Preheating in Higgs–R^2 inflation

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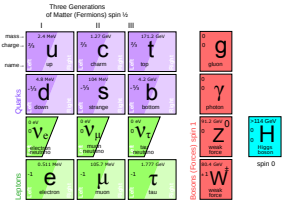
MANCHESTER 1824
The University of Manchester

Lesson from LHC so far – Standard Model is good



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Einstein gravity

- SM works in all laboratory/collider experiments (electroweak, strong)
- LHC 2012 – final piece of the model discovered – Higgs boson
  - Mass measured $\sim$ 125 GeV – weak coupling!
    Perturbative and predictive for high energies
- Add gravity
  - get cosmology
  - get Planck scale $M_{Pl} \sim 1.22 \times 10^{19}$ GeV as the highest energy to worry about

Non-minimal coupling to gravity solves the problem

Quite an old idea

For a scalar field coupling to the Ricci curvature is possible (actually *required* by renormalization)

- [A.Zee'78, L.Smolin'79, B.Spokoiny'84]
- [D.Salopek J.Bond J.Bardeen'89]

Scalar part of the (Jordan frame) action

$$S_J = \int d^4x\sqrt{-g}\left\{ -\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2-v^2)^2 \right\}$$

- h is the Higgs field; $M_P \equiv \frac{1}{\sqrt{8\pi G_N}} = 2.4\times 10^{18}\text{GeV}$
- SM higgs vev $v \ll M_P/\sqrt{\xi}$ – can be neglected in the early Universe
- At $h \gg M_P/\sqrt{\xi}$ *all* masses are proportional to h – scale invariant spectrum!

Bezrukov and Shaposhnikov 2008

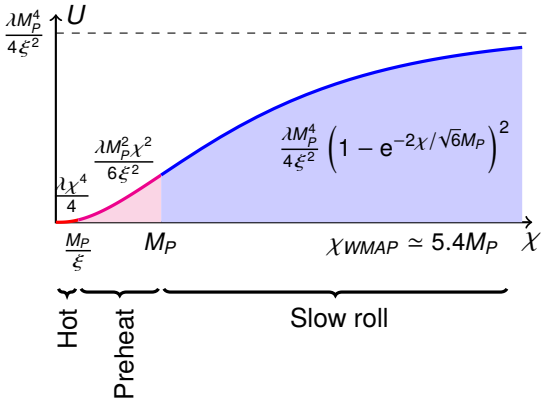
Conformal transformation $\longrightarrow$ flat potential in Einstein Frame

$$S_J = \int d^4x\sqrt{-g}\left\{ -\frac{M_P^2+\xi h^2}{2}R + \frac{(\partial h)^2}{2} - \frac{\lambda h^4}{4}\right\}$$

Change of variables:

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\,, \qquad \Omega^2 \equiv 1 + \frac{\xi h^2}{M_P^2}$$

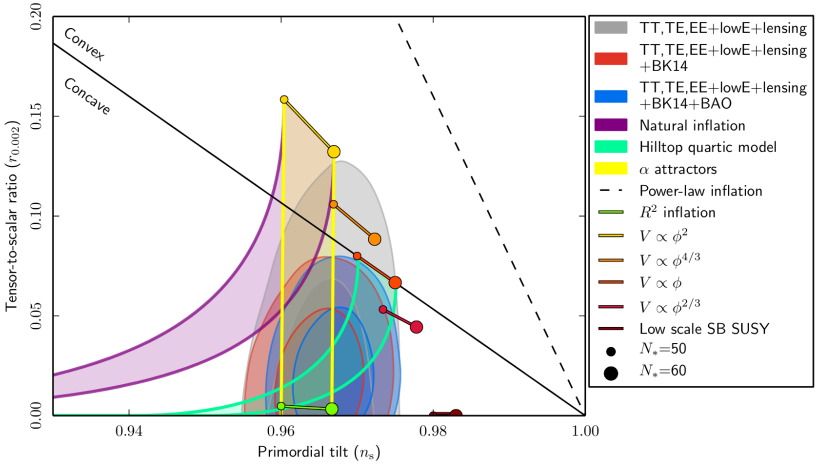
$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{(\partial\chi)^2}{2} - \frac{\lambda}{4}\frac{h(\chi)^4}{\Omega(\chi)^4}\right\}$$



$\delta T/T \sim 10^{-5}$ normalization

$$\frac{\xi}{\sqrt{\lambda}} \simeq 47000$$ – Small λ is traded for large ξ

CMB observations favour flat potentials



PLANCK'18

Consistency

Up to now we neglected the quantum effects, assuming they do not spoil the story.

Is this really the case?

Power counting around vacuum reveals higher dimensional operators suppressed by M_P/ξ

$$V(\chi) = \lambda \frac{h^4}{4\Omega^4} \simeq \lambda \frac{h^4}{4} \simeq \lambda \frac{\chi^4}{4} + \# \frac{\chi^6}{(M_P/\xi)^2} + \cdots$$

Burgess, Lee, and Trott 2009; Barbon and Espinosa 2009; Hertzberg 2010

“Cut off” is background dependent!

Classical background → $\bar{\chi}(t)$ + Quantum perturbations ← $\delta\chi(x,t)$

$$\chi(x,t) = \bar{\chi}(t) + \delta\chi(x,t)$$

leads to background dependent suppression of operators of dim $n > 4$

$$\frac{O_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}}$$

Example

Potential in the inflationary region $\chi > M_P$: $U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - \exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^2$

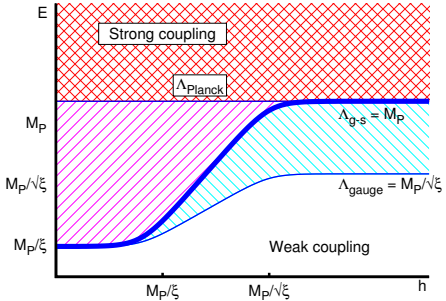
leads to operators of the form: $\frac{O_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}} = \frac{\lambda M_P^4}{\xi^2} e^{-\frac{2\bar{\chi}}{\sqrt{6}M_P}} \frac{(\delta\chi)^n}{M_P^n}$

Leading at high n to the "cut-off" $\Lambda \sim M_P$

Bezrukov, Magnin, et al. 2011; Bezrukov, Gorbunov, and Shaposhnikov 2011

Cut-off grows with the field background

Einstein frame



Relevant scales at inflation

Hubble scale $H \sim \lambda^{1/2} \frac{M_P}{\xi}$

Energy density at inflation $V^{1/4} \sim \lambda^{1/4} \frac{M_P}{\sqrt{\xi}}$

Cut-off grows with the field background

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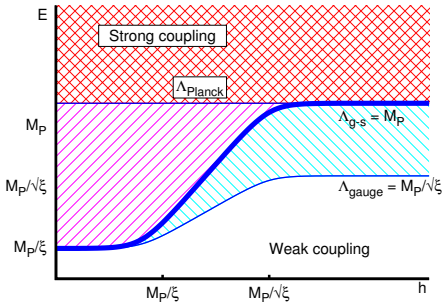

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Energy density at inflation $V^{1/4} \sim \lambda^{1/4} \frac{M_P}{\sqrt{\xi}}$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

Problems during reheating

Study by UV completion (embed into something well behaved)

- R^2-HI Ema 2017; Gorbunov and Tokareva 2019; He, Jinno, Kamada, Park, et al. 2019

$$S = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \frac{\xi h^2}{2}R + \frac{\beta}{4}R^2 - \frac{(\partial h)^2}{2} - \frac{\lambda h^4}{4}\right\}$$

  - R^2 is complete up to M_P
  - If scalaron is lighter than the problematic scale: weakly coupled

$$M^2 = \frac{M_P^2}{6\beta} < 4\pi\frac{M_P^2}{6\xi^2}$$

- More generic additional scalar below M_P/ξ Giudice and Lee 2011

No known UV completion without a state with $M < M_P/\xi$

- See, however:
  - "self-healing" Calmet and Casadio 2014
  - "nonlocal" Koshelev and Tokareva 2020

We are in a large class of similar models

Can we distinguish them?

HI

$$S_J = \int d^4x \left\{ -\frac{M_P + \xi h^2}{2} R + \frac{(\partial h)^2}{2} - \frac{\lambda h^4}{4} \right\}$$

EF potential at inflation

$$U \simeq \frac{\lambda M_P^4}{4\xi^2} \left(1 - e^{-2\chi/\sqrt{6}M_P}\right)^2$$

R^2

$$S_J = \int d^4x \left\{ -\frac{M_P}{2} R + \frac{\beta}{4} R^2 \right\}$$

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EF potential at inflation

$$U = \frac{M_P^4}{4\beta} \left(1 - \mathrm{e}^{-2\chi/\sqrt{6}M_P}\right)^2$$

Expect efficient preheating

larger N_*

larger n_s

Expect inefficient preheating

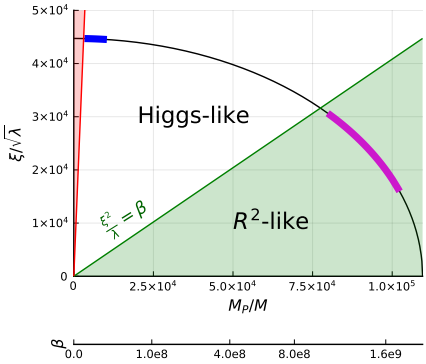
smaller N_*

smaller n_s

We need to study preheating!

Changes to the model properties

- Basic HI
  - Low energy: λ, high order operators controlled by ξ
  - Inflation: perturbations fixed by ξ^2/λ
- R^2-Higgs
  - Low energy: λ, and high order operators controlled by ξ, M
  - Inflation: perturbations fixed by $\xi^2/\lambda + \beta$
  - Note – λ RG running is modified above M_ϕ



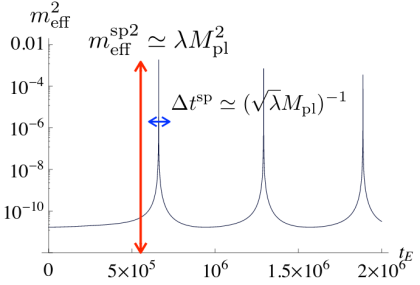
Gorbunov and Tokareva 2019

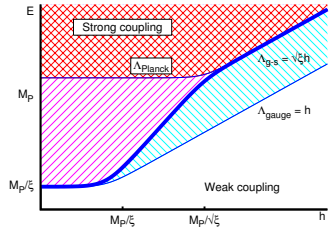
HI: Longitudinal gauge bosons produced beyond tree level unitarity

Seems that at reheating

$$m_W \sim g^2 \frac{M_P |\chi|}{\xi}$$

quite in the region of the validity of the theory





$$S = \int dx \left(-\frac{(F_{\mu\nu})^2}{4} - m_A^2(t) \frac{(A_\mu)^2}{2} \right)$$

For *longitudinal* bosons

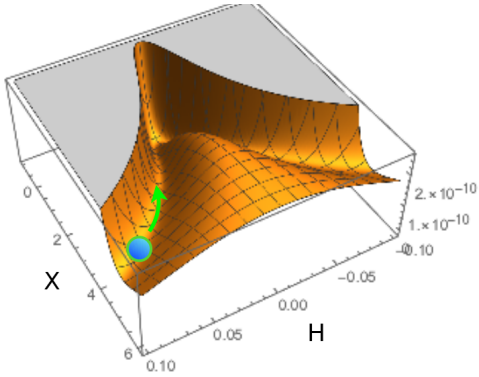
$$m^2_{{\rm eff},L} = m_A^2 - \frac{k^2}{k^2 + m_A^2} \left(\frac{\ddot{m}_A}{m_A} - \frac{3\dot{m}_A^2}{k^2 + m_A^2} \right)$$

Ema et al. 2017

R^2+Higgs inflation – simple UV completion

In Einstein frame: Higgs doublet h, $|h| \equiv \sqrt{hh^\dagger}$, scalaron ϕ

$$S_{EF} = \int d^4x \left[-\frac{M_P^2}{2} R + e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \frac{(\partial h)^2}{2} + \frac{(\partial\phi)^2}{2} \right.$$
$$\left. - \frac{1}{4} e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \left(\lambda |h|^4 + \frac{M_P^4}{\beta} \left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} - 1 - \xi \frac{|h|^2}{M_P^2} \right)^2 \right) \right]$$



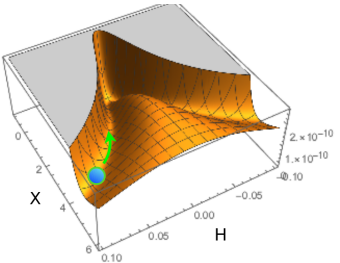
Perturbative up to $E \lesssim M_P$ if

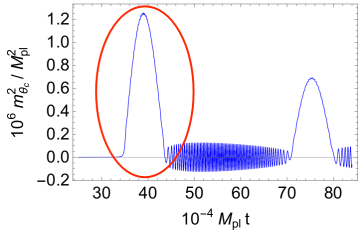
$$\beta \gtrsim \frac{\xi^2}{4\pi}$$

Effectively single field during inflation

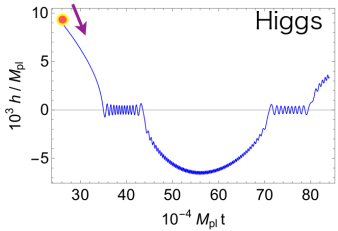
$\xi^2/\lambda + \beta$ defines the potential

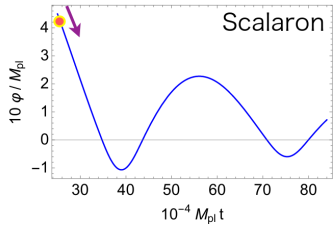
So, how R^2+Higgs reheats?





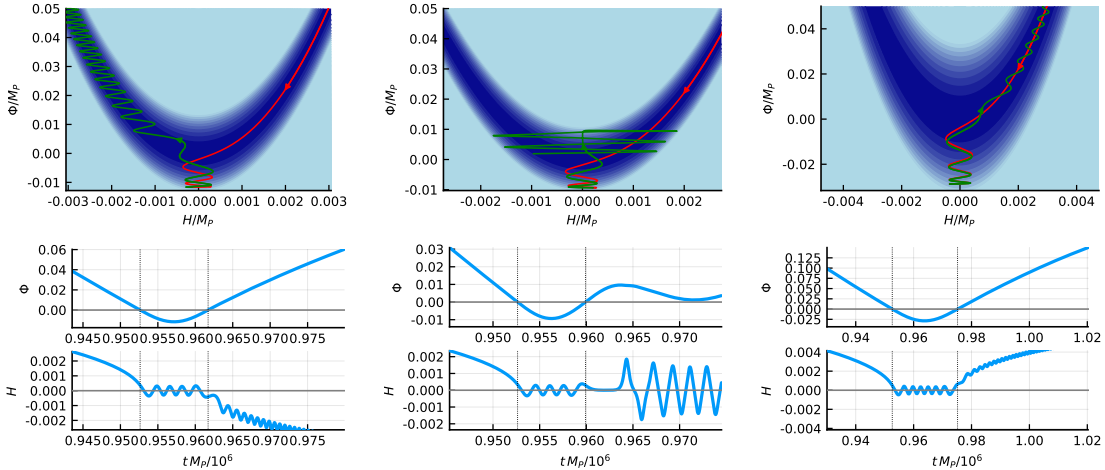
$\lambda = 0.01 \quad \xi \simeq 4089 \quad M_{\rm pl}/M = 3 \times 10^4$





No reheating on the expected mass peak: $\rho_{W_L} \sim \#M_\Phi^{-4} \ll \rho_{\rm infl}$ He, Jinno, Kamada, Park, et al. 2019

So, how it reheats?!



$\beta = 2.9 \times 10^6$ $\beta = 1.869 \times 10^6$ $\beta = 18.0 \times 10^6$

Critical (bifurcation) case depending on model parameters/or background energy.

Bezrukov, Gorbunov, Shepherd, et al. 2019

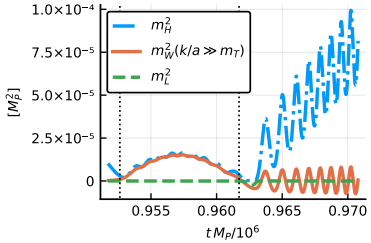
Particles become tachyonic in the critical case

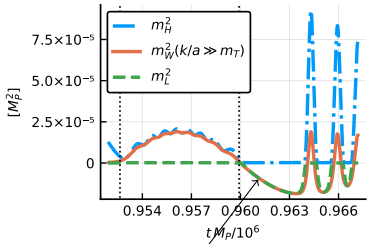
"Higgs" excitation mass

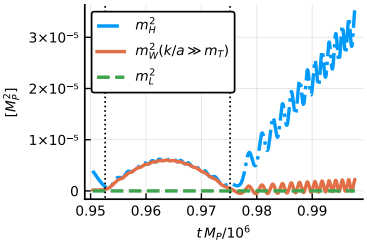
$$m_h^2 \approx 3\left(\lambda + \frac{\xi^2}{\beta}\right)H_0^2 - \frac{\sqrt{2}\xi}{\sqrt{3}\beta}M_P\Phi_0$$

Longitudinal gauge boson energies

$$\omega_W^2(\mathbf{k}) \approx \frac{k^2}{a^2} + m_T^2 - \frac{k^2}{k^2 + a^2 m_T^2}\left(\frac{\ddot{m}_T}{m_T} - \frac{3(\dot{m}_T)^2}{k^2/a^2 + m_T^2}\right).$$





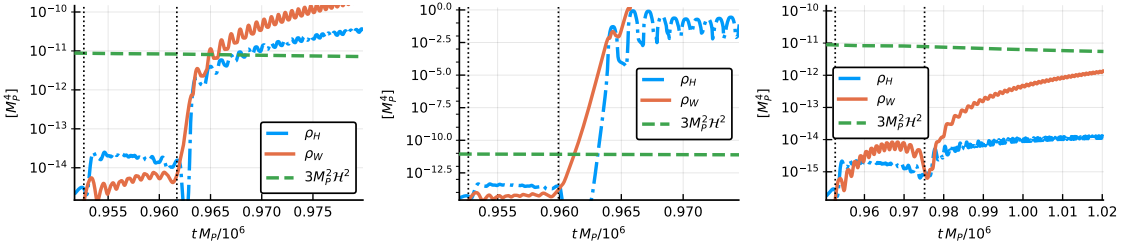


Tachionic!

Very efficient reheating on the “tachyon”

- Write linearized equations for perturbations
- Start with vacuum initial conditions (negative frequency)
- Calculate occupation number of positive frequency modes at the end (i.e. Bogolyubov coefficients)

In the critical case energy in the produced modes grows fast!



Immediate reheating!

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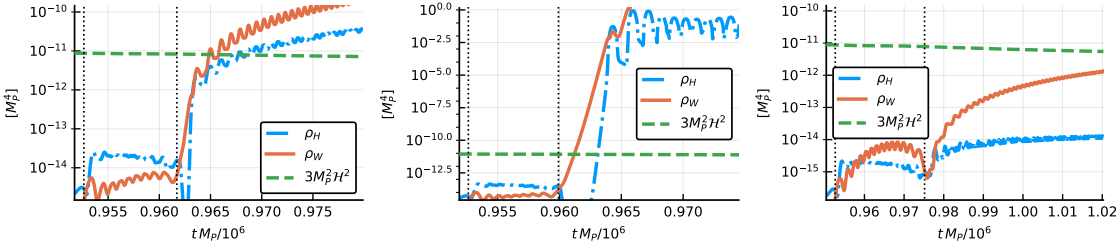
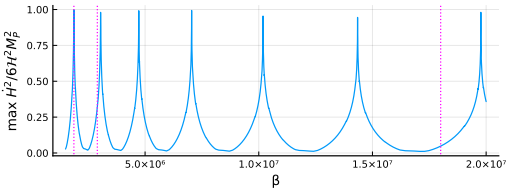
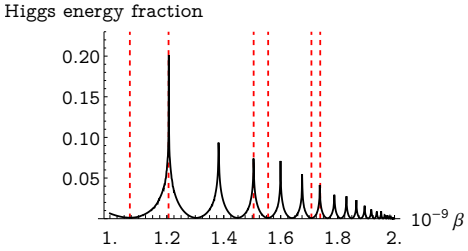


Immediate reheating!

Small print: only if the parameters lead to critical case evolution

What happens in the generic case?





- If β is small (close to $\xi^2/2\pi$) – "Higgs like" case, critical values are relatively frequent.
- Whole range of β is studied in He, Jinno, Kamada, Starobinsky, et al. 2021

Complications

- Can tachyon happen not on the first oscillation?
- Backreaction!
  - Modifies background evolution during the tachyonic regime
  - Modifies background evolution away from tachyonic regime

Semiclassical approach to reheating

- Quantum theory with small coupling constant ß reaches large occupation numbers while still linear in perturbations

Polarski and Starobinsky 1996

- Large occupation numbers ß classical equations of motion can be used

Semiclassical algorithm

- Set Gaussian random initial conditions for all fields $f_{\mathbf{k}}$, giving $n_{\mathbf{k}} = 1/2$
- Evolve the classical equations of motion

Khlebnikov and Tkachev 1996

Complications (starting the simulations)

- Non-canonical kinetic terms
  - Luckily not too relevant for us after slow-roll
  - We use modified version of GABE
    - ★ It can deal with non-canonical kinetic terms, though this turned out to be not that important.

Complications (starting the simulations)

- Non-canonical kinetic terms
  - Luckily not too relevant for us after slow-roll
  - We use modified version of GABE
    - ★ It can deal with non-canonical kinetic terms, though this turned out to be not that important.
- Relevant reheating processes should
  - "fit" on the lattice
    - ★ $\frac{2\pi}{L} < k_{\text{tachyon}}, k_{\text{rescatering}} < \frac{2\pi N}{L}$
  - be in semiclassical regime – i.e. have large occupation number.

“Vacuum oscillations” as initial conditions

- To simulate the parametric tachyonic resonance initial “seed” is required

$$n_{\mathbf{k}} \equiv \frac{a^3}{2\omega_k}\left(|\dot{f}_{\mathbf{k}}|^2 + \omega_k^2 |f_{\mathbf{k}}|^2\right) = \frac{1}{2}$$

Problem of large vacuum oscillations

“vacuum” energy should not be larger, than “tree level” background energy

$$\int_{\text{all lattice momenta}} \frac{\omega_{\mathbf{k}}}{2} d^3\mathbf{k} < V(\phi, h)$$

Or at least

$$\int_{\text{typical tachyonic momenta}} \frac{\omega_{\mathbf{k}}}{2} d^3\mathbf{k} < V(\phi, h)$$

This means that simulations are reliable: small $\lambda \frac{\xi^2}{\lambda\beta}$

Tachyonic dynamics is not semiclassical for small β!

- Realistic simulations: make modes unoccupied above some initial cut-off

$$n_{\mathbf{k}}|_{k<\Lambda_{in}} = 1/2$$
$$n_{\mathbf{k}}|_{k>\Lambda_{in}} = 0$$

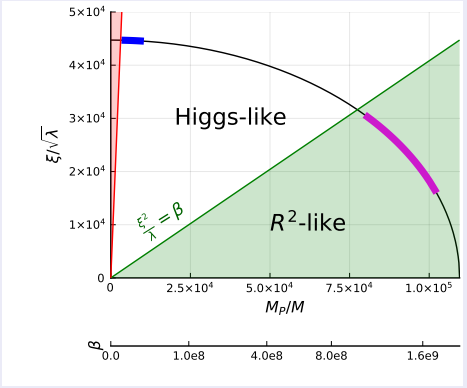
- Large enough lattice to grab tachyonic

$$\frac{2\pi}{L} < k_{\text{tachyonic}} < \Lambda_{in}$$

and late time rescattering evolution

$$k_{\text{rescattering}} < \frac{2\pi N}{L}$$

Possible only for $\beta > 10^{-9}$



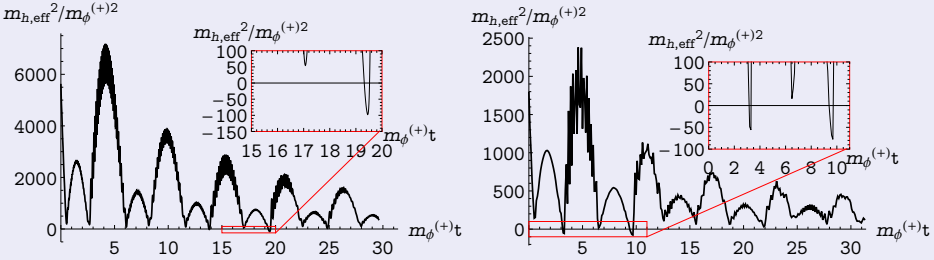
Bezrukov and Shepherd 2020

What changes compared to analytic calculation?

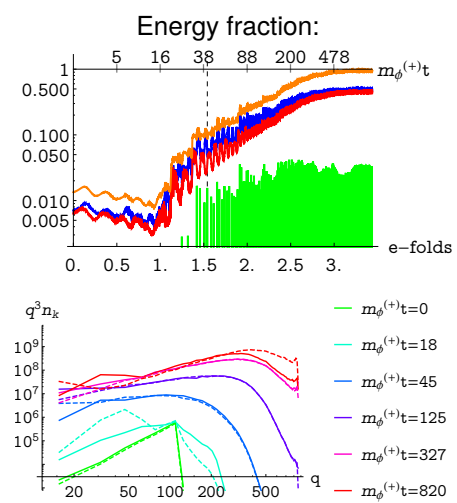
1. Tachyonic behaviour may appear not only on first $\chi = 0$ crossing, but also on consequent ones
   - Depends on field background amplitudes, etc.
2. Created particles *suppress* tachyonic behaviour at late time

$$m_{h,\mathrm{eff}}^2 = -\sqrt{\frac{2}{3}}\frac{\xi}{\beta} M_P\, \phi_{(0)} + \left(\lambda + \frac{\xi^2}{\beta}\right)\left(3h_{(0)}^2 + \langle (h_i - \langle h_i \rangle)^2 \rangle\right).$$

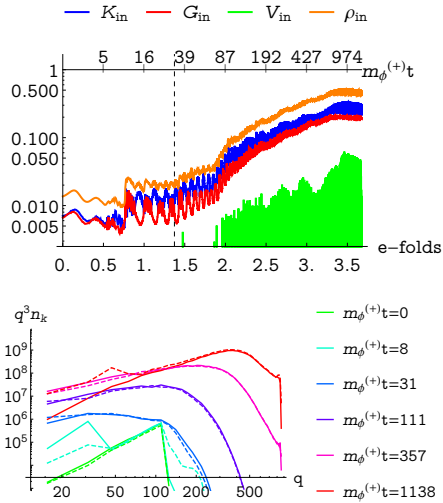
Observation: (1) always happens before (2)!



Reheating stages



$\beta = 1.060 \times 10^9$

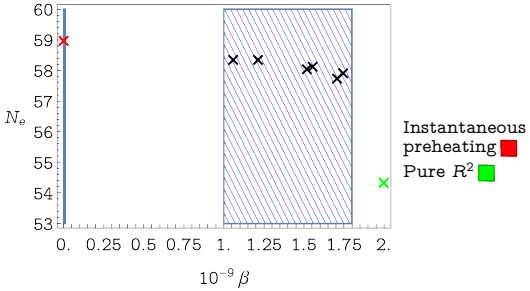


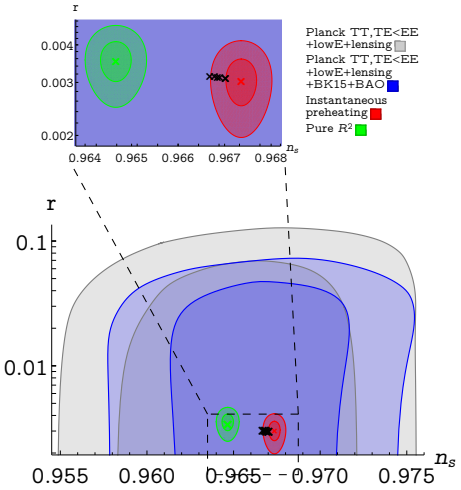
$\beta = 1.745 \times 10^9$

1. Tachyonic
2. scattering on the background
3. perturbative decay of the scalaron (relevant for larger β)

Does it lead to observable effects?

Preheating is fast!





For precise absolute numbers second order in slow-roll is needed, c.f. Gorbunov and Tokareva 2012

Not yet finished after preheating

- We get to the model with long lived heavy scalaron
- Additional entropy release!

He 2021, and soon by FB, Lee, Park, Shepherd, Yeon

Further note on variable choice:

We really need to know how quantum gravity works

- How do we interpret the gravity action:
  - Metric – $g_{\mu\nu}(x)$ is an independent field, Connection – $\Gamma^{\lambda}_{\mu\nu} \equiv \frac{g^{\lambda\rho}}{2}(g_{\rho\mu,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho})$
  - Palatiny – $g_{\mu\nu}(x)$, $\Gamma^{\lambda}_{\mu\nu}(x)$ are independent fields
- Different *classical* dynamics if $\xi \neq 0$
  Can be seen as different transformation under $g_{\mu\nu} \to \Omega(x) g_{\mu\nu}$

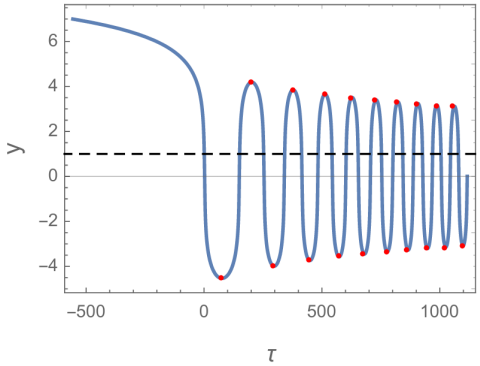
Rather different inflationary predictions!

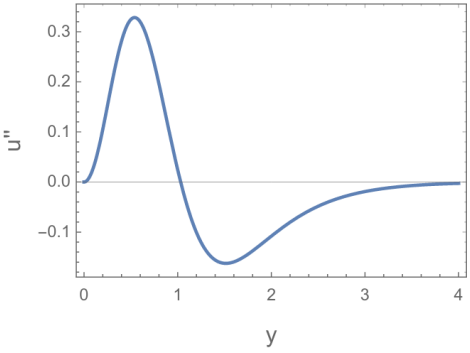
Metric	Palatini
$R \to \Omega^2 R + 6 g^{\mu\nu} \partial_\mu \ln\Omega \partial_\nu \ln\Omega$	$R \to \Omega^2 R$
$\xi \sim 5 \times 10^4 \sqrt{\lambda}$	$\xi \sim 1.5 \times 10^{10} \lambda$
$r \sim 3.2 \times 10^{-3}$	$r \sim 3.5 \times 10^{-14} \lambda^{-1}$

e.g. Rasanen,Wahlman'17; Järv,Racioppi,Tenkanen'17

Another preheating possibilities in HI: Palatini HI

- Fast again, but for a different reason:





Tachyonic regime on maxima of higgs oscillations!

- A bit care for longitudinal gauge bosons may be needed...

Rubio and Tomberg 2019

Conclusions

- To get exact predictions from a theory:
  The theory should be properly defined first!
- Simplest UV completion of Higgs inflation – R^2+HI
  - Reheats nearly immediately for not too light scalaron mass!
  - Tachyonic dynamics is achieved after several scalaron oscillations, and then blocked by backreaction
- Details of reheating for even lighter scalaron – connection with R^2 case – yet to study.

Cut off scale today

Let us work in the Einstein frame

Change of variables: $\frac{d\chi}{dh} = \frac{M_P\sqrt{M_P^2+(\xi+6\xi^2)h^2}}{M_P^2+\xi h^2}$ leads to the higher order terms in the potential (expanded in a power law series)

$$V(\chi) = \lambda\frac{h^4}{4\Omega^4} \simeq \lambda\frac{h^4}{4} \simeq \lambda\frac{\chi^4}{4} + \#\frac{\chi^6}{(M_P/\xi)^2} + \cdots$$

Unitarity is violated at tree level

in scattering processes (eg. $2 \to 4$) with energy above the "cut-off"

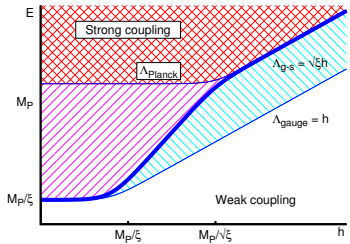
$$E > \Lambda_0 \sim \frac{M_P}{\xi}$$

Hubble scale at inflation is $H \sim \lambda^{1/2}\frac{M_P}{\xi}$ – not much smaller than the today cut-off Λ_0 :(

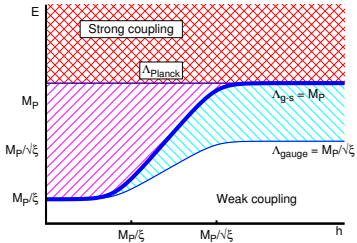
Burgess, Lee, and Trott 2009; Barbon and Espinosa 2009; Hertzberg 2010

Cut-off grows with the field background

Jordan frame



Einstein frame



Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}}\Omega$$

Relevant scales at inflation

Hubble scale $H \sim \lambda^{1/2}\frac{M_P}{\xi}$

Energy density at inflation

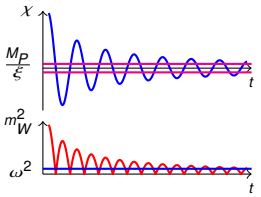
$$V^{1/4} \sim \lambda^{1/4}\frac{M_P}{\sqrt{\xi}}$$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

Problems during reheating

Reheating in Higgs inflation (attempt 1)

- Post-inflationary evolution $\chi < M_P$ ($h < M_P/\sqrt{\xi}$)
  - quadratic potential $U \simeq \frac{\omega^2}{2}\chi^2$ with $\omega = \sqrt{\frac{\lambda}{3}}\frac{M_P}{\xi}$
  - matter domination $a \propto t^{2/3}$
- Resonance
  - gauge masses $m_W^2(\chi) \sim g^2 \frac{M_P|\chi|}{\xi}$
  - generate nonrelativistic W
    - ★ $\sqrt{\langle\chi^2\rangle} \lesssim 23\left(\frac{\lambda}{0.25}\right)\frac{M_P}{\xi}$: resonance creation and annihilation of W
  - Creation of Higgs bosons is less efficient $\sqrt{\langle\chi^2\rangle} \sim 2.6\left(\frac{\lambda}{0.25}\right)^{1/2}\frac{M_P}{\xi}$
  - Radiation-dominated stage starts at χ amplitude

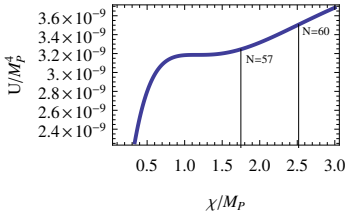


$$\frac{3.0M_P}{\xi}\left(\frac{\lambda}{0.25}\right)^{1/2} < \chi_r < \frac{32.7M_P}{\xi}\left(\frac{\lambda}{0.25}\right)$$

Bezrukov, Gorbunov, and Shaposhnikov 2009, Garcia-Bellido, Figueroa, and Rubio 2009

Another preheating possibilities in HI: Critical HI

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$



- Small $\xi \lesssim 10 - \lambda$ vs. $\delta\lambda$ significant, may give interesting "features" in the potential ("critical inflation", large r)
- Preheating is inefficient for small λ. Both for longitudinal modes, and expected due to transverse modes:

$$\frac{3.0M_P}{\xi}\left(\frac{\lambda}{0.25}\right)^{1/2} < \chi_r < \frac{32.7M_P}{\xi}\left(\frac{\lambda}{0.25}\right)$$

Maybe we can compute everything in HI!

to be confirmed

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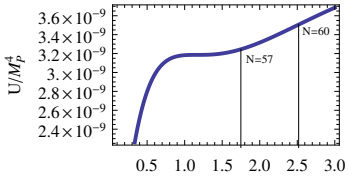

- Small $\xi \lesssim 10$ – λ vs. $\delta\lambda$ significant, may give interesting "features" in the potential ("critical inflation", large r)
- Preheating is inefficient for small λ. Both for longitudinal modes, and expected due to transverse modes:

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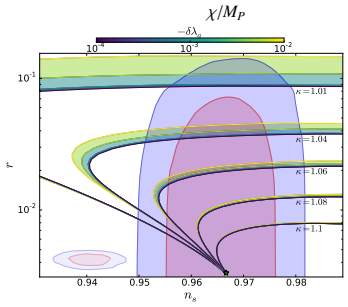
  Maybe we can compute everything in HI!
- However – tend to get both inflation and $\delta\lambda$ "jumps" at the same scale around M_P/ξ
- Loop corrections change result – harder to control

  Bezrukov, Pauly, and Rubio 2018



to be confirmed

Threshold effects at M_P/ξ summarized by two new arbitrary constants $\delta\lambda$, δy_t

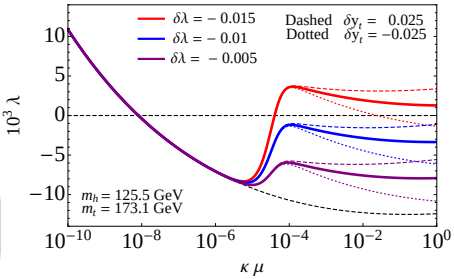
- Low and high scale coupling constants may be different

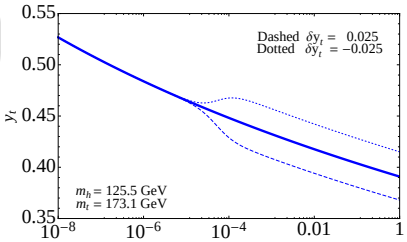
$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \tfrac{1}{3} F'' F \right)^2 - 1 \right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1 \right]$$

Attempts to improve

- UV complete theories
- Scale invariant theories





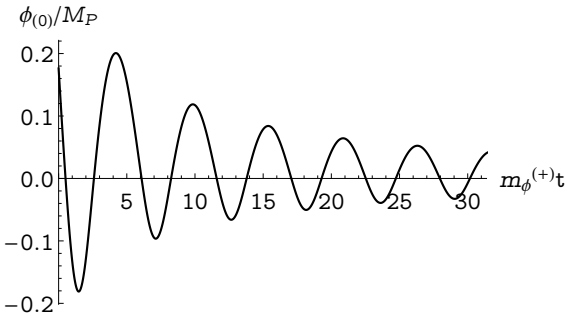
Effective theories at high and low h

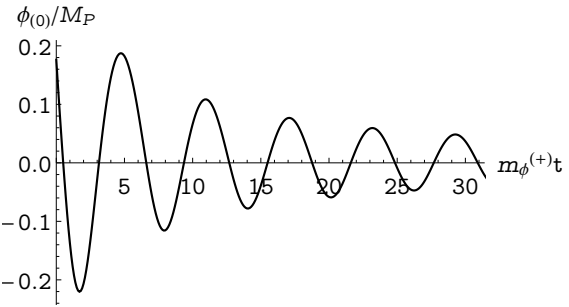
- Below M_P/ξ
  - Renormalizable ϕ^4-like Standard Model
  - + M_P/ξ suppressed operators
- Above $M_P/\sqrt{\xi}$
  - Non-renormalizable, but the potential nicely arranges

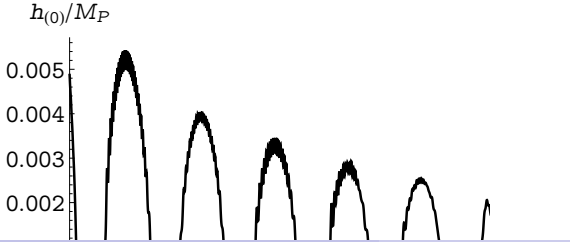
$$\#e^{-\chi/M} + \#e^{-2\chi/M} + \#e^{-3\chi/M} + \cdots$$

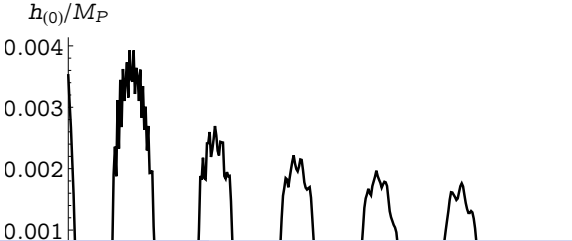
Higher terms are irrelevant

Field evolution

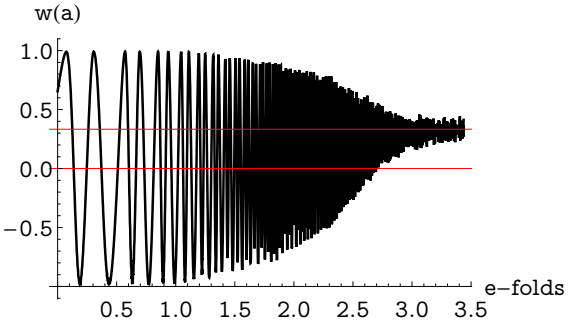


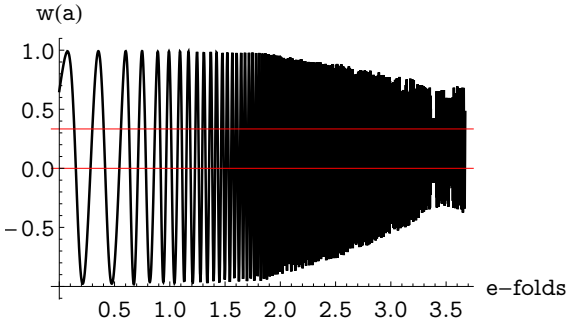




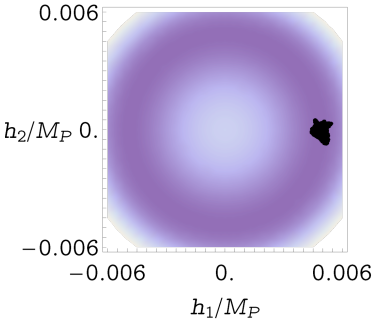


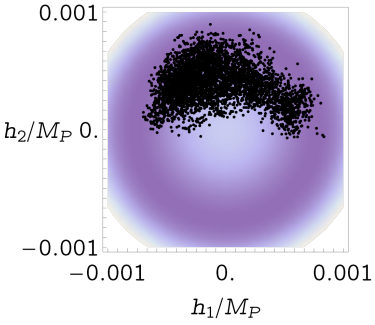
Equation of state

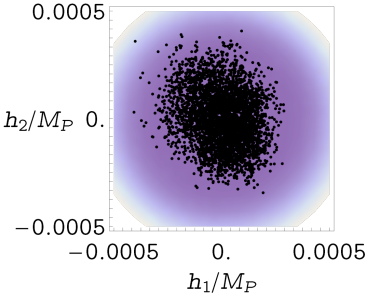




Symmetry restoration







Decay widths

$$\Gamma_\phi \approx \frac{1}{24\pi m_\phi^{(-)}} \left(M_P \frac{\xi}{\beta}\right)^2 \sqrt{1 - 2\frac{m_{h,\text{eff}}^2}{m_\phi^{(-)2}}}.$$

$$m_\phi^{(-)2} = \frac{M_P^2}{6\beta}.$$

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