, which is the case for PBH evaporation.

Inomata et al., arXiv: 2003.10455

$$\frac{1}{\Delta t} = \left|\frac{1}{M}\frac{dM}{dt}\right| = \frac{1}{3(t_{\rm ev} - t)} \gg H \text{ as } t \rightarrow t_{\rm ev}$$



may lead to strong constraints on early PBH dominance model

Constraints on early PBH dominated universe

Domenech, Lin & MS, arXiv:2012.08151
Domenech, Takhistov & MS, arXiv:2105.06816

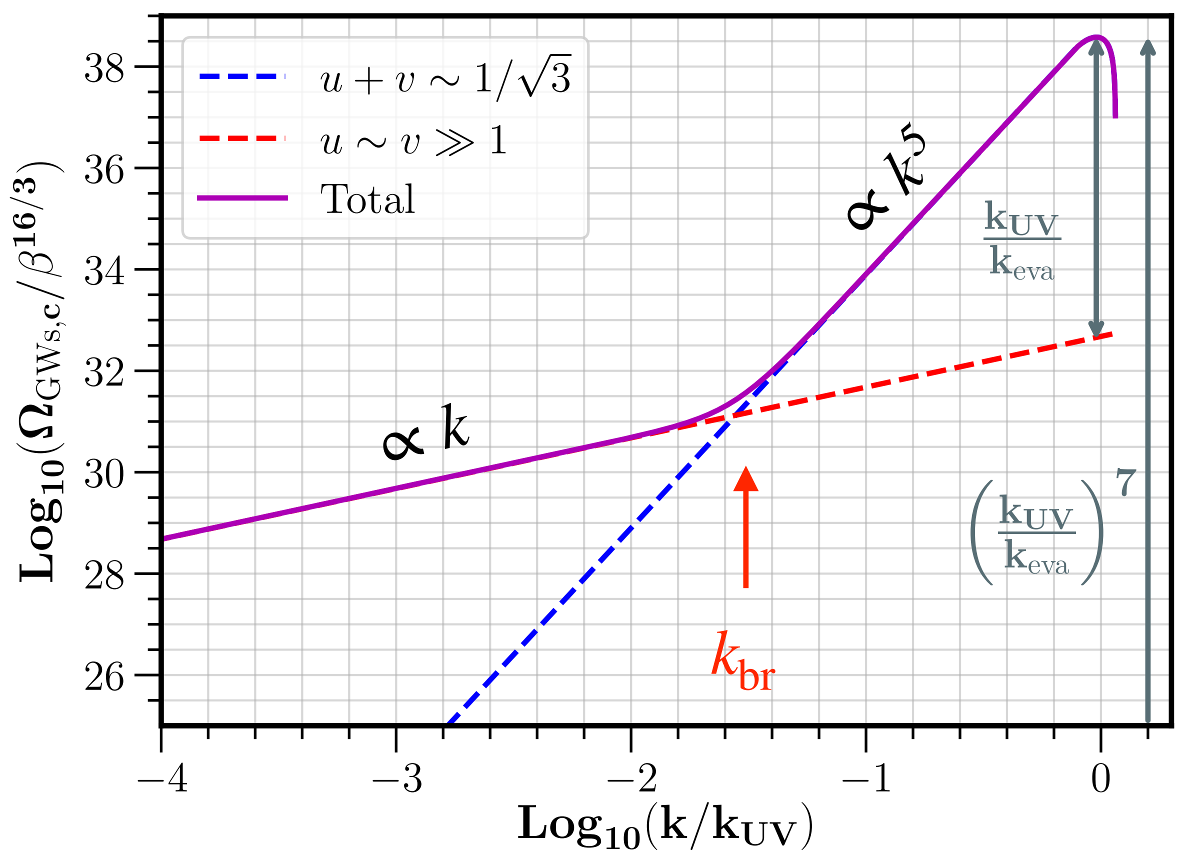
- Assumptions
  - Monochromatic mass function for PBHs.
  - Poisson distribution for $\delta n_{\rm PBH}/n_{\rm PBH}$: $\mathscr{P}_S(k) = \dfrac{2}{3\pi}\left(k/k_{\rm UV}\right)^3; \quad k < k_{\rm UV} = n_{\rm PBH}^{-1/3}$

- Resulting spectrum
  - sharp rise ~ k^5 near the peak.
  - Peak value:

$$\left(\frac{\Omega_{GW,max}}{\Omega_{r,0}}\right) \approx 5\times 10^{34}\,\beta^{16/3}\left(\frac{M}{10^4\,\mathrm{g}}\right)^{14/3}$$

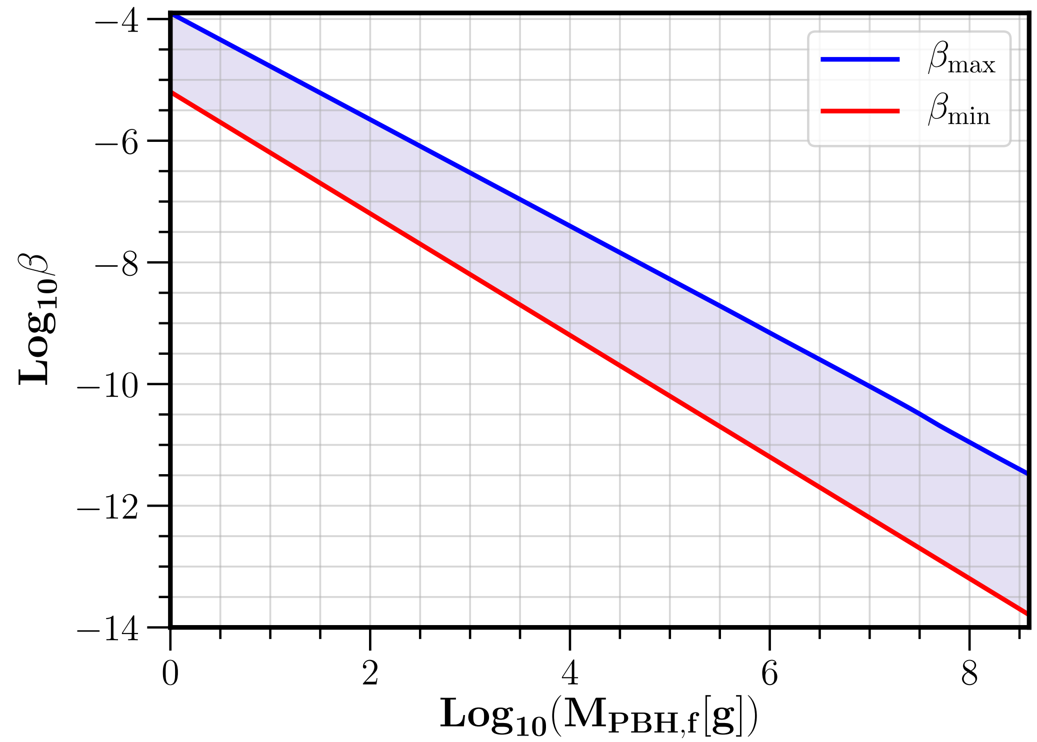
β : PBH fraction at formation

constraints on β



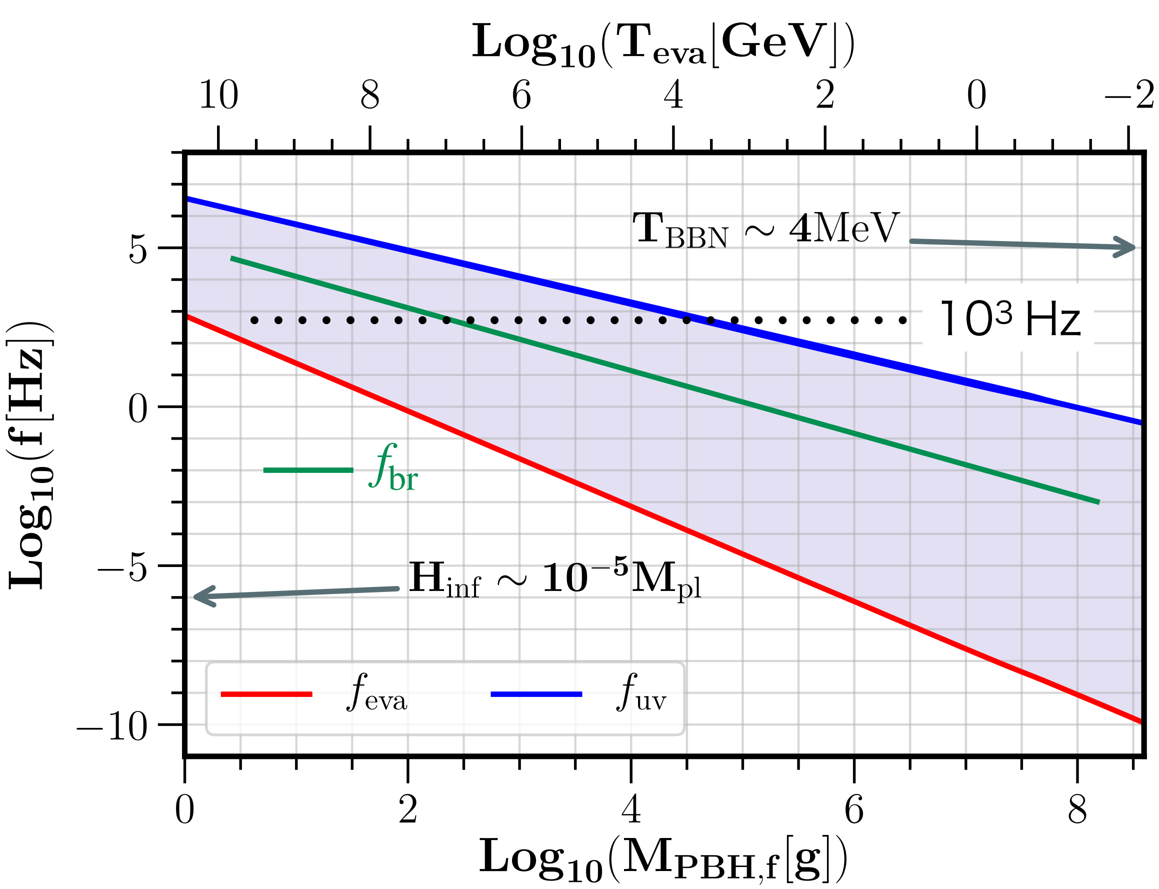
$$k_{\rm br} \approx 0.04\,k_{\rm UV}\left(M_{\rm PBH}/10^4\,\mathrm{g}\right)^{-1/6}$$

Constraints on β and frequencies

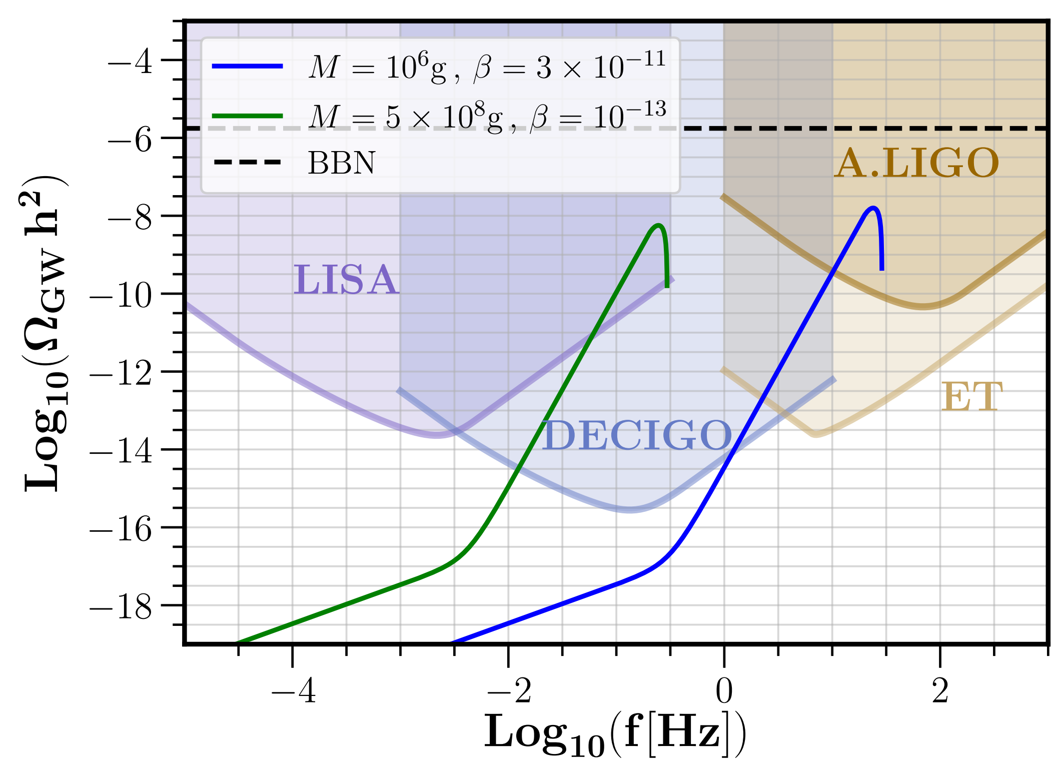


$$\beta_{max} \approx 3 \times 10^{-8} \left(\frac{M_{\rm PBH}}{10^5\,{\rm g}}\right)^{-7/8}$$

$$\beta_{min} \approx 6 \times 10^{-10} \left(\frac{M_{\rm PBH}}{10^5\,{\rm g}}\right)^{-1}$$



frequency range vs M_{PBH}



GW detectors sensitivity curves

Caviat . . .

For the primordial isocurvature perturbation,

$$\mathcal{P}_S(k) = \frac{2}{3\pi}\left(k/k_{\rm UV}\right)^3; \quad k < k_{\rm UV} = n_{\rm PBH}^{-1/3}$$

the resulting curvature perturbation at PBH dominated Universe is

$$\Phi = \frac{3}{4}\left(\frac{k_{\rm eq}}{k}\right)^2 S \sim 0.3\left(\frac{k_{\rm eq}}{k_{\rm UV}}\right)^2\left(\frac{k}{k_{\rm UV}}\right)^{-1/2} \qquad \text{for} \quad k_{\rm eq} < k < k_{\rm UV}$$

➡ The density perturbation becomes nonlinear for $k > k_{\rm NL}$:

$$\frac{\delta\rho}{\rho} = \frac{2}{3}\left(\frac{k}{aH}\right)^2 \Phi \sim 0.1\left(\frac{a_{\rm evap}}{a_{\rm eq}}\right)\left(\frac{k}{k_{\rm UV}}\right)^{3/2} \gtrsim 1$$

$$\text{for} \quad k > k_{\rm NL} \sim 5\left(\frac{a_{\rm evap}}{a_{\rm eq}}\right)^{-2/3} k_{\rm UV}$$

$$\log\left(\frac{a_{\rm evap}}{a_{\rm eq}}\right)^{2/3} \approx 2 + \frac{8}{9}\left(\log\frac{\beta}{10^{-7}} + \log\frac{M}{10^4\,{\rm g}}\right)$$

take-home messages:

- PBHs may play central roles in GW cosmology

→ **PBH-GW Cosmology!**

- (nonlinear) isocurvature perturbations may play important roles in PBH-GW cosmology